Quantum Correlations Group
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Accessible quantification of multiparticle entanglement

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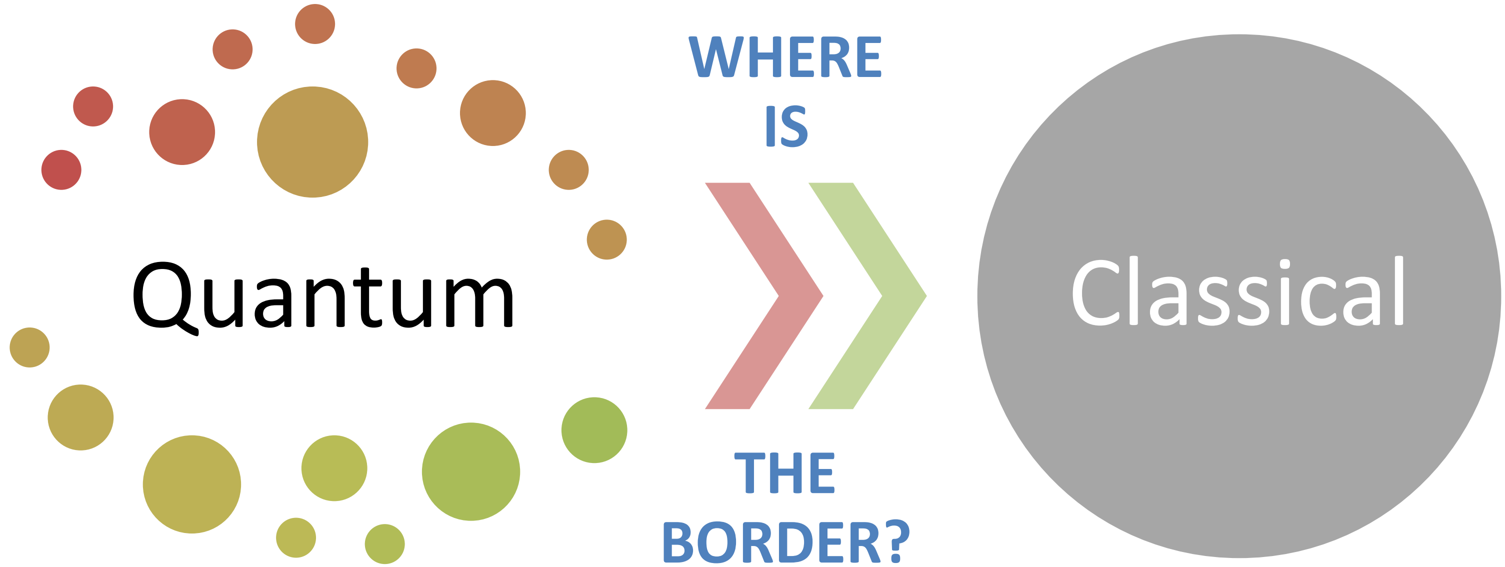
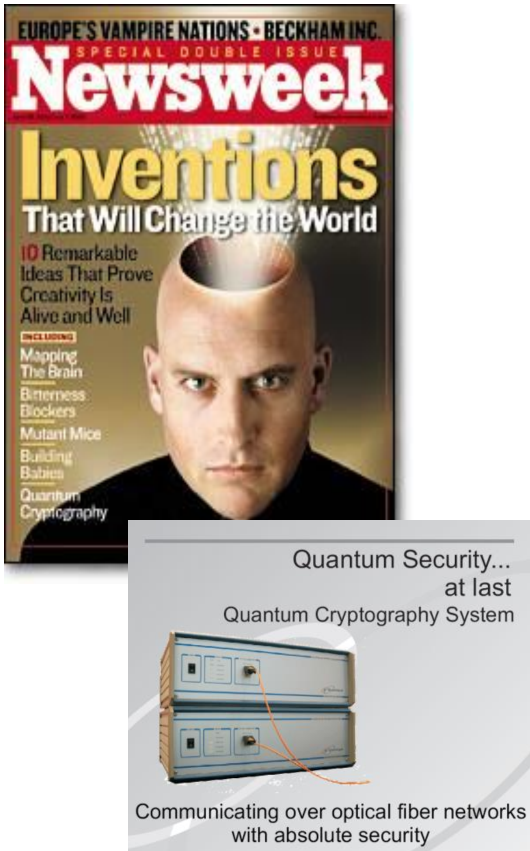
Our research at Nottingham

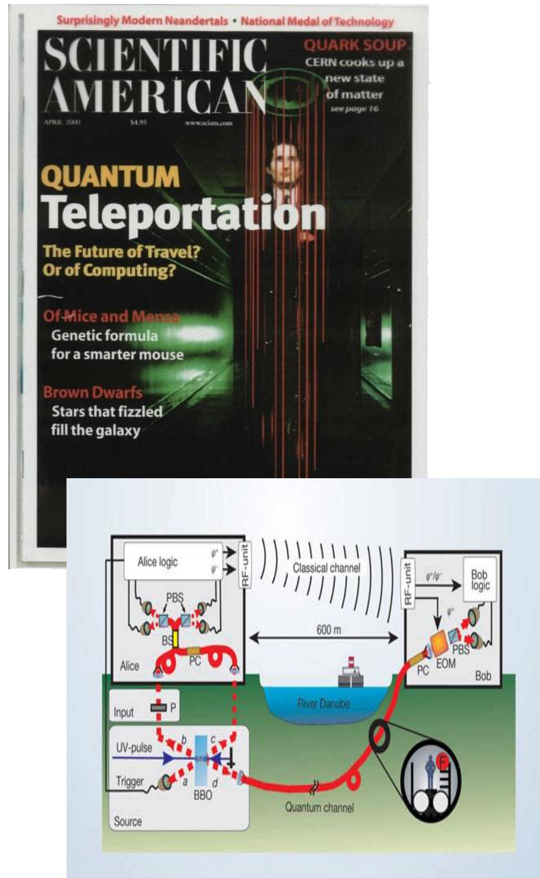


- Identifying quantumness by its most essential and genuine signatures in general composite systems
- Providing novel operational interpretations and satisfactory measures for quantum resources

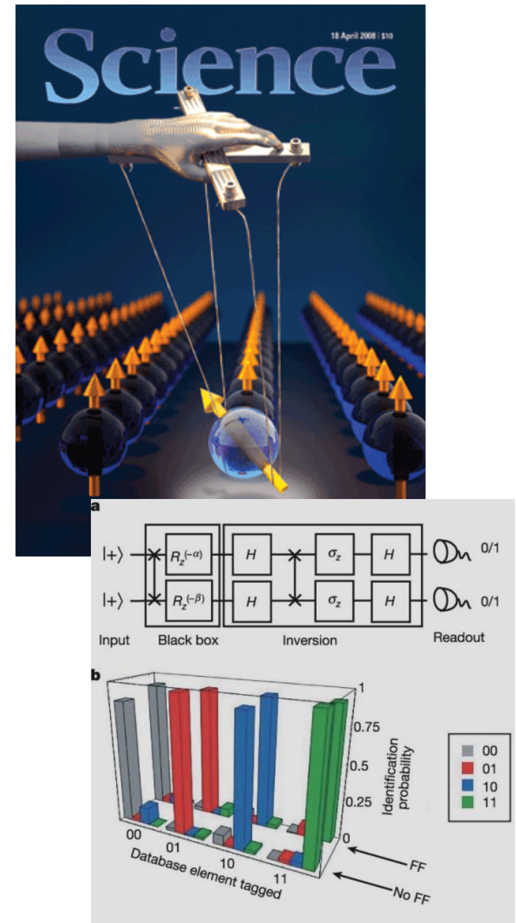
Entanglement: a quantum resource



Quantum cryptography



Quantum teleportation



Quantum computation

Characterising entanglement

Detection ☺

- **Is a state entangled?**
- *NP-hard* but feasible for given classes of states and entanglement types
- Essential to distinguish *useful versus useless* states for quantum applications
- Experimentally *accessible* for bipartite & multipartite systems (e.g. witnesses)

Quantification ☹

- **How entangled a state is?**
- OK for pure bipartite states, *formidable* in general even for a known density matrix
- Essential to determine *how efficiently* a quantum task can be performed
- Experimentally requires *full tomography*, unless the state is partially known

This work

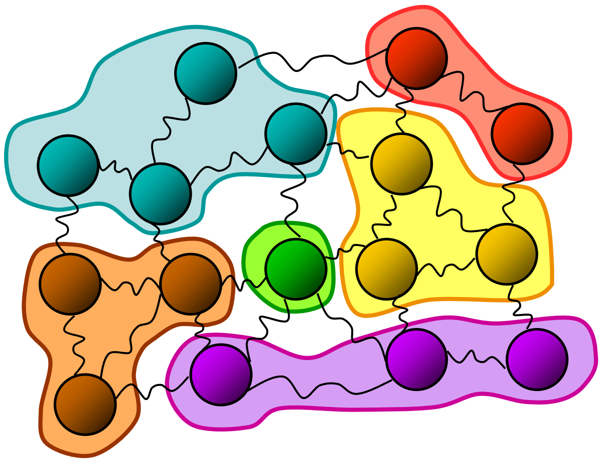
Can we get an ***experimentally-friendly*** *and* ***quantitative*** *assessment of* ***multipartite*** *entanglement*?

Quantification ☹ ⇒ ☺ ?

- ✓ Yes, **exactly** for a family of N-qubit states, and for a general class of entanglement measures
- ✓ Yes, providing **lower bounds** for the global multipartite entanglement of arbitrary states
- ✓ All based on measuring just **three** correlation functions
- ✓ Useful in current **experiments**

Marco Cianciaruso, Thomas Bromley & GA, arXiv:1507.01600 (2015)

Multipartite entanglement

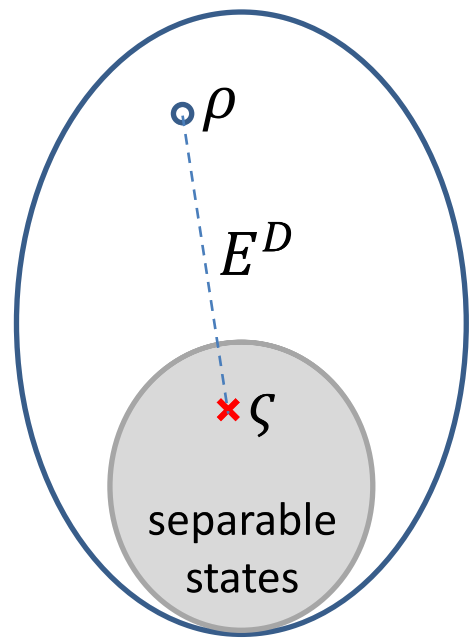


Consider a system of N qubits partitioned into $M \leq N$ subsystems

A **(fully) separable state** with respect to this partition is: $\varsigma = \sum_i p_i \tau_i^{(1)} \otimes \tau_i^{(2)} \otimes \cdots \otimes \tau_i^{(M)}$

Geometric measure of (global) multipartite entanglement: distance D from the set of (fully) separable states, where D is contractive under quantum channels, and jointly convex (e.g. trace distance, Bures distance, relative entropy…)

$$E^D(\rho) = \inf_{\varsigma \text{ separable}} D(\rho, \varsigma)$$

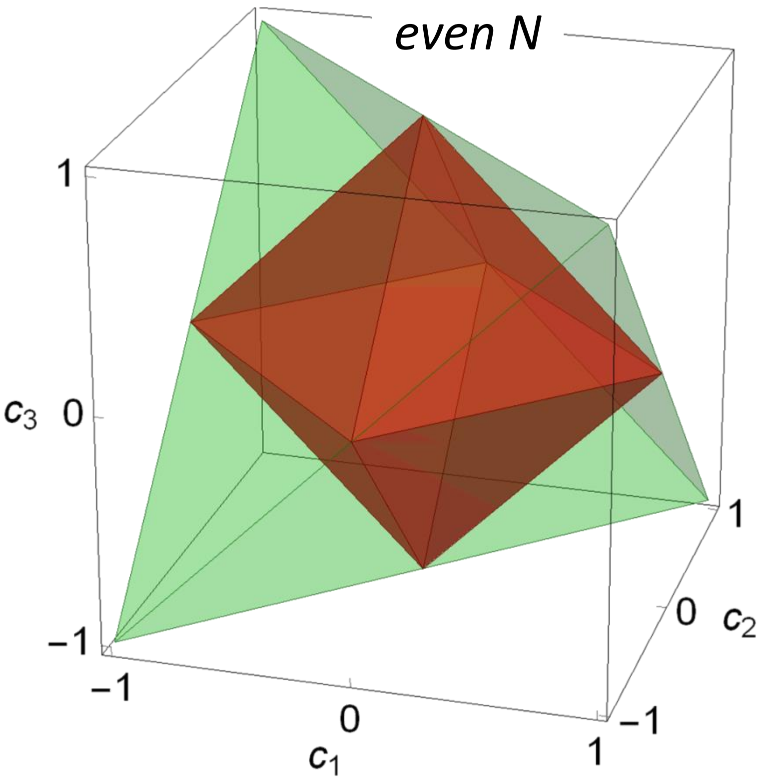


M3N states

M_N^3 states: A family of *N*-qubit mixed states with all maximally mixed marginals, extending the Bell diagonal states of 2 qubits, and defined as

$$\varpi = \frac{1}{2^N}\left(\boldsymbol{I}^{\otimes N} + \sum_{j=1}^{3} c_j \sigma_j^{\otimes N}\right)$$

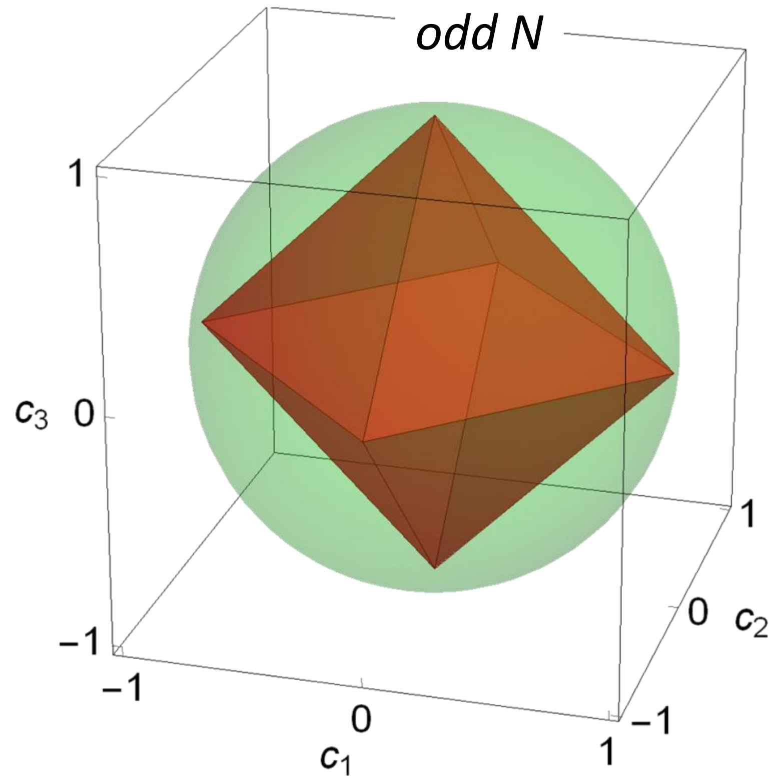
Entirely specified by **three *N*-point correlation functions** $c_j = \langle \sigma_j^{\otimes N} \rangle_{\varpi}$



Green: Set of M_N^3 states

Red: Set of separable M_N^3 states

(for partitions with two or more odd-numbered subsystems)



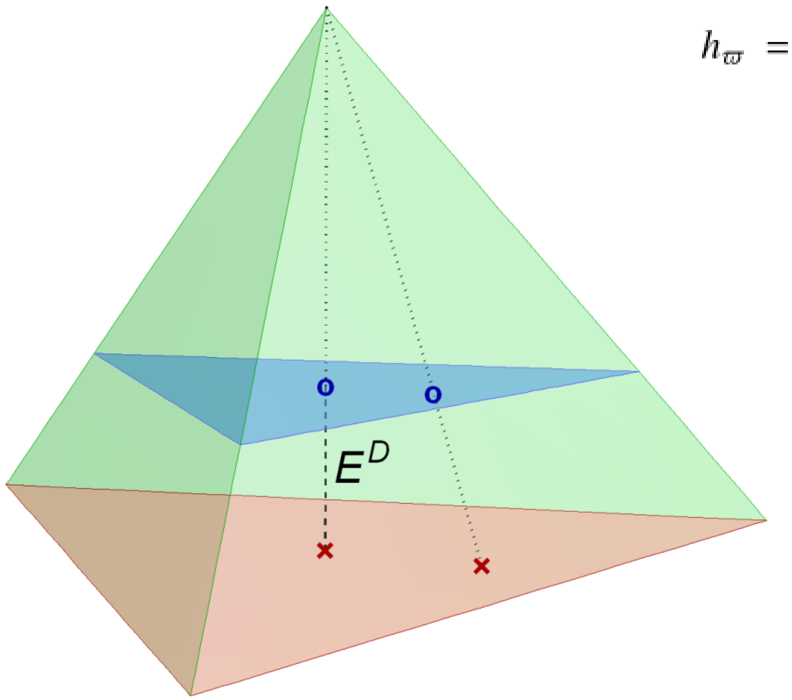
Entanglement of M3N states

M_N^3 states: We evaluate their geometric multipartite entanglement analytically.
For *even N*, it applies to any distance D. For *odd N*, it applies to the trace distance.

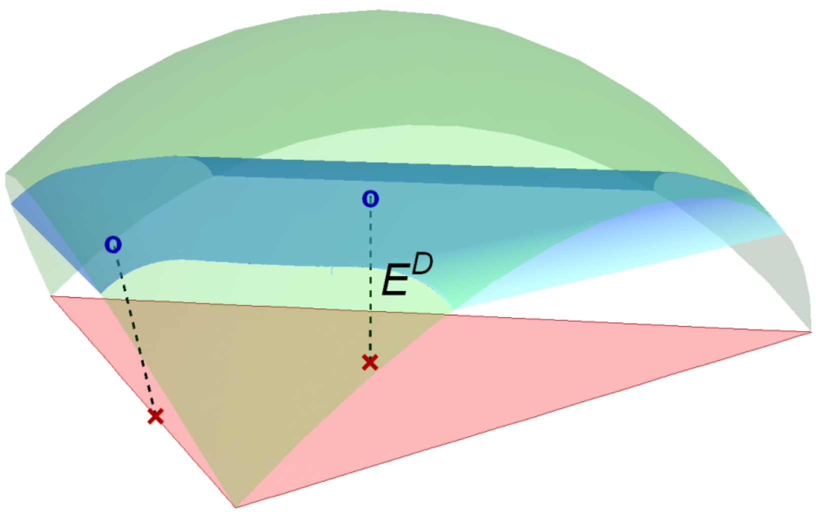
$$E^D(\varpi^{\{c_j\}}) = \begin{cases} 0\,, & h_\varpi \le 0 \text{ or the partition is trivial;} \\ f_D(h_\varpi)\,, & \text{otherwise,} \end{cases}$$

$$E^{D_{\mathrm{Tr}}}(\varpi^{\{c_j\}}) = \begin{cases} 0\,, & h_\varpi \le 0 \text{ or the partition is trivial;} \\ \frac{h_\varpi}{\sqrt{3}}\,, & 0 < h_\varpi \le 3|c_j|/2\ \forall j; \\ \min_j \frac{1}{2}\sqrt{|c_j|^2 + \frac{1}{2}(2h_\varpi - |c_j|)^2}\,, & \text{otherwise} \end{cases}$$

$$h_\varpi = \tfrac{1}{2}\left(\textstyle\sum_{j=1}^{3} |c_j| - 1\right).$$



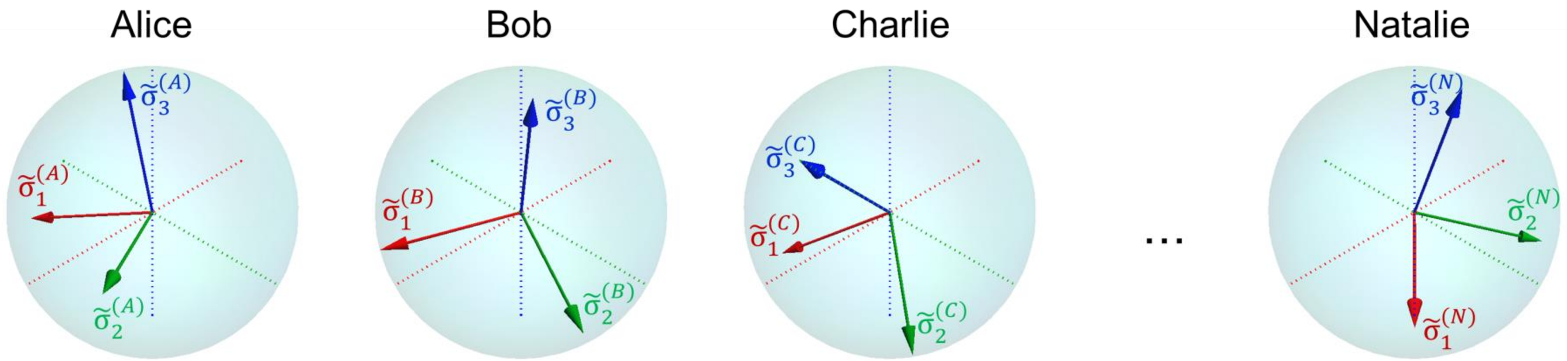
even N



odd N

Extremality of M3N states

- Every N-qubit state can be reduced to a M_N^3 state by a *LOCC operation*
- The M_N^3 states are the *least entangled* among all states with the same $\{c_j\}$
- **The geometric quantities evaluated before give *exact analytical lower bounds* to the global multipartite entanglement of arbitrary *N*-qubit states**
- These can be *accessed experimentally* just by measuring the three $\{c_j\}$
- The bounds can be *optimised* by local unitary operations prior to the LOCC, i.e. accessed by measuring in some optimal rotated Pauli basis on each qubit



Relevant examples

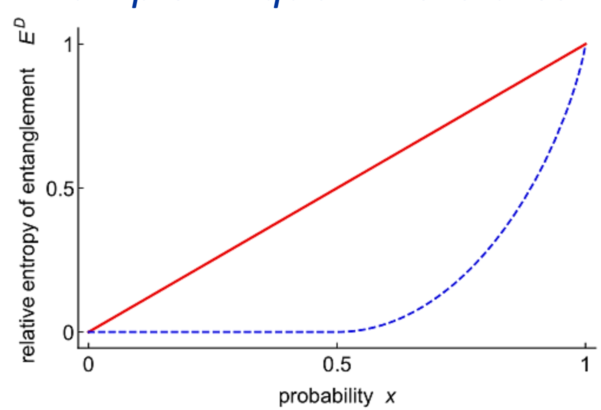
N	State	$\{\tilde{c}_1, \tilde{c}_2, \tilde{c}_3\}$	$\sum_{j=1}^{3} \lvert\tilde{c}_j\rvert$	$\{\theta, \psi, \phi\}$
$N=3$	$\varrho^{(3)}_{GHZ}(p)$	$\left\{-\sqrt{\frac{8}{27}}p, \sqrt{\frac{8}{27}}p, -\sqrt{\frac{8}{27}}p\right\}$	$2\sqrt{\frac{2}{3}}p$	$\left\{\cos^{-1}(\frac{1}{\sqrt{3}}), \frac{5\pi}{30}, \frac{\pi}{4}\right\}$
	$\varrho^{(3)}_{W}(q)$	$\left\{\frac{q}{\sqrt{3}}, -\frac{q}{\sqrt{3}}, \frac{q}{\sqrt{3}}\right\}$	$\sqrt{3}q$	$\left\{\cos^{-1}(\frac{1}{\sqrt{3}}), 0, \frac{\pi}{4}\right\}$
	$\varrho_{H}(\eta)$	$\left\{\frac{g(\eta)}{3}, -\frac{g(\eta)}{3}, \frac{g(\eta)}{3}\right\}$	$g(\eta)$	$\left\{\cos^{-1}(\frac{1}{\sqrt{3}}), 0, \frac{\pi}{4}\right\}$
$N=4$	$\varrho^{(4)}_{GHZ}(p)$	$\{p, p, p\}$	$3p$	$\{0, 0, 0\}$
	$\varrho^{(4)}_{W}(q)$	$\left\{\frac{5q}{9}, \frac{5q}{9}, \frac{5q}{9}\right\}$	$\frac{5q}{3}$	$\left\{\cos^{-1}(\frac{1}{\sqrt{3}}), 0, \frac{\pi}{4}\right\}$
	$\varrho^{(4)}_{Wei}(x)$	$\{x, x, 2x-1\}$	$2x + \lvert 2x-1\rvert$	$\{0, 0, 0\}$
	$\lvert C^{(4)}\rangle$	$\{1, 1, 1\}$	3	$\left\{0, \frac{\pi}{8}, \frac{\pi}{8}\right\}$
$N=5$	$\varrho^{(5)}_{GHZ}(p)$	$\left\{\frac{p}{\sqrt{2}}, \frac{p}{\sqrt{2}}, 0\right\}$	$\sqrt{2}p$	$\left\{0, \frac{\pi}{40}, \frac{\pi}{40}\right\}$
	$\varrho^{(5)}_{W}(q)$	$\left\{\frac{7q}{9\sqrt{3}}, -\frac{7q}{9\sqrt{3}}, \frac{7q}{9\sqrt{3}}\right\}$	$\frac{7q}{3\sqrt{3}}$	$\left\{\cos^{-1}(\frac{1}{\sqrt{3}}), 0, \frac{\pi}{4}\right\}$
	$\varrho^{(5)}_{Wei}(x)$	$\left\{\frac{x}{\sqrt{2}}, \frac{x}{\sqrt{2}}, 0\right\}$	$\sqrt{2}x$	$\left\{0, \frac{\pi}{40}, \frac{\pi}{40}\right\}$
$N=6$	$\varrho^{(6)}_{GHZ}(p)$	$\{p, -p, p\}$	$3p$	$\{0, 0, 0\}$
	$\varrho^{(6)}_{Wei}(x)$	$\{x, -x, 2x-1\}$	$2x + \lvert 2x-1\rvert$	$\{0, 0, 0\}$
$N=7$	$\varrho^{(7)}_{GHZ}(p)$	$\left\{\frac{p}{\sqrt{2}}, -\frac{p}{\sqrt{2}}, 0\right\}$	$\sqrt{2}p$	$\left\{0, \frac{\pi}{56}, \frac{\pi}{56}\right\}$
	$\varrho^{(7)}_{Wei}(x)$	$\left\{\frac{x}{\sqrt{2}}, -\frac{x}{\sqrt{2}}, 0\right\}$	$\sqrt{2}x$	$\left\{0, \frac{\pi}{56}, \frac{\pi}{56}\right\}$
$N=8$	$\varrho^{(8)}_{GHZ}(p)$	$\{p, p, p\}$	$3p$	$\{0, 0, 0\}$
	$\varrho^{(8)}_{Wei}(x)$	$\{x, x, 2x-1\}$	$2x + \lvert 2x-1\rvert$	$\{0, 0, 0\}$

The lower bound is nontrivial if $\sum_j \lvert\tilde{c}_j\rvert \geq 1$

The last column contains the optimised local settings

Useful bounds are found for all relevant states (e.g. GHZ for any *N*, W, cluster…)

Example: N-qubit Wei states



Relevant examples (experiments)

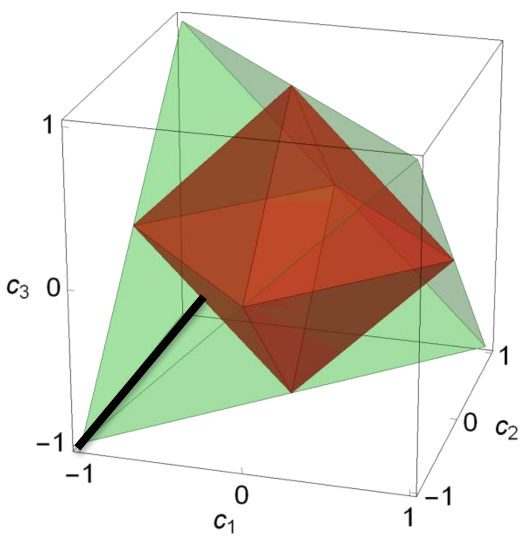
Smolin states are bound entangled states useful for information concentration and locking
Generalised Smolin states are M3N states: we quantify their entanglement exactly!

PRL **105,** 130501 (2010) PHYSICAL REVIEW LETTERS week ending 24 SEPTEMBER 2010

Experimental Bound Entanglement in a Four-Photon State

Jonathan Lavoie, Rainer Kaltenbaek, Marco Piani, and Kevin J. Resch

exp. values	$\langle \sigma_z^{\otimes 4} \rangle = 0.3966 \pm 0.0075$	$\langle \sigma_x^{\otimes 4} \rangle = 0.4005 \pm 0.0043$	$\langle \sigma_y^{\otimes 4} \rangle = 0.3621 \pm 0.0043$



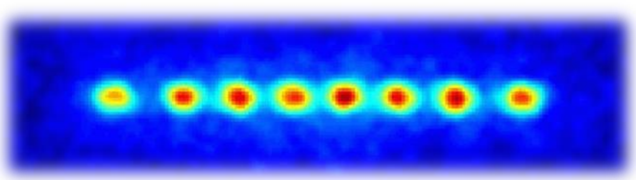
$$\Rightarrow h_{\varpi} = 0.080 \pm 0.005$$
$$\Rightarrow E^{D_{\text{Rel.Ent.}}} = 0.0046 \pm 0.0006$$

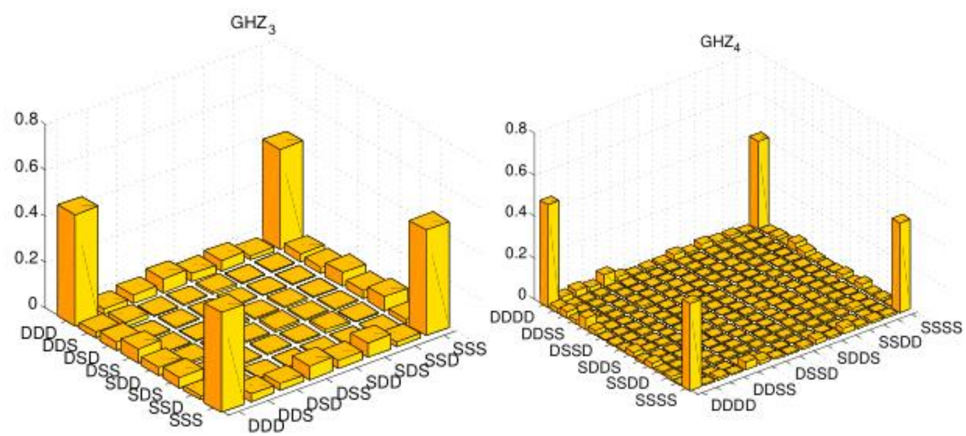
Relevant examples (experiments)

Noisy W and GHZ states of trapped ions (data by T Monz *et al* at Innsbruck, Blatt's group)

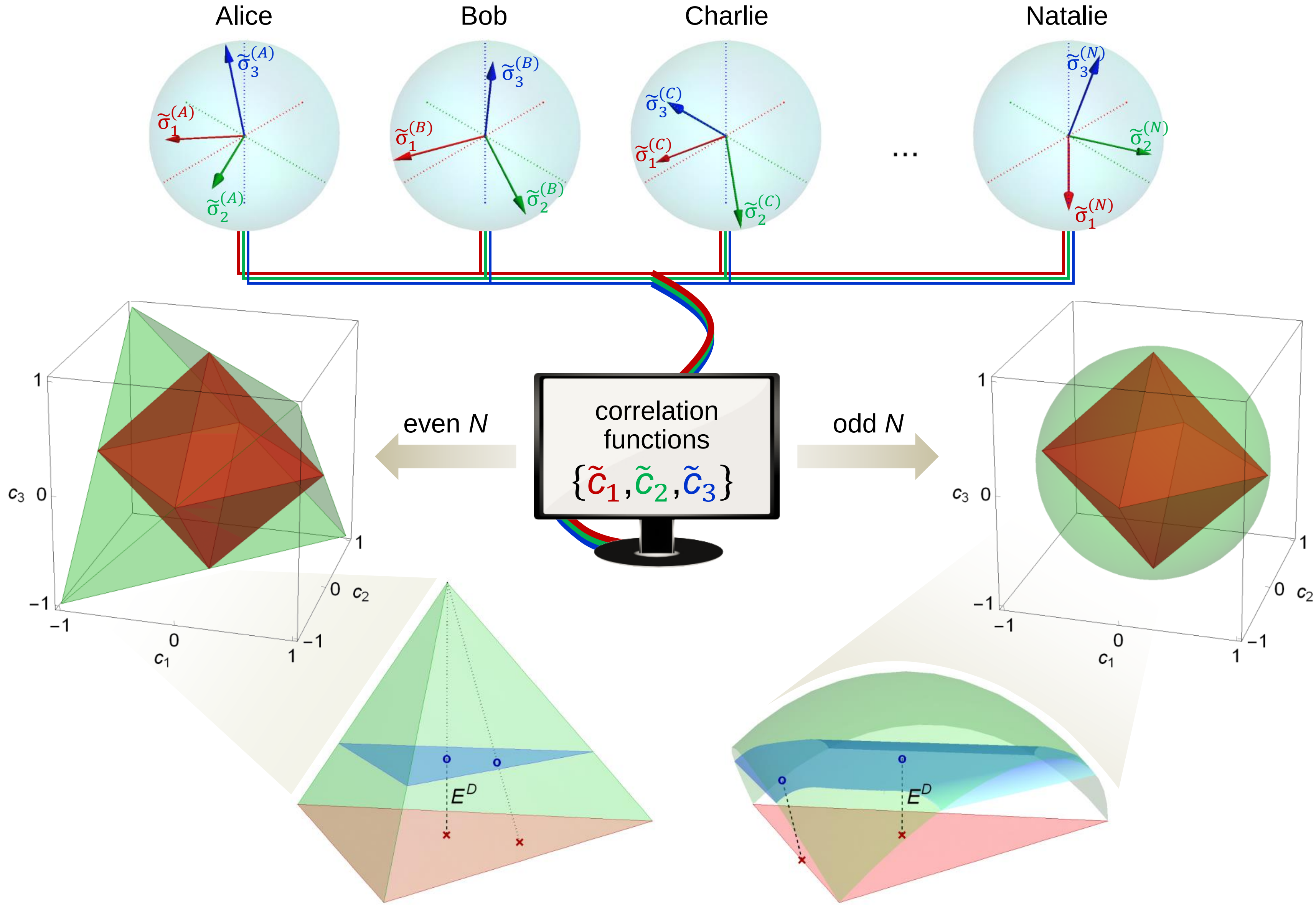
State	Fidelity	Before optimisation: $\{c_1, c_2, c_3\}$	$\sum_{j=1}^{3} \lvert c_j \rvert$	After optimisation: $\{\tilde{c}_1, \tilde{c}_2, \tilde{c}_3\}$	$\sum_{j=1}^{3} \lvert \tilde{c}_j \rvert$	$E^{D_{\mathrm{Tr}}}$
$\varrho^{(4)}_{W_A}$	19.4%	$\{-0.00469, 0.0113, -0.722\}$	0.738	$\{-0.404, 0.454, -0.378\}$	1.24	0.0589
$\varrho^{(4)}_{W_B}$	31.4%	$\{0.0174, 0.0132, -0.807\}$	0.838	$\{0.472, -0.468, -0.446\}$	1.39	0.0963
$\varrho^{(3)}_{GHZ}$	87.9%	$\{0.830, 0.235, -0.0100\}$	1.07	$\{0.474, 0.483, -0.488\}$	1.44	0.128
$\varrho^{(4)}_{GHZ}$	80.3%	$\{0.663, 0.683, 0.901\}$	2.25	$\{0.868, 0.852, 0.915\}$	2.64	0.409

We obtain nontrivial lower bounds to global entanglement even for highly mixed states

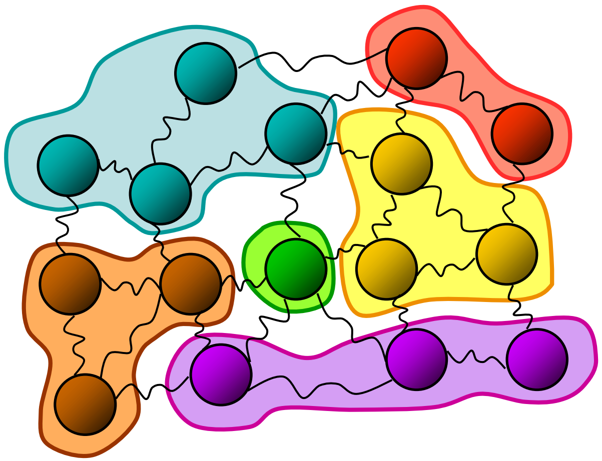




Summary of the procedure



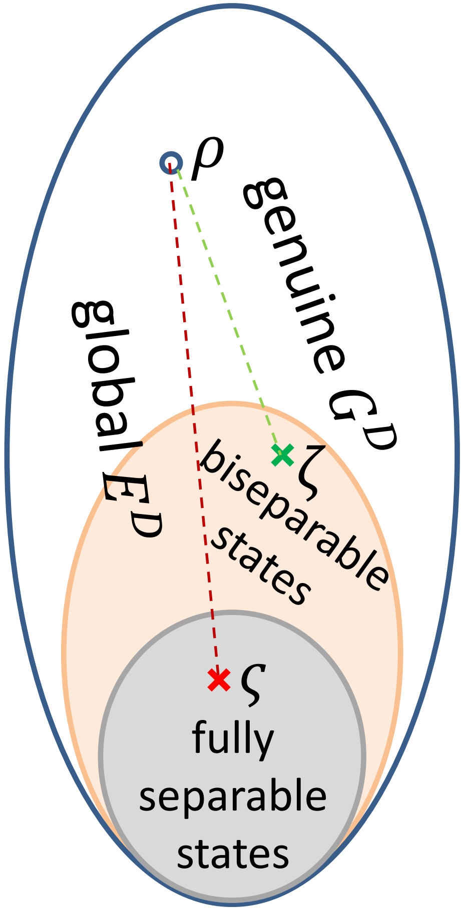
Outlook & work in progress



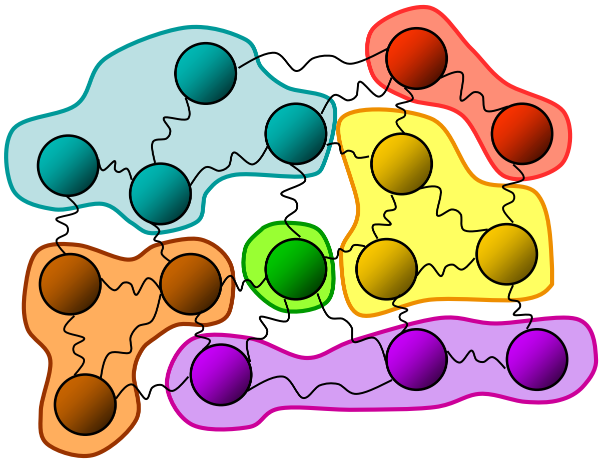
Can we quantify ***genuine multipartite entanglement*** (as opposed to global?)

Geometric measure of *genuine* multipartite entanglement: distance D from the set of *biseparable states* (i.e. mixtures of states, each separable according to some bipartition),

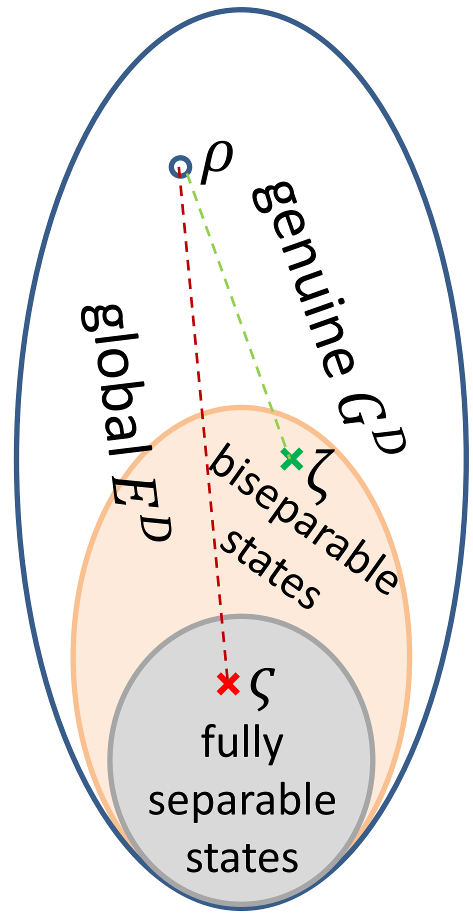
$$G^D(\rho) = \inf_{\zeta \text{ biseparable}} D(\rho, \zeta)$$



Outlook & work in progress



Can we quantify ***genuine multipartite entanglement*** (as opposed to global?)



NO with the M_N^3 states as they are always biseparable

But, we are looking for a *larger set* (depending on more than 3 correlation functions) rich enough to allow for genuine entanglement, yet still tractable analytically

TO BE CONTINUED…

Thank you

The University of Nottingham

Marco Cianciaruso, Thomas Bromley & GA, arXiv:1507.01600 (2015)

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