Nano-friction in cavity quantum electrodynamics

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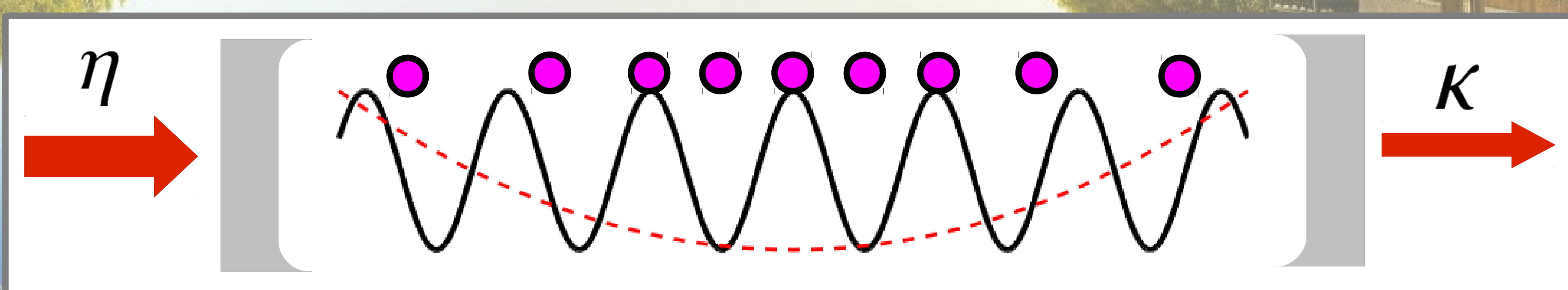
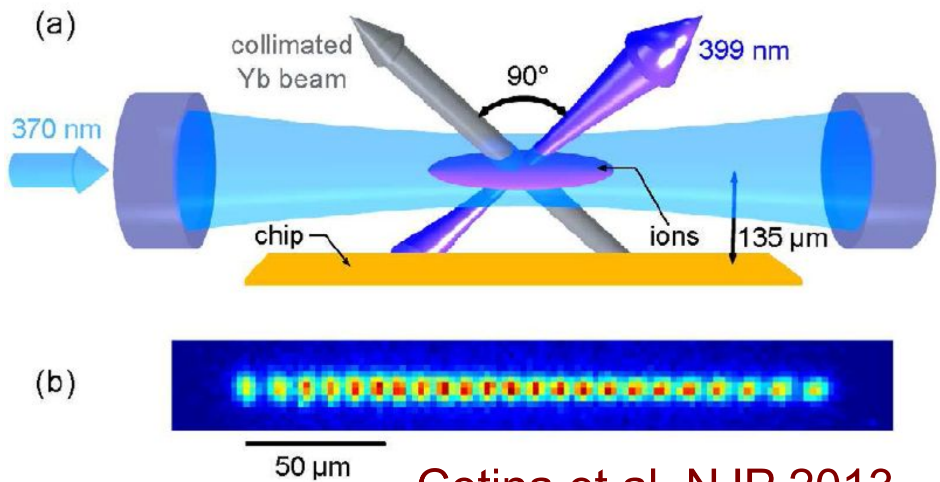


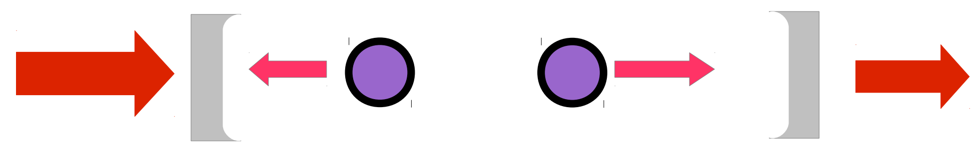
arXiv:1504.00275

Ion crystal in a high finesse cavity



Cetina et al, NJP 2013

Long-range interactions
mediated by the cavity



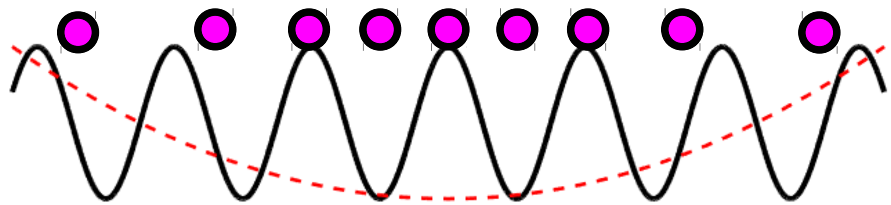
- Ion chain forms an elastic crystal
- Optical lattice of the cavity mode forms a substrate potential

Mismatch between the ordering of the ions due to the Coulomb force and the periodicity of the cavity-light field

Self-organisation in the presence of frustration

→ Leads to localization and **nano-friction**

Ions in a lattice



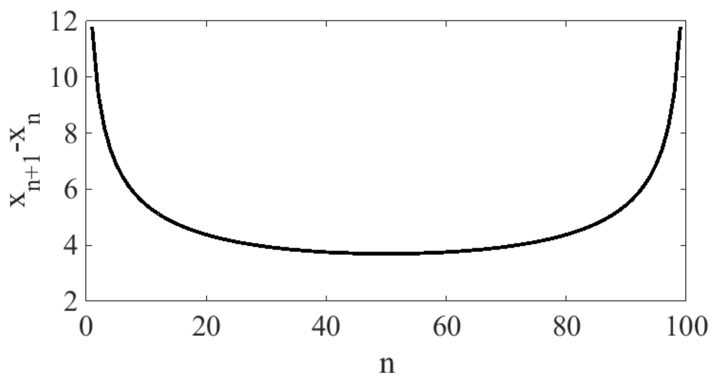
The ions

$$H_{ions} = \sum_{j=1}^{N} \left[\frac{p_j^2}{2m} + \frac{1}{2} m\omega^2 x_j^2 + \sum_{k=j+1}^{N} \frac{q^2}{4\pi\epsilon_0} \frac{1}{|x_j - x_k|} \right]$$

The substrate potential

$$H_{lattice} = V \sum_j \cos^2(kx_j)$$

The trapping potential ensures that the ions have a non-uniform ion density

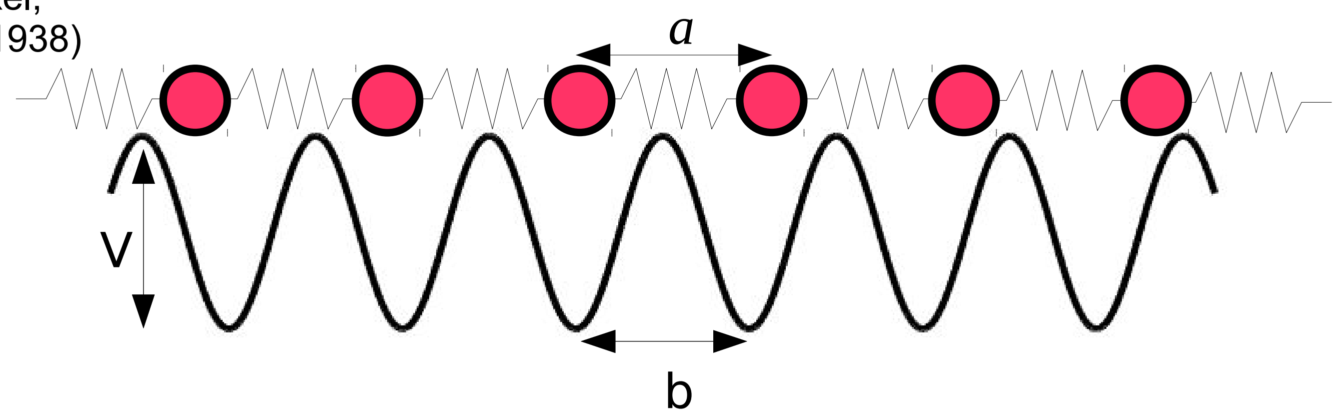


Inherent mismatch between wavelength and particle spacing

Can simulate stick-slip motion and friction – **Frenkel-Kontorova model**!

The Frenkel Kontorova model

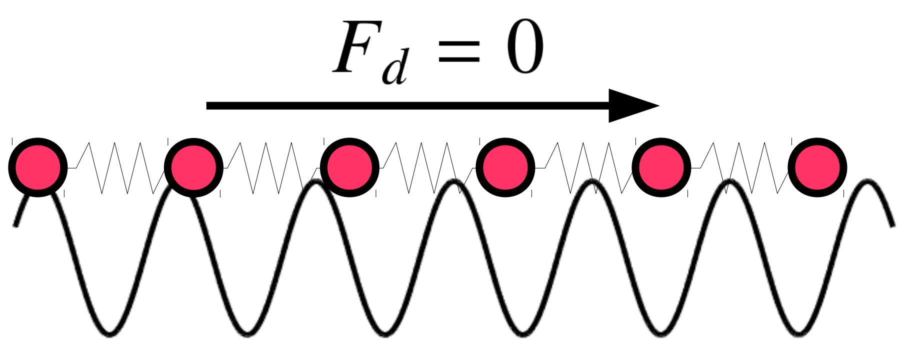
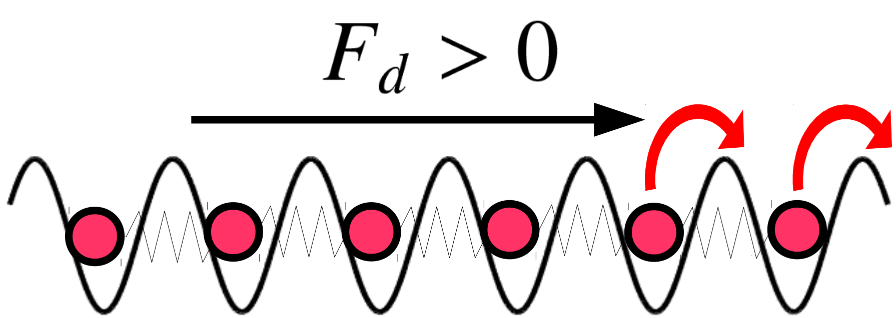
T. Kontorova & J. Frenkel,
Zh. Eksp. Teor. Fiz. **8** (1938)



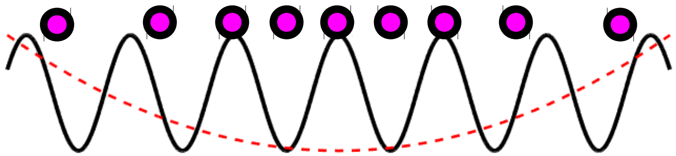
$$H = \sum_n \left[\frac{1}{2}(x_{n+1} - x_n - a)^2 - \frac{V}{(2\pi)^2} \cos\left(\frac{2\pi}{b} x_n \right) \right]$$

Impose a mismatch →

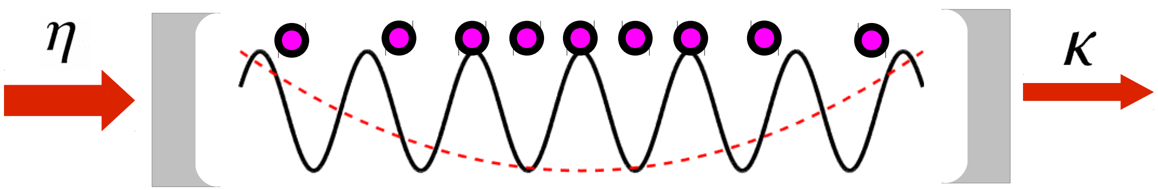
$$\frac{a}{b} = \frac{\sqrt{5} + 1}{2}$$

$V < V_C$ Sliding phase		**Frictionless**
$V > V_C$ Pinned phase		**Emergence of static friction**

The cavity field



$$H_{lattice} = V \sum_j \cos^2(kx_j)$$



$$H_{lattice} = U_0 \hat{n} \sum_j \cos^2(kx_j)$$

Number of photons dynamically fluctuating variable!!

At steady state:

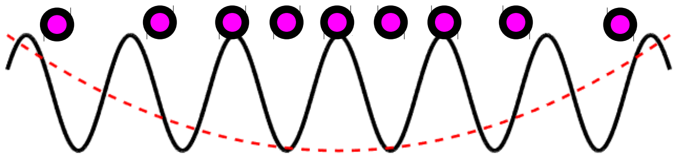
$$\bar{n} = <\hat{n}> = \frac{|\eta|^2}{\kappa^2 + (\Delta_{eff}\{\bar{x}_j\})^2}$$

$$\Delta_{eff}\{\bar{x}_j\} = \Delta_c - \kappa C \frac{\sum_j \cos^2(k\bar{x}_j)}{N}$$

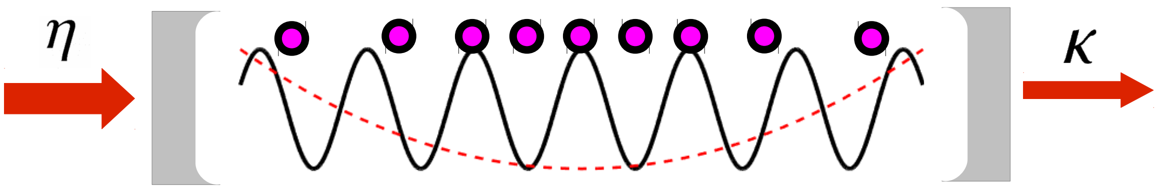
Number of photons depends nonlinearly on the ions positions

$B_N(\{x_j\})$ **Bunching Parameter**

The cavity field



$$H_{lattice} = V \sum_j \cos^2(kx_j)$$



$$H_{lattice} = U_0 \hat{n} \sum_j \cos^2(kx_j)$$

$$\Delta_{eff}(\{x_j\}) = \Delta_c - \kappa C B_N(\{x_j\})$$

Effective cavity potential

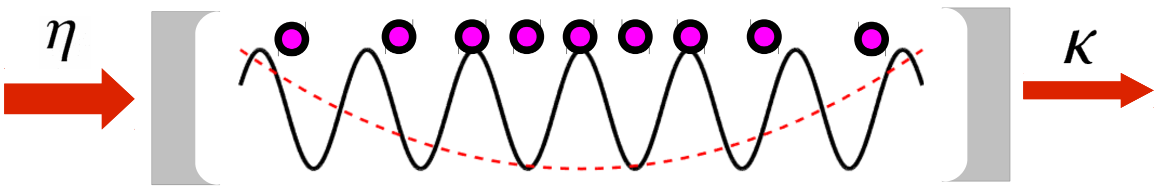
$$V_{cav} = -\frac{\hbar|\eta|^2}{\kappa} \arctan\left(\frac{\Delta_{eff}(\{x_j\})}{\kappa}\right)$$

Substrate depth dependent on:

Pump strength	η
Cooperativity	$C = NU_0/\kappa$

Nonlinearity depends on Cooperativity

The cavity field

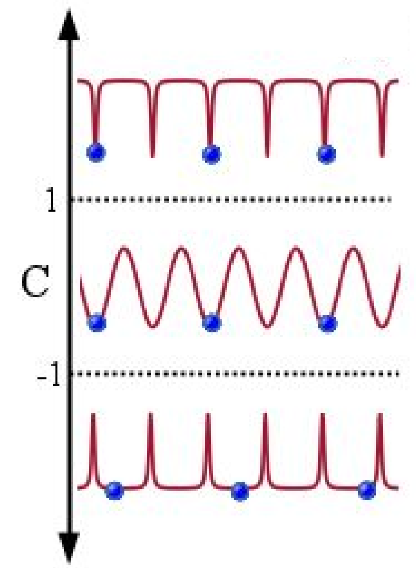


$$V_{cav} = -\frac{\hbar|\eta|^2}{\kappa}\arctan\left(\frac{\Delta_{eff}(\{x_j\})}{\kappa}\right)$$

$|C| \ll 1$ the cavity potential is

$$V_{cav} \propto |\eta|^2 C \sum_j \cos^2(kx_j)$$

$|C| \geq 1$ then the potential gives rise to an effective **long-range force** between the ions



Sign of C changes the effect of the long range interaction!

Cavity field acts as a **deformable potential** due to scattering of photons

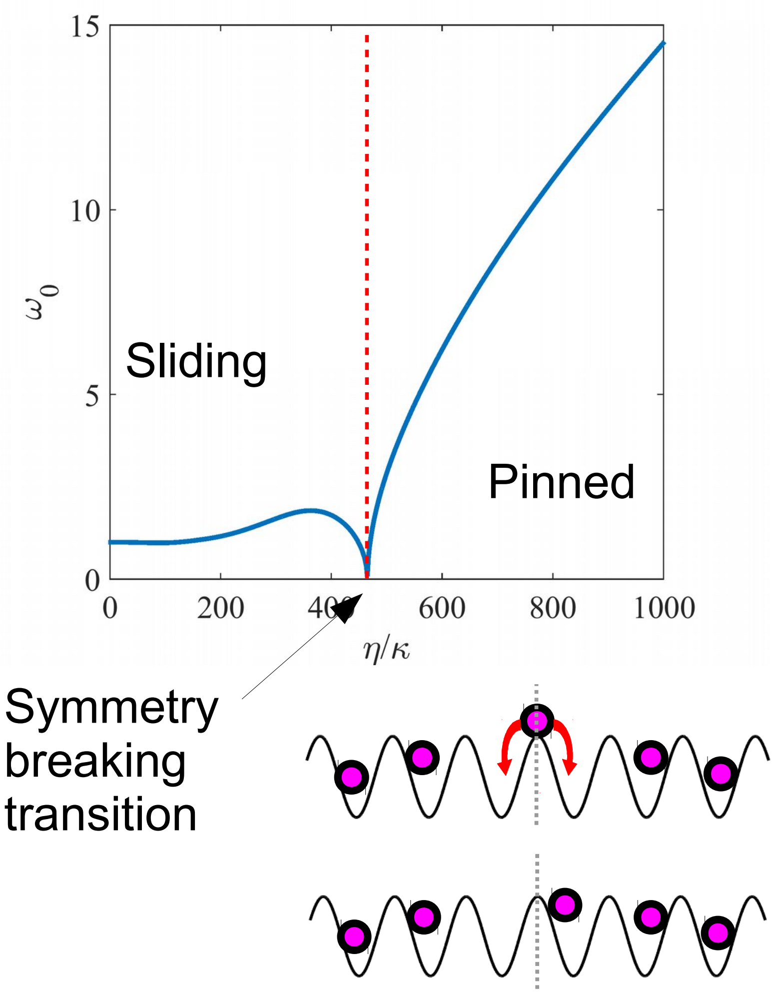
There is a **multi-body long range interaction** mediated by the cavity!

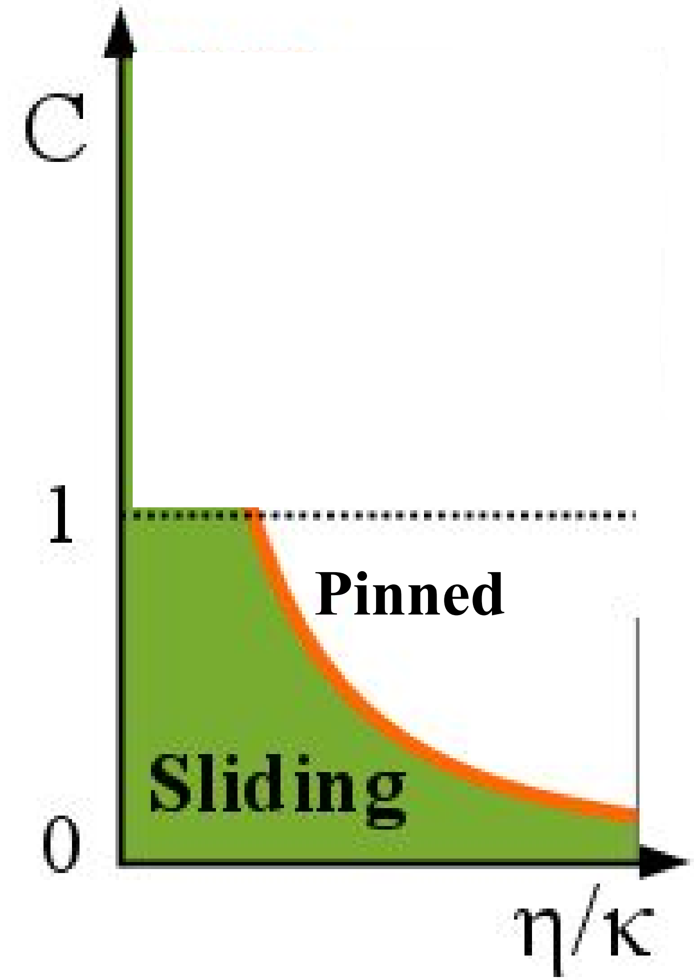
Phase Diagrams

$$|C| \ll 1$$

Vanishing phonon gap

Sliding to pinned transition!

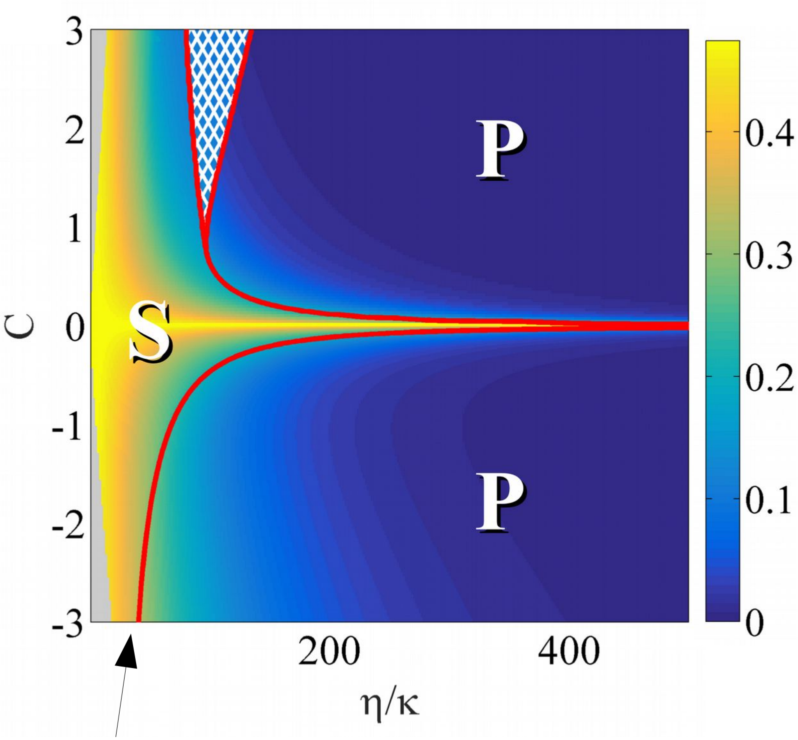




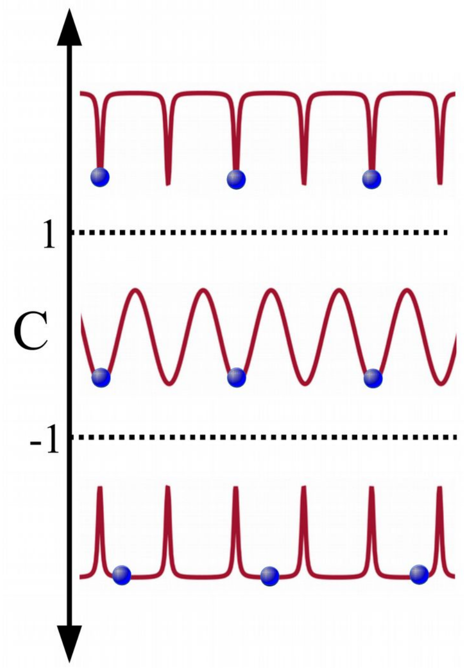
Phase Diagrams

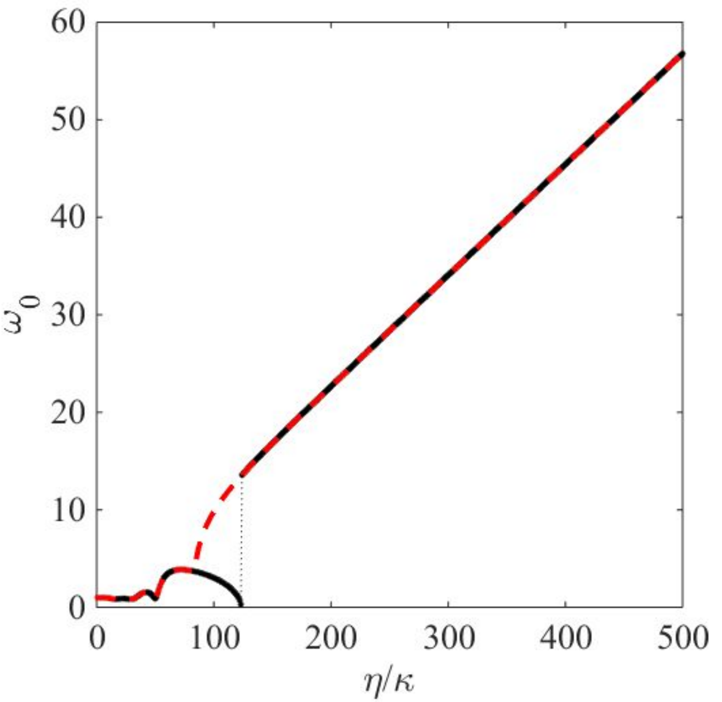
$$B_N = \frac{\sum_j \cos^2(kx_j)}{N}$$

$\Delta_c/\kappa = 0$



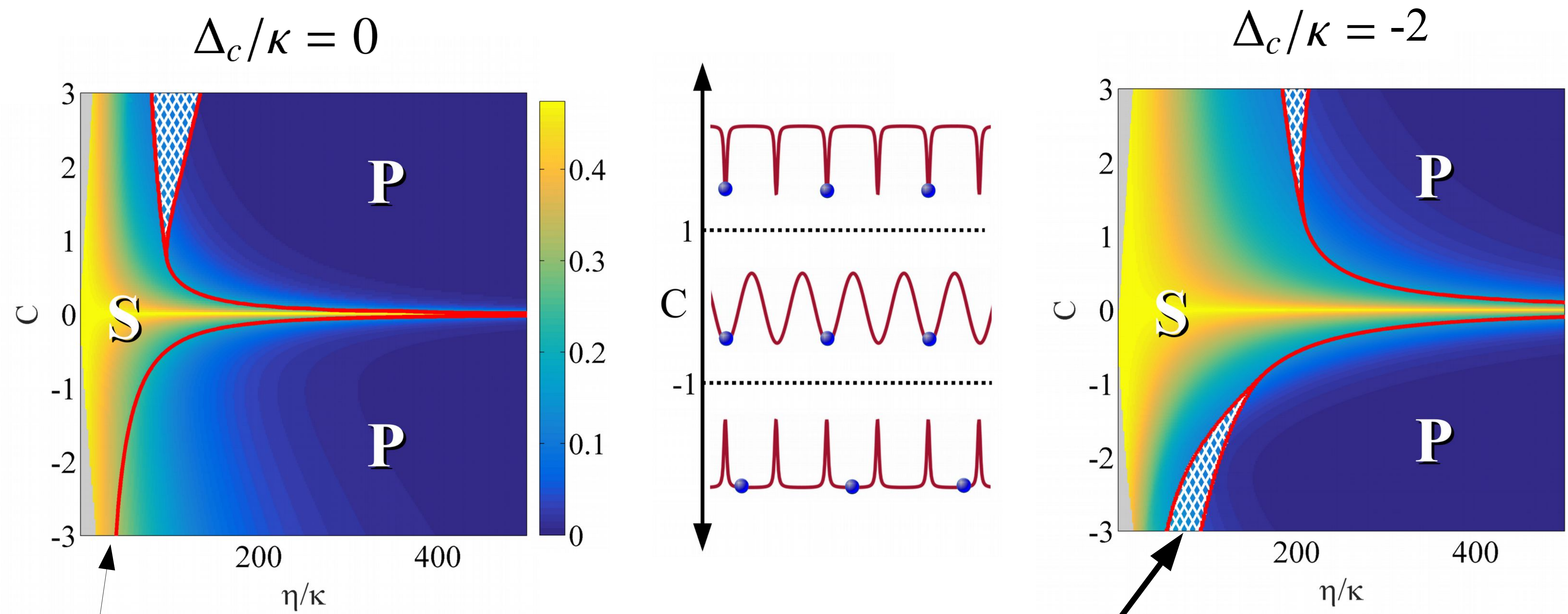
Symmetry breaking transition





Phase Diagrams

$$B_N = \frac{\sum_j \cos^2(kx_j)}{N}$$

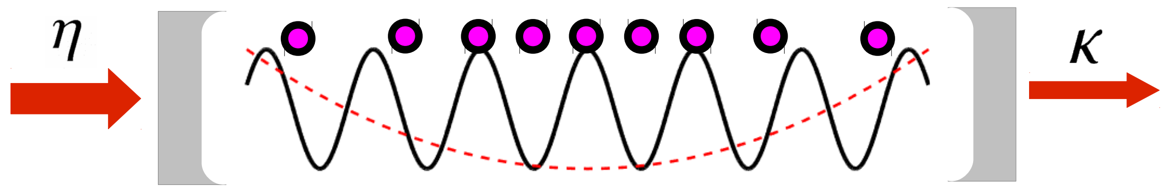


Symmetry breaking transition

Optomechanical resonance

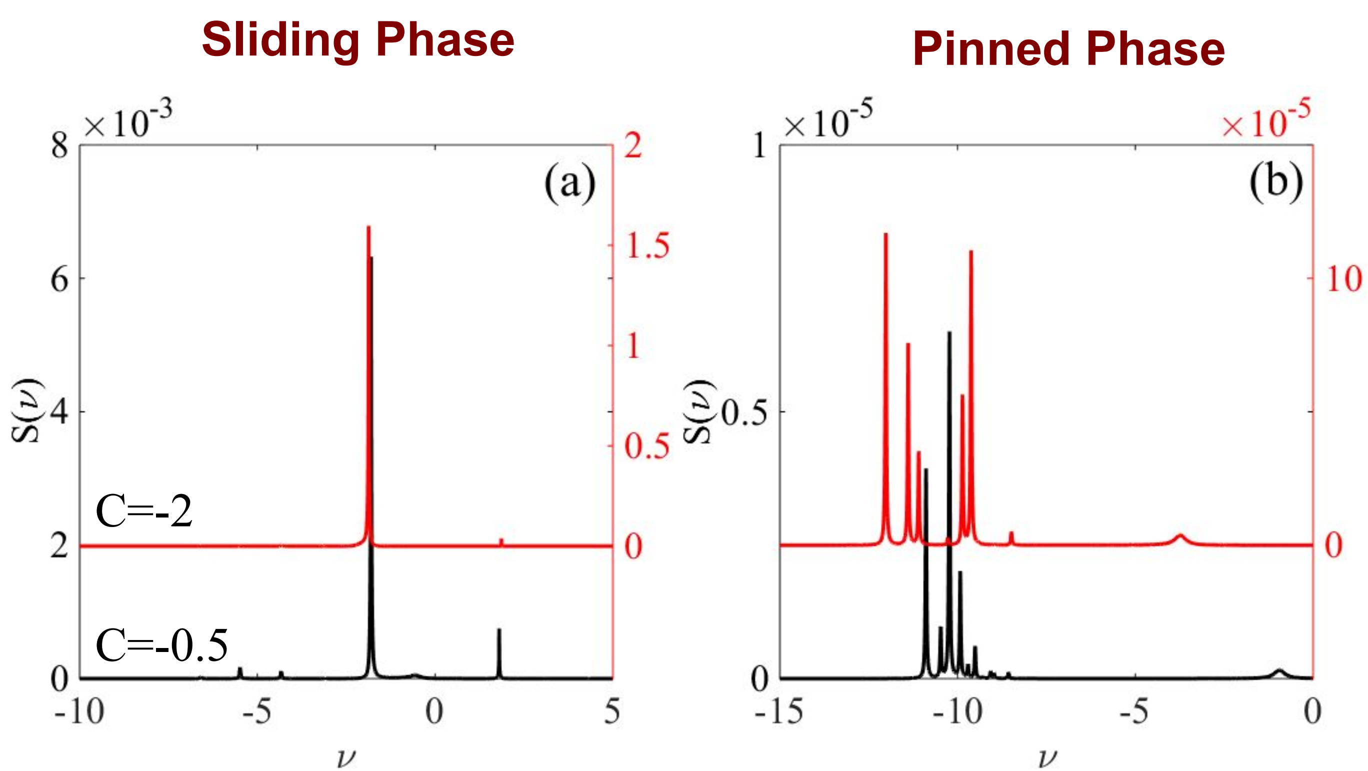
$$\Delta_{eff}(\{x_j\}) = \Delta_c - \kappa C B_N(\{x_j\})$$

Spectrum at cavity output



Light fluctuations strongly coupled to chain vibrations
- chain vibrations imprinted in output light field

Calculated with input-output theory



In Conclusion

- **Bistable states**: coexistence of sliding and pinned phases
- **Detection**: Spectrum at cavity output
- **Cooling**: The substrate acts as a tunable thermal reservoir
- **Entanglement**: Photons and ions are entangled at the transition

Theoretical Quantum Physics @ Saarland University

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