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Quantum Simulations of QED beyond Quantum Link models

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Quantum simulation of gauge theories

QUANTUM SIMULATOR: a special purpose Quantum Computer

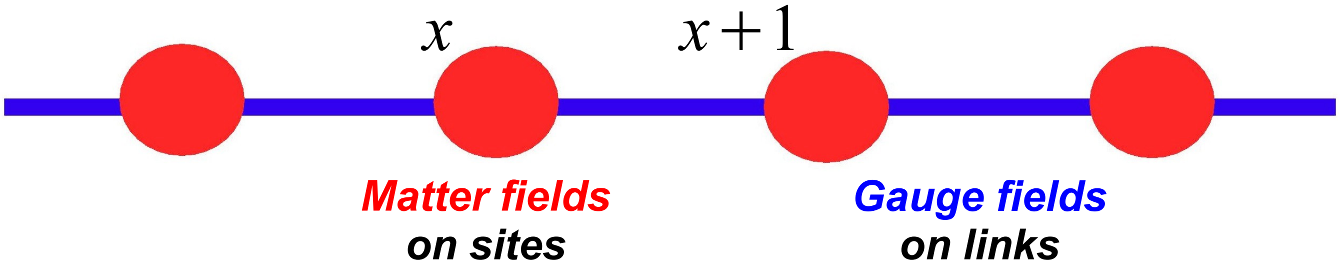
- Exponential speedup with respect to classical computation
- Oriented at one specific (complex) physical system

↓

Quantum simulation of lattice gauge theories

including

Quantum Electrodynamics on a one-dimensional lattice



Lewenstein *et al* 2013,
Cirac *et al* 2015

One-dimensional Lattice QED

$$H = -t \sum_x \left(\psi_x^\dagger e^{-i A_{x,x+1}} \psi_{x+1} + \text{H.c.} \right) + m \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x (E_{x,x+1})^2$$

Parallel transporter

$A_{x,x+1}$ vector potential

Staggering

Wilson 1974
Kogut and Susskind 1975

Fermionic matter fields

$$\{\psi_x, \psi_{x'}\} = 0, \qquad \{\psi_x^\dagger, \psi_{x'}\} = \delta_{x,x'}$$

Canonical COMMUTATION RELATIONS for gauge fields

$$[E_{x,x+1}, A_{x',x'+1}] = i\,\delta_{x,x'}$$

$$[E_{x,x+1}, e^{-i A_{x',x'+1}}] = \delta_{x,x'} e^{-i A_{x,x+1}}$$

Invariance under local U(1) transformations

$$\psi_x \rightarrow \psi_x e^{i\alpha_x}, \qquad e^{-i A_{x,x+1}} \rightarrow e^{i\alpha_x} e^{-i A_{x,x+1}} e^{-i\alpha_{x+1}}, \qquad E_{x,x+1} \rightarrow E_{x,x+1}$$

One-dimensional Lattice QED

Gauge transformation

$$O \rightarrow \prod_x e^{-i\alpha_x G_x} O \prod_y e^{i\alpha_y G_y}$$

GENERATOR

$$G_x = \psi_x^\dagger \psi_x + \frac{(-1)^x - 1}{2} - (E_{x,x+1} - E_{x-1,x})$$

Charge (particle-hole picture) — $\psi_x^\dagger \psi_x + \frac{(-1)^x - 1}{2}$

Divergence of *E* — $(E_{x,x+1} - E_{x-1,x})$

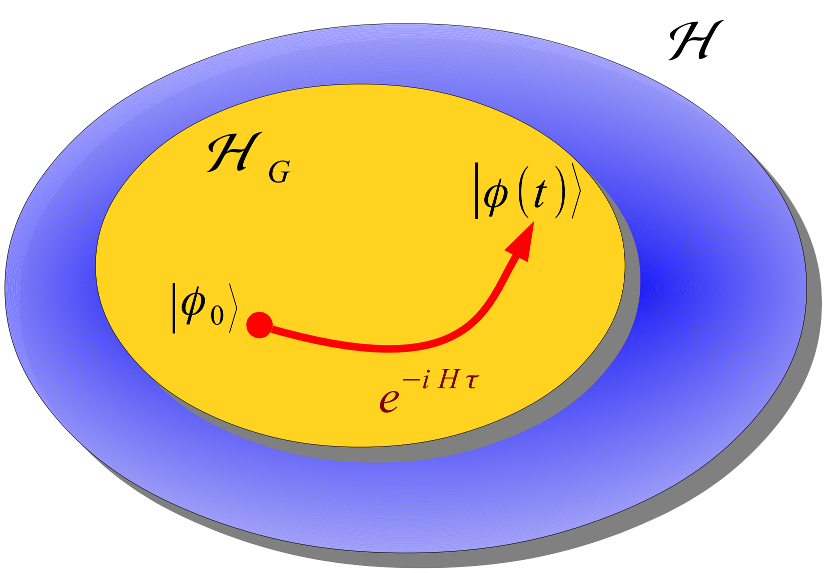
Gauge-invariant states
satisfy the discretized **Gauss law**

$$G_x |\phi\rangle = 0$$

Physical subspace

$$\mathcal{H}_G = \{ |\phi\rangle \in \mathcal{H}, G_x |\phi\rangle = 0 \text{ for all sites } x \}$$

$$[H, G_x] = 0$$ → Constraint on dynamics



Finite link spaces: algebra vs group

Encoding in condensed-matter systems:

- The number of degrees of freedom of links is *finite*
- The microscopic dynamics is *not* gauge-invariant

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QUANTUM LINK models:

the gauge fields are represented by spin-*s* operators on links

$$\dim \mathcal{H}_{\text{link}} = 2s + 1$$

$$E \to S^3, \qquad e^{\mp i A} \to S^{\pm} = S^1 \pm i S^2$$

Horn 1981
Orland and Rohrlich 1990

$$(S^+)^\dagger S^+ = S^- S^+ \neq 1$$

Algebra relations are preserved

(related to **infinitesimal** transformation)

$$[(S^3)_{x,x+1}, (S^+)_{x',x'+1}] = \delta_{x,x'} (S^+)_{x,x+1}$$

Recall $[E_{x,x+1}, e^{-i A_{x',x'+1}}] = \delta_{x,x'} e^{-i A_{x,x+1}}$

BUT the structure of the gauge coupling is altered:
the parallel transporter is **no longer unitary**

Finite link spaces: algebra vs group

Our approach: preserving the *general* Weyl group relation

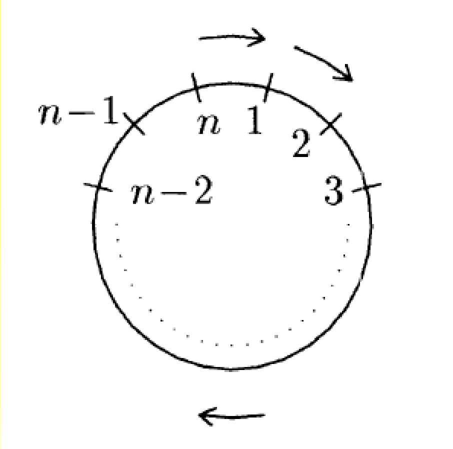
$$e^{-i\eta A} e^{-i\xi E} = e^{i\xi\eta} e^{-i\xi E} e^{-i\eta A} \quad \text{with } \xi, \eta \in \mathbb{R}$$

$$\Downarrow$$

Generalization to *unitary* operators on an n-dimensional space

Conjugated unitary operators

$$U|u_k\rangle = e^{-\frac{2\pi i}{n}k}|u_k\rangle, \qquad V|v_l\rangle = e^{-\frac{2\pi i}{n}l}|v_l\rangle$$



Cyclic permutations

$$U|v_k\rangle = |v_{k+1}\rangle \quad \text{for } k < n, \quad U|v_n\rangle = |v_1\rangle$$

$$V|u_k\rangle = |u_{k-1}\rangle \quad \text{for } k > 1, \quad V|u_1\rangle = |u_n\rangle$$

Schwinger-Weyl group relation

$$U^l V^k = e^{\frac{2\pi i}{n}kl} V^k U^l \quad \text{with } k, l \in \mathbb{Z}$$

Correspondence

$$e^{-iE} \to V, \qquad e^{-iA} \to U$$

A Schwinger-Weyl QED model

$$e^{-iE} \to V, \qquad e^{-iA} \to U \quad \Rightarrow$$

$|v_k\rangle$ electric field basis

$|u_k\rangle$ vector potential basis

$$H = -t\sum_x \left(\psi_x^\dagger U_{x,x+1}\psi_{x+1} + \text{H.c.}\right) + m\sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g_n^2}{2}\sum_x f\left(V_{x,x+1}\right)$$

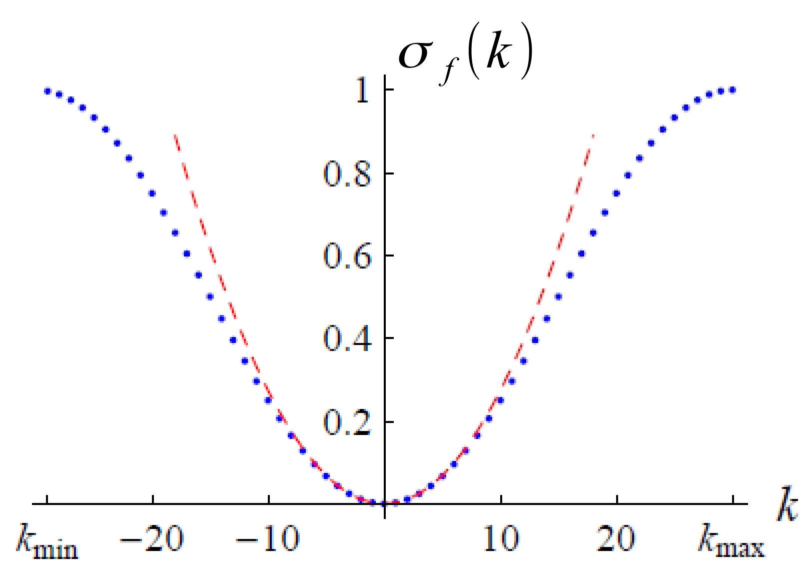
Cyclic permutation of the electric field basis
Correlated with fermion hopping

Electric field energy
$\sim E^2$ for large n

$$f(V) = \frac{1}{4}(V - \mathbb{1})(V^\dagger - \mathbb{1})$$

- **Well defined on the circle**
- **Quadratic spectrum at low energy**

$$\sigma_f(k) = \left(\sin\frac{\pi k}{n}\right)^2$$



A Schwinger-Weyl QED model

Gauge transformation $$O \rightarrow \prod_x (T_x)^{-\nu_x} O \prod_y (T_y)^{\nu_y} \quad \text{with } \nu_x \in \mathbb{Z}$$

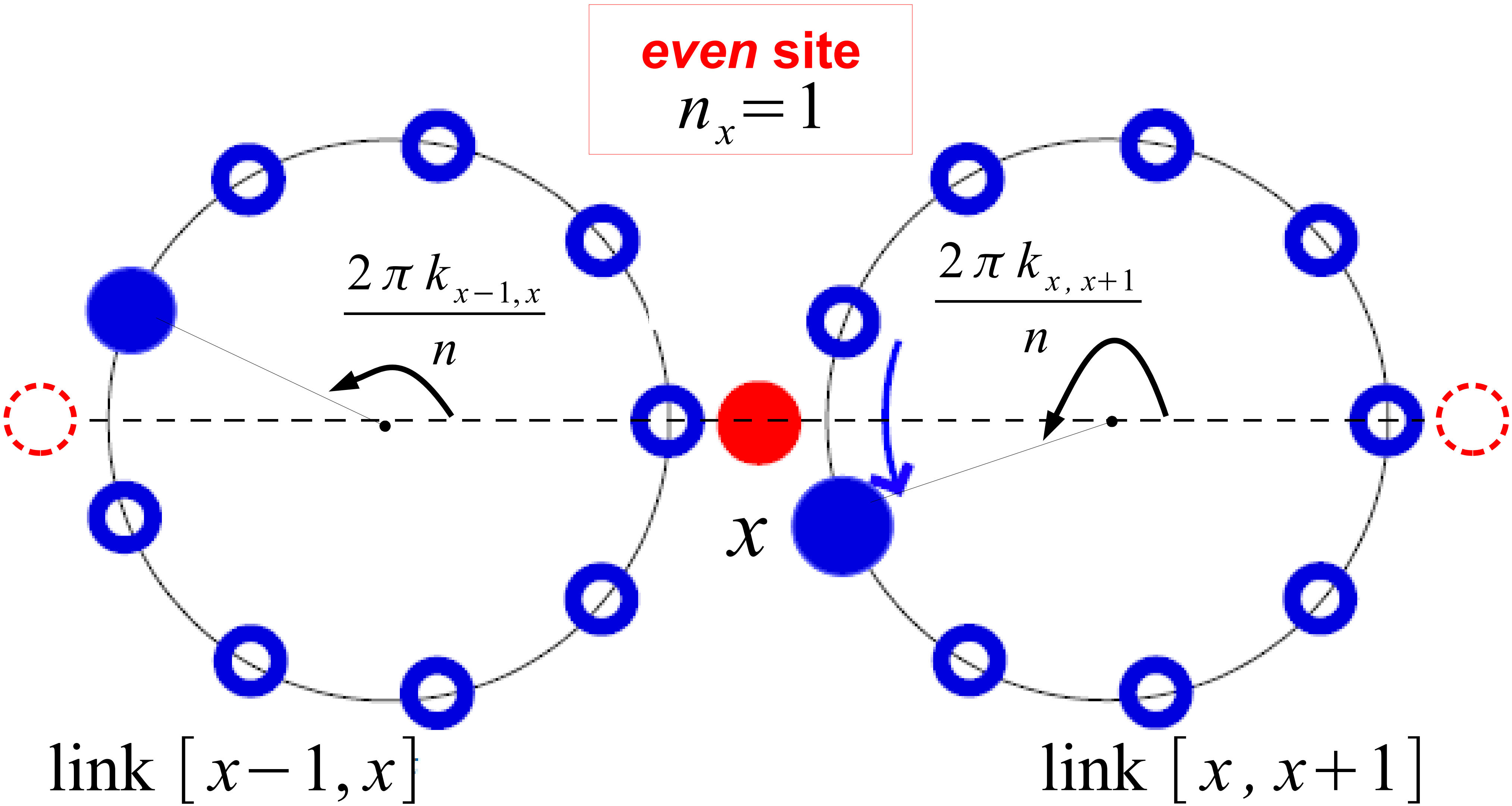
$$T_x = \exp\left[\frac{2\pi i}{n}\left(\psi_x^\dagger \psi_x + \frac{(-1)^x - 1}{2}\right)\right] V_{x,x+1} V_{x-1,x}^+, \qquad (T_x)^n = \mathbb{1}$$

$\mathbb{Z}_n$ GAUGE GROUP

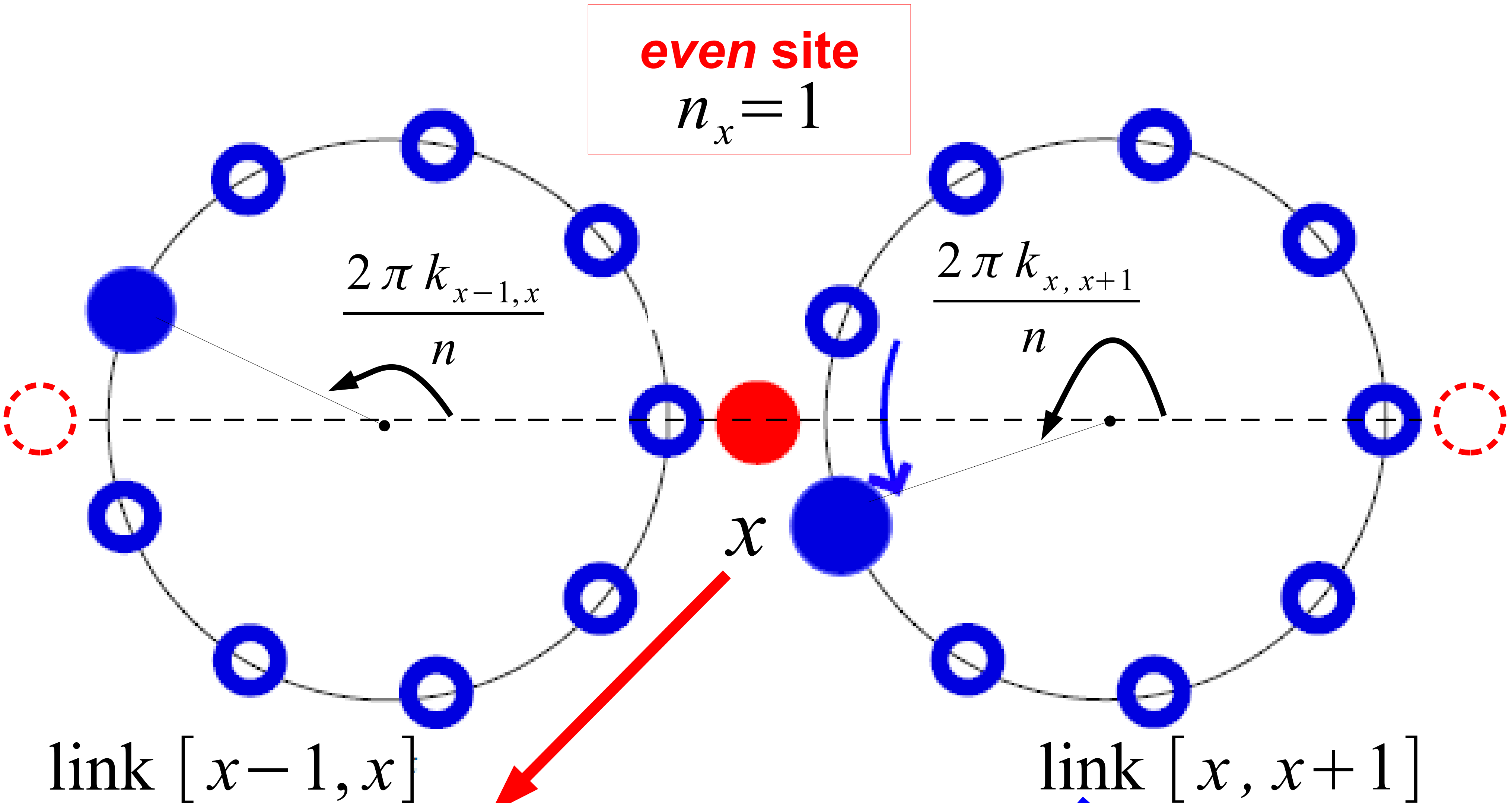
Physical subspace $$\mathcal{H}_T = \left\{ |\phi\rangle \in \mathcal{H}, T_x |\phi\rangle = |\phi\rangle \ \text{for all sites } x \right\}$$

Generalized Gauss law $$e^{\frac{2\pi i}{n}\left(n_x + \frac{(-1)^x - 1}{2} - k_{x,x+1} + k_{x-1,x}\right)} = 1$$

Physical states



Physical states

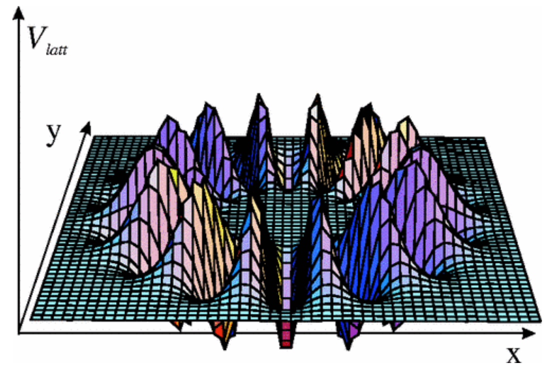


Fermionic atom

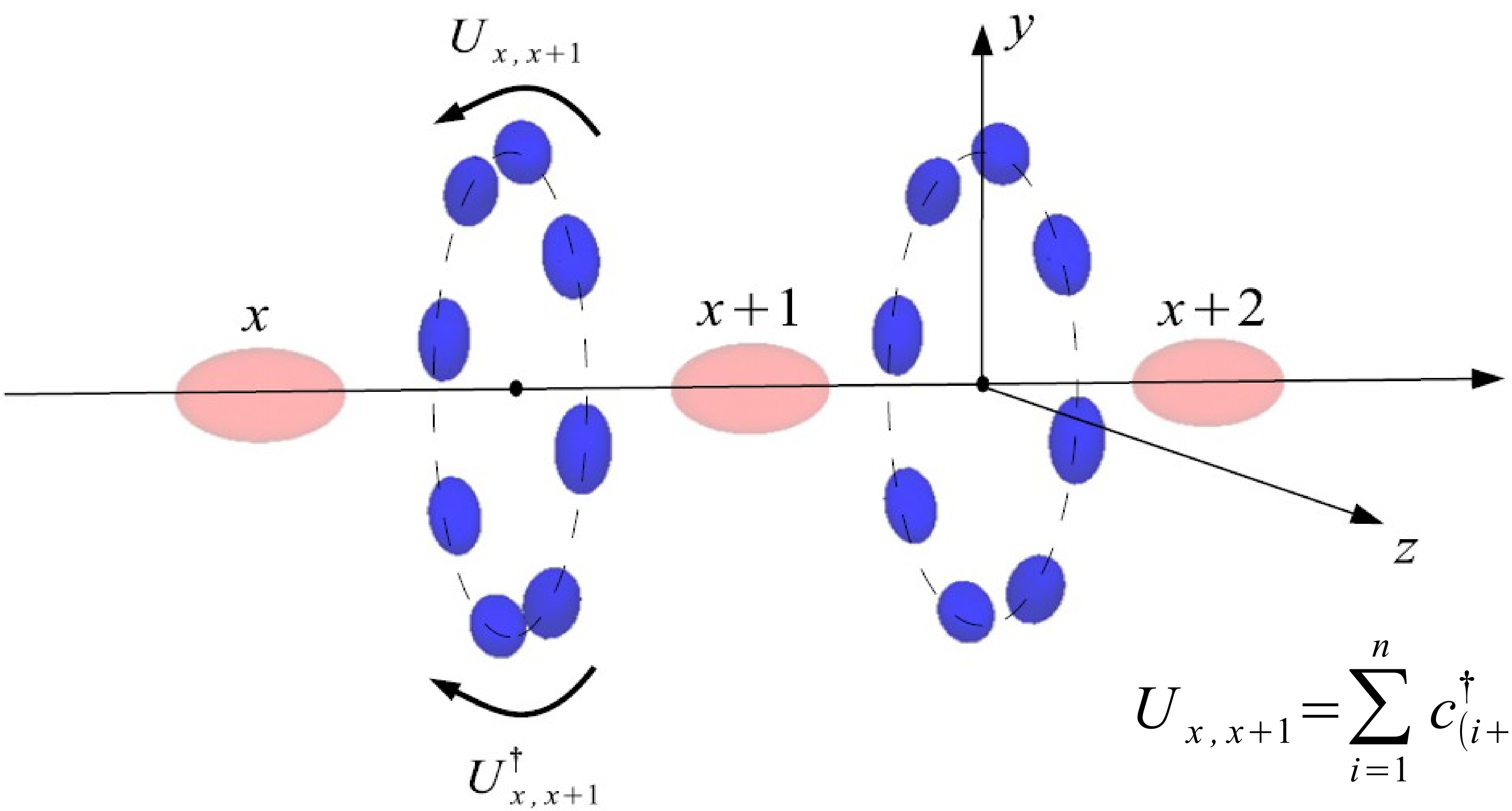
***n*-level (bosonic) atom?**
Potential wells on a circle?

Implementation of links

Ring-shaped optical lattices can be obtained from the interference of a plane wave with a Laguerre-Gauss mode (Amico, Osterloh and Cataliotti 2005)



A possible tool to implement link dynamics



$$U_{x,x+1} = \sum_{i=1}^{n} c^\dagger_{(i+1)x,x+1} c_{(i)x,x+1}$$

Implementation: effective dynamics

Encoding in condensed-matter systems:

- The microscopic dynamics is *not* gauge-invariant

Constrain the dynamics in the physical subspace

Zohar and Reznik 2011, Banerjee, Dalmonte, Mueller, Rico, Stebler, Wiese, Zoller 2012
Stanniger, Hauke, Marcos, Hafezi, Diehl, Dalmonte, Zoller 2014

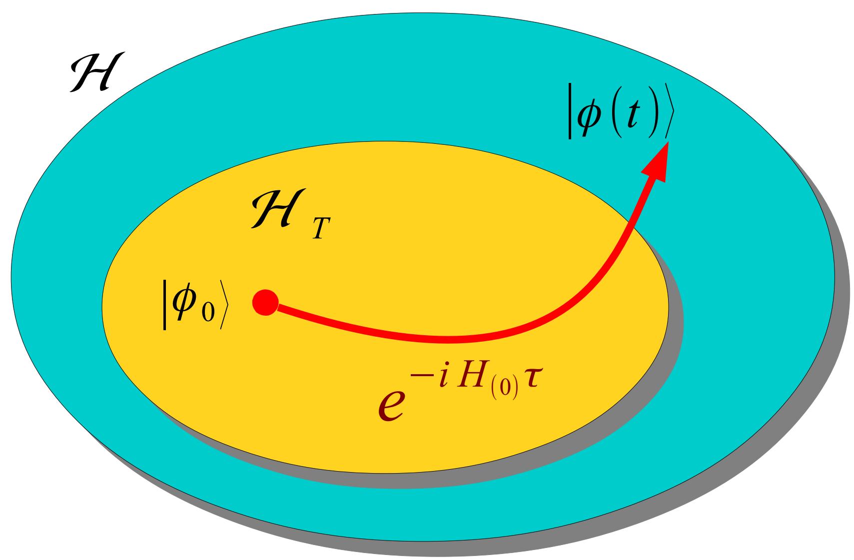
$$\Gamma = \sum_x (T_x - \mathbb{1})(T_x^\dagger - \mathbb{1}) \geq 0 \qquad |\phi\rangle \in \mathcal{H}_T \Longleftrightarrow \Gamma|\phi\rangle = 0$$

$$H_{(1)} = H_{(0)} + u\,\Gamma$$

$H_{(0)}$: *Microscopic Hamiltonian*

$u\,\Gamma$: ***Energy penalty to non-physical states***

$$[H_{(0)}, T_x] \neq 0$$



Implementation: effective dynamics

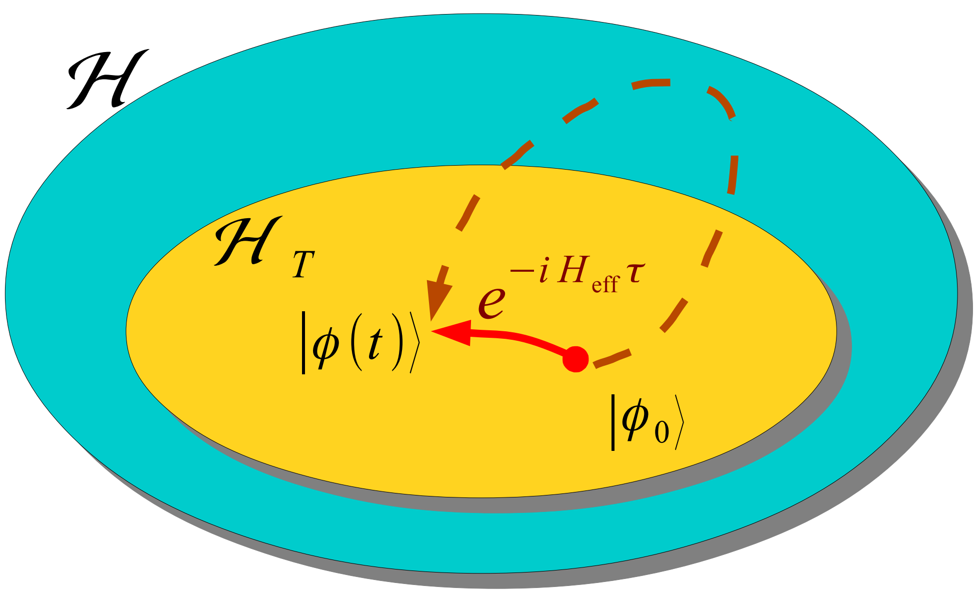
$$H_{(0)} = -\tilde{t} \sum_x \left(\psi_x^\dagger \psi_{x+1} + \psi_{x+1}^\dagger \psi_x \right) - \tilde{w} \sum_x \left(U_{x,x+1} + U_{x,x+1}^\dagger \right) + H_d$$

Fermion hopping — On-link Rabi coupling

H_d includes the gauge-preserving terms:

- Fermion mass term
- Electric field energy
- *counterterms*

$$H_{\text{eff}} = -t \sum_x \left(\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \psi_{x+1}^\dagger U_{x,x+1}^\dagger \psi_x \right) + H_d$$



Summary and outlook

- We propose a $\mathbb{Z}_n$ approximation to lattice QED, based on the discretization of U(1) *group* properties.

- The unitary structure of the gauge coupling in the original model is preserved.

  ↳ **Unitarity ensures the correct properties of transition amplitudes**

- Effective dinamics can represent a path to experimental realizations.

Further research:

- Identification of suitable atomic systems for implementation.

- Extension to non-Abelian gauge groups.

Thank you
for your attention