Mean-Field Dissipative Dynamics in Infinite Quantum Systems

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Outline

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Motivation and Aim

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- Some many-body effects are well approximated by unitary mean-field evolutions ($H_N \sim \frac{1}{N}\sum_{k,h} x^{(k)} x^{(h)}$ as in the BCS model)
- Analogous dissipative evolutions represent a natural generalization of the unitary case

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- Analogous dissipative evolutions represent a natural generalization of the unitary case

Aim

- Find the time-behaviour of different classes of observables;
- Discuss physical results.

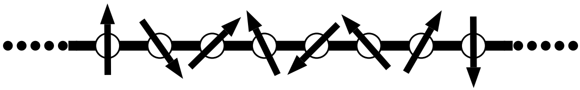
Outline

INFINITE NUMBER OF D-LEVEL SYSTEMS

Description of Quantum Systems:

- Algebra containing the observables
- State of the system

INFINITE NUMBER OF D-LEVEL SYSTEMS



ALGEBRA

- One d-level system: $\mathcal{A}_p = M_d(\mathbb{C})$
  $\{v_\alpha\}_{\alpha=1}^{d^2}$ is an hermitian basis of $\mathcal{A}_p$.
- For finite number of particles: $\mathcal{A}_N = \bigotimes_{k=-N}^{N} \mathcal{A}_p^{(k)}$.
  When considering large systems, one takes the limit:

$$\mathcal{A} = \overline{\lim_{N\to\infty} \bigotimes_{k=-N}^{N} \mathcal{A}_p^{(k)}}^{\|\cdot\|}$$

Infinite Number of d-Level Systems

State

- States are needed to compute averages; for finite systems:

$$\mathrm{Tr}\,(\rho_N A) = \langle A \rangle \quad \forall\, A \in \mathcal{A}_N$$

- For infinite systems, no density operator formalism. The state is a positive, linear, and normalized functional, providing mean values:

$$\omega(A) = \langle A \rangle \quad \forall\, A \in \mathcal{A}$$

INFINITE NUMBER OF D-LEVEL SYSTEMS

STATE: Relevant Thermodynamical Assumptions

- Translation Invariance:

$$\omega\left(\tau^k[A]\right) = \omega\left(\tau^h[A]\right), \ \forall k, h$$

- Cluster Property:

$$\lim_{|h-k|\to\infty} \omega\left(\tau^k[A]\,\tau^h[B]\right) = \omega(A)\,\omega(B)$$

$\forall A, B \in \mathcal{A}$, with $\tau^h[x^{(k)}] = x^{(k+h)}$.

Outline

N Particle Systems

Mean-Field Hamiltonians: $H \to \frac{1}{N}\sum_{k,h=1}^{N}\left(x^{(k)}y^{(h)} + h.c\right)$

Examples

- BCS model:

$$H_N = \epsilon \sum_{k=1}^{N} \sigma_z - \frac{1}{N}\sum_{k,h=1}^{N}\left(\sigma_x^{(k)}\sigma_x^{(h)} + \sigma_y^{(k)}\sigma_y^{(h)}\right)$$

- Mean-Field Bose Gas:

$$H_N = \sum_k \epsilon a_k^\dagger a_k + \frac{\lambda}{2N}\left(\sum_k a_k^\dagger a_k\right)^2$$

N PARTICLE SYSTEMS

DISSIPATIVE MEAN-FIELD INTERACTION

$$\mathbb{L}_N[X] = \frac{1}{N}\sum_{k,h=1}^{N}\sum_{\mu,\nu=1}^{d^2} C_{\mu\nu}\left(v_\mu^{(k)} X v_\nu^{(h)} - \frac{1}{2}\left\{v_\mu^{(k)} v_\nu^{(h)}, X\right\}\right)$$

($\{v_\alpha\}_{\alpha=1}^{d^2}$ hermitian basis), $C \geq 0$.

Evolution map:

$$\Lambda_t^N = e^{t\mathbb{L}_N},\ t \geq 0.$$

Mean-Field Generators

N particles system, in interaction with environment:

$$H_N = H_N^S \otimes \mathbf{1} + \mathbf{1} \otimes H^E + \gamma \sum_{\alpha} V_{\alpha}^N \otimes B_{\alpha}$$

Mean-field dissipative generators come from weak-coupling procedure when:

$$V_{\alpha}^N = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} v_{\alpha}^{(k)}$$

- Collective particle-invariant interaction with the environment
- For clustering states, the scaling is the only one giving a meaningful evolution in the limit.

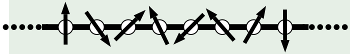
Outline

Macroscopic Observables

Infinite Spin Chain

Spin-$\frac{1}{2}$ chain

$$\omega_+(X) = \langle \uparrow_1 \uparrow_1 \ldots \uparrow_1 |X| \uparrow_1 \uparrow_1 \ldots \uparrow_1 \rangle$$



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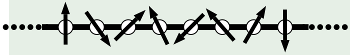

$$S_\alpha^N = \frac{1}{N} \sum_{k=1}^{N} s_\alpha^{(k)}$$

$$\begin{aligned} S_1 &= \omega(s_1)\mathbf{1} \\ S_2 &= \omega(s_2)\mathbf{1} \\ S_3 &= \omega(s_3)\mathbf{1} \end{aligned} \quad \xrightarrow{\omega_+} \quad \begin{aligned} S_1 &= \tfrac{1}{2} \\ S_2 &= 0 \\ S_3 &= 0 \end{aligned}$$

In general:

$$\lim_{N\to\infty} \left\| \left[S_\alpha^N, S_\beta^N \right] \right\| = 0$$

Macroscopic Observables

Classical Description of Systems

On *clustering states*, macroscopic observables are multiples of the identity

Macroscopic Observables' Dynamics

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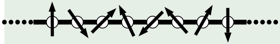
Spin-$\frac{1}{2}$ chain



$$\omega_+(X) = \langle \uparrow_1 \uparrow_1 \ldots \uparrow_1 | X | \uparrow_1 \uparrow_1 \ldots \uparrow_1 \rangle$$

$$S_\alpha^N(t) = \Lambda_t^N \left[\frac{1}{N} \sum_{k=1}^{N} s_\alpha^{(k)} \right]$$

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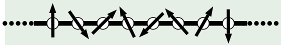

$$S_\alpha^N(t) = \Lambda_t^N \left[\frac{1}{N} \sum_{k=1}^{N} s_\alpha^{(k)} \right]$$

$$\mathbb{L}_N[X] = \frac{1}{N} \sum_{k,h=1}^{N} \left(\ell^{(k)} X \ell^{\dagger(h)} - \frac{1}{2} \left\{ \ell^{(k)} \ell^{\dagger(h)}, X \right\} \right), \quad \ell = s_1 + i s_2$$

$$\begin{aligned} \frac{d}{dt} S_1 &= S_1 S_3 \\ \frac{d}{dt} S_2 &= S_2 S_3 \\ \frac{d}{dt} S_3 &= -(S_1^2 + S_2^2) \end{aligned} \quad \xrightarrow{t \to \infty} \quad \begin{aligned} S_1 &= 0 \\ S_2 &= 0 \\ S_3 &= -\frac{1}{2} \end{aligned}$$

Macroscopic Observables' Dynamics

Define:

$$\omega_{v_\alpha}(t) = \lim_{N\to\infty} \omega\left(\Lambda_t^N\left[\frac{1}{N}\sum_{k=1}^{N} v_\alpha^{(k)}\right]\right) \; ;$$

Evolution of Macroscopic Observables

$$\frac{d}{dt}\omega_{v_\alpha}(t) = i\sum_{\mu,\nu=1}^{d^2} B_{\mu\nu}\,\omega_{v_\nu}(t)\,\omega_{[v_\mu,v_\alpha]}(t), \qquad B = \Im(C) \; .$$

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Local Evolution

Microscopic Operators: O on a finite support.

- Characteristic relaxation time goes like N;
- What can be left?

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Microscopic Dynamics

$$\lim_{N\to\infty} \omega\left(X\Lambda_t^N[O]\,Y\right) = \lim_{N\to\infty} \omega\left(XU_N^\dagger(t)OU_N(t)Y\right)$$

$\forall\, X, Y \in \mathcal{A}$.

where:

$$H_N^\omega(t) = \sum_{\mu,\nu=1}^{d^2} B_{\mu\nu}\omega_{v_\nu}(t) \sum_{k=1}^{N} v_\mu^{(k)}, \qquad B = \Im(C)\,,$$

$$U_N(t) = \overrightarrow{T} \exp\left(-i\int_0^t ds\, H_N^\omega(s)\right).$$

The Unitary Evolution

Comments:

- From a purely dissipative to unitary evolution
- The generator is non-local in time, showing memory of the initial moment of the dynamics

  D. Chruscinski, A. Kossakowski, *Phys. Rev. Lett.* **104**, 070406 (2010)

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From time $s \to t$:

$$\Lambda^N_{(t-s)}[\cdot] \longrightarrow \mathcal{U}^N_{(t-s)}[\cdot] \equiv U^\dagger_N(t-s) \cdot U_N(t-s)$$

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Semigroup Composition Law of $\Lambda^N_{(t-s)}$:

$$\Lambda^N_{(t-s)} \circ \Lambda^N_s = \Lambda^N_t, \qquad t > s > 0$$

to non composing unitaries

$$\mathcal{U}^N_{(t-s)} \circ \mathcal{U}^N_s \neq \mathcal{U}^N_t, \qquad t > s > 0$$

The Unitary Evolution

Due to the non-locality of the generator:

$$\frac{d}{dt}\mathcal{U}^N_{(t-s)}[X] = \mathcal{U}^N_{(t-s)}\left[i\left[H^\omega_N(t-s), X\right]\right]$$

The Unitary Evolution

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$$\frac{d}{dt}\mathcal{U}^N_{(t-s)}[X] = \mathcal{U}^N_{(t-s)}\left[i\left[H^\omega_N(t-s), X\right]\right]$$

Remark

Recall that in the usual time-dependent Schroedinger equation we have:

$$\frac{d}{dt}\mathcal{U}_{t,s}\left[X\right] = \mathcal{U}_{t,s}\left[i\left[H(t), X\right]\right]$$

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Fluctuation Operators

Classification of Observables:

- Microscopic Observables
  E.g. Observable of the k-th particle $x^{(k)}$
- Macroscopic Observables
  E.g. Mean average over the system

$$\lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}x^{(k)}$$

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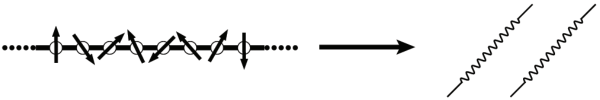
$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} x^{(k)}$$

- Mesoscopic Observables
  E.g. Fluctuations about the mean value

$$F_{v_\alpha} = \lim_{N\to\infty} F^N_{v_\alpha} = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \left(v_\alpha^{(k)} - \omega(v_\alpha^{(k)}) \right)$$

The $N \to \infty$ Mapping



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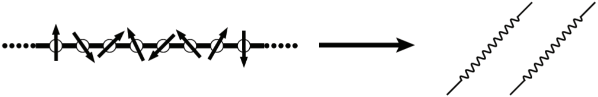
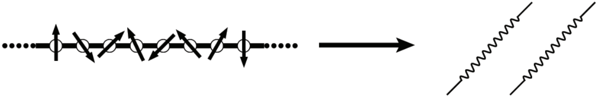


- $\{F^N_{v_\alpha}\}_\alpha$

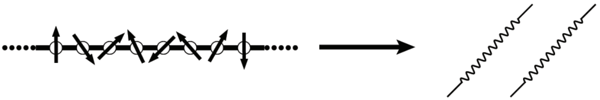
$\{F_{v_\alpha}\}_\alpha$
Bose Field Operators

The $N \to \infty$ Mapping



- $\{F^N_{v_\alpha}\}_\alpha$ — $\{F_{v_\alpha}\}_\alpha$ Bose Field Operators
- $e^{ix_\alpha F^N_{v_\alpha}} e^{ix_\beta F^N_{v_\beta}}$ — $e^{i\left(x_\alpha F_{v_\alpha} + x_\beta F_{v_\beta}\right)} e^{-i\frac{x_\alpha x_\beta}{2}\Omega_{\alpha\beta}}$ Weyl Operators

The $N \to \infty$ Mapping



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  Weyl Operators
- ω — $\tilde{\omega}(e^{ix_\alpha F_{v_\alpha}}) = e^{-\frac{x_\alpha^2}{2}\sigma^2_{v_\alpha}}$
  Gaussian Bosonic State

D. Goderis *et al.*, *Prob. Th. Rel. Fields* **82** (1989) 527; *Commun. Math. Phys.* **128** (1990) 533

A. Verbeure, *Many-Body Boson Systems* (Springer, London, 2011)

Outline

Definition of the Time-Evolution

From the local evolution:

$$O(t) = \lim_{N\to\infty} \mathcal{U}_t^N[O]$$

We define the map:

$$\Gamma_t^N[X] = e^{t\mathbb{L}_N} \circ \left(\mathcal{U}_t^N\right)^{-1}[X]$$

Remark

If the dynamics of fluctuations were the same as the local one, in the limit Γ_t^N would become the identity map.

EVOLUTION OF FLUCTUATION OPERATORS

TIME-EVOLUTION ON FLUCTUATIONS

$$\Gamma_t\left[e^{i(\xi,F)}\right] = e^{i(\xi,X_t\,F)}\;e^{-\frac{1}{2}(\xi,Y_t\,\xi)}$$

- non-trivial Gaussian evolution;
  (on fluctuations the dynamics is richer)
- as the microscopic one, it does not compose as a semi-group.

SUMMARY

We defined a class of Lindblad generators and studied its thermodynamical convergence on different observables:

At the microscopic level: time-dependent unitary dynamics, with signature of non-Markovianity (non-local in time generator);

At the macroscopic level: classical evolution from non-linear differential equations;

At the mesoscopic level: dissipative quasi-free time-evolution.

Different Dynamics

Dissipation is only affecting the mesoscopic level of the system. At this level, the dynamics is "richer", proving the existence of long-range correlations, not visible microscopically.

Thank You

Thank You For Your Attention!!