

Test of quantum gravity effects with mechanical oscillators

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HUMOR

Heisenberg Uncertainty Measured with Opto-mechanical Resonators

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DI OTTICA



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UNIVERSITÀ DEGLI STUDI
DI TRENTO



Planck units

Special Relativity



c

Quantum Mechanics



\hbar

Newton Theory



G

It is interesting to note that with the help of the [above constants] it is possible to introduce units [...] which [...] remain meaningful for all times and also for extraterrestrial and non-human cultures, and therefore can be understood as 'natural units'.

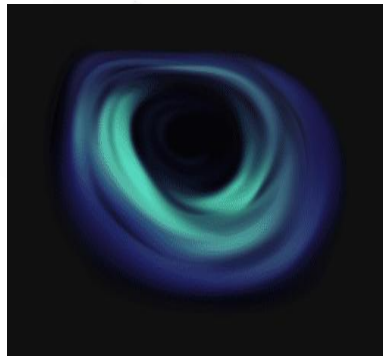
Max Planck, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin (1899), p. 479

$$m_p = \sqrt{\frac{\hbar c}{G}} \simeq 2.18 \times 10^{-8} \text{ kg} \sim 1.2 \times 10^{19} \text{ GeV}$$

$$l_p = \sqrt{\frac{\hbar G}{c^3}} \simeq 1.62 \times 10^{-35} \text{ m}$$

$$t_p = \sqrt{\frac{\hbar G}{c^5}} \simeq 5.39 \times 10^{-44} \text{ s}$$

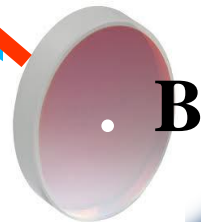
Limits to distance measurements



A

$$\Delta v \sim \frac{\hbar}{2m\Delta x}$$

$$D = c T$$



B

A mass m compressed in a sphere of radius D must be

$$m < \frac{Dc^2}{G}$$

(Schwarzschild limit)

BLACK HOLE

$$\Delta D \sim \delta x + \frac{\hbar T}{2m\Delta x} \quad \longrightarrow \quad \Delta D_{min} = \sqrt{\frac{\hbar T}{2m}} = \sqrt{\frac{\hbar D}{mc}}$$

$$\Delta D_{min} \geq \sqrt{\frac{\hbar G}{c^3}} = L_p$$

Quantum gravity phenomenology

Quantum Mechanics + Gravity \longrightarrow Minimal measurable length

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left(1 + \beta_0 \left(\frac{L_p \Delta p}{\hbar} \right)^2 \right) \quad \Delta x_{min} = \sqrt{\beta_0} L_p$$

Physical interpretation

1. The high energies used to probe Planck-scale distances disturb the spacetime geometry ('Induced' spacetime fuzziness) \longrightarrow High-energy experiments
2. Quantum space-time fluctuations related to the creation and annihilation of particles (Spacetime is 'intrinsically' discrete) \longrightarrow Low energy experiments

GUP can be associated to a modified canonical commutator

$$[x, p] = i\hbar \left(1 + \beta_0 \left(\frac{p}{M_p c} \right)^2 \right)$$

MODIFIED SPECTRUM OF A QUANTUM SYSTEM

MODIFIED DYNAMICS \longrightarrow MORE STRINGENT LIMITS

Phenomenological quantum gravity

- General 'remark': one cannot determine a position with an accuracy better than the Planck length $L_p = \sqrt{\hbar G/c^3} = 1.6 \cdot 10^{-35} \text{ m}$
- Generalized Heisenberg uncertainty relations (GUP)
- Generalized commutators between p e q
- Modified quantum physics



Detecting signatures of Planck scale-physics in highly-sensitive metrological systems

Basic assumptions:

$$\frac{d\hat{O}}{dt} = \frac{1}{i\hbar}[\hat{O}, H]$$

Heisenberg dynamics

$$[x, p] = i\hbar \left(1 + \beta_0 \left(\frac{p}{M_p c} \right)^2 \right)$$

Deformed commutation relations

$$\text{from } \Delta x \Delta p \geq \frac{\hbar}{2} \left(1 + \beta_0 \left(\frac{\Delta p}{M_p c} \right)^2 \right)$$

$$H = \frac{\hbar\omega_0}{2} (X^2 + P^2)$$

Solution:

$$\beta = \beta_0 \frac{\hbar m \omega_0}{M_p^2 c^2}$$

$$X = X_0 \left[\sin(\tilde{\omega}t) + \frac{\beta}{8} X_0^2 \sin(3\tilde{\omega}t) \right]$$

3° harmonic

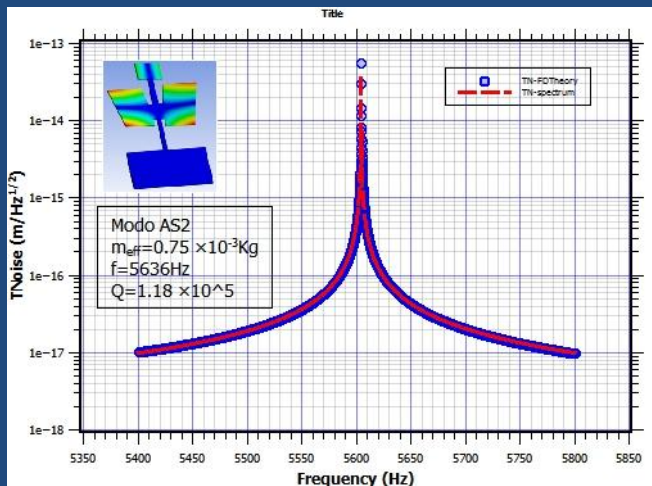
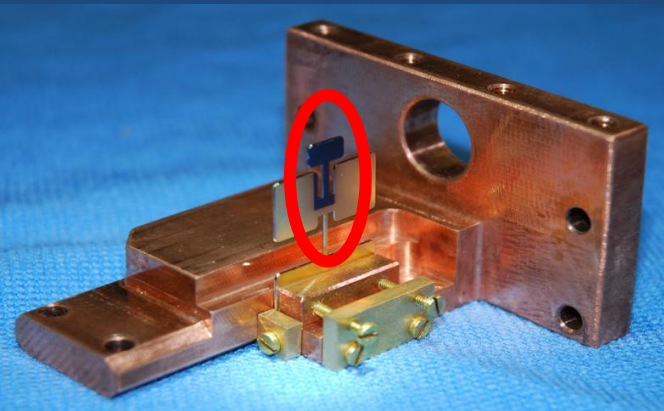
$$\tilde{\omega} = \left(1 + \frac{\beta}{2} X_0^2 \right) \omega_0$$

Freq. shift

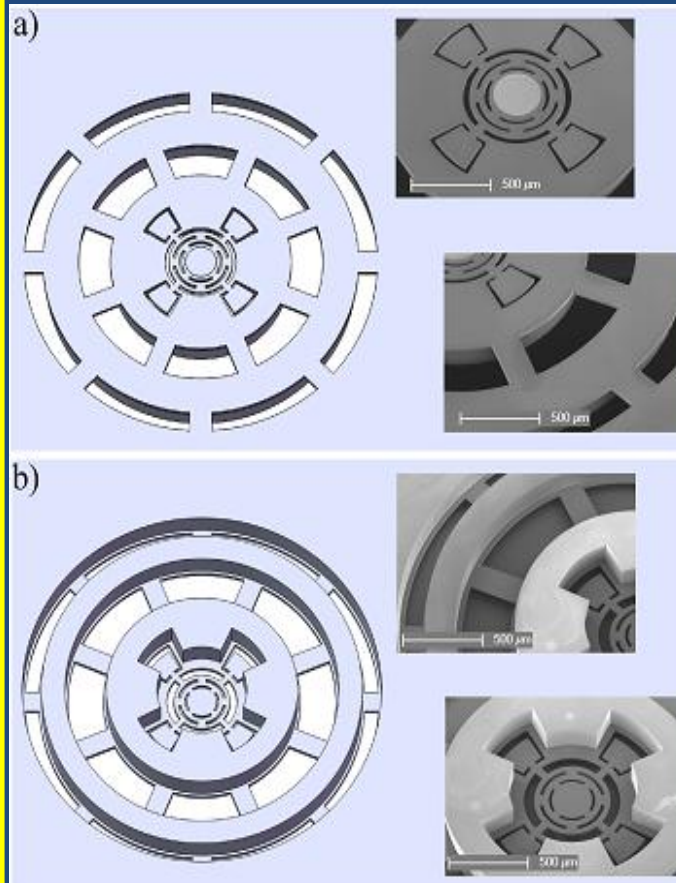
(First order in βX_0^2)

- Test on a wide mass range
- High mechanical quality factor → 'isolated' oscillators
- Exploit the slow decay to obtain frequency/3° harmonic vs amplitude curves

1° oscillator:
 $m \cong 1 \text{ g}$

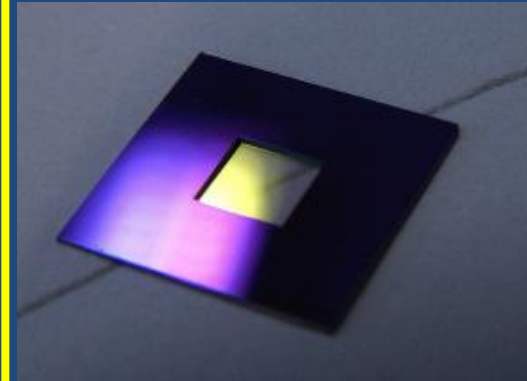


2° oscillator:
 $m \cong 100 \mu\text{g}$

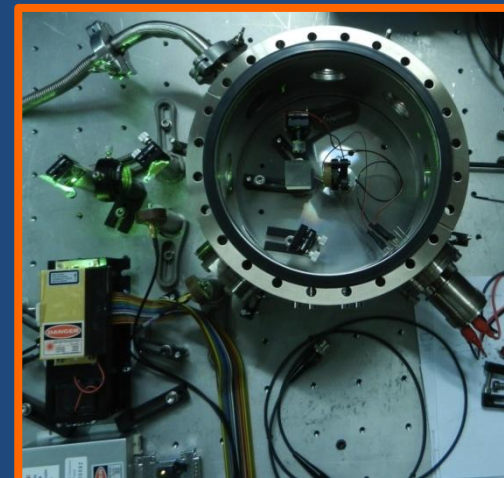


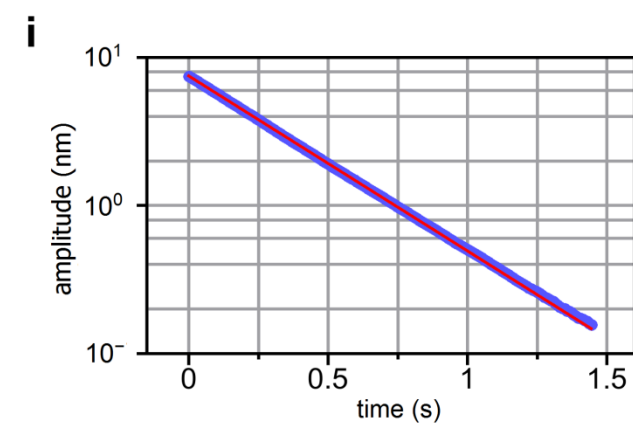
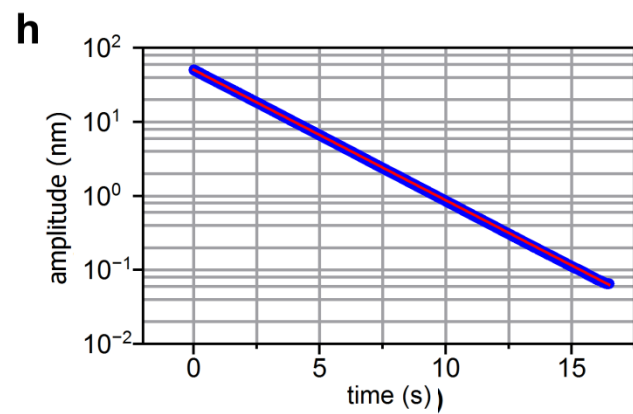
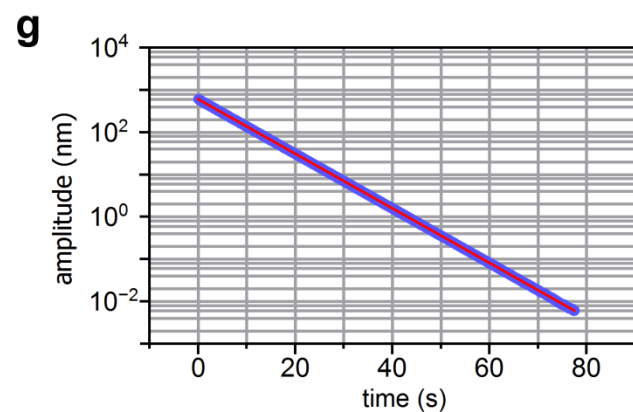
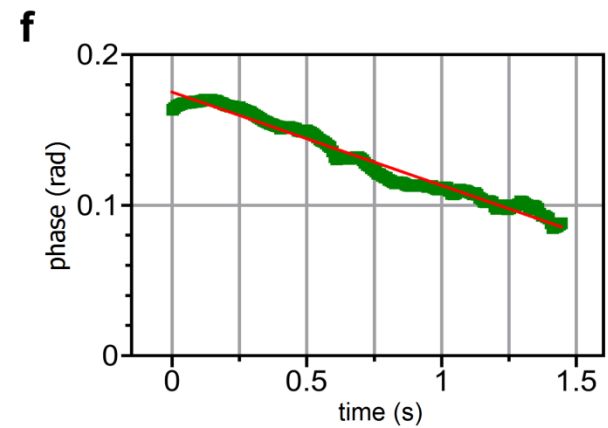
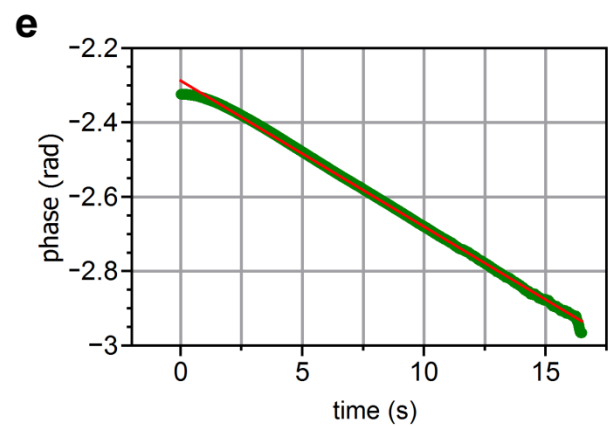
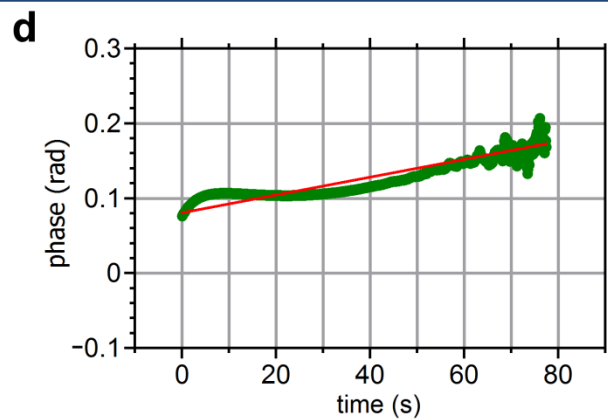
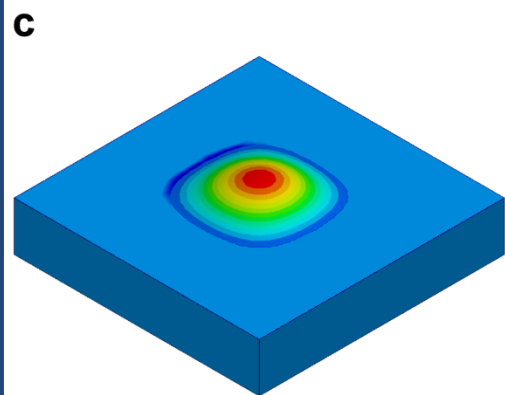
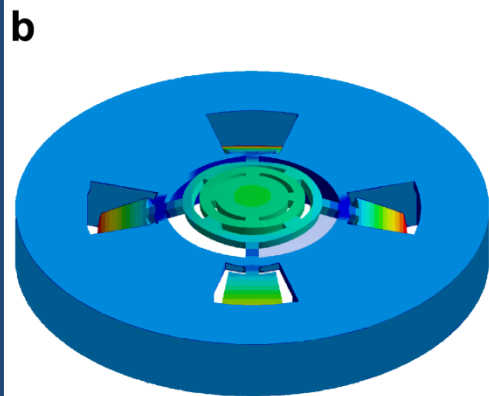
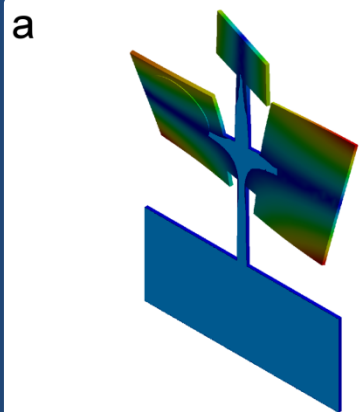
$m = 20 \mu\text{g}$
 $f_m = 141 \text{ KHz}$
 $Q = 1.2 \times 10^6$
 $T = 4.3 \text{ K}$

3° oscillator:
 $m \cong 100 \text{ ng}$

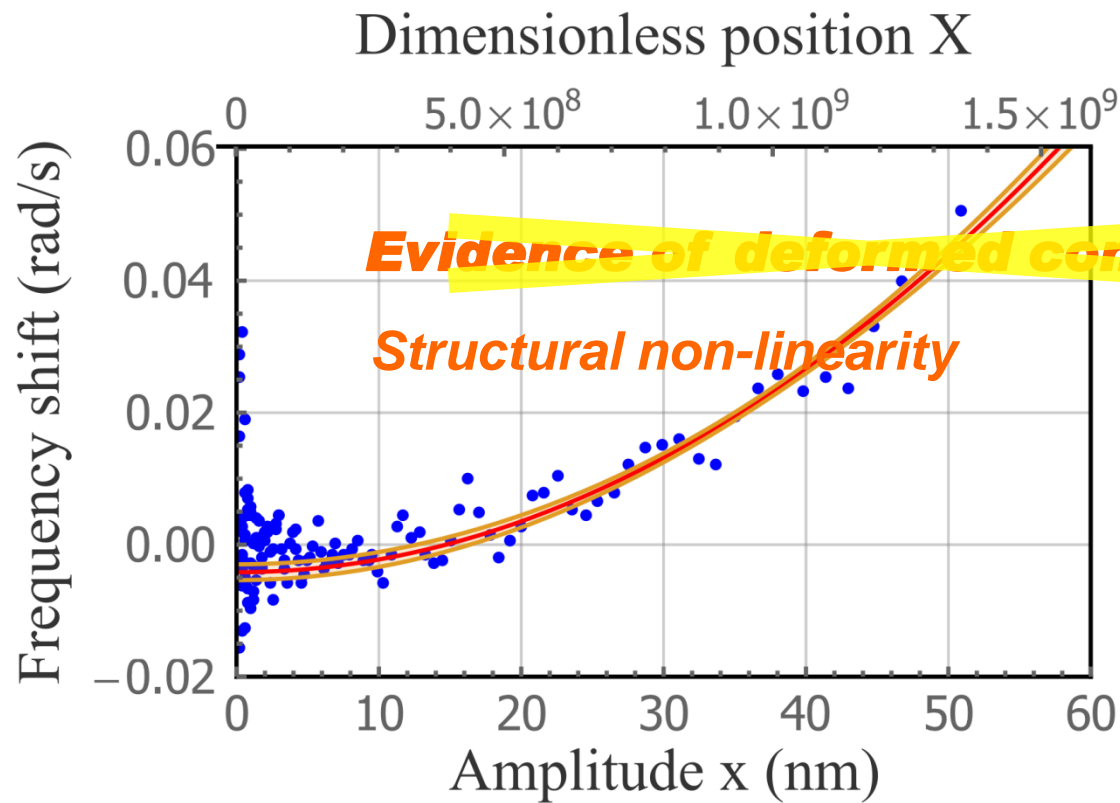
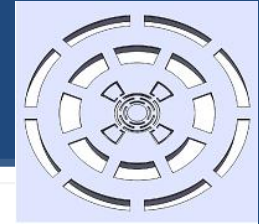


SiN membrane
 $0.5 \times 0.5 \text{ mm}^2 \times 50 \text{ nm}$
 mass = 135 ng
 $Q = 8.6 \times 10^5$





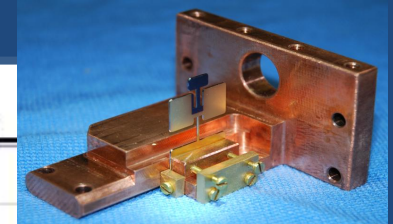
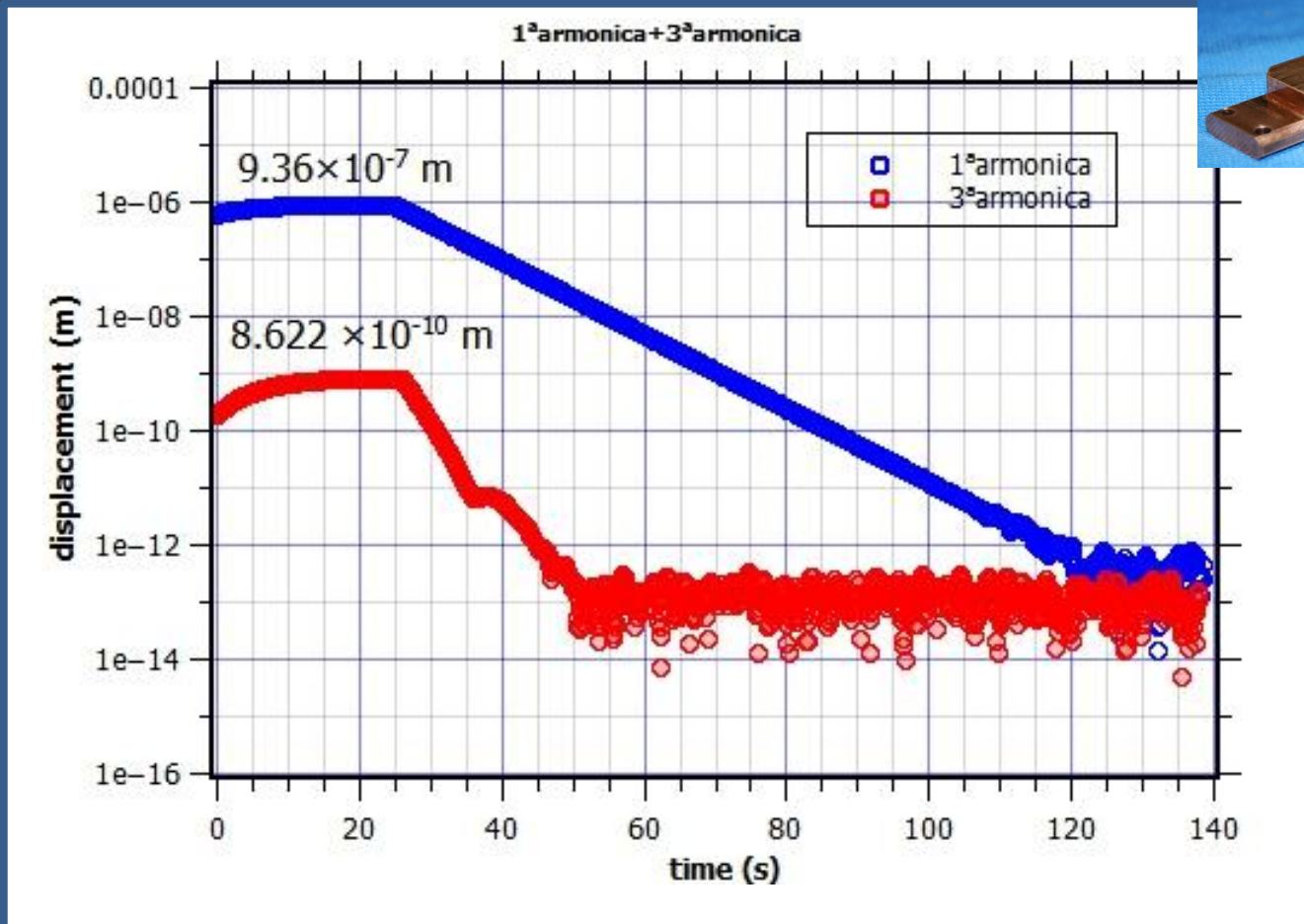
$$\tilde{\omega} = \left(1 + \frac{\beta}{2} X_0^2\right) \omega_0$$

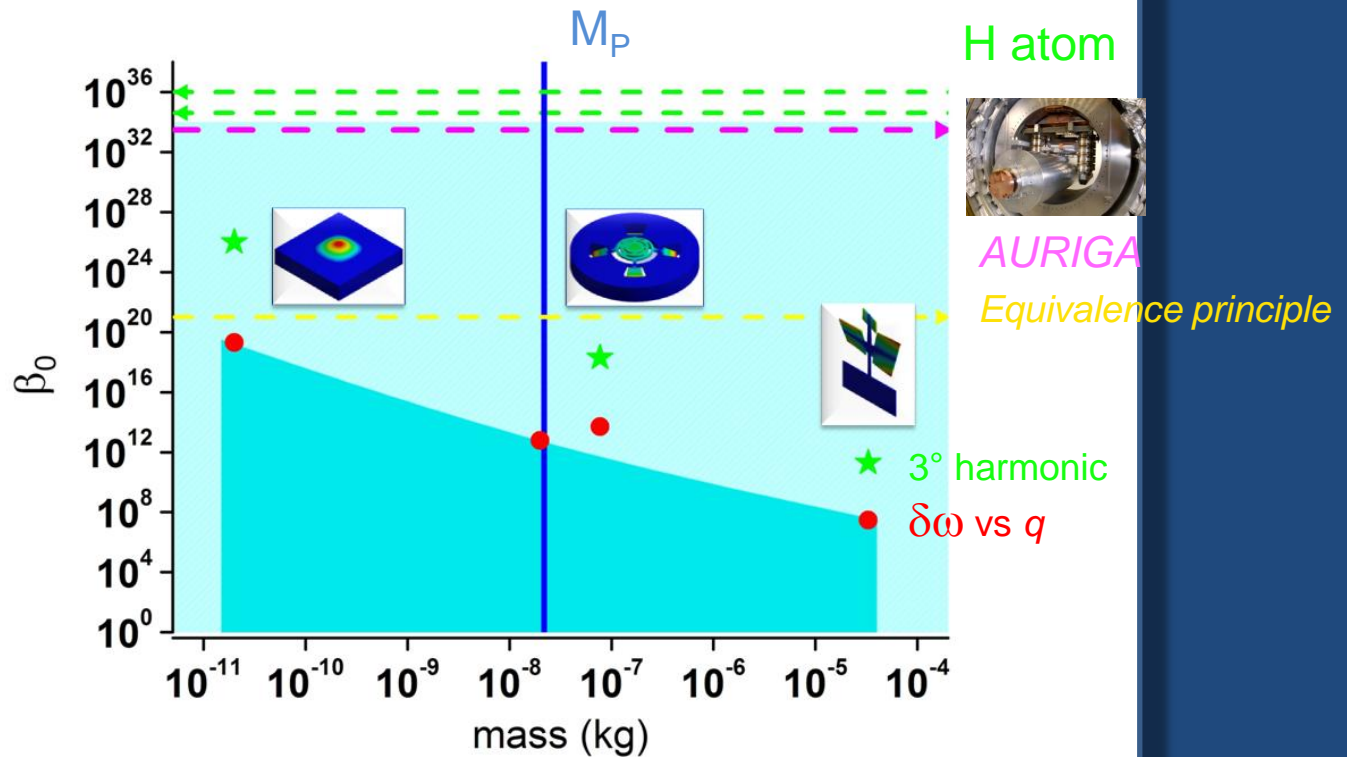


'Model-independent' limits

Mass (kg)	Frequency (Hz)	Max. ampl. (nm)	Max. Q_0	Max. $\Delta\omega/\omega_0$
3.3×10^{-5}	5.64×10^3	600	6×10^{10}	4×10^{-7}
2×10^{-8}	1.42×10^5	55	7×10^8	6×10^{-8}
2×10^{-11}	7.47×10^5	7.5	7×10^6	4×10^{-8}

$$X = X_0 \left[\sin(\tilde{\omega}t) + \frac{\beta}{8} X_0^2 \sin(3 \tilde{\omega}t) \right]$$





If a deformed commutator applies to the coordinates of a *fundamental constituent*, then its effect on a macroscopic object (composed of N such constituents) should decrease as $1/N$

- what is a fundamental constituent?
- geometrical properties of space-time → property of each particle
- not in the spirit of quantum mechanics: the position and momentum of an oscillator c.m. should be THE meaningful (i.e., measurable) quantities. Uncertainty relation between p and q is tightly related to the general problem of quantum measurements (see Braginsky, Caves, etc.)

LETTER

10-1038/nature10461



Focus on peculiar quantum properties

Deriving bounds on the deformation parameter in the QUANTUM regime.

Are there alternative, more “quantum” ways that can be applied in the quantum regime ?

Modified commutators and deformed uncertainty relations could be directly seen by looking at **second order moments (covariance matrix)**.

Straightforward way: deformation seen as **NONGAUSSIAN modification of the steady state of the system** (possible estimators from higher-order correlations in homodyne measurement data); bounds for β derived from upper bounds of a nonGaussianity parameter

Less straightforward:

- i) use more general measure of non-Gaussianity (M. Genoni et al., PRA A **78**, 060303(R) (2008))
- ii) Detecting signature of the deformation parameter β from the dynamics of variances (it is already known that deformed commutators yield a small quadrature squeezing)

Testing non-local EFT with macroscopic quantum objects?

$$\epsilon = \frac{m\omega}{\hbar\Lambda^2}$$

Suppose

$$m = 1\mu g = 10^{-9} Kg$$

and

$$\omega \approx 5 \cdot 10^4 Hz.$$

Then our parameter will be

$$\epsilon \approx 5 \cdot 10^{29} l_{nl}^2$$

that means

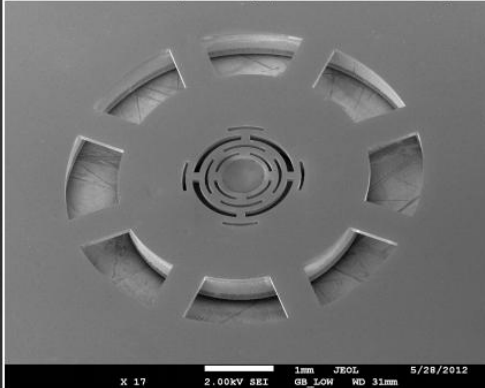
$$\epsilon \ll 1 \Leftrightarrow l_{nl} \ll \sqrt{2} \cdot 10^{-14} m$$

The expansion is justified for small ϵ but for it to be within experimental reach one wants macroscopic quantum objects.
Best case scenario: macroscopic quantum oscillators? (or alternative lighter but better developed BEC?)

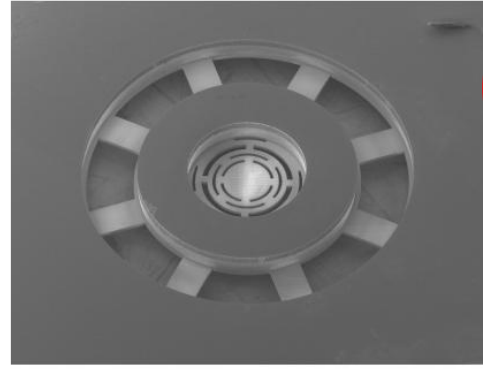
HUMOR

Heisenberg Uncertainty Measured with Opto-mechanical Resonators

1 - Front view of DWO (SEM image) with the central coating



2 - Back view DWO (SEM image) with the insulation wheel



Collaboration SISSA gravity group
with with F. Marin, F. Marino, A.
Ortolan.

Determine evolution of

$$\langle x \rangle = \langle \psi | x | \psi \rangle$$

$$\langle x^2 \rangle \text{ and correlators...}$$

See
F. Marin talk...

Others on gravity + macro quantum oscillator

PRL 112, 210404 (2014)

PHYSICAL REVIEW LETTERS



Proposal for a Noninterferometric Test of Collapse Models in Optomechanical Systems

M. Bahrami,¹ M. Paternostro,² A. Bassi,^{1,3} and H. Ulbricht⁴

PRL 110, 170401 (2013)

PHYSICAL REVIEW LETTERS



Macroscopic Quantum Mechanics in a Classical Spacetime

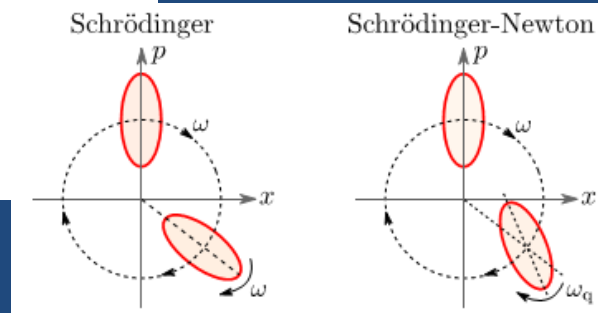
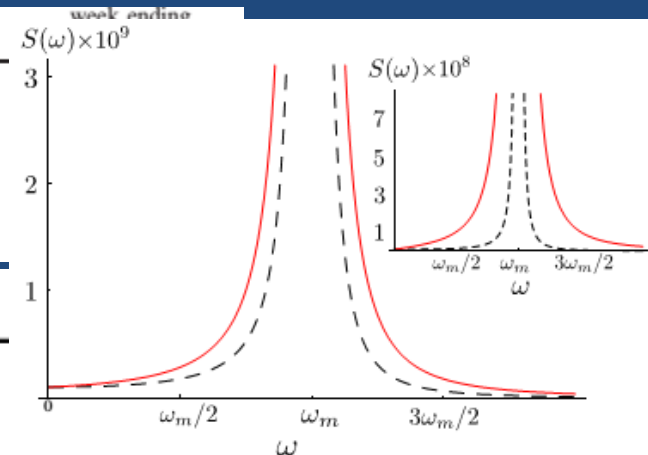
Huan Yang,¹ Haixing Miao,¹ Da-Shin Lee,^{2,1} Bassam Helou,¹ and Yanbei Chen¹

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(Received 9 October 2012; revised manuscript received 19 January 2013; published 22 April 2013)

We apply the many-particle Schrödinger-Newton equation, which describes the coevolution of a many-particle quantum wave function and a classical space-time geometry, to macroscopic mechanical objects.



Gravity-related spontaneous wave function collapse in bulk matter

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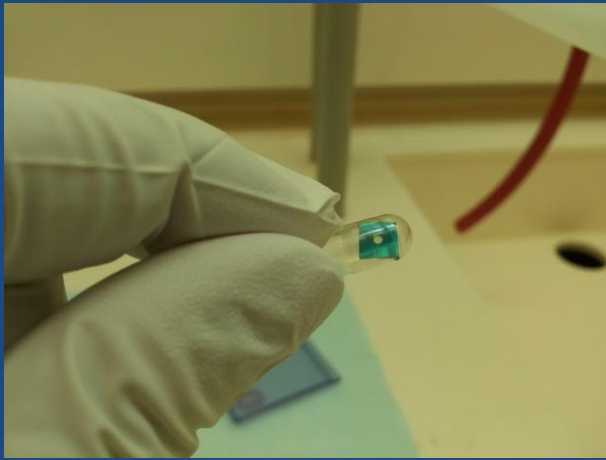
Abstract

In the DP-model, gravity-related spontaneous wave function collapses suppress Schrödinger cat states which are conceptually problematic especially for gravity and space-time. We derive the equations of the model for the hydrodynamic-elastic (acoustic) modes in a bulk. Two particular features are discussed: the universal dominance of spontaneous collapses at large wavelengths, and the reduction of spontaneous heating by a slight refinement of the DP-model.

... toward a quantum mechanical oscillator

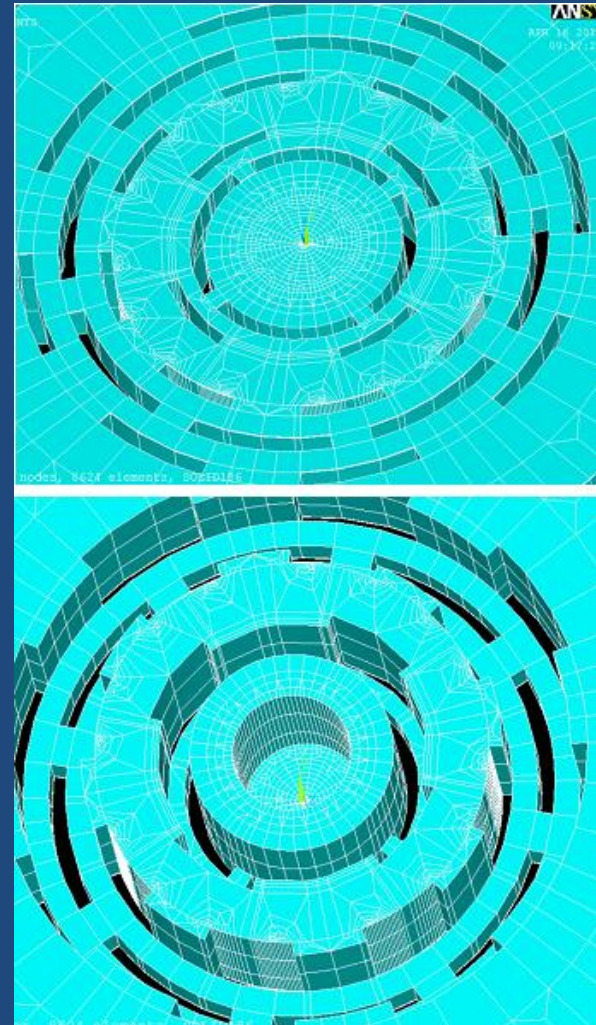
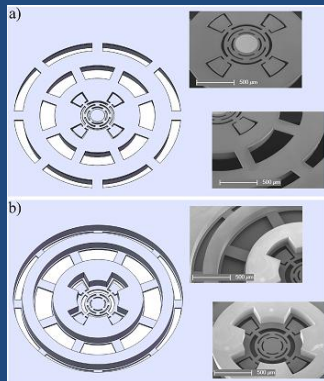
(quantum fluctuations prevails over thermal fluctuations)

- ❖ nanoparticle trapped in optical potential
- ❖ SiN nano-membranes

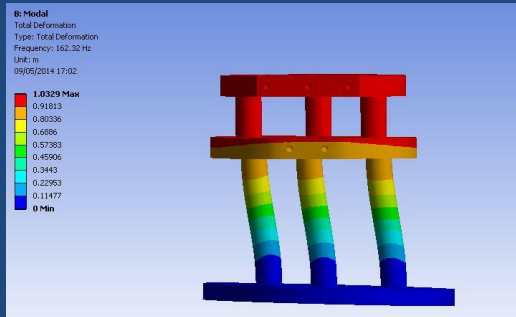
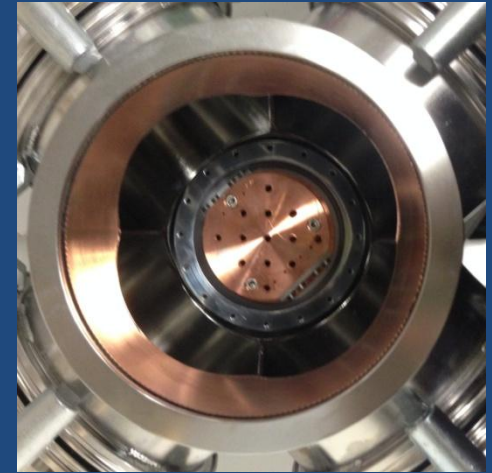
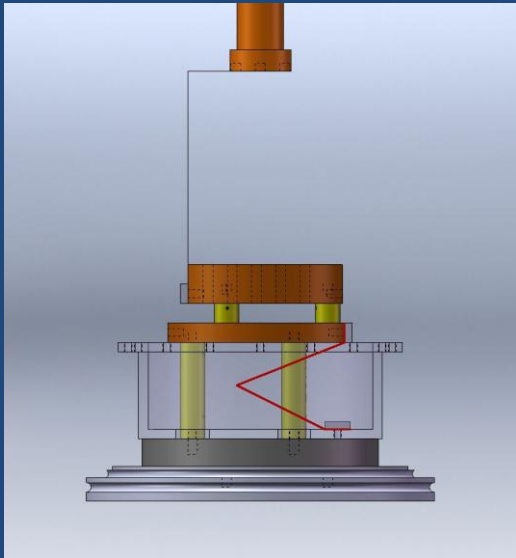


Fabricated SiN membranes by deep-DRIE

The membranes will be integrated on isolation systems realized from SOI



Designed and realized a low-noise cryostat



... tests on quantum oscillators coming soon