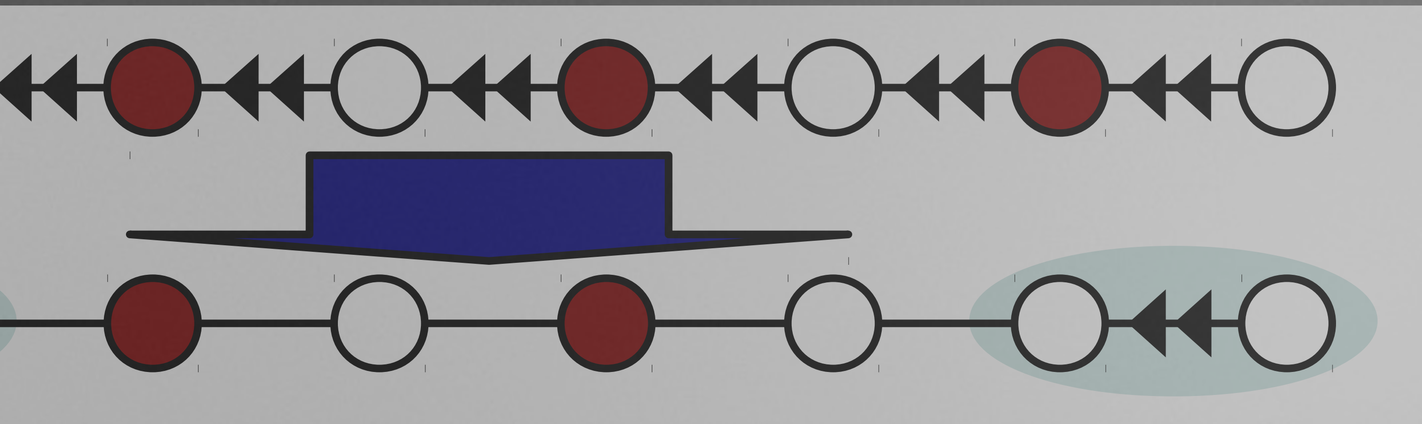
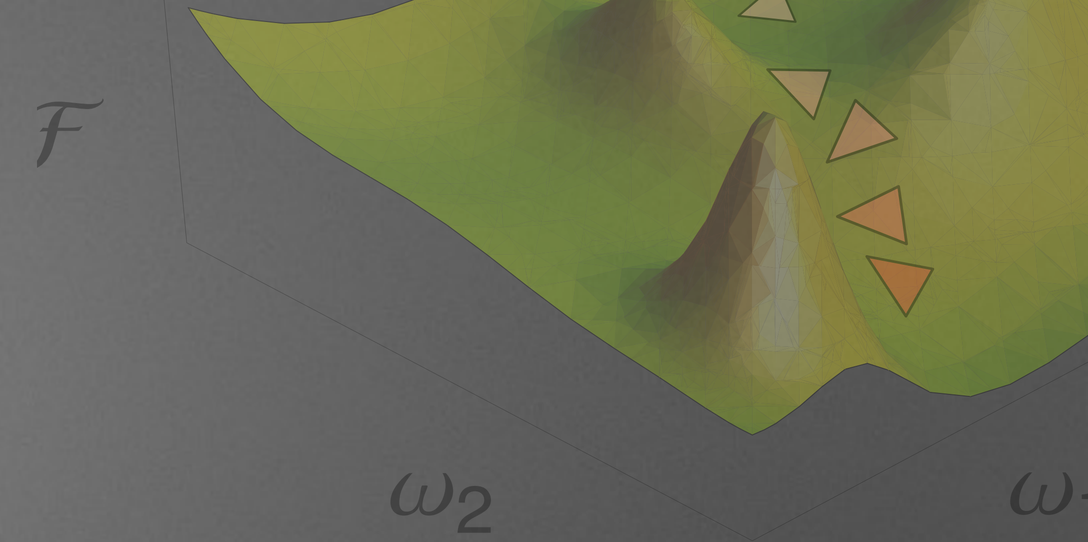
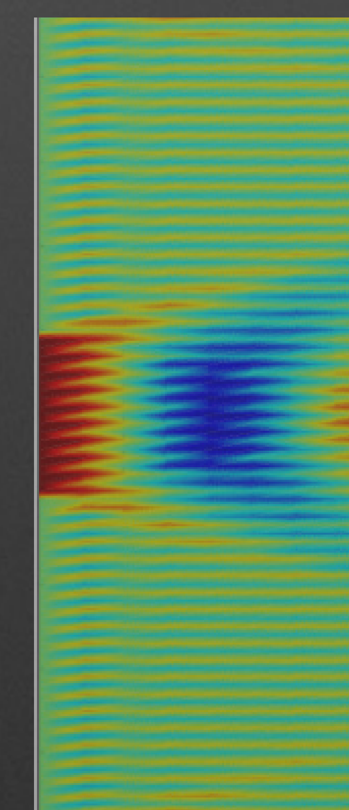


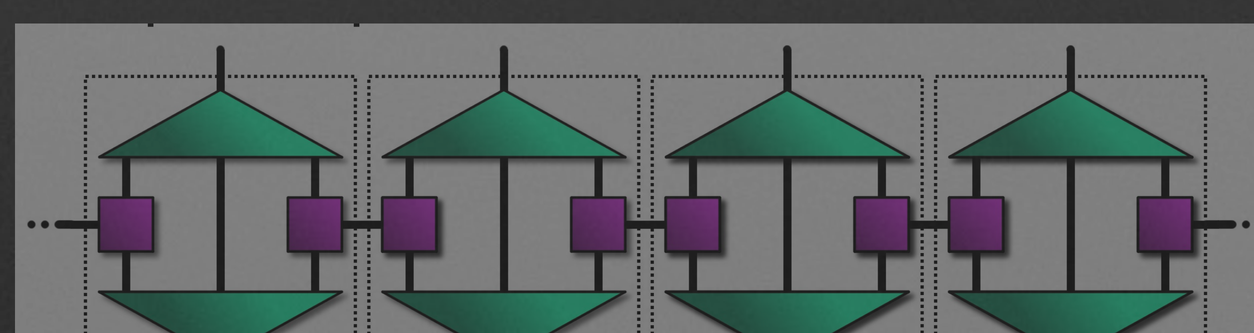


Lattice Gauge Tensor Networks

Simone Montangero
Institute for Complex Quantum Systems
&
Institute for Integrated Quantum Science and Technologies
Ulm University

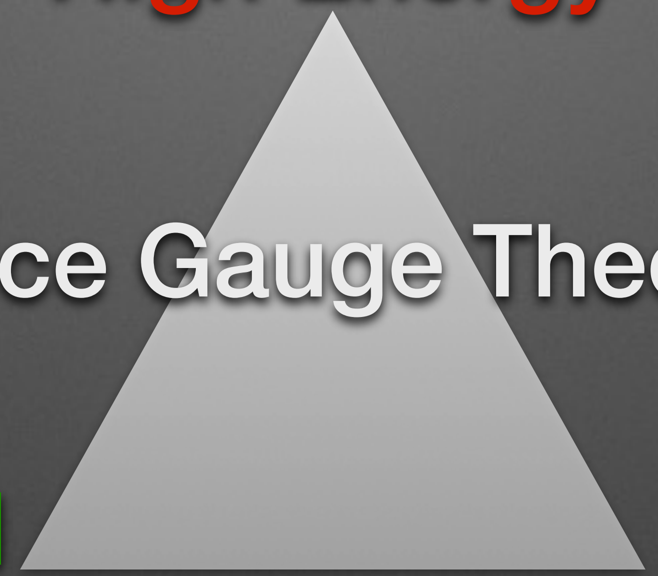


5/5/15 GGI - Firenze



Non-perturbative formulation
of high-energy models

High Energy



Condensed
Matter

Quantum
Information

Strongly correlated quantum
many body systems

Exotic phases of matter
to support QI processing

Traditional numerical methods
(Quantum Monte Carlo,...)

High Energy

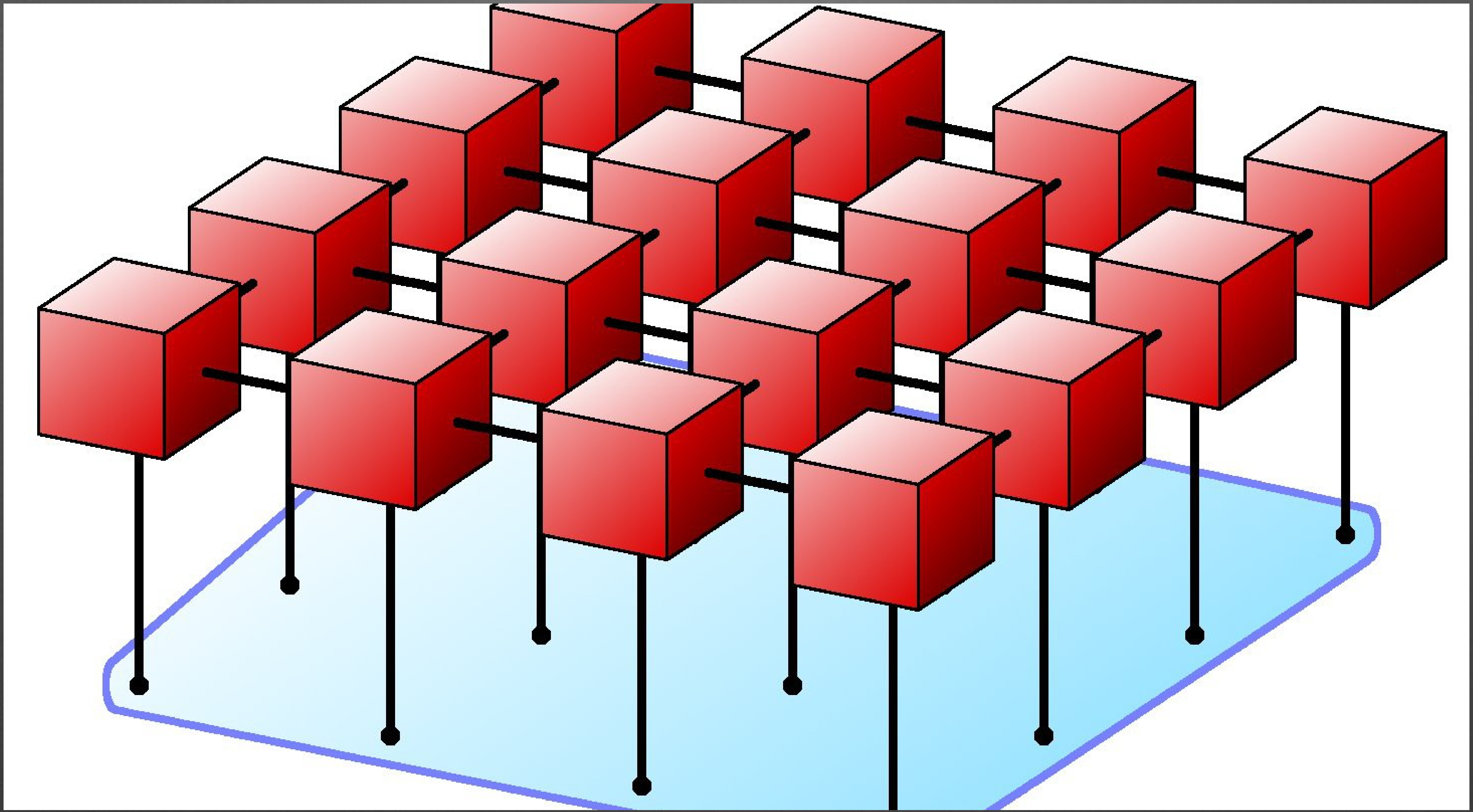
Quantum Technologies for Lattice Gauge Theories

Condensed
Matter

Quantum
Information

Tensor networks
(classical simulations)

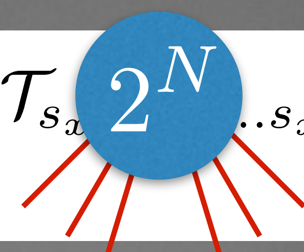
AMO physics
(quantum simulations)



Tensor Networks

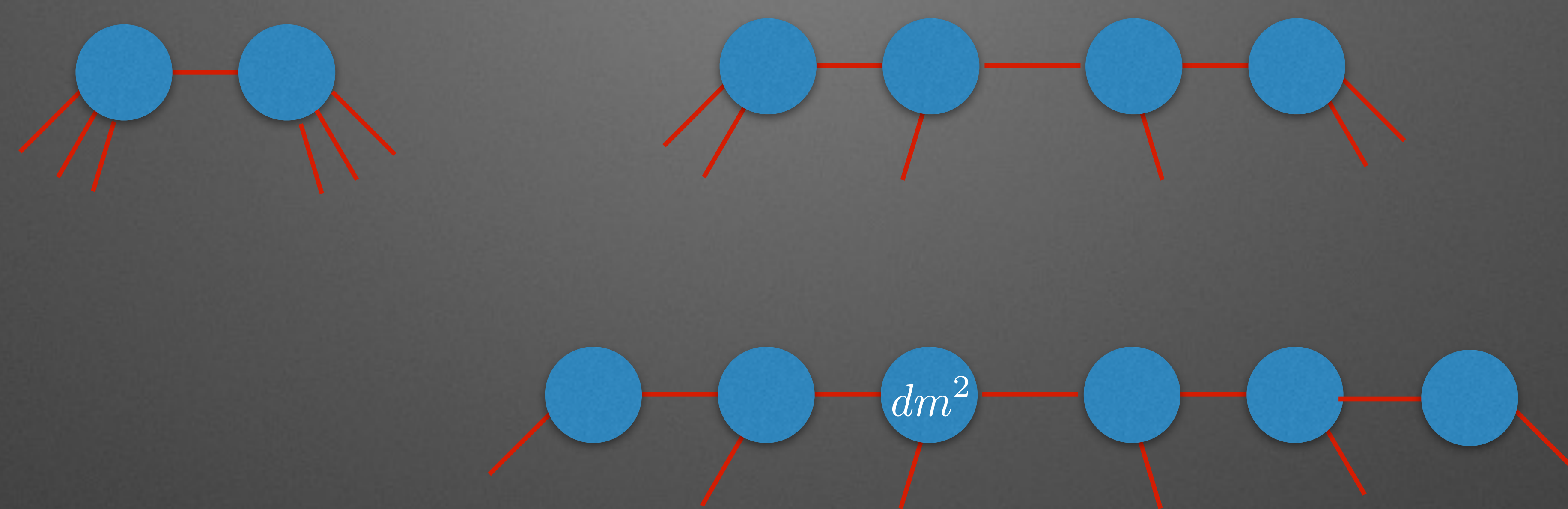
Brief Introduction

$$|\Psi_{\text{many-body}}\rangle = \sum_{s_{x_1}, s_{x_2} \dots s_{x_N}} \mathcal{T}_{s_{x_1} \dots s_{x_N}} |s_{x_1}, s_{x_2} \dots s_{x_N}\rangle$$



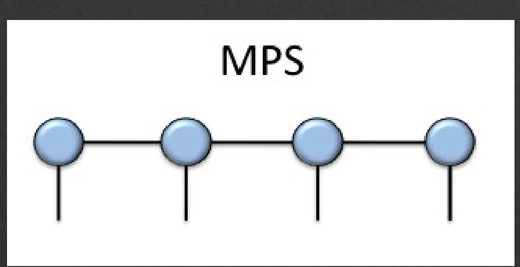
SVD

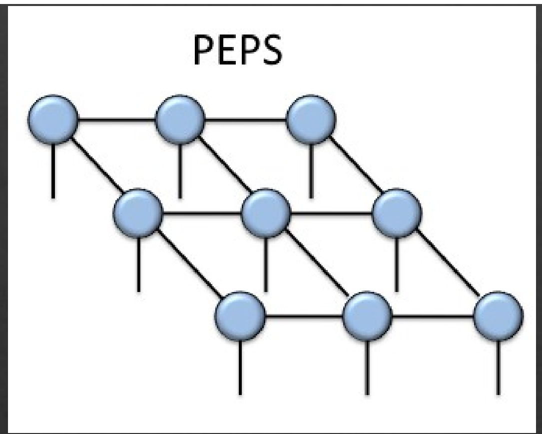
Contractions

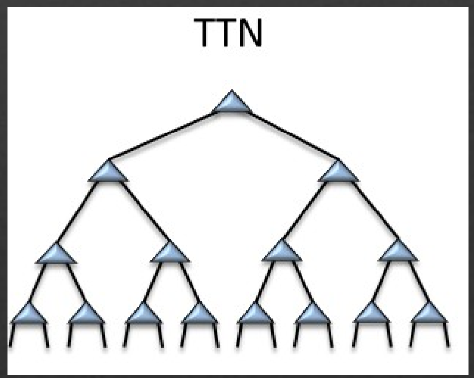


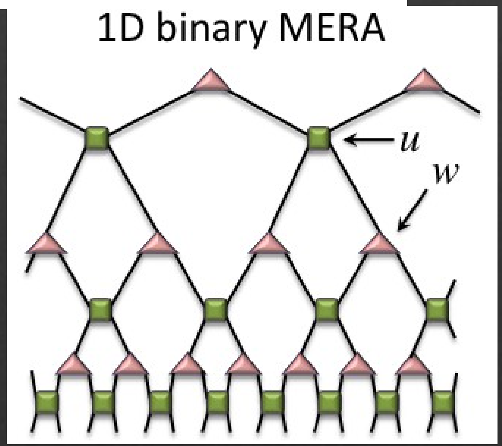
Assume:

$$|\Psi_{\text{MPS}}\rangle = \sum_{\{s_i\},\{\alpha_i\}} A^{(s_1)}_{\alpha_1} A^{(s_2)}_{\alpha_1,\alpha_2} \cdots A^{(s_N)}_{\alpha_{N-1}} |s_1, s_2, \cdots, s_N\rangle$$









U. Schwollöck, Rev. Mod. Phys. (2005)

“Is all that useful at all?”

Tensor network algorithms

Describe ground states (finite chemical potential) and real time dynamics of (1D, short range) many-body quantum systems.

$$H = \sum_{i,j} H_{i,j} + H(t)$$

Access to local quantities, correlators (space and time), entanglement

$$\langle \hat{O}_i \rangle \qquad \langle \hat{O}_i \hat{O}_j \rangle \qquad \mathrm{Tr}|\Psi\rangle\langle\Psi|$$

Extension to open quantum systems dynamics

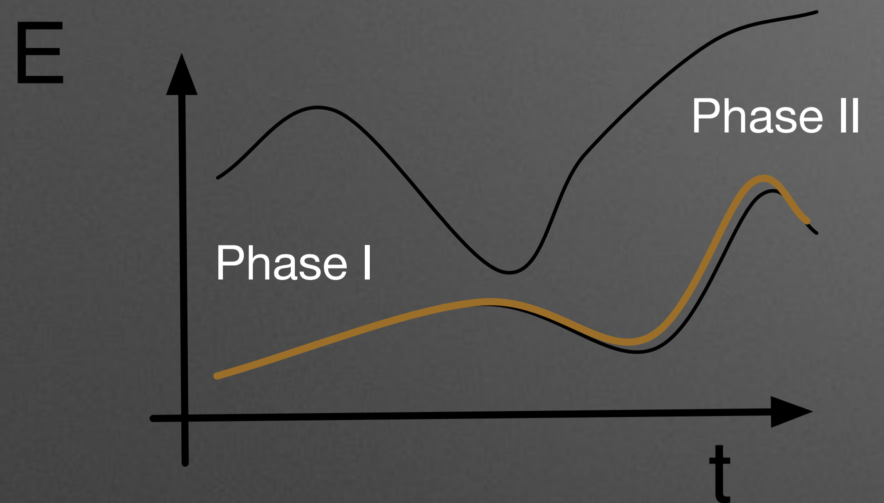
Q. trajectories, MPO, and A. Werner et.al. 1412.5746

Optimal control of many-body quantum system

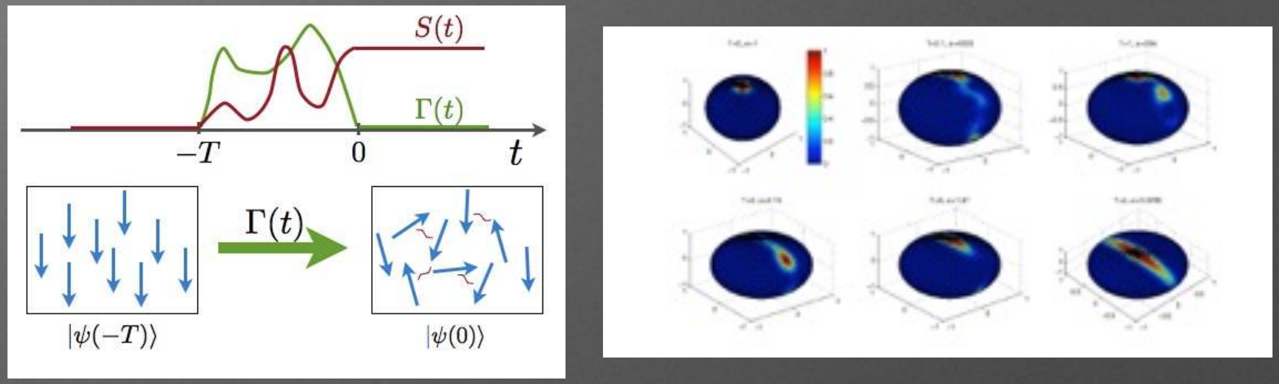
$$H = H_D + \lambda(t) H_C \qquad \max_{\lambda(t)} F(\lambda, H, \dots)$$

P. Doria, T. Calarco, SM PRL (2010), S. Lloyd, SM PRL (2014)

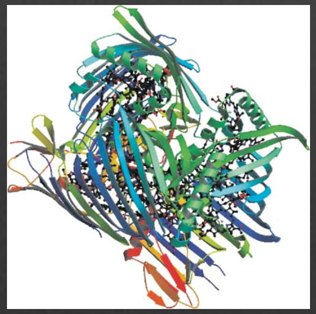
Applications



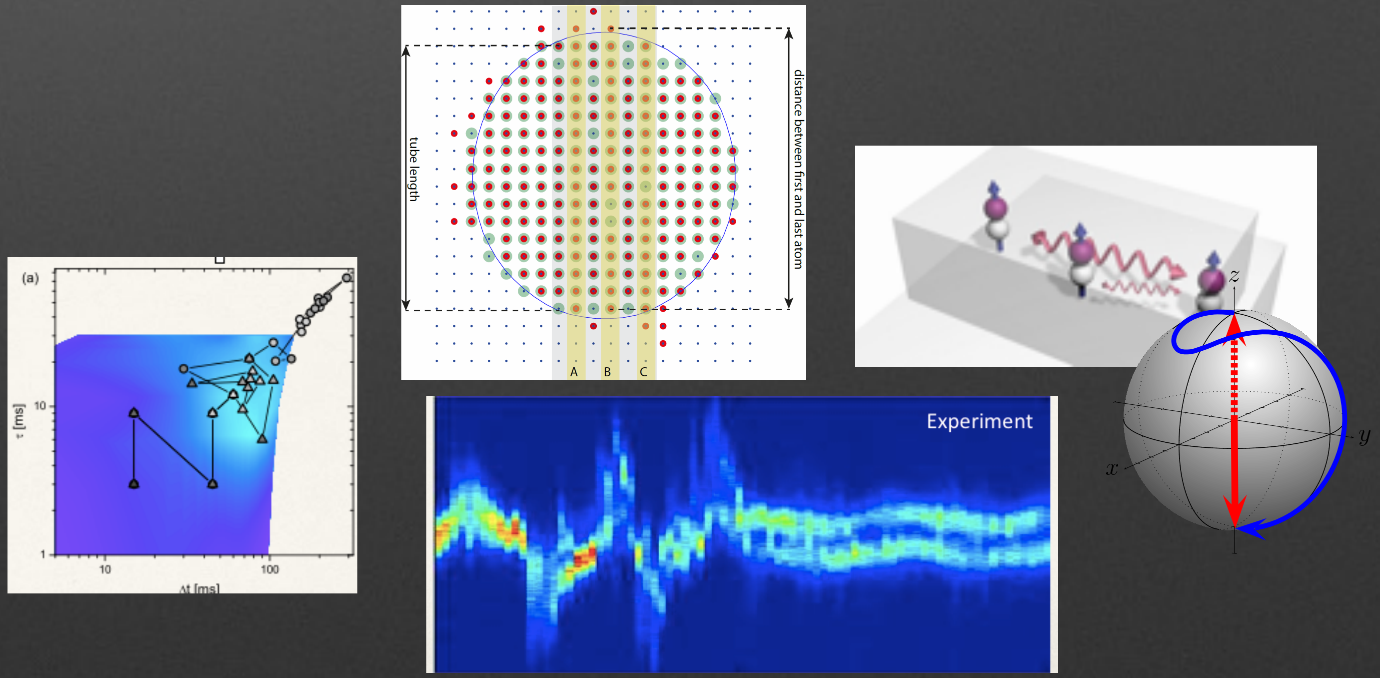
Quantum Phase Transition dynamics



Entanglement/Squeezing manipulation



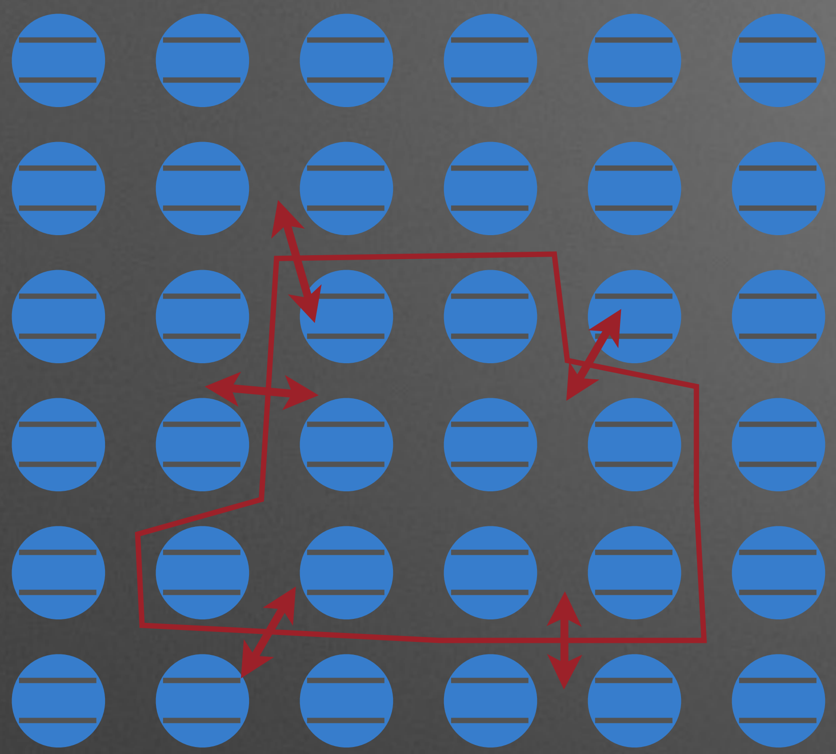
Light-harvesting dynamics



Optimal experimental protocols

“How is all that possibly working?”

Area laws



Quantum correlations

$$S = \mathrm{Tr}\{\rho_A \log \rho_A\}$$
$$S \propto N^{D-1}(\log N)$$

Representable entanglement in MPS states

$$S \propto \log m \qquad m \propto O(Poly(N))$$

Ground states of local Hamiltonians obey area laws

J.Eisert, M. Cramer, M.B. Plenio Rev. Mod. Phys. 2009

Time evolution

Highly excited dynamics: $S \propto t$

P. Calabrese and J. Cardy, J. Stat. Mech. (2005)

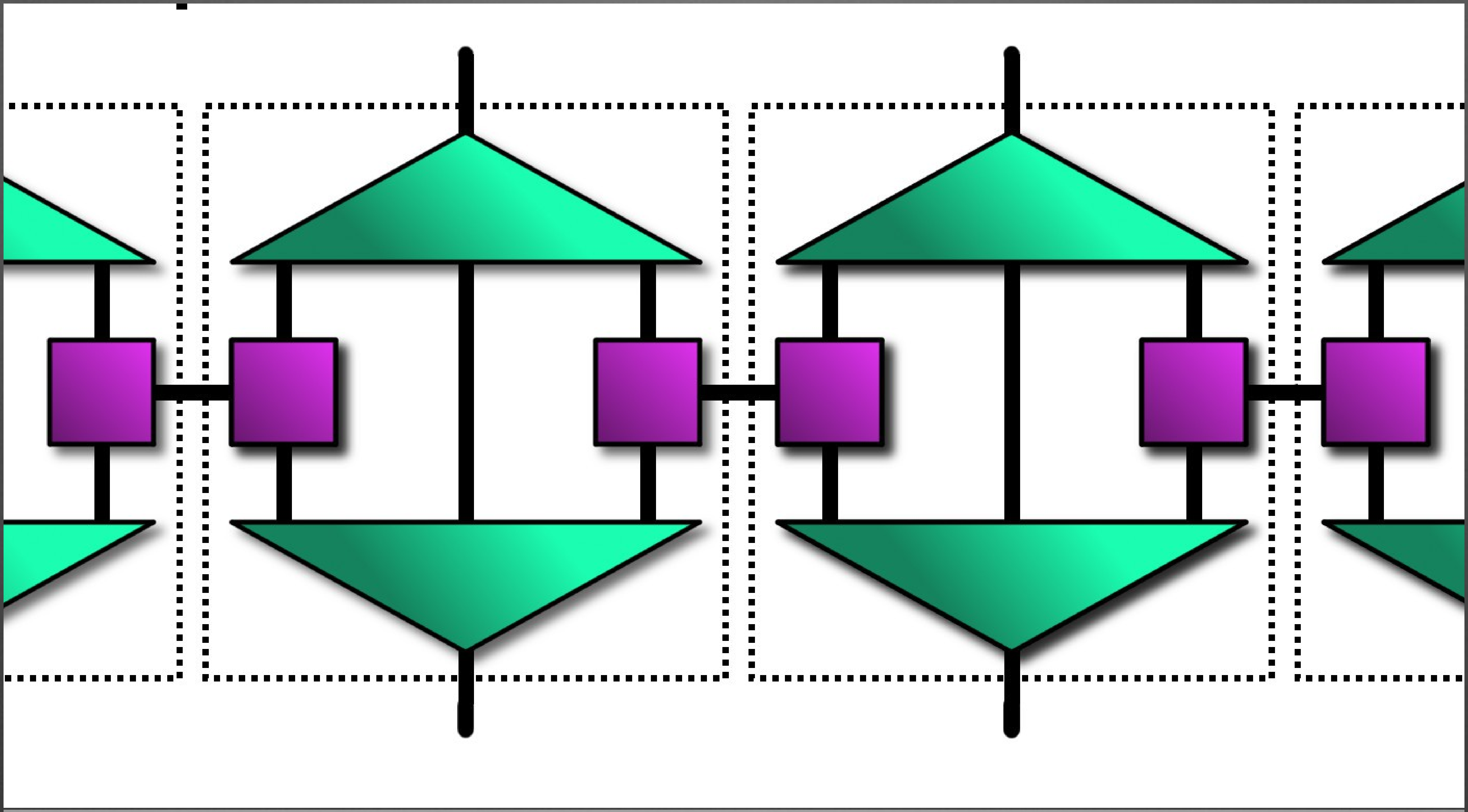
+ localization effects: $S \propto \log(t)$

G. De Chiara, SM, P. Calabrese, R. Fazio, J.Stat.Mech. (2006)

Adiabatic or quasi-adiabatic dynamics: $S \sim \mathrm{const}$

What can be simulated can be controlled

S. Lloyd, SM PRL (2014)



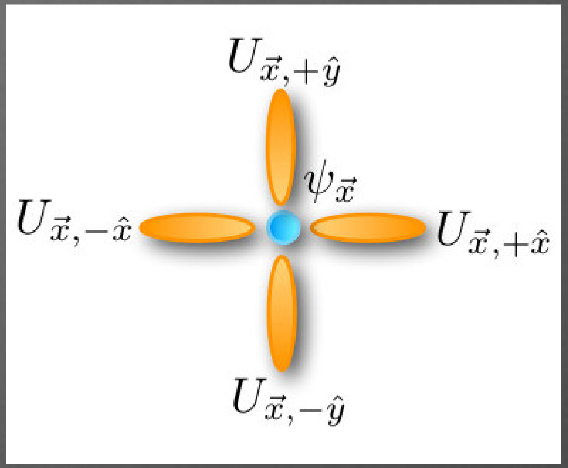
Lattice Gauge Tensor Networks

Models

Local degrees of freedom

$$[\psi_x^a, U_{x,x+\mu_x}^{ab}]$$

matter field gauge field



Gauge symmetry generator $[G_x^\nu]$

Gauge invariant dynamics

$$H = J \sum_x \left(\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \text{h.c.}\right)$$

abelian

$$H = t \sum_{x,a,b} [\psi_x^{a\dagger} U_{x,x+1}^{ab} \psi_{x+1}^b + \text{h.c.}]$$

non abelian

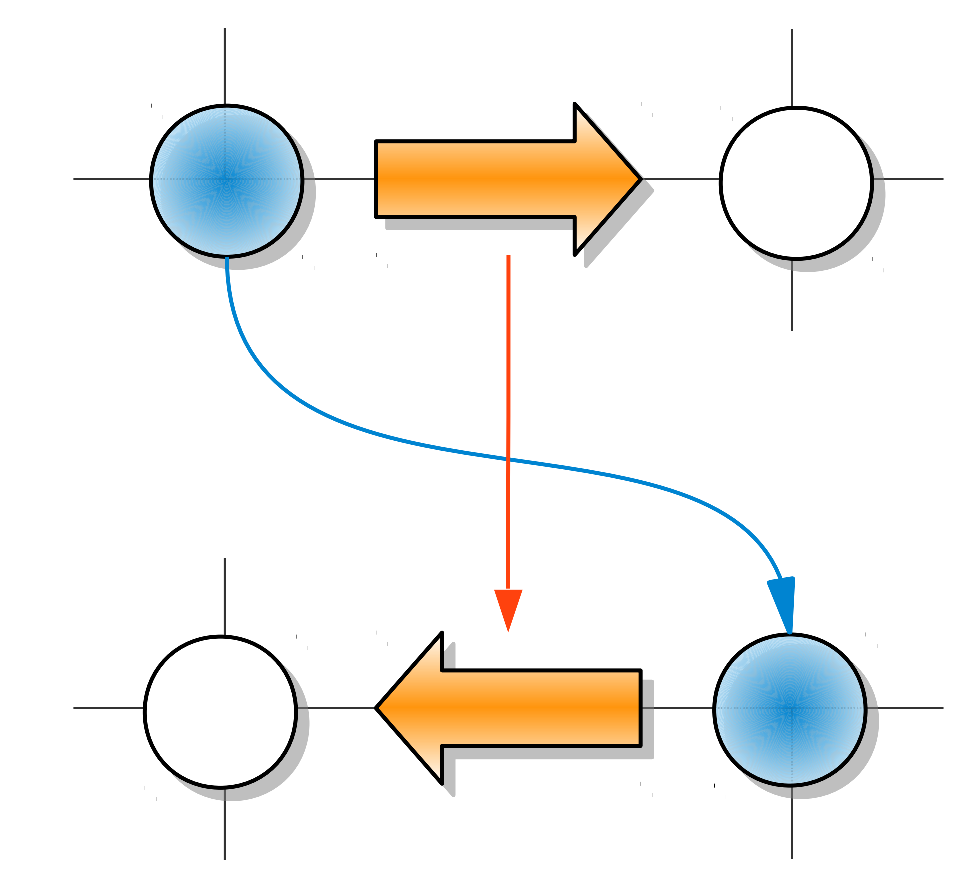
- Kogut-Susskind Hamiltonian formulation of LGT
- Condensed matter models
- Benchmarking of quantum simulations

- **Charge field**: spinless fermions/bosons

- **Electric field**: two-level system (qubit)

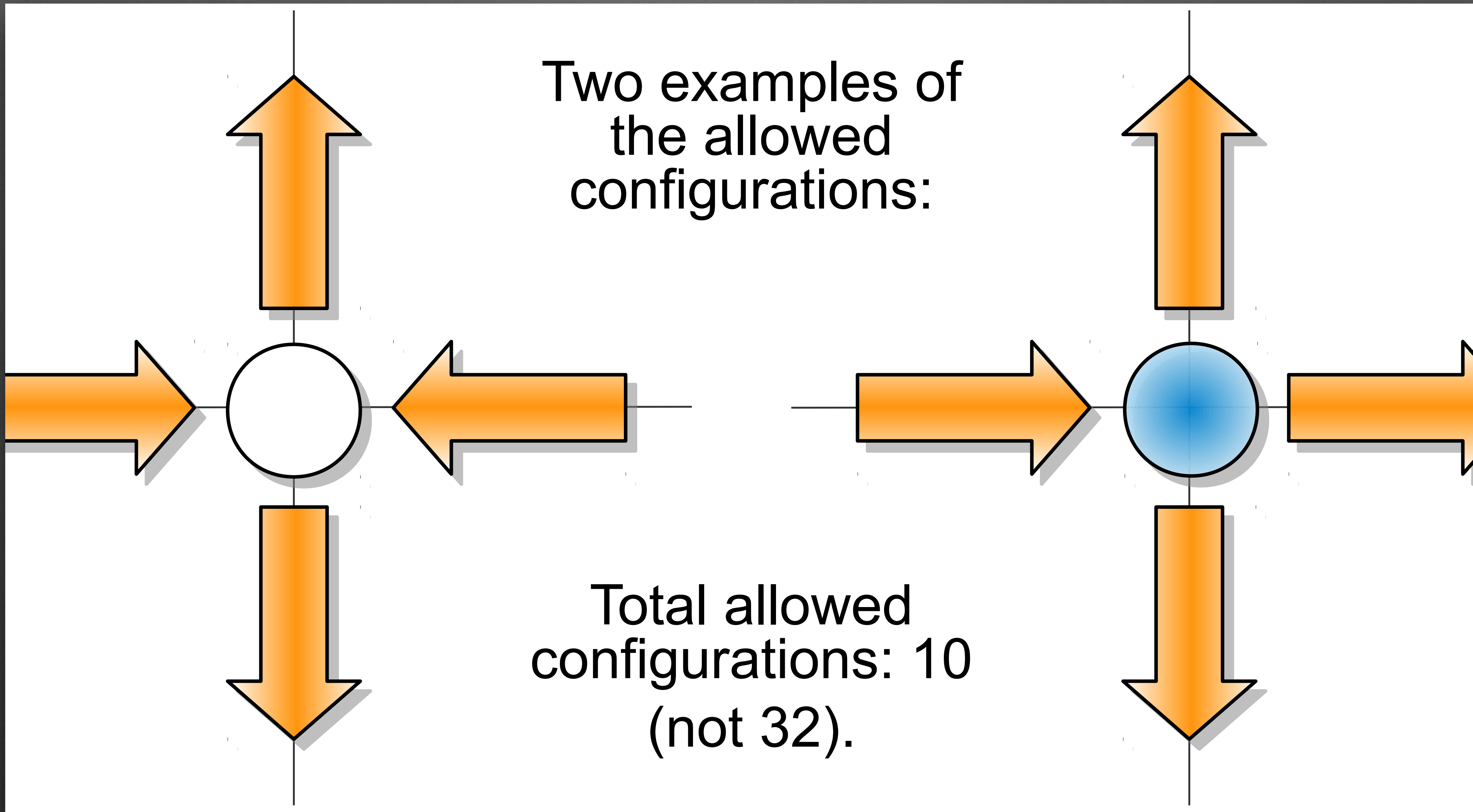
Effective move:

Matter jumps right/left, gauge field flips to the left/right.



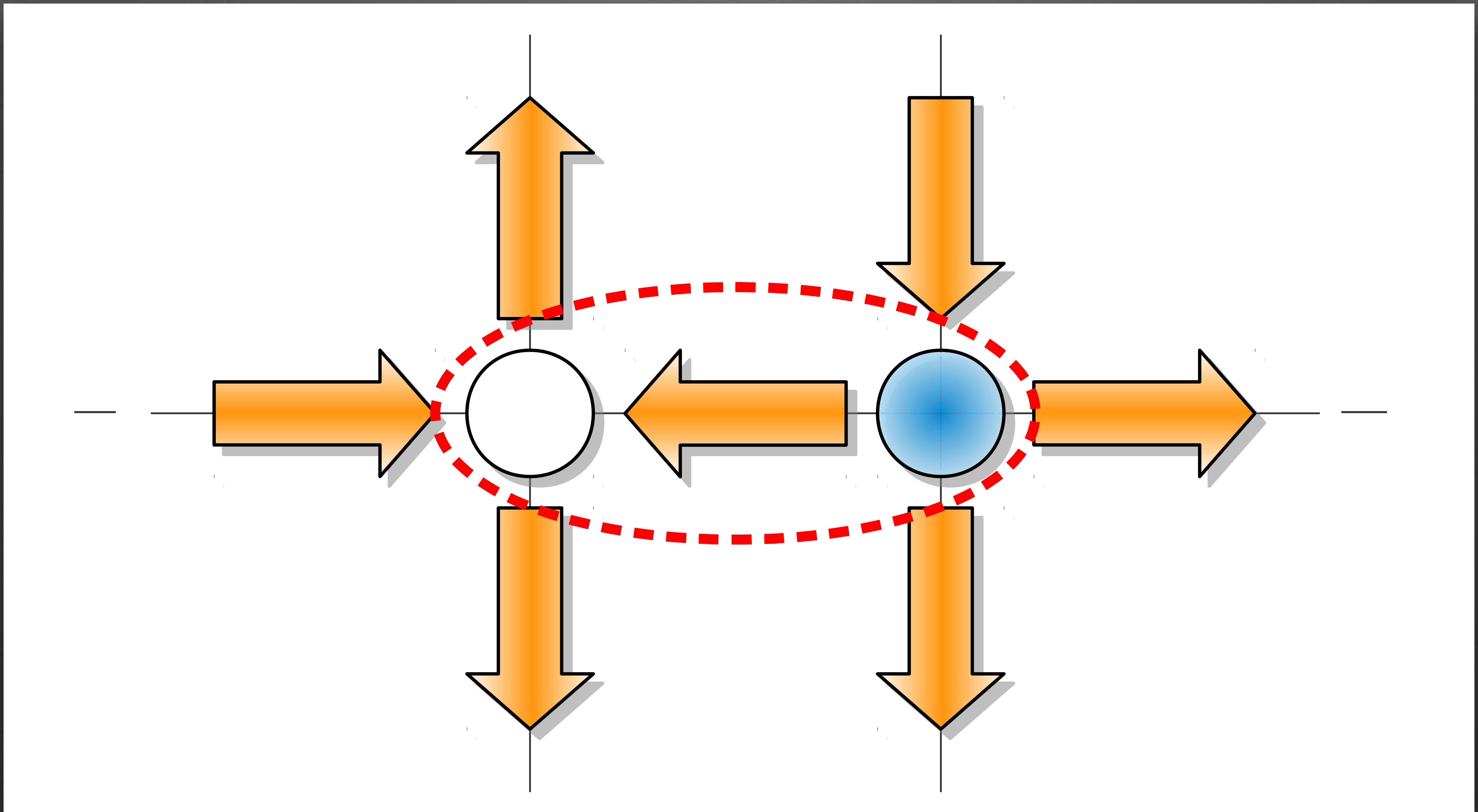
$$H_{\mathrm{int}}^{[\mathrm{QED}]} = \sum_{x,\vec{a}} \psi_x^\dagger \, \sigma_{x,x+\vec{a}}^{+} \, \psi_{x+\vec{a}} + (-1)^q \, \psi_x \, \sigma_{x,x+\vec{a}}^{-} \, \psi_{x+\vec{a}}^\dagger$$

Simplest example



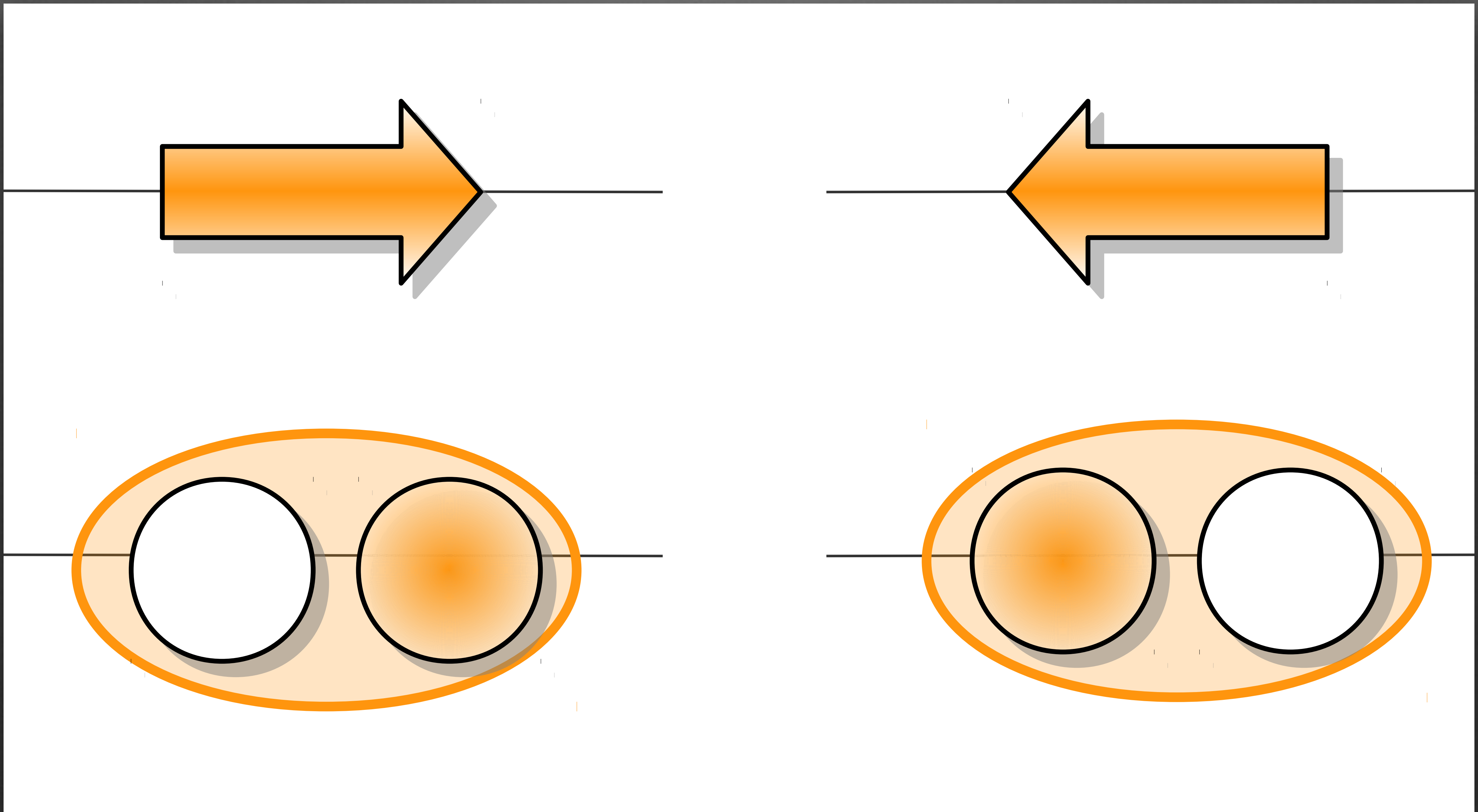
$$\rho - \vec{\nabla} \cdot \vec{E} = 0 \quad \Longrightarrow \quad G_x|\varphi_{\rm phys}\rangle = \left(\psi_x^\dagger\,\psi_x - \tfrac{1}{2}\sum_{\vec{a}}\sigma^z_{x,x+\vec{a}}\right)|\varphi_{\rm phys}\rangle = 0$$

Lattice gauge generator (Gauss' law)



Dynamics commutes with the gauge symmetry

$$\left[H_{\text{int}}^{[\text{QED}]}, G_x\right] = 0 \quad \forall\, x$$

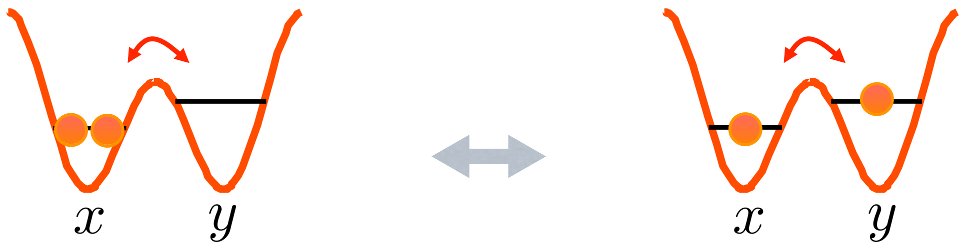


Quantum link formulation

Schwinger representation

$$U_{x,x+\vec{a}} \longrightarrow c_{x,\vec{a}}\, c^{\dagger}_{x+\vec{a},-\vec{a}}$$

Schwinger represenation



Link operator $$U_{x,y} \equiv S^{+}_{x,y} = c^{\dagger}_{y} c_{x}$$

Electric field [U(1) generator] $$E_{x,y} \equiv S^{(3)}_{x,y} = \frac{1}{2}\left[c^{\dagger}_{y} c_{y} - c^{\dagger}_{x} c_{x}\right]$$

$\{c_x, c^{\dagger}_{y}\} = \delta_{x,y}$ Schwinger fermions (rishons)

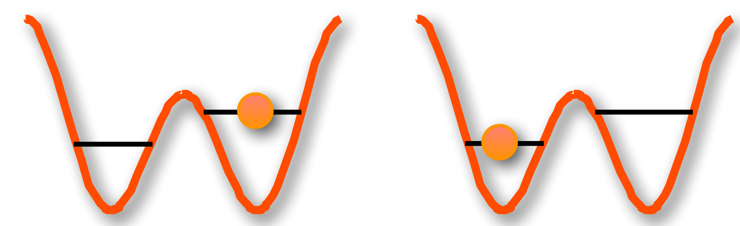
$[c_x, c^{\dagger}_{y}] = \delta_{x,y}$ Schwinger bosons

Spin representation:

$$N_{x,y} = c^{\dagger}_{y} c_{y} + c^{\dagger}_{x} c_{x} \qquad \left[\vec{S}_{x,y}\right]^{2} \equiv \frac{N_{x,y}}{2}\left[\frac{N_{x,y}}{2} + 1\right]$$

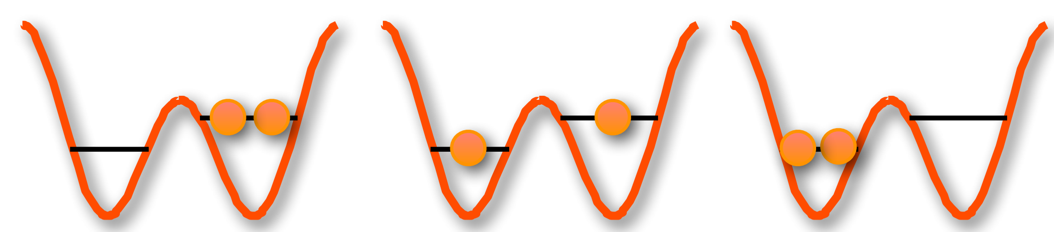
Spin-½:

E=1/2 E=-1/2



Spin-1:

E=+1 E=0 E=-1



Quantum link + Tensor Networks

- Standard approach with additional symmetry
- Gauge satisfied via local Hilbert space definition (also non abelian):

Reduced Hilbert space size

- Additional Abelian symmetry (rishons per site) that can be exploited for computational speedup.

Speed up increase with theory complexity

Gauge invariant TN

Gauge invariant local bases

$$G_x|i\rangle = 0$$

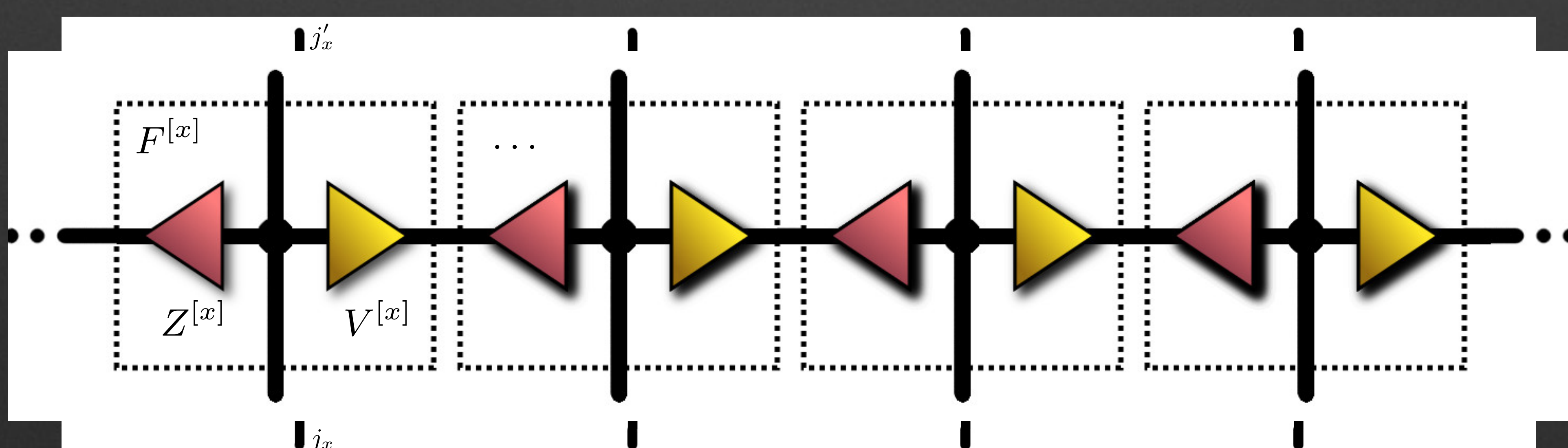
$$|j_x\rangle_r = \sum_{s_\psi, s_R, s_L} A^{[x]j}_{s_R, s_\psi, s_L} |s_R, s_\psi, s_L\rangle_x$$

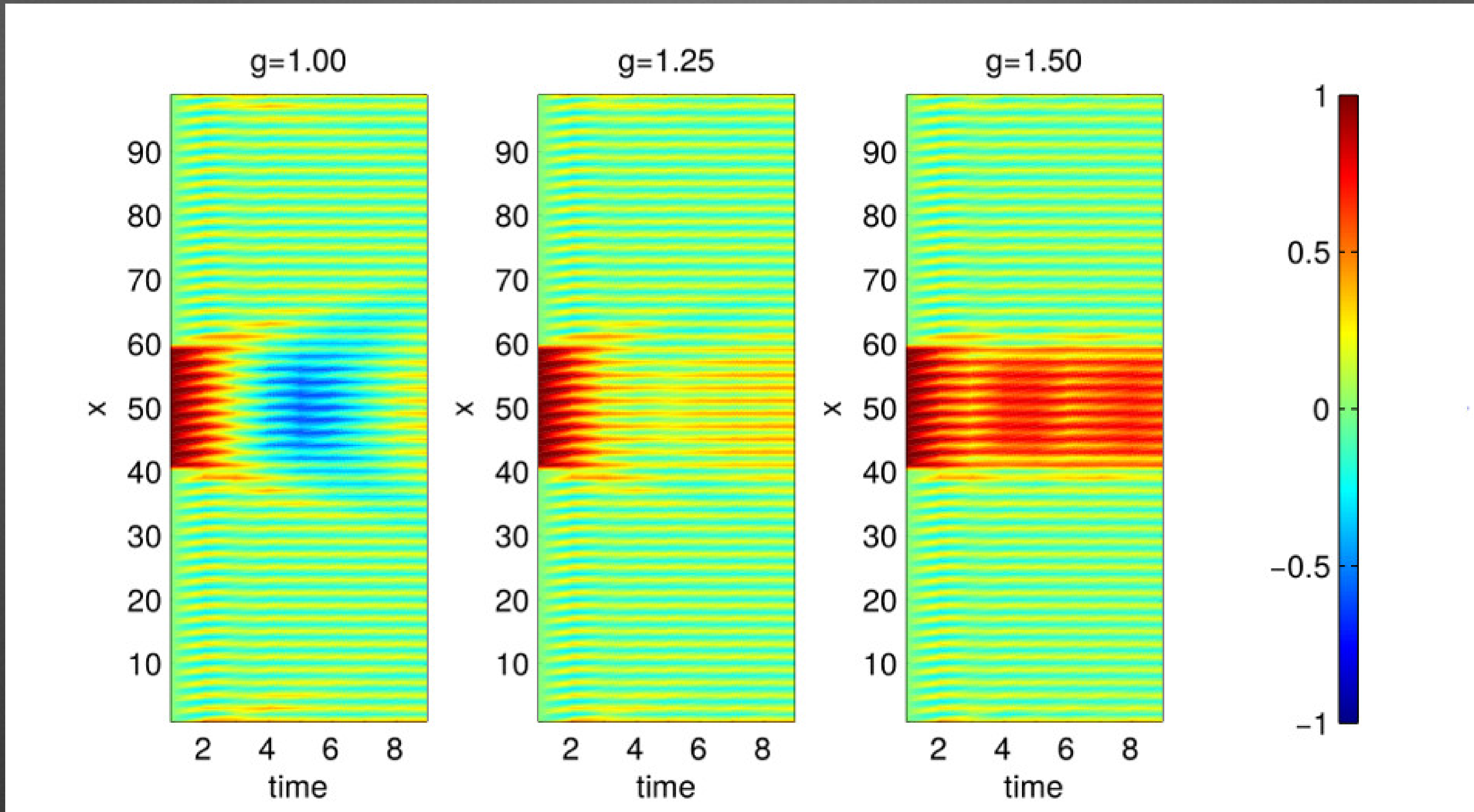
Projection over rishons number

$$Q_{x,x+1} = \sum_{s_R, s_L, q} B^{[x]}_{s_L,q} C^{[x+1]}_{q,s_R} \, |s_L\rangle\langle s_L|_x \otimes |s_R\rangle\langle s_R|_{x+1}$$

$$F^{[x]q_{x-1},q_x}_{j_x,j'_x} = \delta_{j_x,j'_x} \cdot Z^{[x]q_{x-1},j_x} \cdot V^{[x]q_x,j_x}$$

MPO





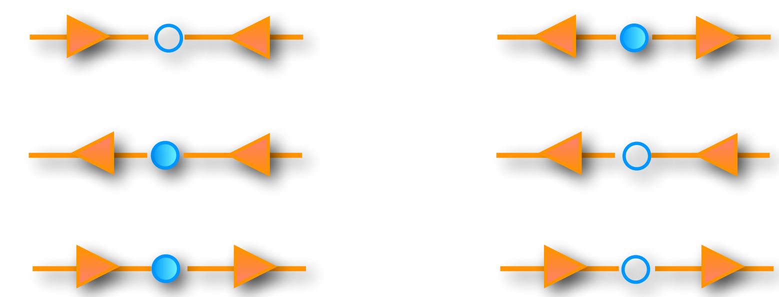
Numerical Results

Phase diagram (cartoon)

Spin-1/2 representation:

$|0\rangle$ $|1\rangle$ $|-1/2\rangle$ $|+1/2\rangle$

Gauge invariant states:



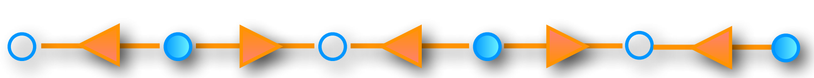
odd-sites even-sites

Vacuum (reference) state:

$$H = \mu \sum_x (-1)^x \, \psi_x^\dagger \psi_x$$

Staggered mass term

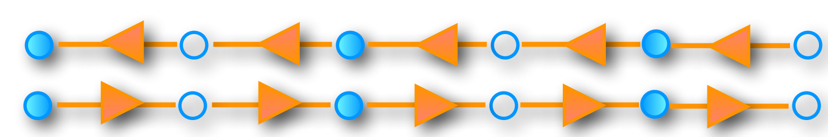
$\mu < 0:$ $\mathcal{E} = \sum_x E_{x,x+1} = 0$



$2x-1$ $2x$ $2x+1$

Total electric flux

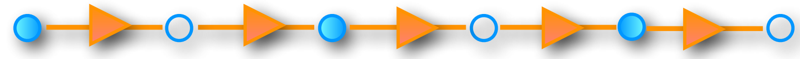
$\mu > 0:$ $\mathcal{E} = \sum_x E_{x,x+1} \neq 0$



$$|\mu| \gg t$$

Phase diagram II

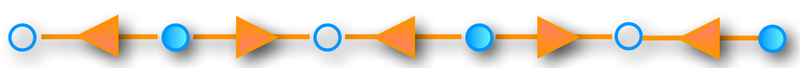
$$H = \mu \sum_x (-1)^x \, \psi_x^\dagger \psi_x + \sum_x \psi_x^\dagger S_{x,x+1}^+ \psi_{x+1} + \text{h.c.}$$

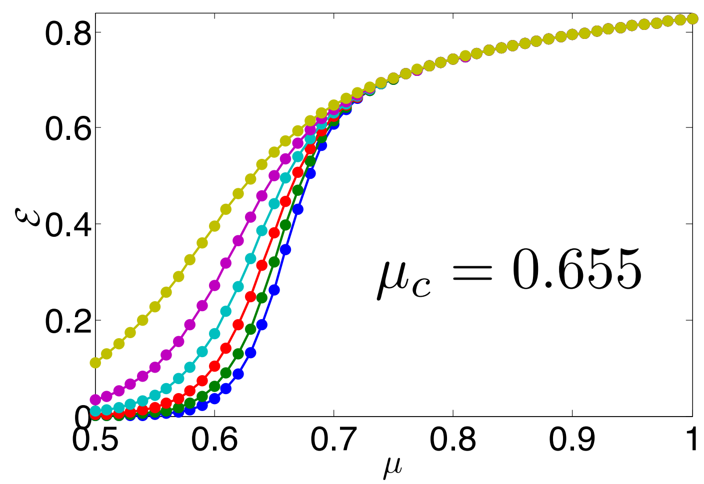


$$\mu > 0: \quad \mathcal{E} = \sum_x E_{x,x+1} \neq 0$$

Total electric flux

$$\mu < 0: \quad \mathcal{E} = \sum_x E_{x,x+1} = 0$$

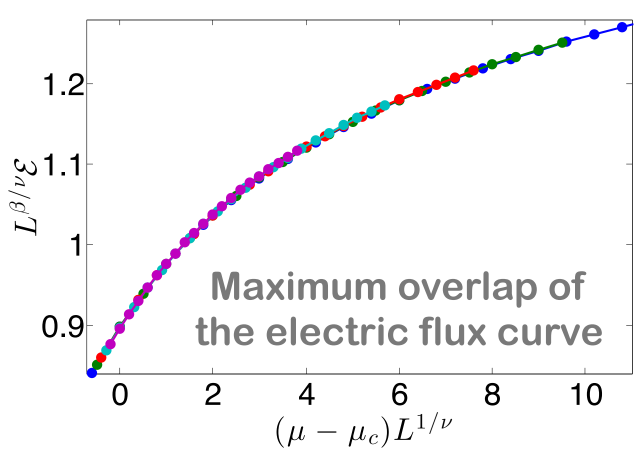


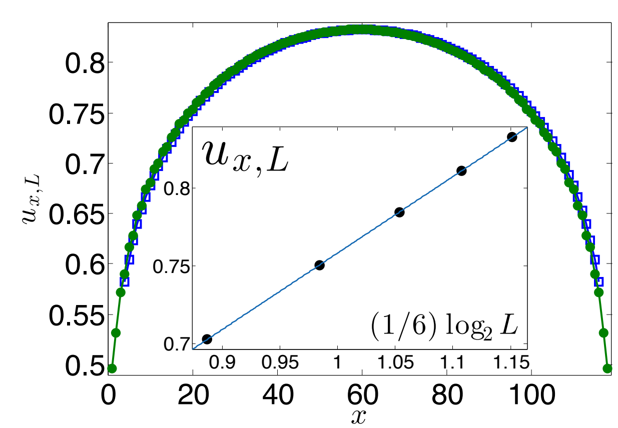


Second order phase transition: Parity and charge conjugation spontaneously broken (Ising universality class)

Critical exponents:

$$\beta = \frac{1}{8} \qquad \nu = 1$$





Central charge:

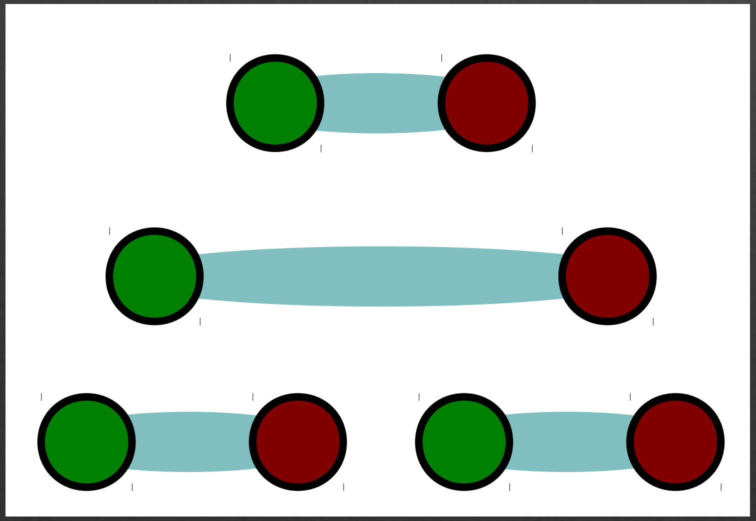
$$c = \frac{1}{2}$$

T. Pichler, E. Rico, M. Dalmonte, P. Zoller, SM PRL (2014)

Real-time dynamics

Spin 1 representation

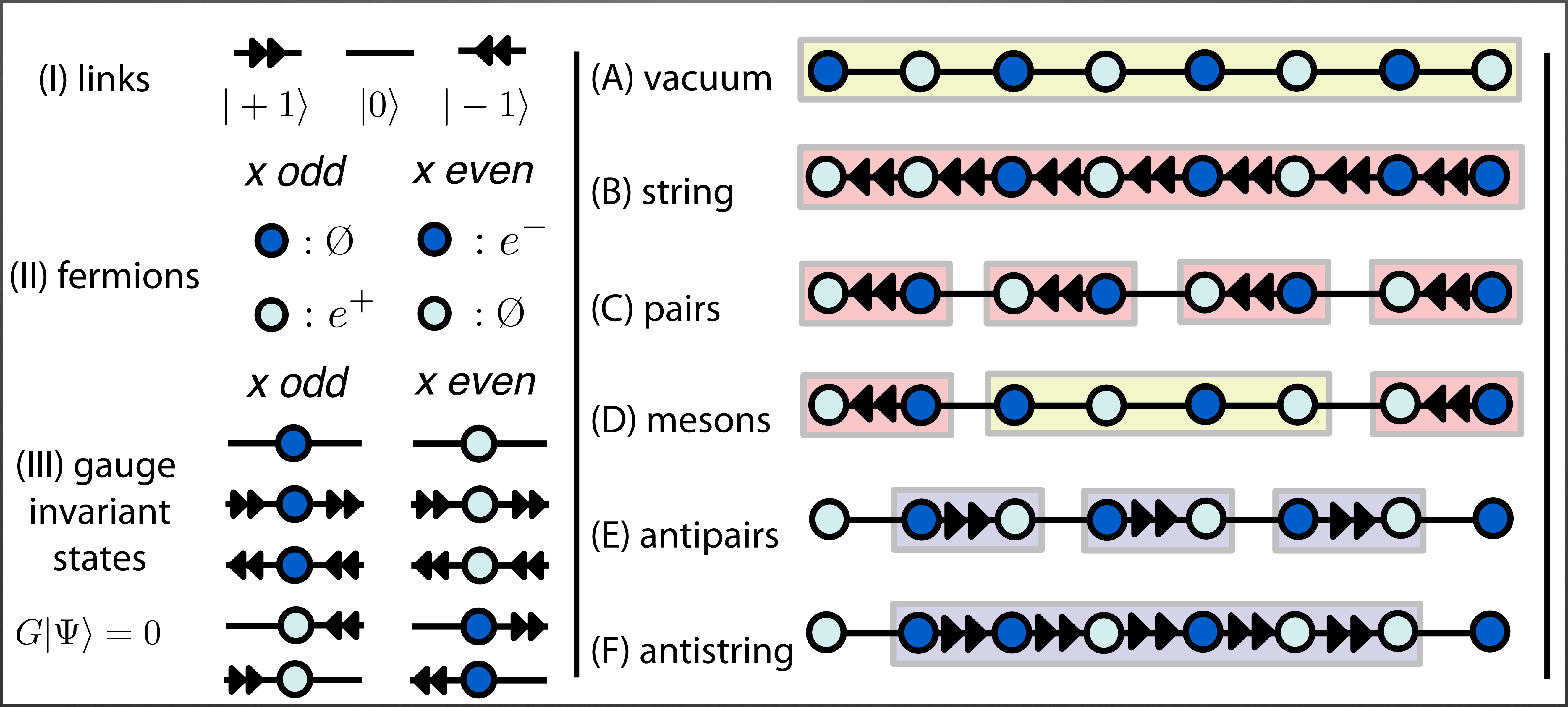
$$
\begin{aligned}
H &= -t\sum_x \left(\psi_x^\dagger S^+_{x,x+1}\psi_{x+1} + \psi_{x+1}^\dagger S^-_{x,x+1}{}^\dagger \psi_x\right) \\
&\quad + m\sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2}\sum_x (S^3_{x,x+1})^2,
\end{aligned}
$$



Exhibits string-breaking

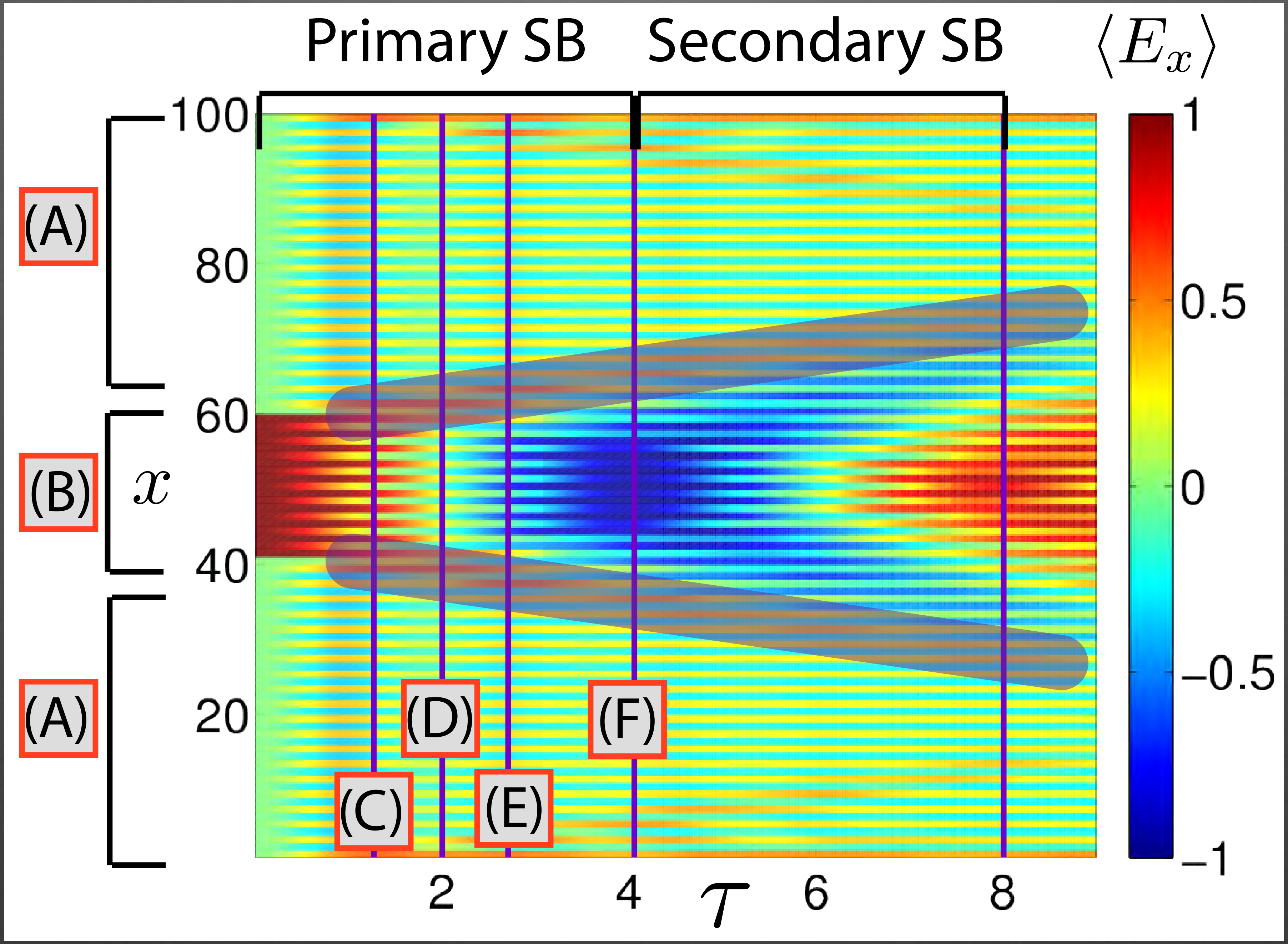
T. Pichler, E. Rico, M. Dalmonte, P. Zoller, SM 1505.????

Gauge invariant Hilbert space



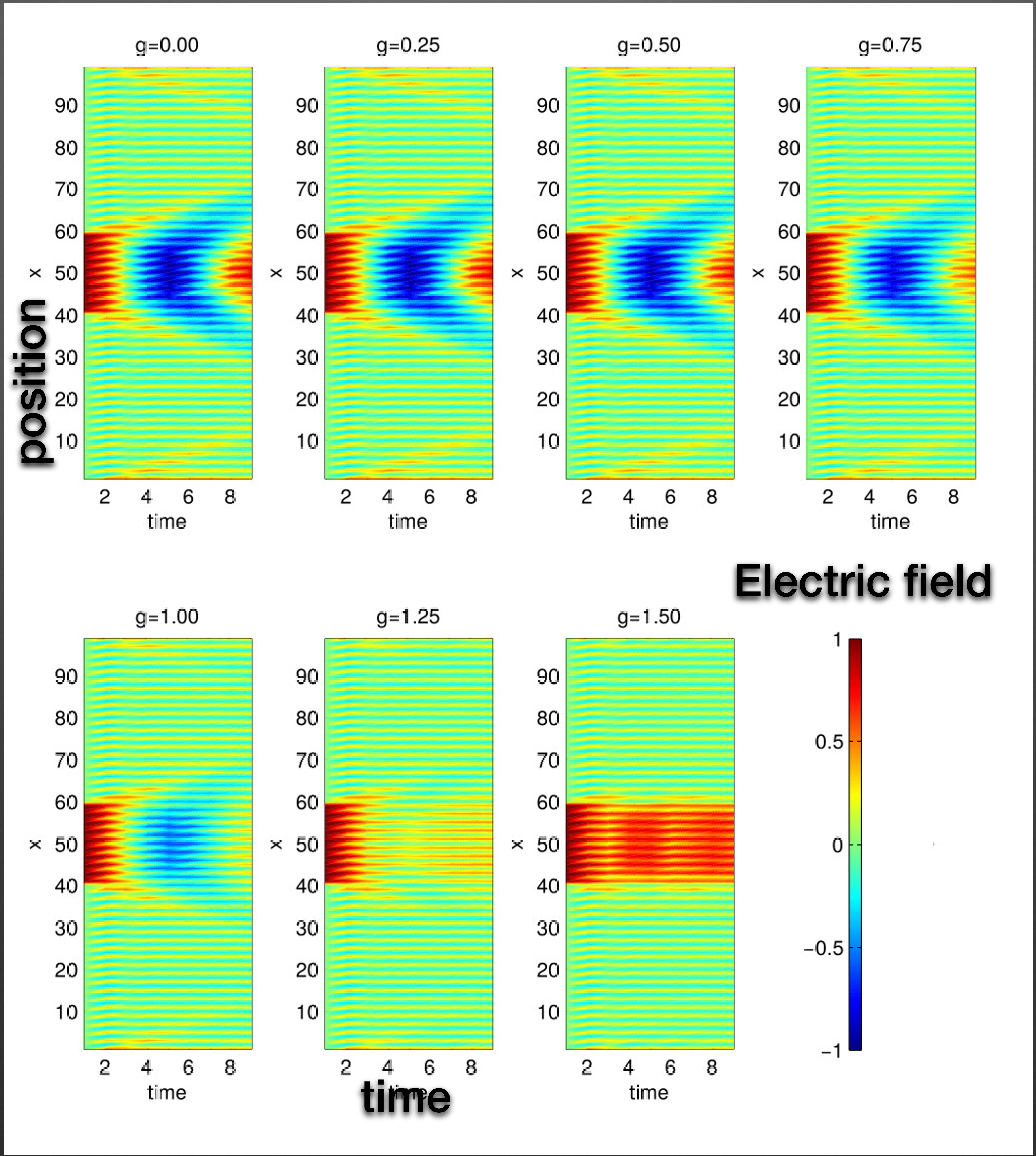
Local states

Many-body classical states



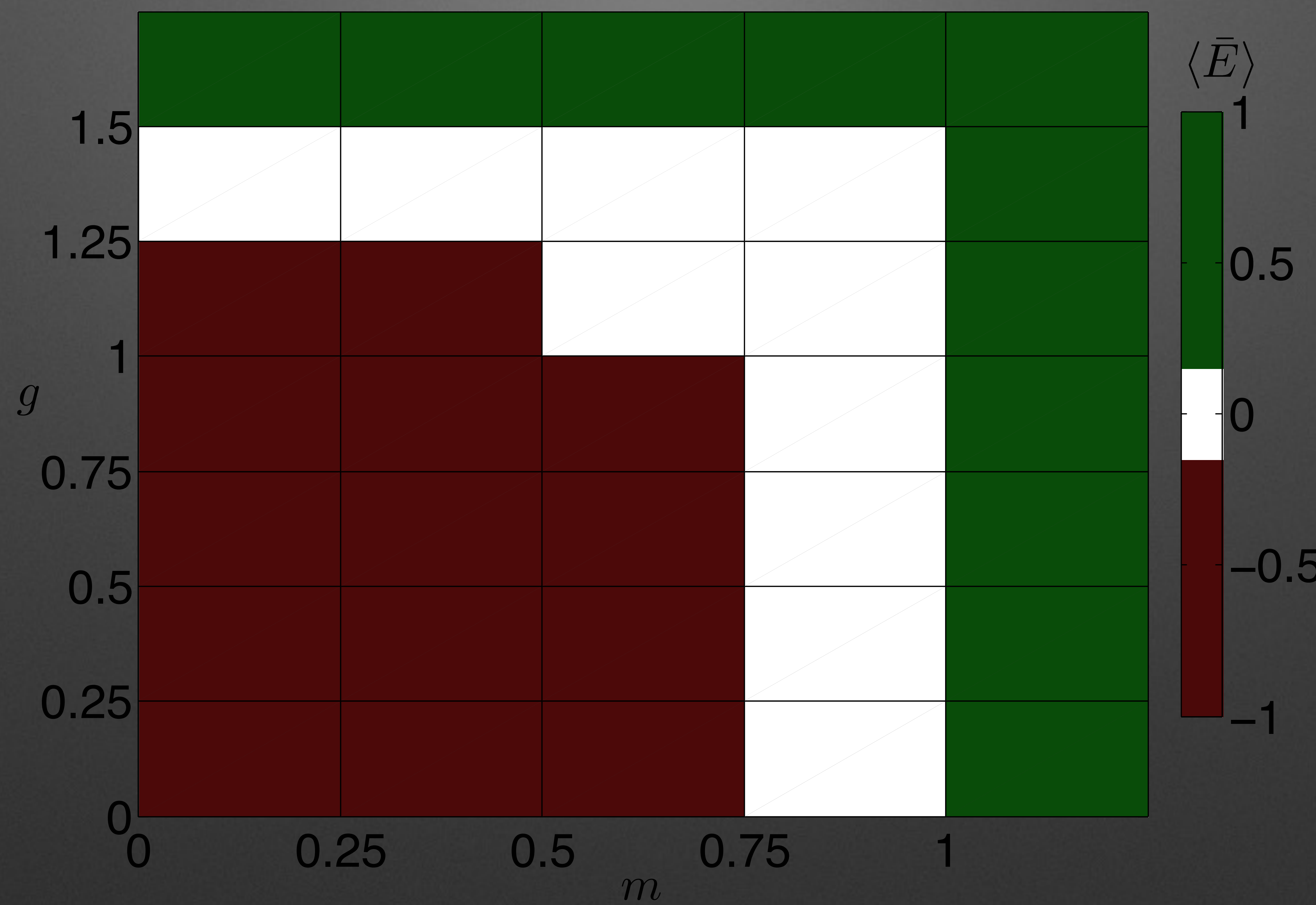
String breaking

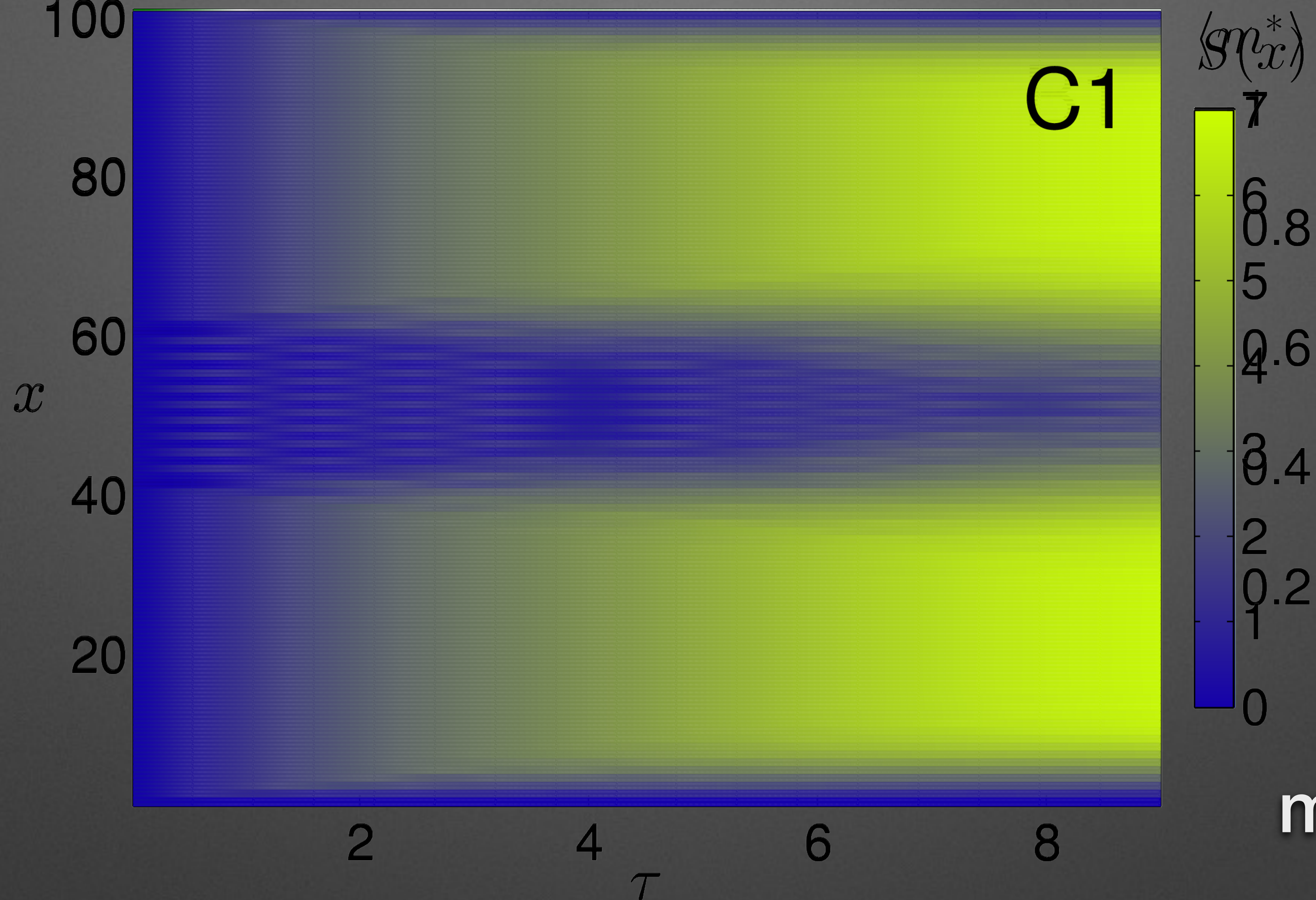
Real-time dynamcis


g=0.00
g=0.25
g=0.50
g=0.75
g=1.00
g=1.25
g=1.50
position
x
time
Electric field
1
0.5
0
−0.5
−1

m=0, t=1

State diagram

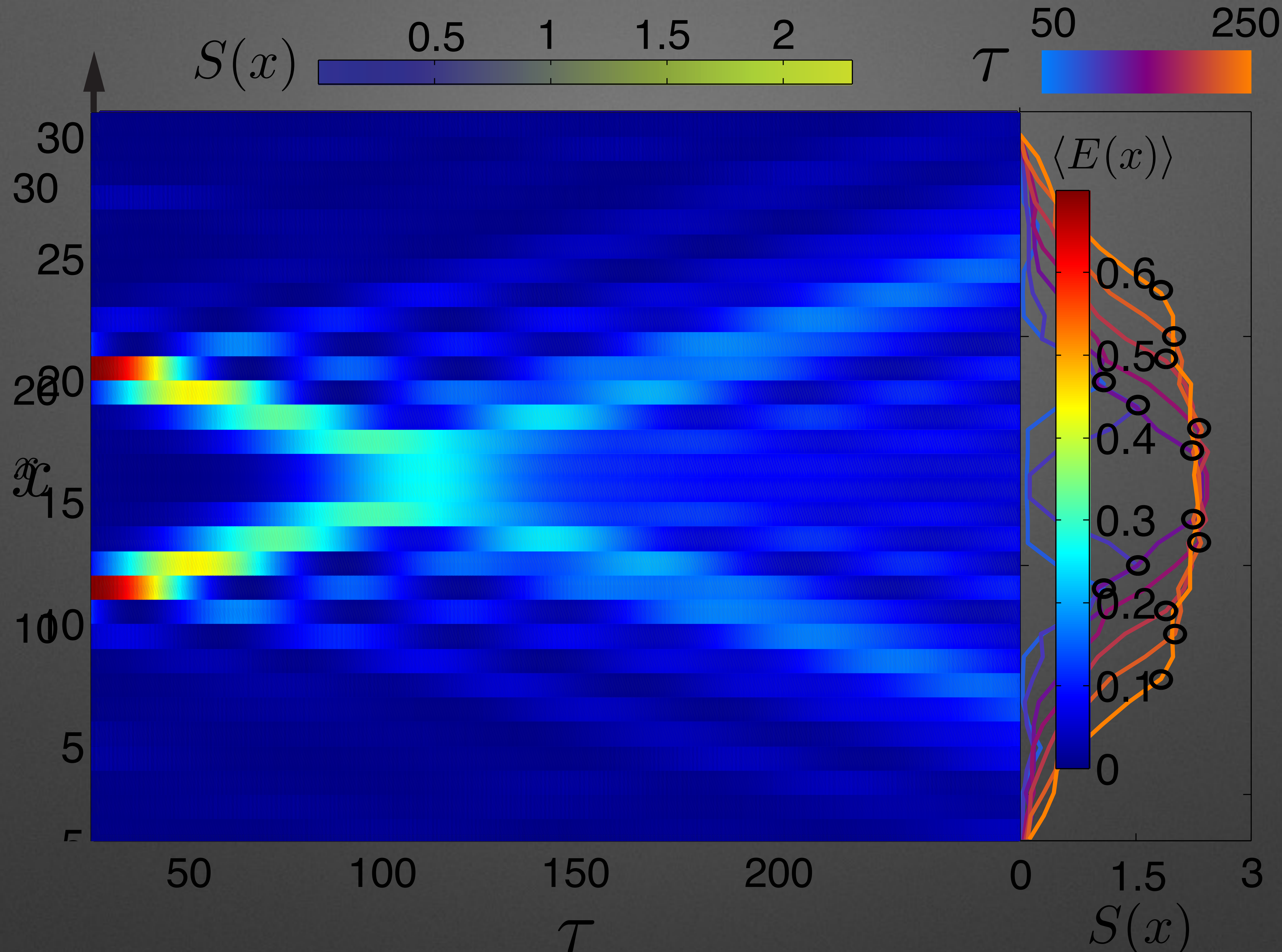




m=0

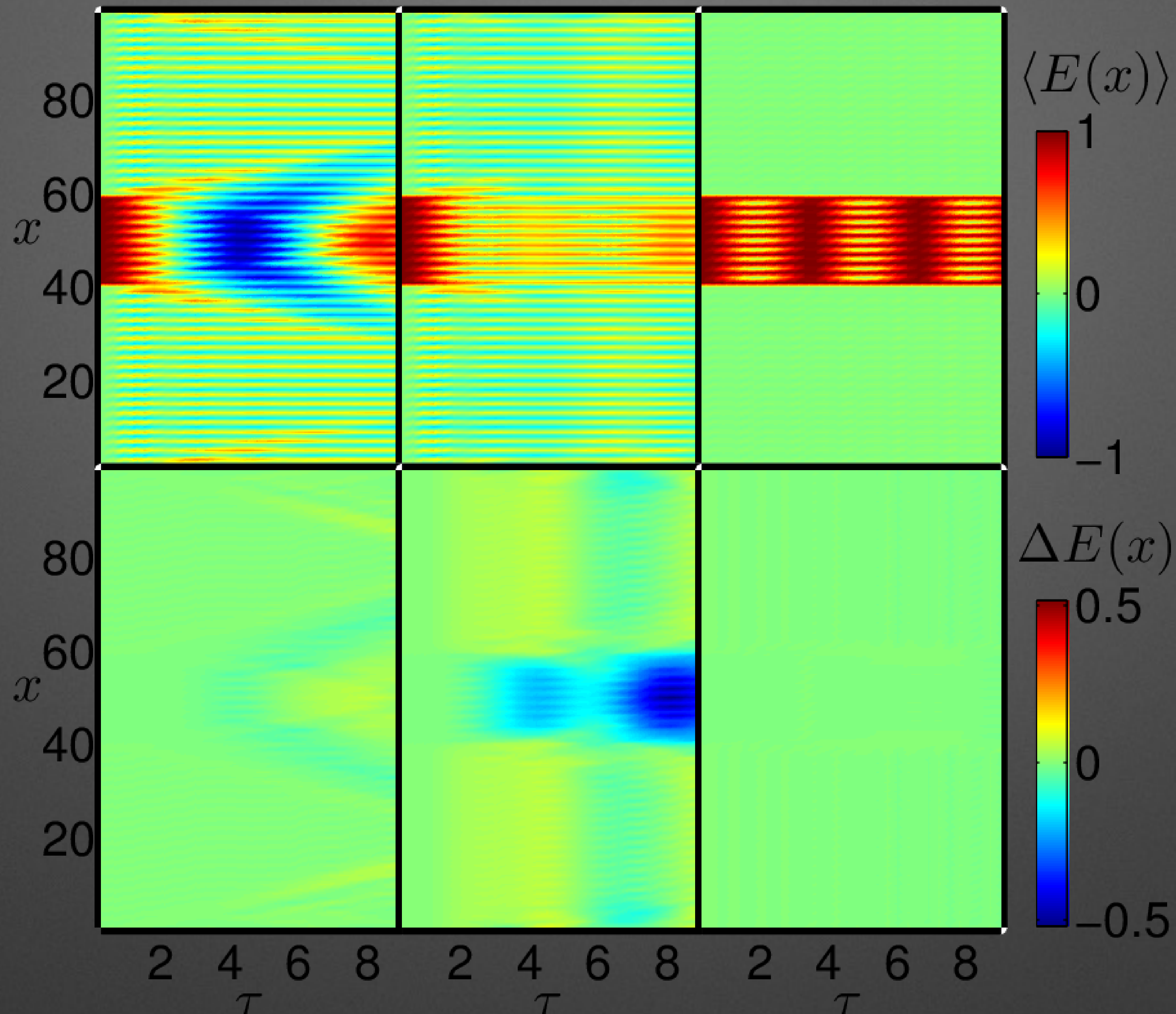
Schwinger mechanism

z



Mesons scattering

real-time simulation



Robustness

against gauge-invariant errors in quantum simulation platforms

$$H_I = \xi \sum_x n_x \left(S^z_{x-1,x} + S^z_{x,x+1}\right)$$

Conclusions & Outlook

- Versatile tools that enables real-time lattice gauge investigations (1+1d)
- Non abelian LGT: SU(2) in progress
- Tensor network extensions in 2+1d
- Condensed matter models
- Real-time scattering dynamics

Thank you for your attention!

Tommaso Calarco
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Thomas Pichler
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Daniel Jaschke

Peter Zoller
Marcello Dalmonte

Enrique Rico

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IP-DIADEMS
IP-RYSQ

Horizon 2020

Numerics:
BW-Grid

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