LANDAUER'S PRINCIPLE IN MULTIPARTITE OPEN QUANTUM SYSTEM DYNAMICS

Salvatore Lorenzo

- Dipartimento di Fisica e Chimica, Università degli Studi di Palermo, Italy
- Dipartimento di Fisica, Università della Calabria, Italy
- INFN - Gruppo collegato di Cosenza

in collaboration with: F. Ciccarello, R. McCloskey, M. Paternostro & G. M. Palma

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in collaboration with

University of Palermo

F. Ciccarello

G. M. Palma

Queen's University Belfast

R. McCloskey

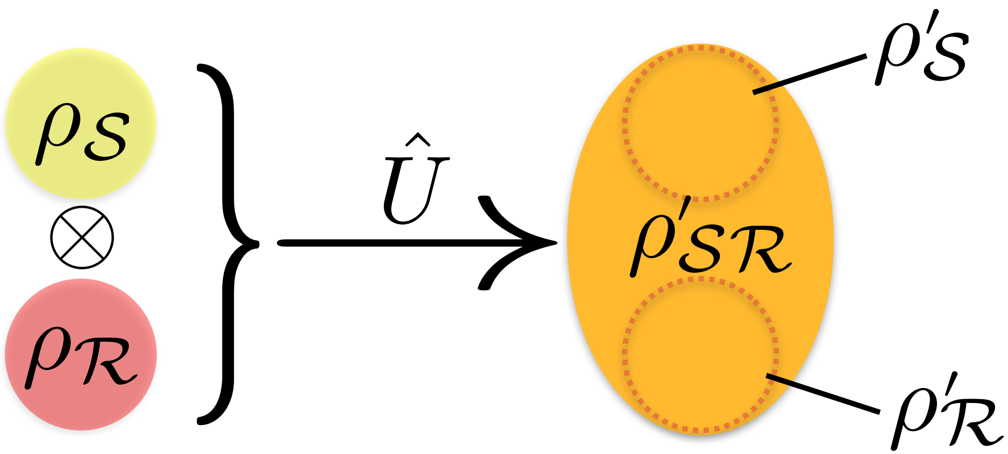
M. Paternostro

Outline

- Landauer's Principle
- Standard Quantum Collision Model
- Landauer's Principle for Fluxes
    - Recycling environment
    - Indirect erasure scheme
- Conclusions

Landauer's Principle

D. Reeb and M. Wolf, New J. Phys. 16, 103011 (2014)



β inverse temperature of $\mathcal{R}$

ΔQ heat transferred to $\mathcal{R}$

$\Delta\tilde{S}$ entropy *decrease* of $\mathcal{S}$

$$\rho_{\mathcal{SR}} = \rho_{\mathcal{S}} \otimes \rho_{\mathcal{R}}$$

$$\rho_{\mathcal{R}} = e^{-\beta H} / \mathrm{Tr}[e^{-\beta H}]$$

$$\rho'_{\mathcal{SR}} = \hat{U} \rho_{\mathcal{SR}} \hat{U}^{\dagger}$$

$$\Delta\tilde{S} = S(\rho_{\mathcal{S}}) - S(\rho'_{\mathcal{S}})$$

$$\Delta Q = \mathrm{Tr}[H \rho'_{\mathcal{R}}] - \mathrm{Tr}[H \rho_{\mathcal{R}}]$$

R. Landauer, IBM J. of Res. and Develop. 5 183 (1961)
A. Berut, Nature 483, 187 (2012)
J.P.P. Silva et al., arXiv: 1412.6490
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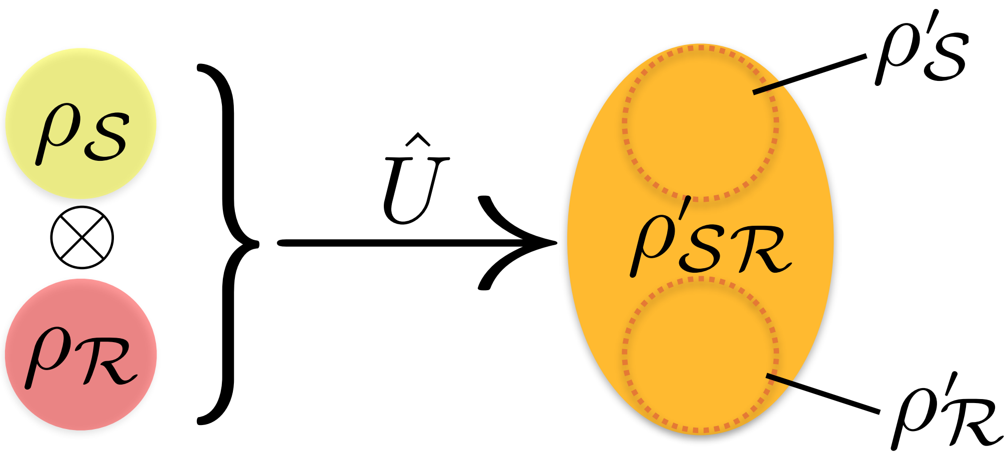
Landauer's Principle

D. Reeb and M. Wolf, New J. Phys. 16, 103011 (2014)



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$$\rho_{\mathcal{R}} = e^{-\beta H} / \mathrm{Tr}[e^{-\beta H}]$$
$$\rho'_{\mathcal{SR}} = \hat{U} \rho_{\mathcal{SR}} \hat{U}^{\dagger}$$

$$\beta \Delta Q \geq \Delta \tilde{S}$$

no Hamiltonian, no temperature for $\mathcal{S}$ needed

$$\Delta\tilde{S} = S(\rho_{\mathcal{S}}) - S(\rho'_{\mathcal{S}})$$
$$\Delta Q = \mathrm{Tr}[H \rho'_{\mathcal{R}}] - \mathrm{Tr}[H \rho_{\mathcal{R}}]$$

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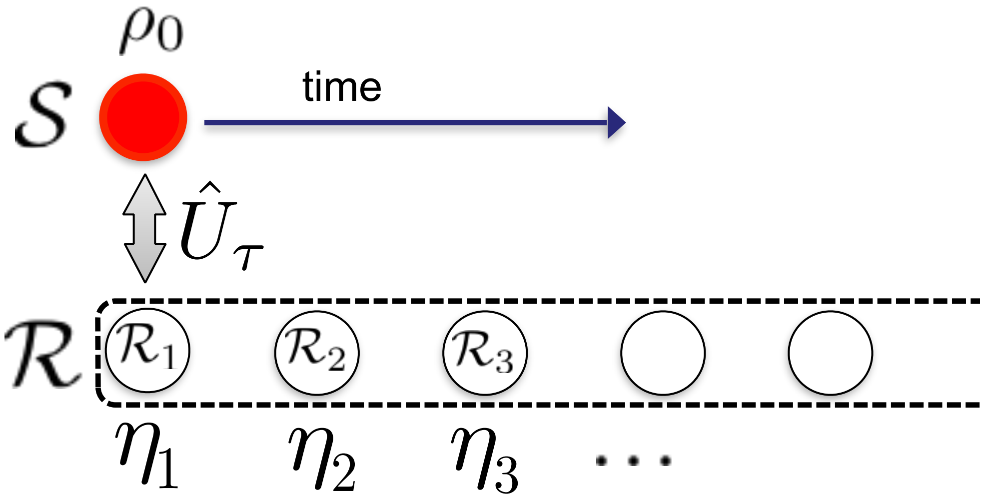
Quantum Collision Model



τ=collision time

g=coupling strength

$$\hat{U}_\tau = e^{-igV\tau}$$

$$V = \sum_k \hat{S}_k \otimes \hat{R}_k$$

time

$$\rho_\tau = \mathrm{Tr}_{\mathcal{R}_1}[\hat{U}_\tau(\rho_0 \otimes \eta_1)\hat{U}_\tau^\dagger] = \mathcal{E}_1^\tau[\rho_0]$$

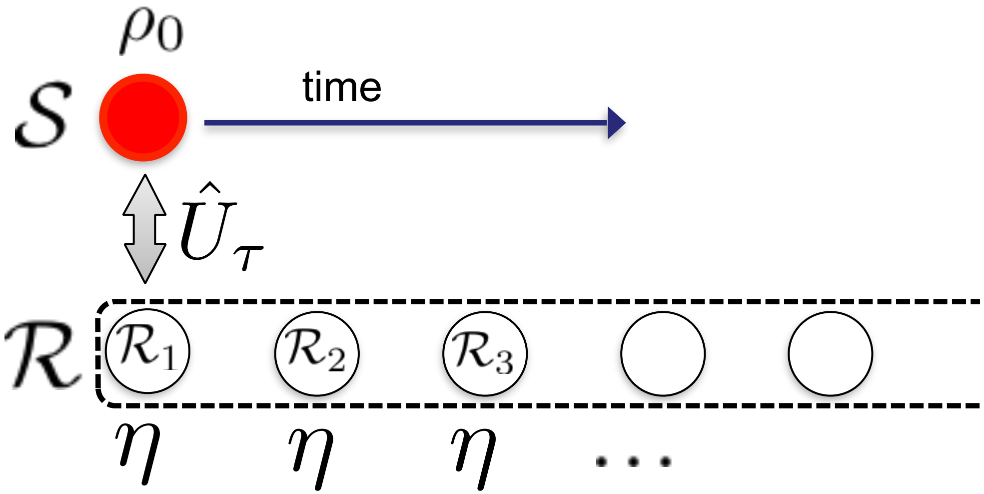
$$\rho_{2\tau} = \mathrm{Tr}_{\mathcal{R}_2}[\hat{U}_\tau(\rho_\tau \otimes \eta_2)\hat{U}_\tau^\dagger] = \mathcal{E}_2^\tau[\rho_\tau]$$

$$\vdots$$

$$\rho_{n\tau} = \mathrm{Tr}_{\mathcal{R}_n}[\hat{U}_\tau(\rho_{(n-1)\tau} \otimes \eta_n)\hat{U}_\tau^\dagger] = \mathcal{E}_n^\tau[\rho_{(n-1)\tau}]$$

J. Rau, Phys. Rev. **129,** 1880 (1963)
M. Ziman et al., PRA **65** , 042105 (2002); V. Scarani et al., PRL **88**, 97905 (2002)
M. Ziman, P. Stelmachovic, and V. Buzek, Open Syst. and Inform. Dynam. **12**, 81 (2005)

Quantum Collision Model



τ=collision time

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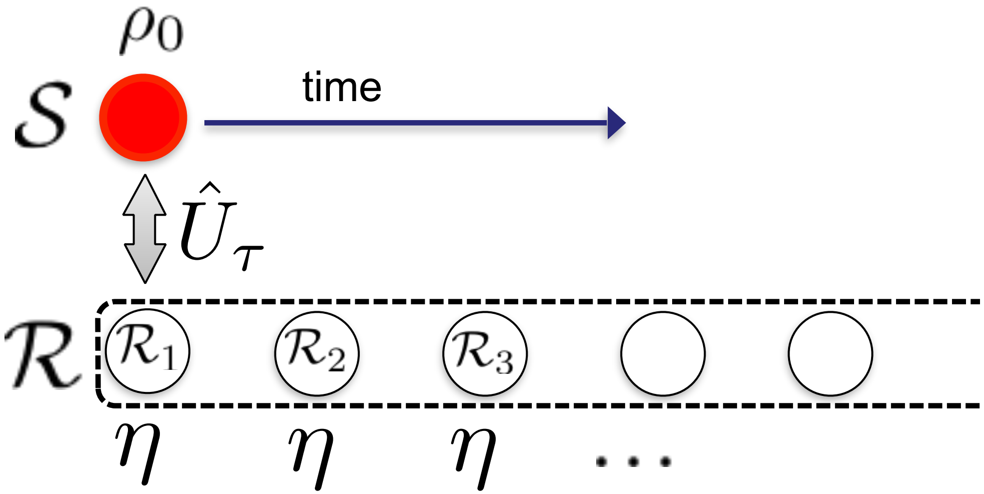
$$\hat{U}_\tau = e^{-igV\tau}$$

$$V = \sum_k \hat{S}_k \otimes \hat{R}_k$$

$\eta_k = \eta \quad \forall k$ → $\mathcal{E}_k^\tau := \mathcal{E}^{k\tau}$ → $\mathcal{E}_k^\tau \mathcal{E}_j^\tau = \mathcal{E}^{(k+j)\tau}$ **One parameter semigroup**

Quantum Collision Model



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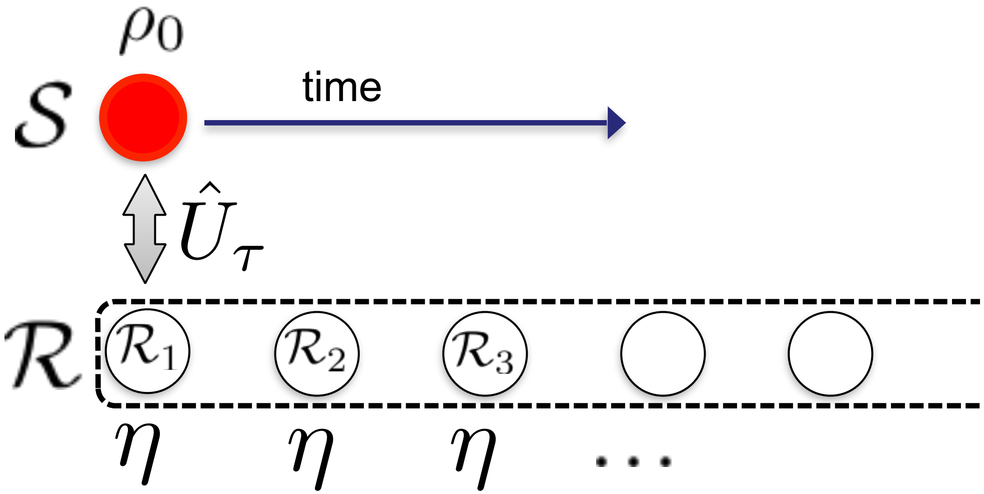
$$\mathcal{E}_k^\tau := \mathcal{E}^{k\tau} \quad \rightarrow \quad \mathcal{E}_k^\tau \mathcal{E}_j^\tau = \mathcal{E}^{(k+j)\tau}$$

One parameter semigroup

$$\rho_{n\tau} = \mathrm{Tr}_\mathcal{R}[\hat{U}_\tau(\rho_{(n-1)\tau} \otimes \eta)\hat{U}_\tau^\dagger] = \mathcal{E}^\tau[\rho_{(n-1)\tau}]$$

$$\Delta\rho_{n\tau} = \rho_{n\tau} - \rho_{(n-1)\tau} = (\mathcal{E}^\tau - \mathbb{I})[\rho_{(n-1)\tau}]$$

QCM in the continuous time limit



τ=collision time

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$$\hat{U}_\tau = e^{-igV\tau}$$

$$V = \sum_k \hat{S}_k \otimes \hat{R}_k$$

TAKING THE LIMIT

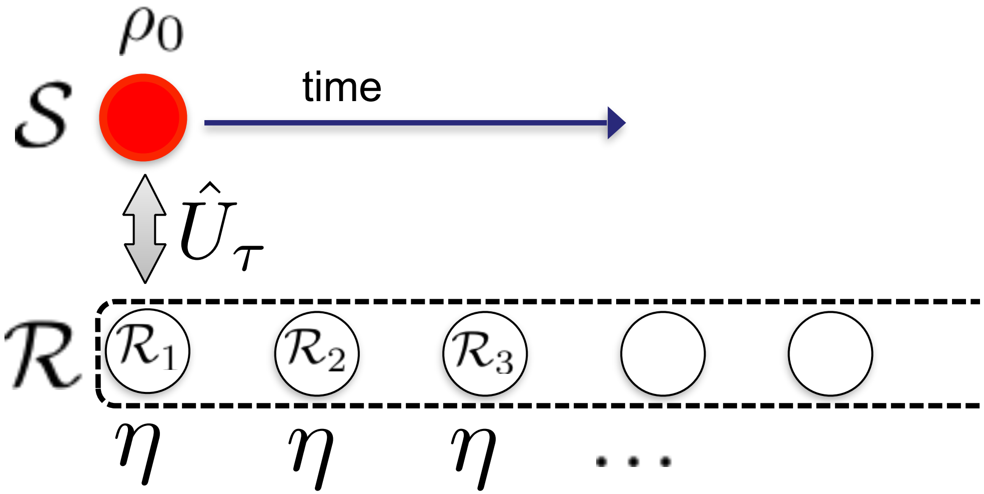
$$n \to \infty$$
$$\tau \to 0$$

IN SUCH WAY

THAT

$$n\tau \to t$$
$$g^2\tau \to \gamma$$

QCM in the continuous time limit



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TAKING THE LIMIT

$$n \to \infty$$
$$\tau \to 0$$

IN SUCH WAY THAT

$$n\tau \to t$$
$$g^2\tau \to \gamma$$

$$\frac{\Delta\rho_{n\tau}}{\tau} = \frac{(\mathcal{E}_\tau - \mathbb{I})}{\tau}[\rho_{(n-1)\tau}]$$

$$\frac{d\rho(t)}{dt} = \gamma \sum_k \langle R_k R_{k'} \rangle_\eta \left(S_k \rho(t) S_{k'} - \frac{1}{2}\{S_{k'} S_k, \rho(t)\} \right)$$ <u>Lindblad ME</u>

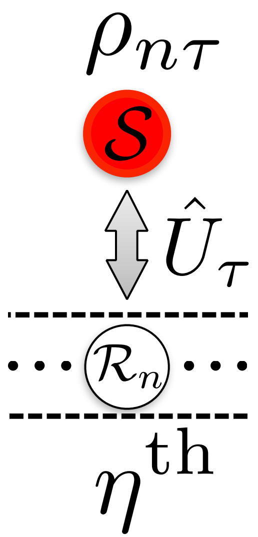
Quantum Collision Models

Advantages:

1. Intuitive & easy to handle
2. Lindblad-type MEs arise naturally
3. Promising tools for tackling NM dynamics

T. Rybar, S. N. Filippov, M. Ziman, and V. Buzek, J. Phys. B 45, 154006 (2012)
N. K. Bernardes, A. R. R. Carvalho, C. H. Monken, M. F. Santos, Phys. Rev A 90, 032111 (2014)
V. Giovannetti, and G. M. Palma, Phys. Rev. Lett. 108, 040401 (2012)
F. Ciccarello, G. M. Palma, and V. Giovannetti, Phys. Rev. A 87, 040103(R) (2013)
F. Ciccarello and V. Giovannetti, Phys. Scrip. T153, 014010 (2013)
R. McCloskey and M. Paternostro, Phys. Rev. A 89, 052120 (2014)
A. Bodor, L. Diosi, Z. Kallus, and T. Konrad, Phys. Rev. A 87, 052113 (2013)
J. Jin, V. Giovannetti, R. Fazio, F. Sciarrino, P. Mataloni, A. Crespi, and R. Osellame, PRA 91, 012122 (2015)
N. K. Bernades et al., arXiv: arXiv:1504.01602

QCM and Landauer's Principle



$\eta^{\mathrm{th}} = e^{-\beta H_{\mathcal{R}}} / \mathrm{Tr}[e^{-\beta H_{\mathcal{R}}}]$

τ=collision time

g=coupling strength

$\hat{U}_\tau = e^{-igV\tau}$

$V = \sum_k \hat{S}_k \otimes \hat{R}_k$

$$\rho_{n\tau} = \mathrm{Tr}_{\mathcal{R}}[\hat{U}_\tau(\rho_{(n-1)\tau} \otimes \eta^{\mathrm{th}})\hat{U}_\tau^\dagger] = \mathcal{E}^\tau[\rho_{(n-1)\tau}]$$

$$\eta'_n = \mathrm{Tr}_{\mathcal{S}}[\hat{U}_\tau(\rho_{(n-1)\tau} \otimes \eta^{\mathrm{th}})\hat{U}_\tau^\dagger] = \Lambda_{(n-1)\tau}[\eta^{\mathrm{th}}]$$

FLUXES

$$\Delta\tilde{S}_{n\tau} = S(\rho_{(n-1)\tau}) - S(\rho_{n\tau})$$

$$\Delta E_{n\tau} = \mathrm{Tr}\left[\hat{H}_{\mathcal{S}}(\mathcal{E}^\tau - \mathbb{I})[\rho_{(n-1)\tau}]\right]$$

$$\Delta Q_{n\tau} = \mathrm{Tr}\left[\hat{H}_{\mathcal{R}}(\Lambda_{(n-1)\tau} - \mathbb{I})[\eta^{\mathrm{th}}]\right]$$

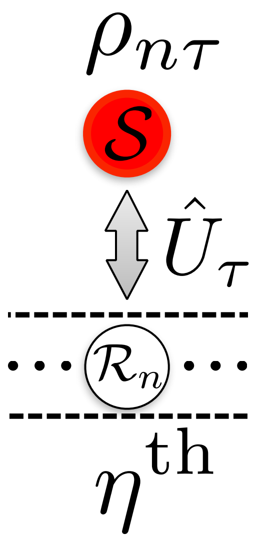
…continuous time limit →

$$\dot{\tilde{S}}(t)$$

$$\dot{E}(t)$$

$$\dot{Q}(t)$$

QCM and Landauer's Principle



$$\eta^{\mathrm{th}} = e^{-\beta H_{\mathcal{R}}} / \mathrm{Tr}[e^{-\beta H_{\mathcal{R}}}]$$

τ=collision time

g=coupling strength

$$\hat{U}_\tau = e^{-igV\tau}$$

$$V = \sum_k \hat{S}_k \otimes \hat{R}_k$$

$$\rho_{n\tau} = \mathrm{Tr}_{\mathcal{R}}[\hat{U}_\tau(\rho_{(n-1)\tau} \otimes \eta^{\mathrm{th}})\hat{U}_\tau^\dagger] = \mathcal{E}^\tau[\rho_{(n-1)\tau}]$$

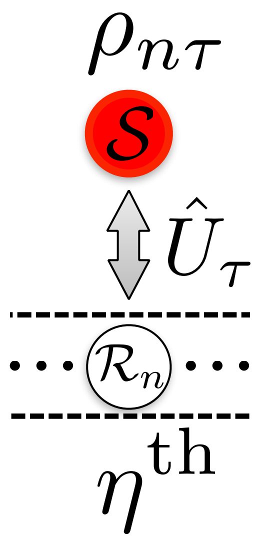
$$\eta_n' = \mathrm{Tr}_{\mathcal{S}}[\hat{U}_\tau(\rho_{(n-1)\tau} \otimes \eta^{\mathrm{th}})\hat{U}_\tau^\dagger] = \Lambda_{(n-1)\tau}[\eta^{\mathrm{th}}]$$

FLUXES

$$\dot{E}(t) = \gamma \sum_{k,j} \langle \hat{R}_k \hat{R}_j \rangle_{\eta^{\mathrm{th}}} \langle \hat{S}_k \hat{H}_{\mathcal{S}} \hat{S}_j - \tfrac{1}{2}\{\hat{S}_k \hat{S}_j, \hat{H}_S\} \rangle_{\rho(t)}$$

$$\dot{Q}(t) = \gamma \sum_{k,j} \langle \hat{S}_k \hat{S}_j \rangle_{\rho(t)} \langle \hat{R}_k \hat{H}_{\mathcal{R}} \hat{R}_j - \tfrac{1}{2}\{\hat{R}_k \hat{R}_j, \hat{H}_R\} \rangle_{\eta^{\mathrm{th}}}$$

QCM and Landauer's Principle



$\eta^{\mathrm{th}} = e^{-\beta H_{\mathcal{R}}} / \mathrm{Tr}[e^{-\beta H_{\mathcal{R}}}]$

THERMALIZATION PROCESSES

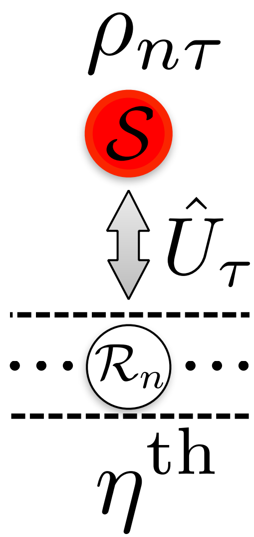
$$\left[\hat{U}_\tau, \hat{H}_{\mathcal{S}} + \hat{H}_{\mathcal{R}}\right] = 0$$

$$\rho^{eq} \equiv \rho(\infty) = e^{-\beta H_{\mathcal{S}}} / \mathrm{Tr}[e^{-\beta H_{\mathcal{S}}}]$$

$$\Delta Q_{n\tau} = -\Delta E_{n\tau} \longrightarrow \dot{Q}(t) = -\dot{E}(t)$$

$$\underbrace{\dot{S}(\rho(t)|\rho^{eq})}_{\leq 0 \text{ for CPT maps}} = \mathrm{Tr}[\dot{\rho}(t)(\ln \rho(t) - \ln \rho^{eq})] = \dot{\tilde{S}} + \beta \dot{E}$$

QCM and Landauer's Principle



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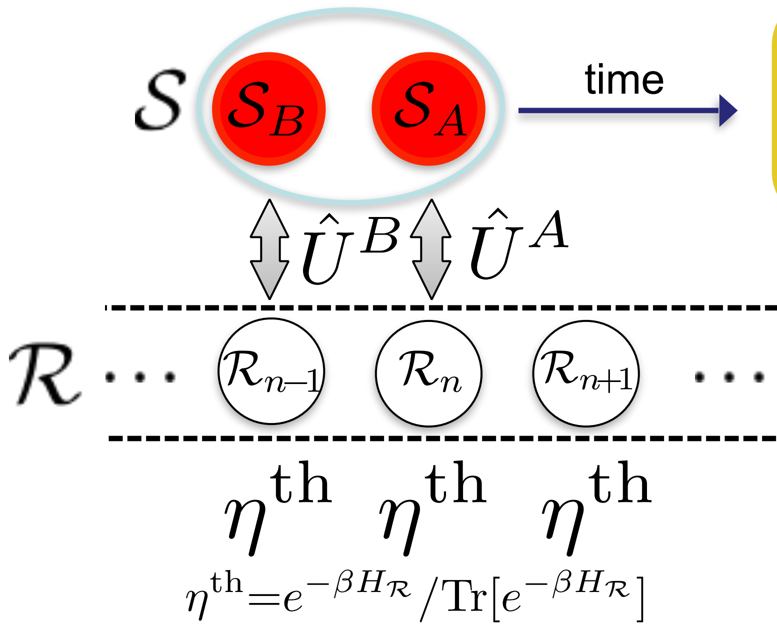
Landauer's principle for fluxes

$$\beta \dot{Q}(t) \geq \dot{\tilde{S}}(\rho(t))$$

SL, R. McCloskey, F. Cicarello, M. Paternostro & G. M. Palma, to appear on PRL

Recycling environment



τ=collision time

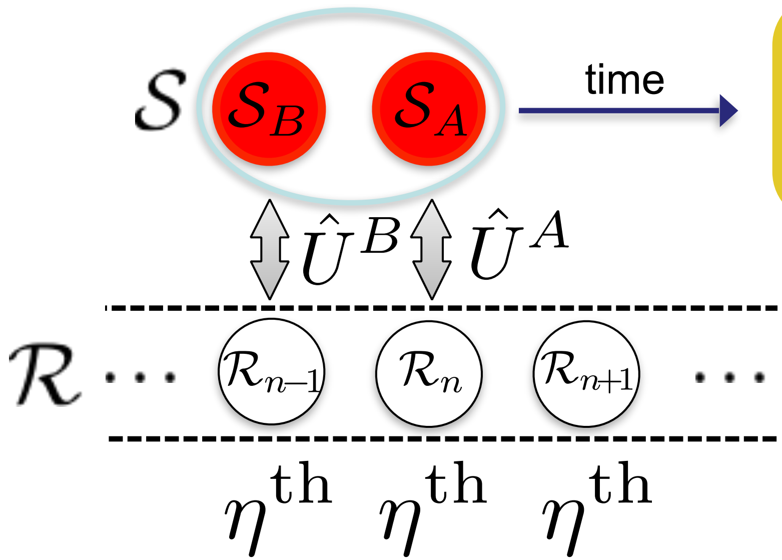
$$\hat{U}_\tau = \hat{U}^B \hat{U}^A$$

V. Giovannetti, and G. M. Palma, PRL. 108, 040401 (2012)
SL et al., PRA **91**, 022121 (2015)

$$\dot{\rho}(t) = \sum_{X=A,B} \mathcal{L}_X[\rho(t)] + \mathcal{D}^{\rightarrow}_{AB}[\rho(t)]$$

$$\rho^{eq} = \frac{e^{-\beta H_A}}{Z_A} \otimes \frac{e^{-\beta H_B}}{Z_B}$$

Recycling environment



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$$\rho^{eq} = \frac{e^{-\beta H_A}}{Z_A} \otimes \frac{e^{-\beta H_B}}{Z_B}$$

Landauer's principle for fluxes

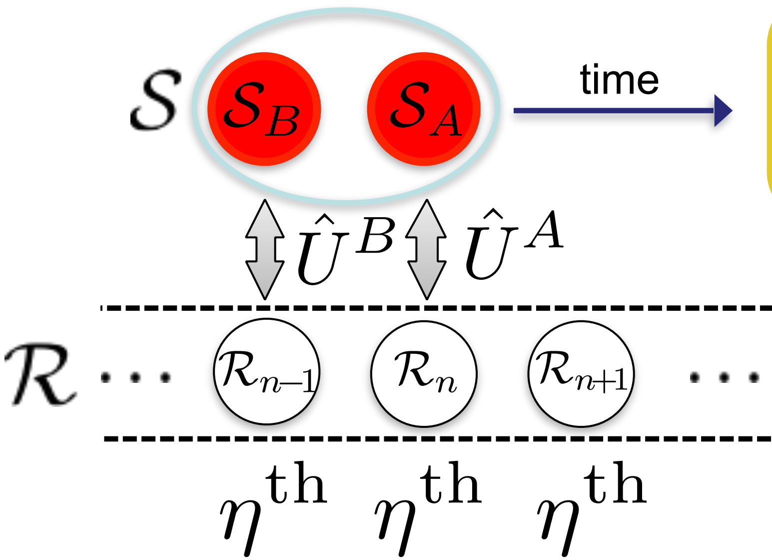
$$\beta \dot{Q} \geq \dot{\tilde{S}}_{AB}$$

$$\beta \dot{Q} \geq 2\dot{\tilde{S}}_A + \dot{S}_{A|B} - \dot{S}_{B|A} + \dot{I}(A{:}B)$$

independent erasure bound

intra-system correlations

Recycling environment



$$\eta^{\rm th}=e^{-\beta H_{\mathcal{R}}}/{\rm Tr}[e^{-\beta H_{\mathcal{R}}}]$$

τ=collision time

$$\hat{U}_\tau=\hat{U}^B\hat{U}^A$$

$$H_{\mathcal{S}}=(\omega/2)\sigma_A^z+(\omega/2)\sigma_B^z$$

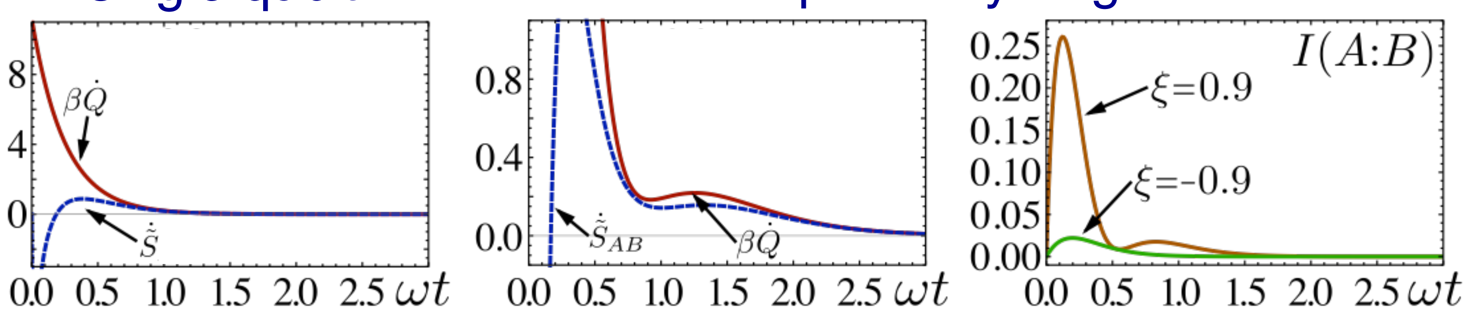
$$H_{\mathcal{R}_n}=(\omega/2)\sigma_n^z$$

$$V_X=\sigma_X^x\sigma_n^x+\sigma_X^y\sigma_n^y$$

$$\rho_0=|\uparrow\uparrow\rangle_{AB}\langle\uparrow\uparrow| \qquad \eta^{\rm th}=\begin{pmatrix}\frac{1-\xi}{2} & 0\\ 0 & \frac{1+\xi}{2}\end{pmatrix}$$

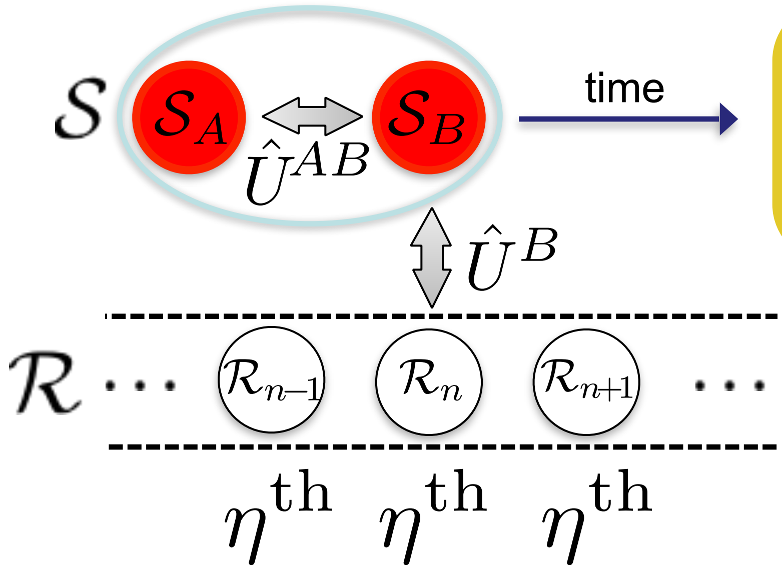
single qubit

2-qubit recycling scheme



SL, R. McCloskey, F. Ciccarello, M. Paternostro & G. M. Palma, to appear on PRL

Indirect Erasure



$$\eta^{\mathrm{th}} = e^{-\beta H_{\mathcal{R}}} / \mathrm{Tr}[e^{-\beta H_{\mathcal{R}}}]$$

τ=collision time

$$\hat{U}_\tau = \hat{U}^B \hat{U}^{AB}$$

SL, F. Ciccarello and G. M Palma, in preparation (2015)

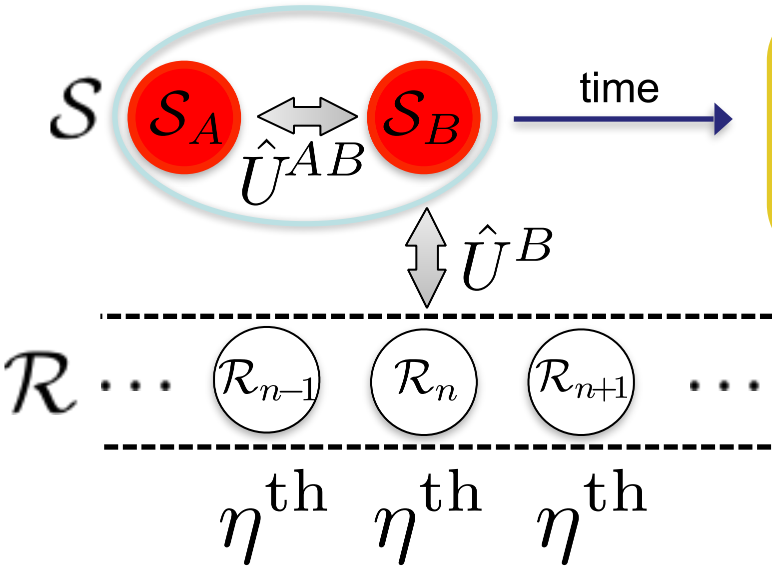
$$\dot{\rho}(t) = -i[H_{AB}, \rho(t)] + \mathcal{L}_B[\rho(t)]$$

$$\rho^{eq} = \frac{e^{-\beta H_A}}{Z_A} \otimes \frac{e^{-\beta H_B}}{Z_B}$$

Landauer's principle for fluxes

$$\beta \dot{Q} \geq \dot{\tilde{S}}_{AB}$$

Indirect Erasure



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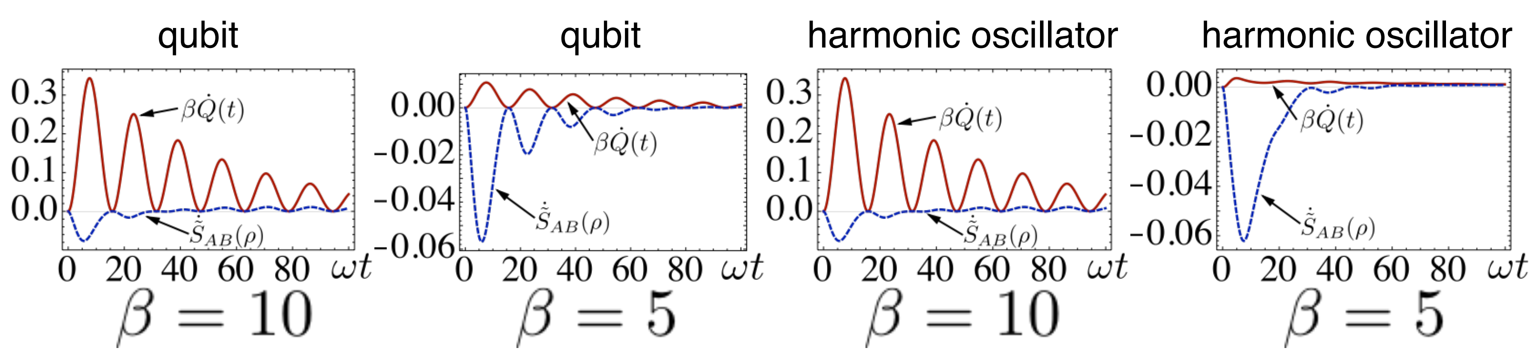
instance

$\mathcal{S}_A$	qubit
$\mathcal{S}_B$	qubit / harmonic oscillator
$\mathcal{R}_n$	qubits

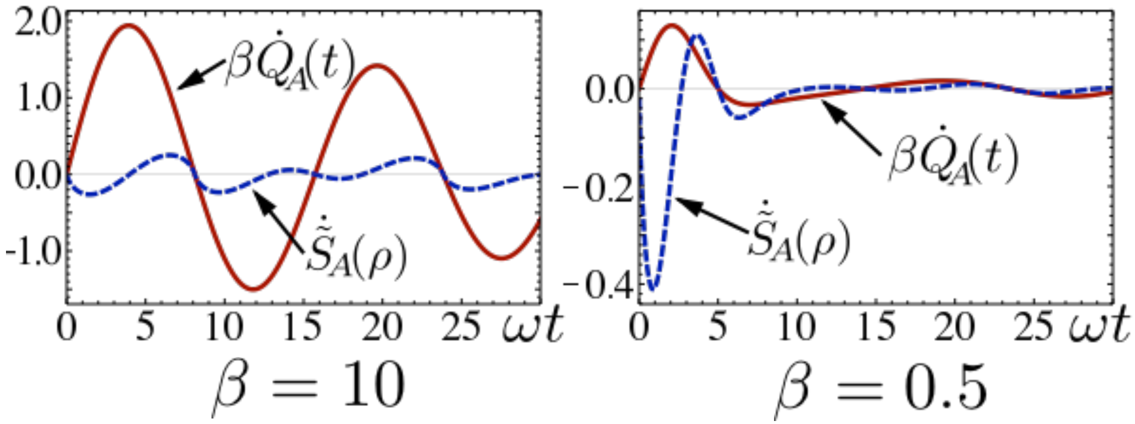
Jaynes-Cummings-like couplings



SL, R. McCloskey, F. Ciccarello, M. Paternostro & G. M. Palma, to appear on PRL

Indirect Erasure

…looking at $\mathcal{S}_A$ only:



Landauer's principle violation

CONCLUSIONS

Landauer’s principle from a microscopic system-bath quantum model

Landauer’s principle for fluxes of open quantum system

Link between Erasure process and intra-system correlations

preprint:

SL, R. McCloskey, F. Ciccarello, M. Paternostro & G. M. Palma, **arXiv: 1503.07837**

thank you!