University of the Basque Country

ulm university universität uulm

The simplest SU(2) lattice gauge model in 1D

Emergent chirality and beyond

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Pietro Silvi

in collaboration with E. Rico, M. Dalmonte, P. Zoller, T. Calarco, and S. Montangero

Motivation:

Understanding QCD:

Long Shot → QCD is a non-solved theory, which can not be addressed perturbatively.

= *Big Open Issue*

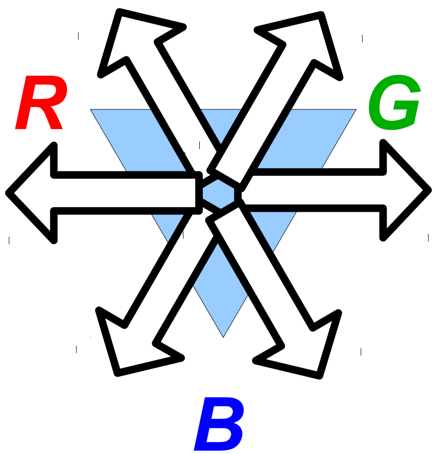
Closer Target →

- First non-abelian application of our LGT Tensor Network scheme[1,2].
- Search for conjectured phases in simpler scenarios.
- Tool for calibrating an atomic quantum simulator.

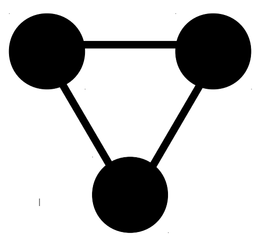
1) P. Silvi. E. Rico, T. Calarco, S. Montangero; New J. Phys. **16** 103015 (2014)
2) E. Rico, T. Pichler, M. Dalmonte, P. Zoller, S. Montangero, PRL **112**, 201601 (2014)

SU(3) and SU(2), not really the same:

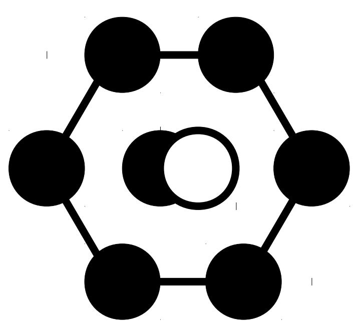
SU(3)



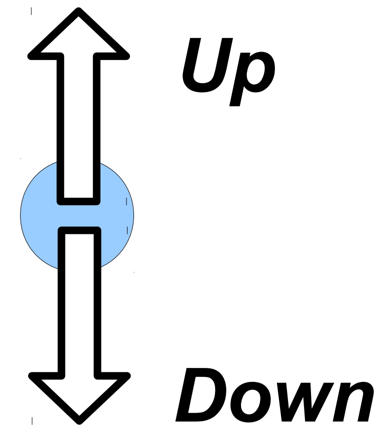
- **Quark**



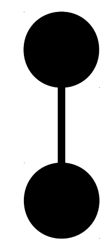
- **Gluon**



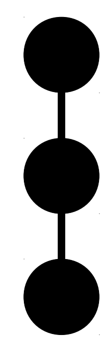
SU(2)



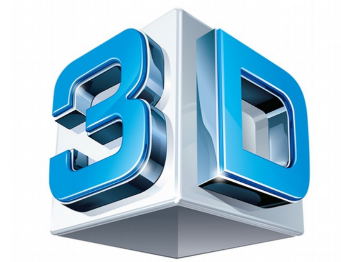
- **Spin 1/2**



- **Spin 1**



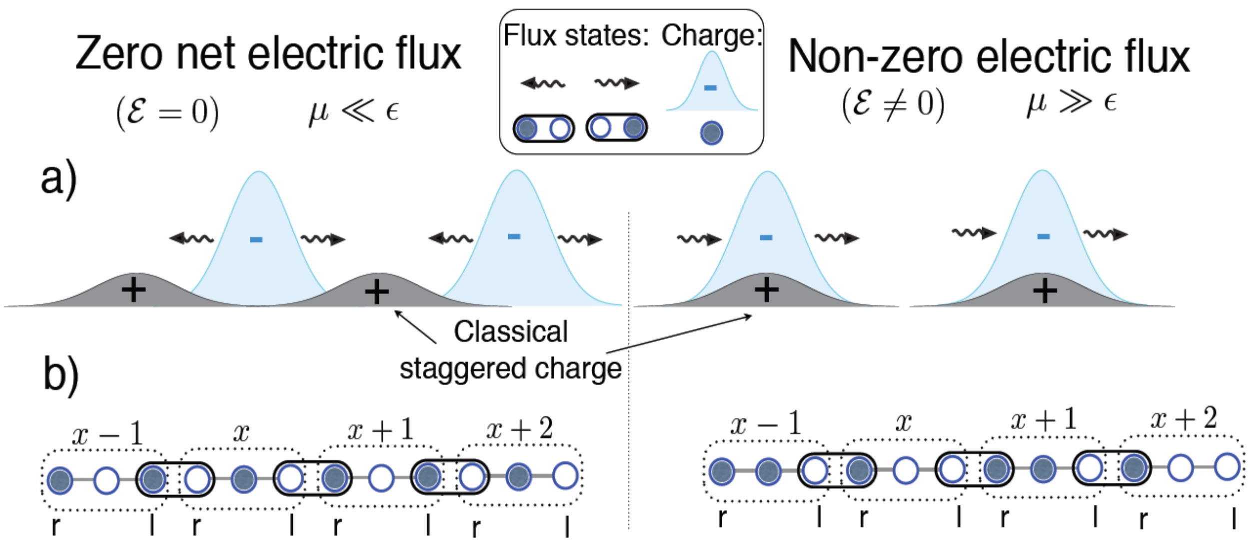
...also not the same dimensionality



→ 1D

And still we can address important questions

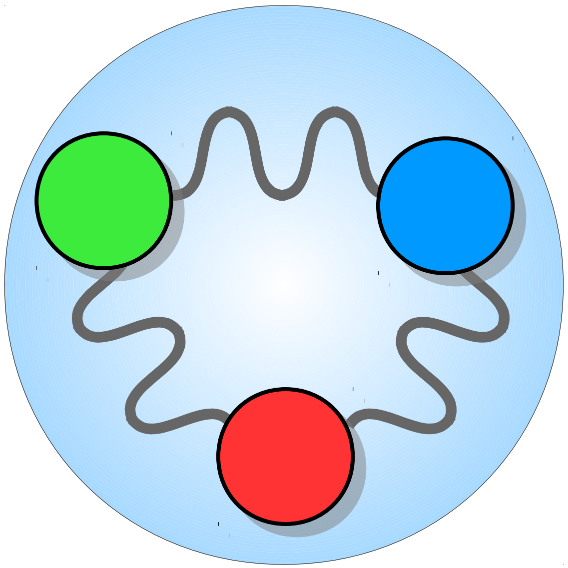
such as **Chiral symmetry breaking**



In QED^2 (abelian LGT) charge density wave, is induced by staggered charge background

For SU(2) LGT, can the chiral symmetry be broken spontaneously?

importance of **Chiral order**



Chiral symmetry breaking allows quarks to arrange in localized triplets: **Baryons**.

Basically 99% of the (stable) matter in the universe.

Let us see if we find it in the simplest SU(2) LGT in 1D.

Other reasons why SU(2) LGT are important:

3) I. Affleck, Z. Zou, T. Hsu, P. W. Anderson, PRB **38**, 745 (1988)

4) E. Dagotto, E. Fradkin, A. Moreo, PRB Rapid 38, 2926 (1988)

Simplest SU(2) lattice gauge model in 1D

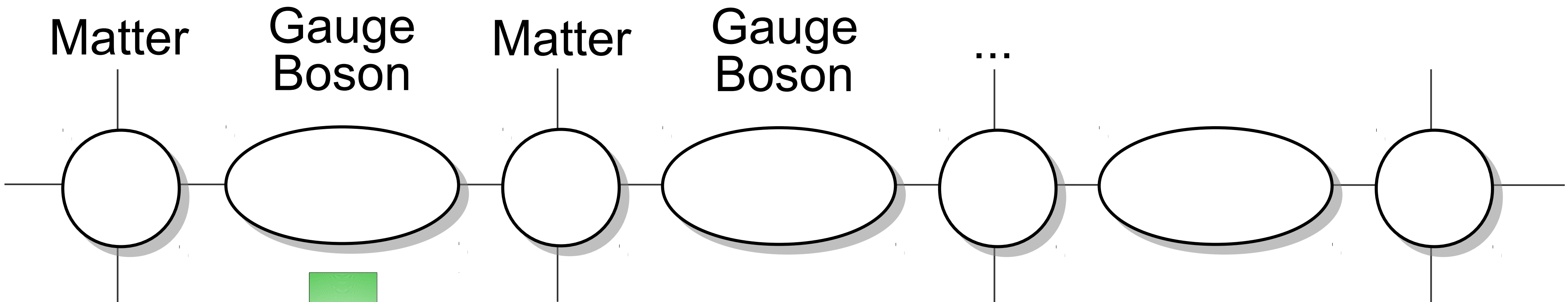
in Quantum Link formulation

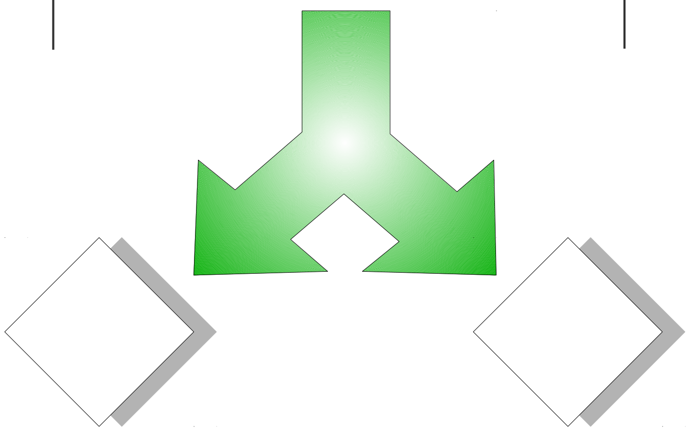
$$
\begin{aligned}
H = \quad & t \sum_j \sum_{a,b=\uparrow,\downarrow} \psi^\dagger_{j,a}\, c^{[L]}_{j,a}\, c^{[R]\dagger}_{j+1,b}\, \psi_{j+1,b} + \psi_{j,a}\, c^{[L]\dagger}_{j,a}\, c^{[R]}_{j+1,b}\, \psi^\dagger_{j+1,b} \\
& + \tilde{g} \sum_j \left(n^{[L]}_{j,\uparrow} + n^{[L]}_{j,\downarrow} - 2 n^{[L]}_{j,\uparrow} n^{[L]}_{j,\downarrow} \right) + \left(n^{[R]}_{j,\uparrow} + n^{[R]}_{j,\downarrow} - 2 n^{[R]}_{j,\uparrow} n^{[R]}_{j,\downarrow} \right) \\
& + \epsilon \sum_j c^{[L]\dagger}_{j,\uparrow}\, c^{[L]\dagger}_{j,\downarrow}\, c^{[R]}_{j+1,\downarrow}\, c^{[R]}_{j+1,\uparrow} + c^{[L]}_{j,\downarrow}\, c^{[L]}_{j,\uparrow}\, c^{[R]\dagger}_{j+1,\uparrow}\, c^{[R]\dagger}_{j+1,\downarrow} \\
& + \sum_j (\mu + (-1)^j m) \left(n^{[M]}_{j,\uparrow} + n^{[M]}_{j,\downarrow} \right) .
\end{aligned}
$$

Puzzling

This will not do. Let me give more insight.

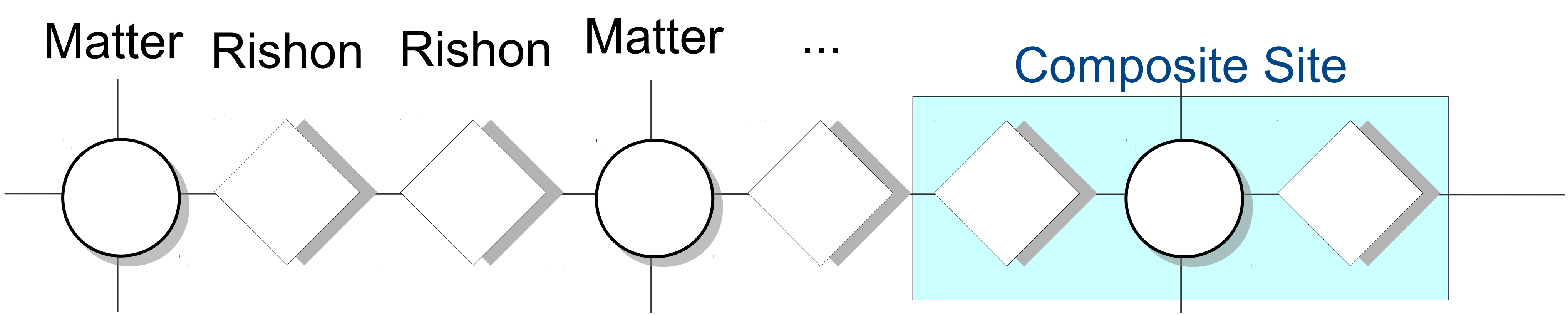
Degrees of freedom 1 - Lattice



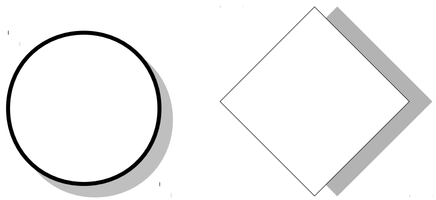


Quantum Link Prescription:

we **split** the gauge boson mode into two “Rishon” modes

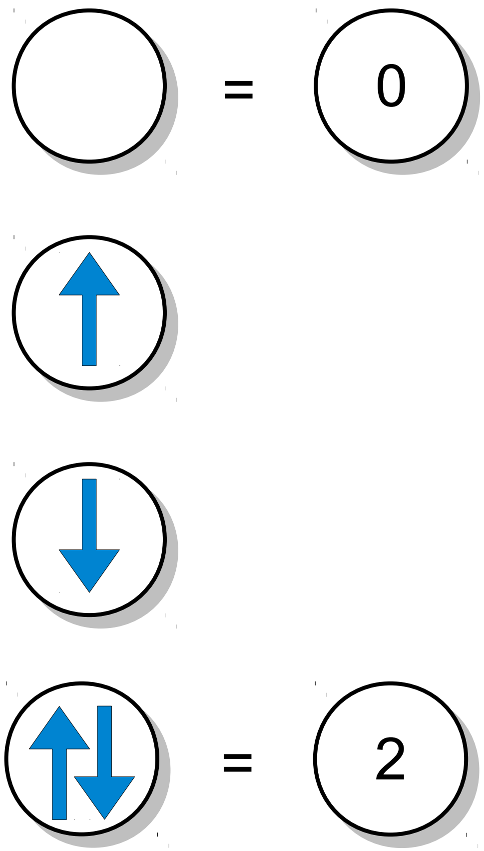


Degrees of freedom 2 – Spin ½ Fermions



Simplest model:

both matter and rishon are
spin ½ fermionic modes!



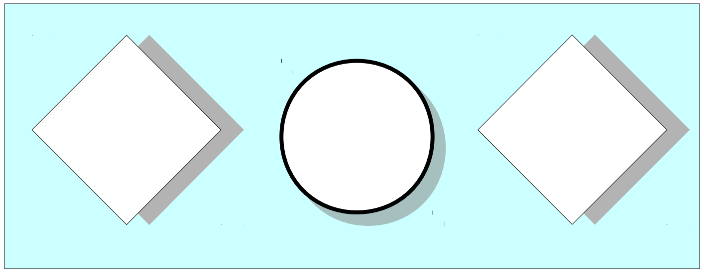
Empty

Singly Occupied – Spin up

Singly Occupied – Spin down

Doubly Occupied (total spin = zero)

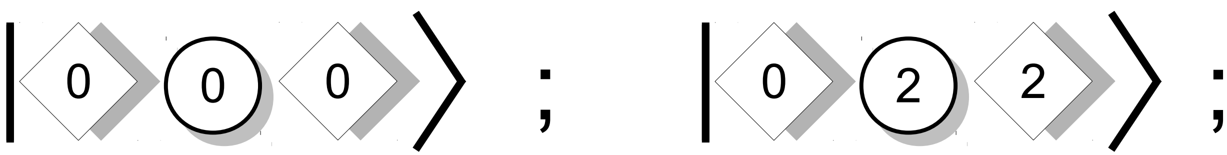
Gauge Invariance 1 – Gauss' Law



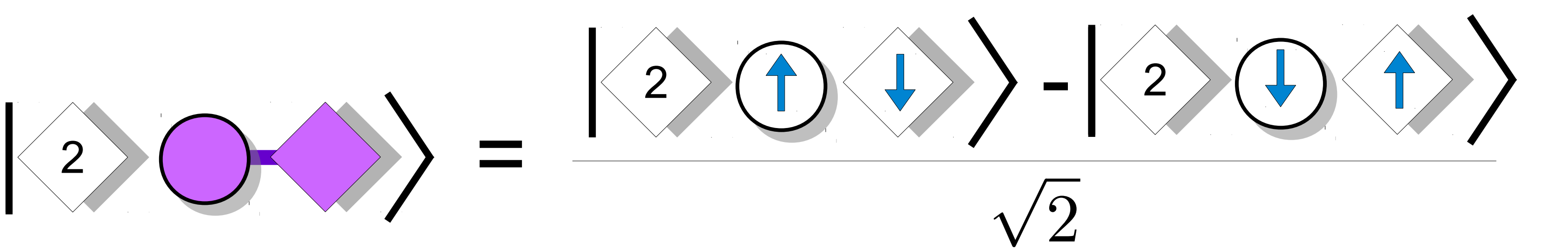
SU(2) Lattice Gauge constraint:

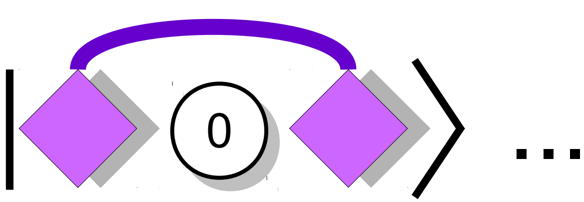
The total spin of a ***composite site*** must be ZERO !!

Examples of allowed states:



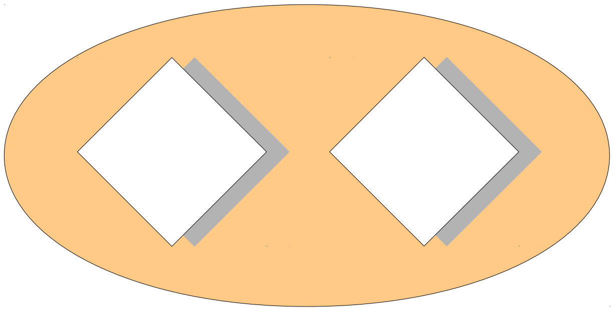
Try yourself to find them all





In total the allowed states are: **14**

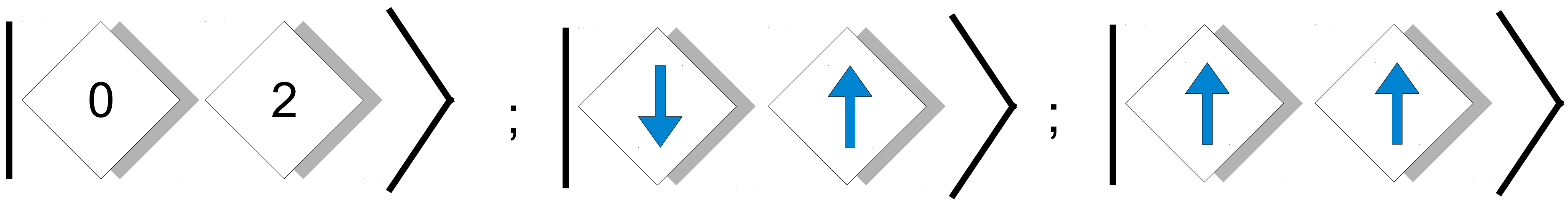
Gauge Invariance 2 – Link constraint



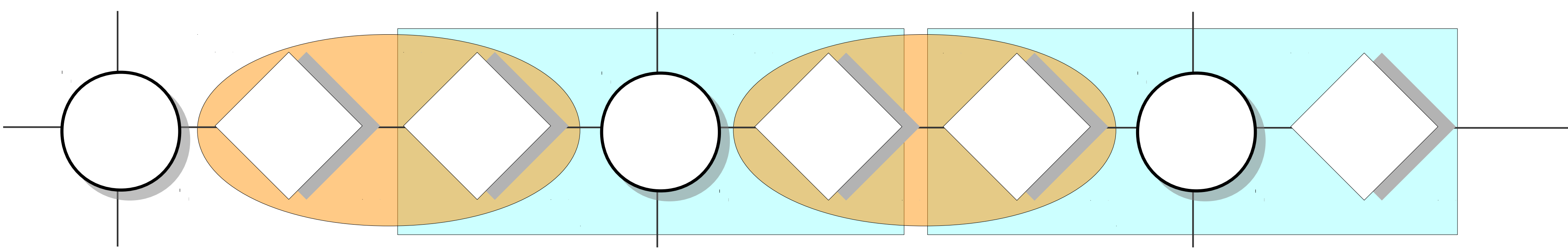
Quantum link constraint:

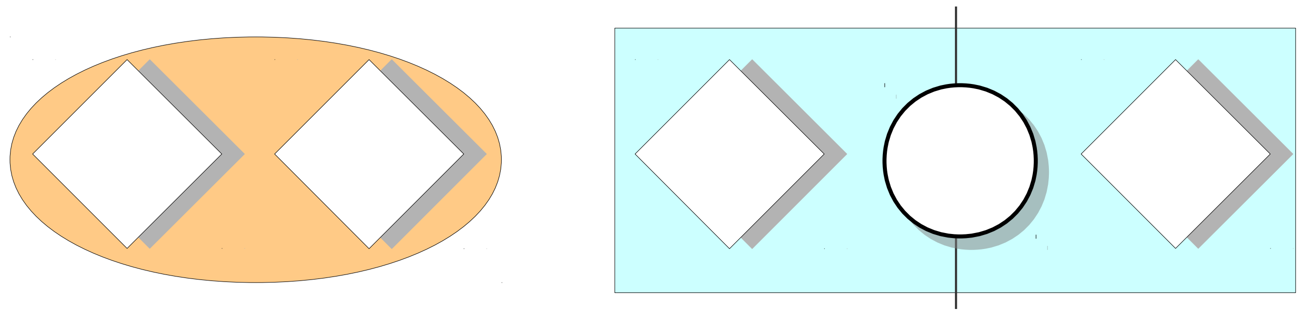
The total *number* of rishons on a link must be **2** !!

Some allowed pairs:



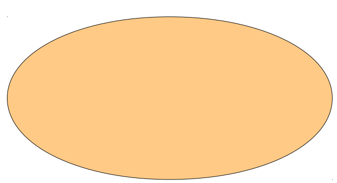
All these constraints mutually commute:





ALSO these constraints are symmetries (they commute with the Hamiltonian) and thus are preserved.

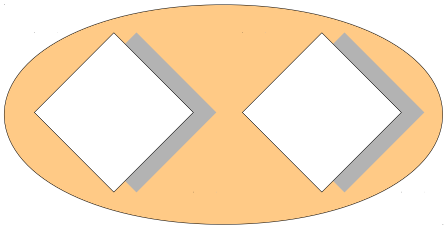
Defines an effective local computational basis, of d = **14** states.



Imposes a (diagonal) selection rule for nearest neighbor states.

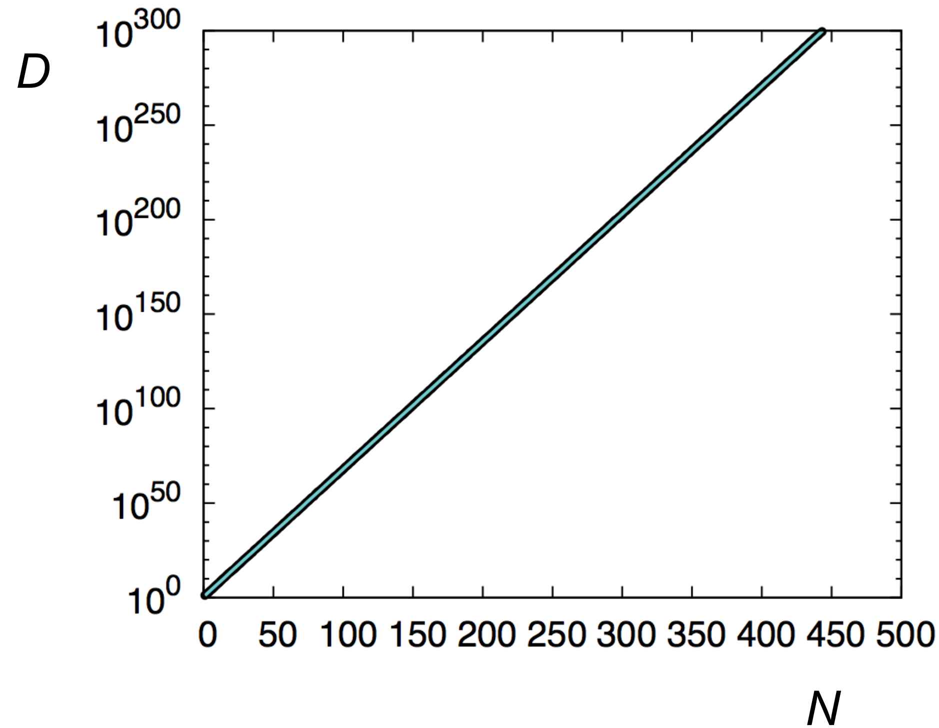
We have defined the Gauge invariant Hilbert in a comfortable way

Computational complexity of the problem



The link condition **reduces dramatically** the growth of the Hilbert space with size.

Sites	Total states
1	14
2	66 (< 196)
3	312 (<< 2744)
...	...
N	$D \sim b^N$



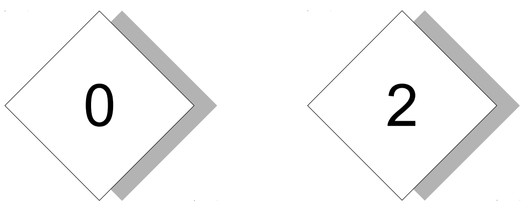
$b = 4.7320508$

Complexity: easier than a spin-2 model in 1D. This is nice!

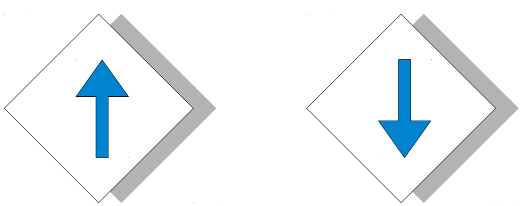
Hamiltonian 1 – Free field energy

$$H_{\text{free}} = \tilde{g} \sum_j \left(n_{j,\uparrow}^{[L]} + n_{j,\downarrow}^{[L]} - 2 n_{j,\uparrow}^{[L]} n_{j,\downarrow}^{[L]} \right) + \left(n_{j,\uparrow}^{[R]} + n_{j,\downarrow}^{[R]} - 2 n_{j,\uparrow}^{[R]} n_{j,\downarrow}^{[R]} \right)$$

- Is completely local
- Acts on the rishon modes only
- Plays a role equivalent to the energy density in electromagnetism → $\sim \dfrac{E^2 + B^2}{8\pi}$



→ These “cost” 0



→ These “cost” $\tilde{g}$

Hamiltonian 2 – Matter chemical potentials

$$H_{\text{chem}} = \sum_j (\mu + (-1)^j m) \left(n_{j,\uparrow}^{[M]} + n_{j,\downarrow}^{[M]} \right) .$$

- Is completely local
- Acts on the matter modes only

- μ is used to tune the matter filling (the Hamiltonian conserves total matter)

- m comes from the bare matter mass in the continuous theory. It becomes a *staggered* potential when casting the gauge theory on a lattice.

Our current simulations → massless $m = 0$

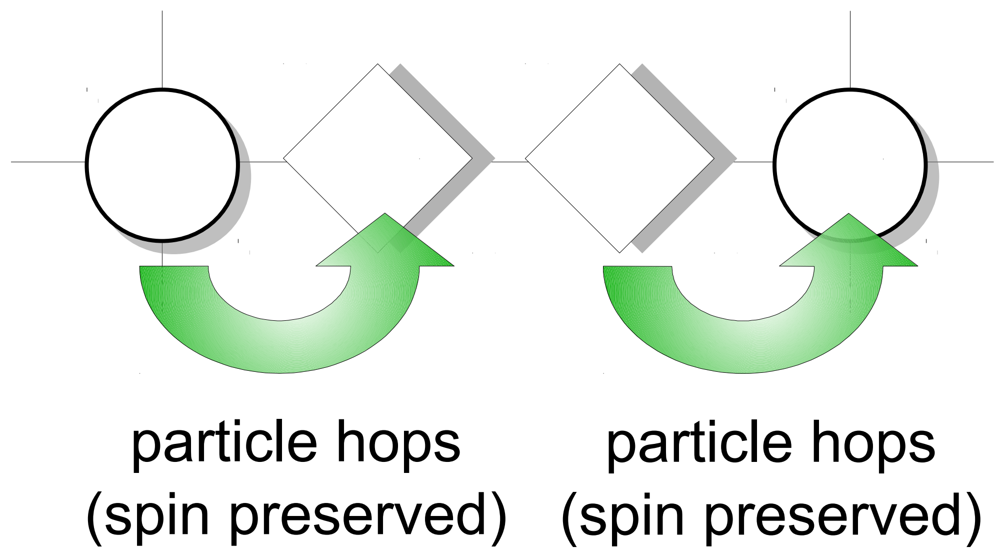
Our simulations take into account global symmetries: we can work at fixed filling → μ not needed

Hamiltonian 3 – Coupling

$$H_{\mathrm{int}} = t \sum_{j} \sum_{a,b=\uparrow,\downarrow} \psi^{\dagger}_{j,a}\, c^{[L]}_{j,a}\, c^{[R]\dagger}_{j+1,b}\, \psi_{j+1,b} + \psi_{j,a}\, c^{[L]\dagger}_{j,a}\, c^{[R]}_{j+1,b}\, \psi^{\dagger}_{j+1,b}.$$

- Defines the interaction between matter and gauge field.
- It is four-body (actually four-operators) which makes it hard.

It acts like this:



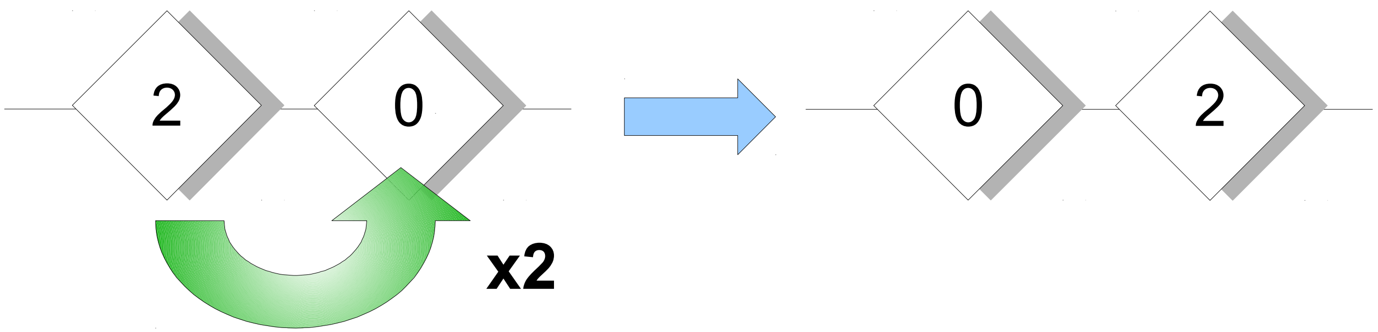
Hamiltonian 4 – U(1) Breaker

$$H_{\text{break}} = \epsilon \sum_j c^{[L]\dagger}_{j,\uparrow}\, c^{[L]\dagger}_{j,\downarrow}\, c^{[R]}_{j+1,\downarrow}\, c^{[R]}_{j+1,\uparrow} + c^{[L]}_{j,\downarrow}\, c^{[L]}_{j,\uparrow}\, c^{[R]\dagger}_{j+1,\uparrow}\, c^{[R]\dagger}_{j+1,\downarrow}.$$

- This term has *no physical origin* (it is artificial).

- Without it, the LGT would be U(2) invariant (too strong symmetry!), but we want SU(2), so we need it.

It acts like this:



rishon *pair* hops

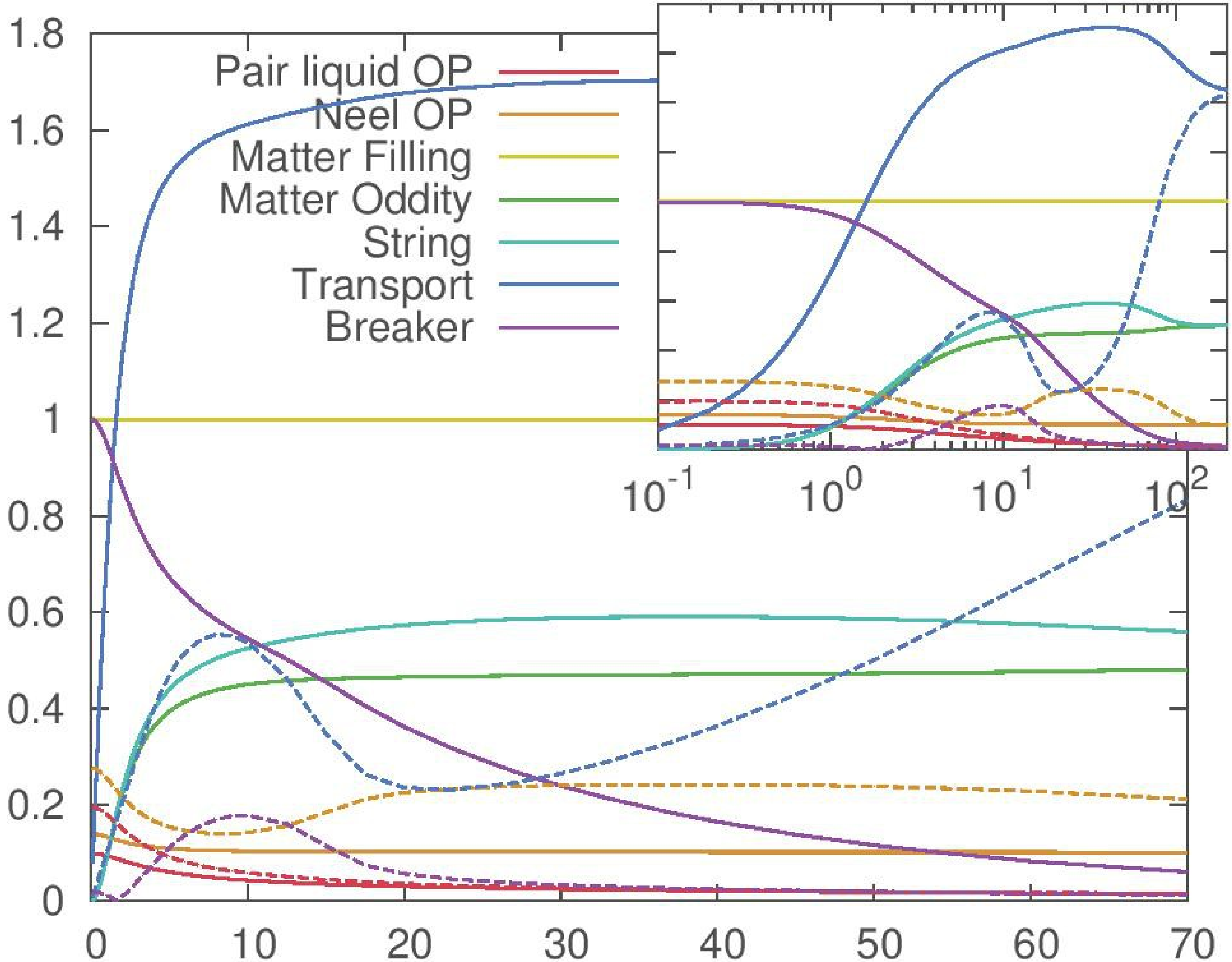
Back to the full Hamiltonian

$$H = t\sum_j \sum_{a,b=\uparrow,\downarrow} \psi^\dagger_{j,a}\, c^{[L]}_{j,a}\, c^{[R]\dagger}_{j+1,b}\, \psi_{j+1,b} + \psi_{j,a}\, c^{[L]\dagger}_{j,a}\, c^{[R]}_{j+1,b}\, \psi^\dagger_{j+1,b}$$
$$+ \tilde{g}\sum_j \left(n^{[L]}_{j,\uparrow} + n^{[L]}_{j,\downarrow} - 2n^{[L]}_{j,\uparrow}n^{[L]}_{j,\downarrow}\right) + \left(n^{[R]}_{j,\uparrow} + n^{[R]}_{j,\downarrow} - 2n^{[R]}_{j,\uparrow}n^{[R]}_{j,\downarrow}\right)$$
$$+ \epsilon\sum_j c^{[L]\dagger}_{j,\uparrow}\, c^{[L]\dagger}_{j,\downarrow}\, c^{[R]}_{j+1,\downarrow}\, c^{[R]}_{j+1,\uparrow} + c^{[L]}_{j,\downarrow}\, c^{[L]}_{j,\uparrow}\, c^{[R]\dagger}_{j+1,\uparrow}\, c^{[R]\dagger}_{j+1,\downarrow}.$$

Now we understand it. Time to solve it.

Characterize its complete phase diagram

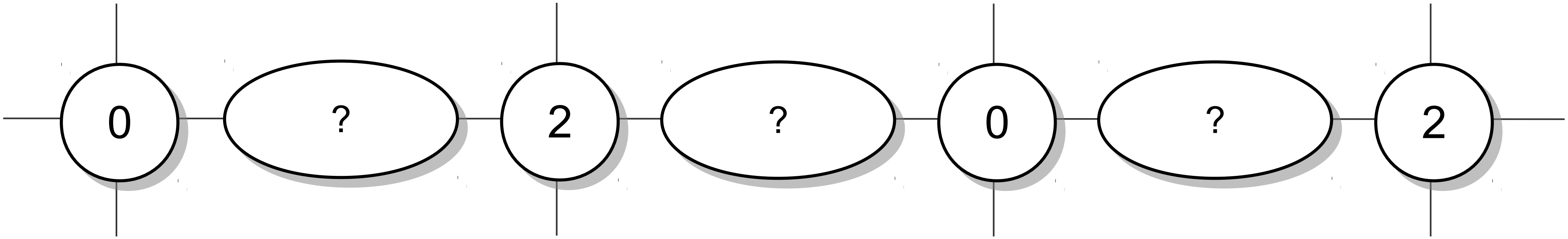
Lots of quantities to investigate: effort-taking phase detection.



FOCUS: Can we find a phase with ***chiral order***?

YES!

Chiral-Nèel Order Parameter

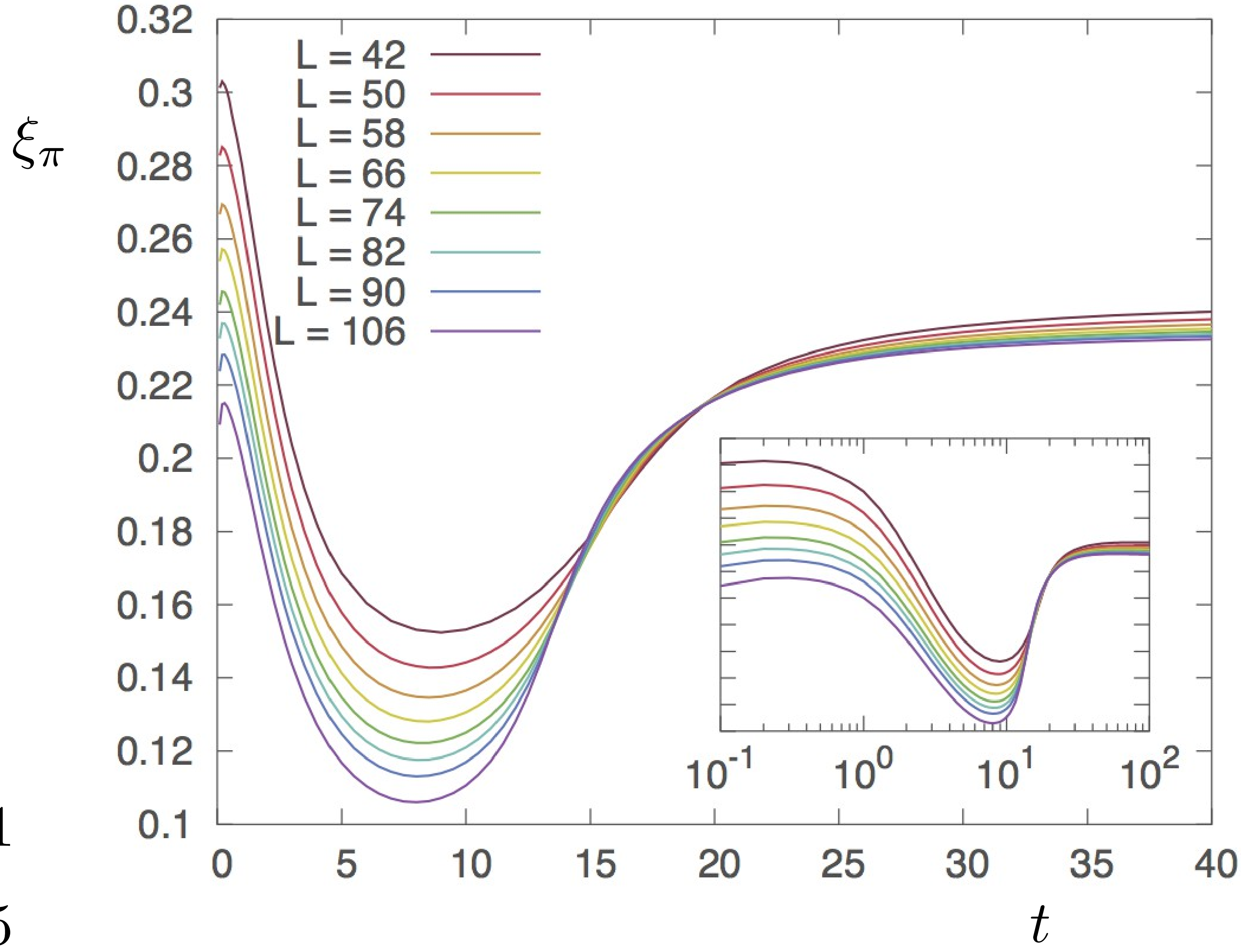


The state is Chiral-Nèel ordered when matter condenses in a crystal of Mesons (charge-density wave, or ising antiferromagnet of pairs).

The symmetry being broken is the particle hole (or spatial inversion). The order parameter is calculated via structure factor:

$$\xi_\pi = \sqrt{\frac{1}{L(L-1)} \left| \sum_{j \neq j'} (-1)^{(j-j')} \langle (n_{j,\uparrow}^{[M]} + n_{j,\downarrow}^{[M]} - 1)(n_{j',\uparrow}^{[M]} + n_{j',\downarrow}^{[M]} - 1) \rangle \right|},$$

Look for a regime where it converges to a finite value at TD-limit.



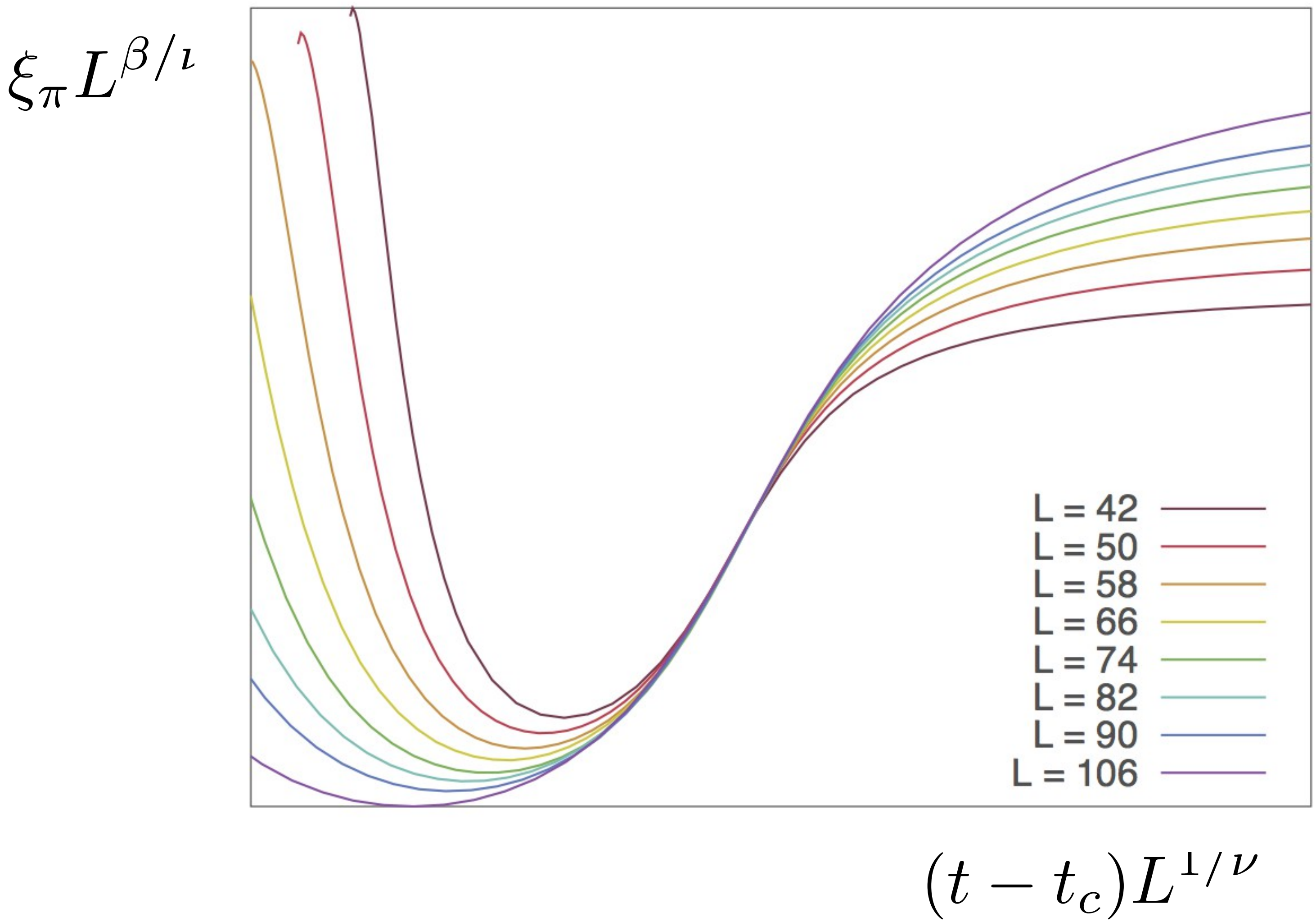
$\tilde{g} = 1$

$\epsilon = 5$

… and then perform a finite-size scaling study.

$$\tilde{g} = 1$$

$$\epsilon = 5$$



$$\beta = 1/4$$

$$\nu = 1$$

- Chiral order is found at $t > t_c$ = 11.5 +/- 0.2.
- The critical exponents suggest XXZ model-like transition

Beyond the chiral phase

Can we find a phase which is XXZ-gapless like?

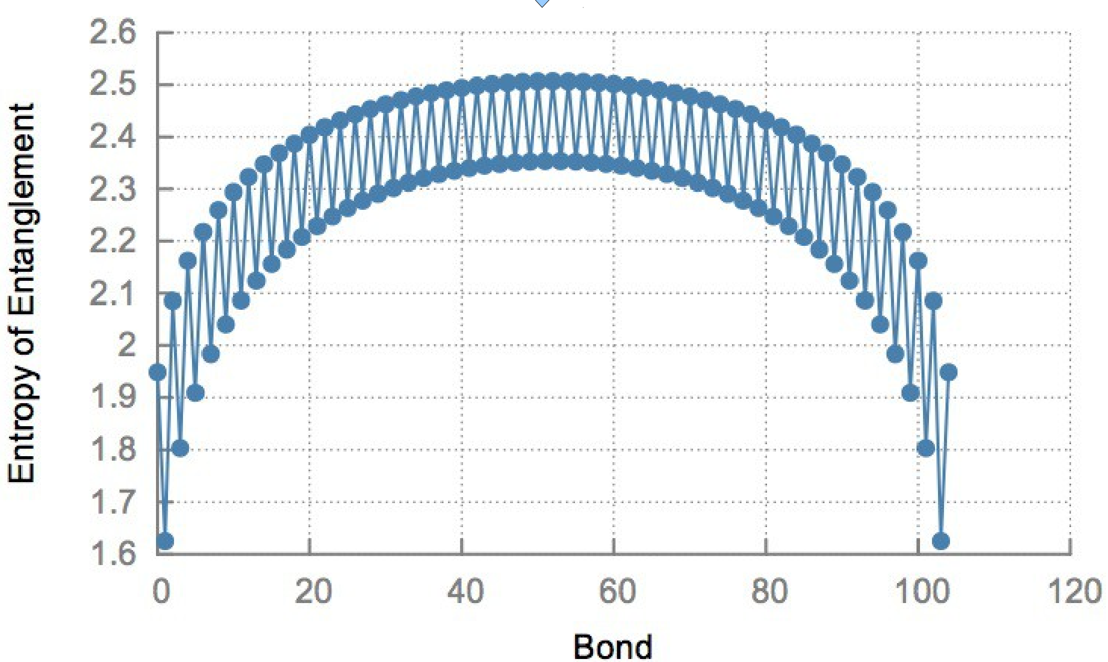
• Suggested by the critical exponents we found.

• Second order (degenerate) perturbation theory in small *t* provides exactly the XXZ model.

• The entanglement entropy has a critical profile $c \simeq 1$

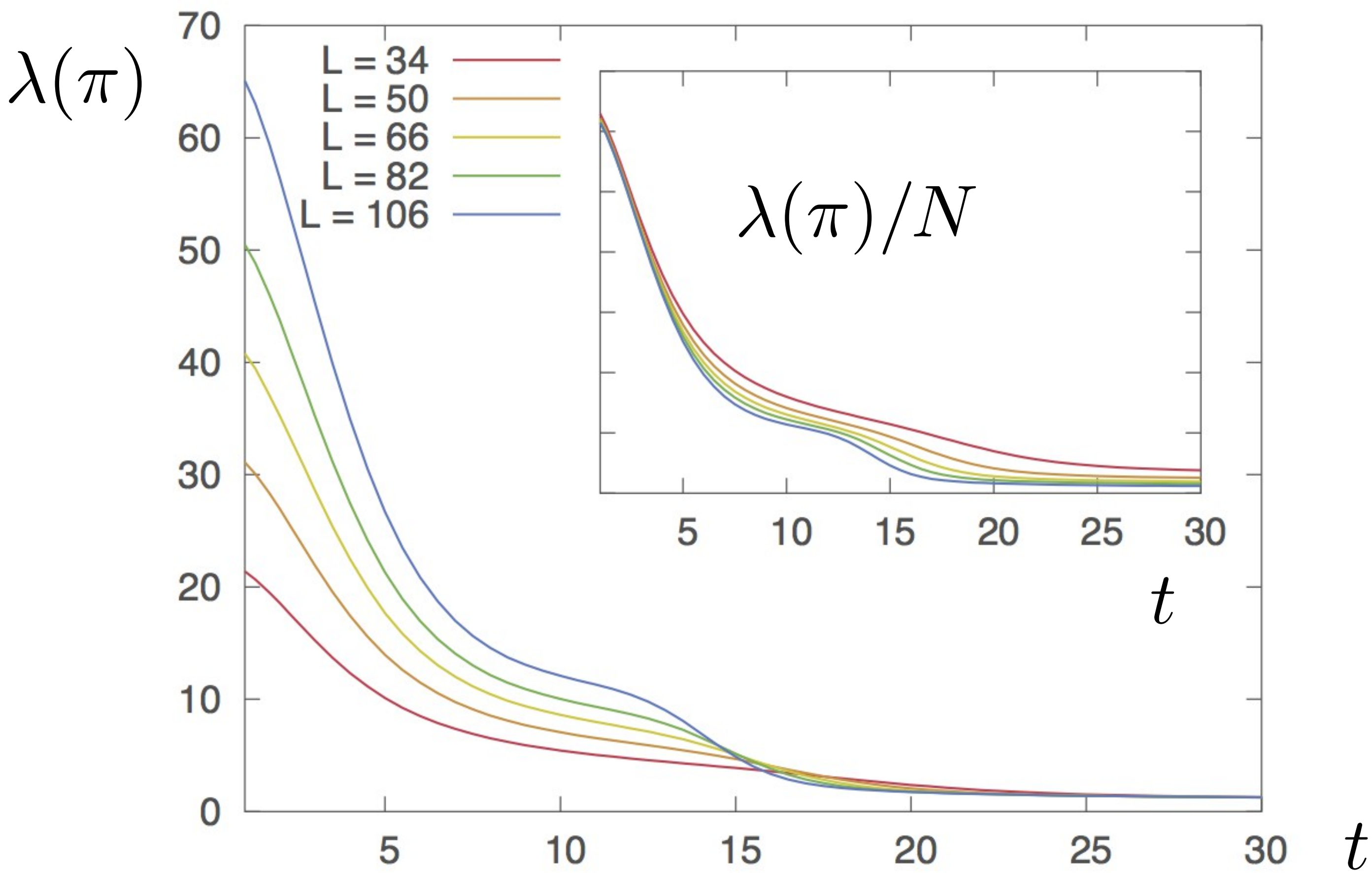
Meson superfluid
quasi-long range
order parameter

$$\langle \psi_{j,\uparrow} \psi_{j,\downarrow} \rangle$$



Investigate divergence of the correlation length

$$\lambda(k) = \frac{\sum_\ell e^{ik\ell}\ell^2 C_\ell}{\sum_\ell e^{ik\ell} C_\ell} \quad \text{where} \quad C_\ell = \langle \psi^\dagger_{j,\downarrow}\psi^\dagger_{j,\uparrow}\psi_{j,\uparrow}\psi_{j,\downarrow}\rangle$$

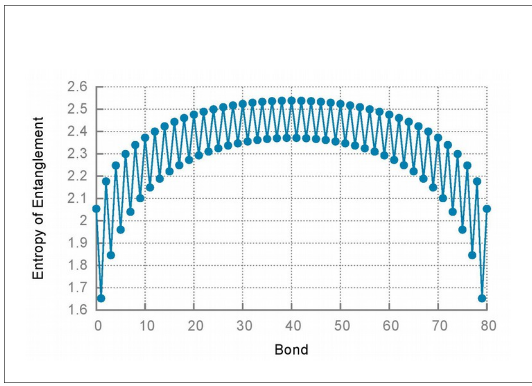


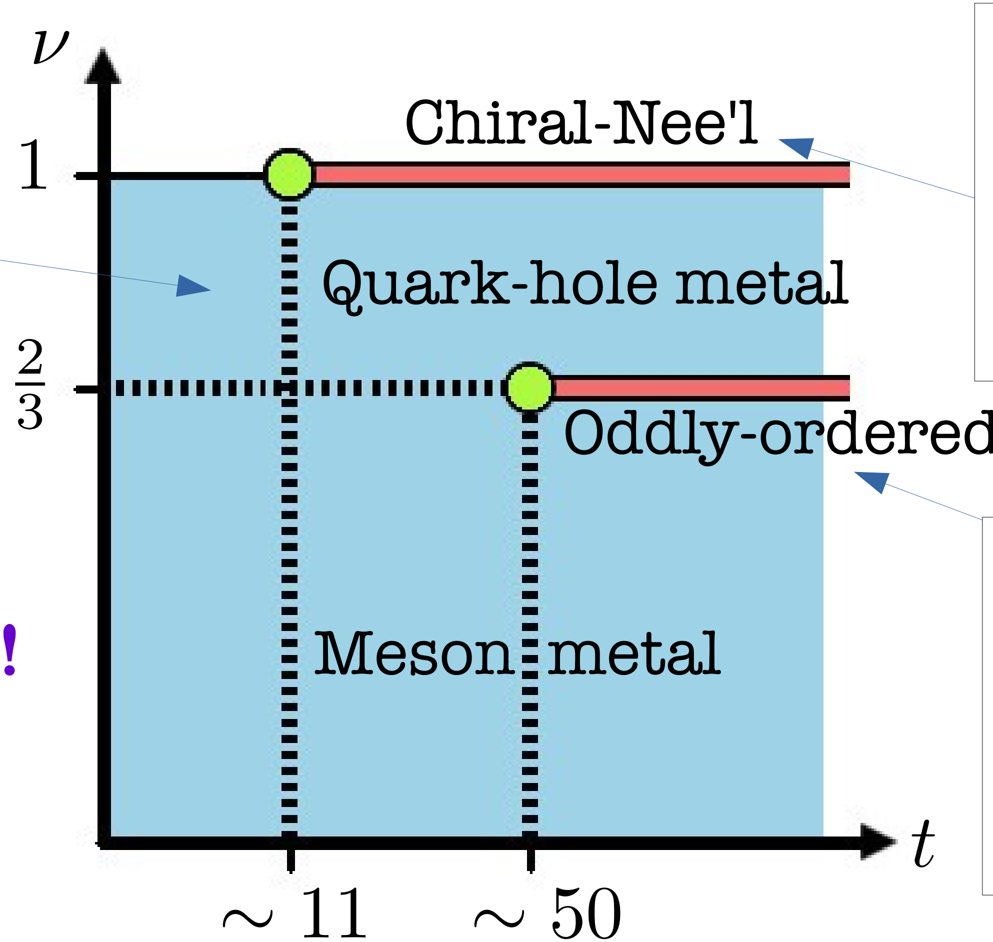
Luttinger Metal phase found!

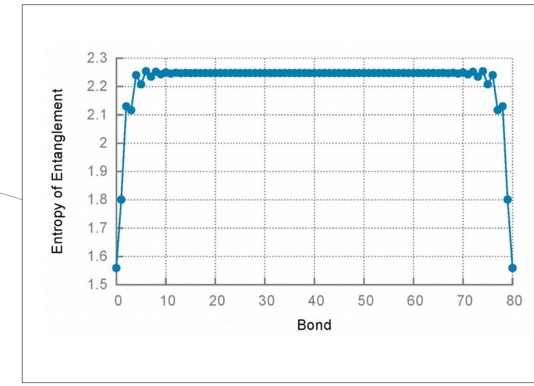
What now?

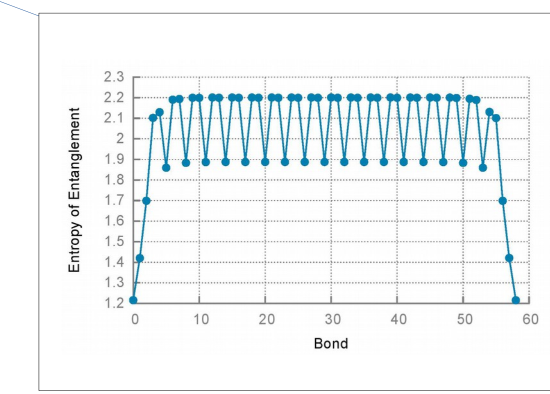
$\tilde{g} = 1 \qquad \epsilon = 5$

Capturing the phase diagram (here in t and the filling $\nu = N/L$)









Rich Physics !!

Plans for the future

- Out-of-equilibrium dynamics, open dynamics, larger LG groups ...