Quantum Cellular Automata, Quantum Fields and Deformed Special Relativity

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in collaboration with:

QUIT group in Pavia

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Outline

Motivation

QCA (QW) for the free fields evolution

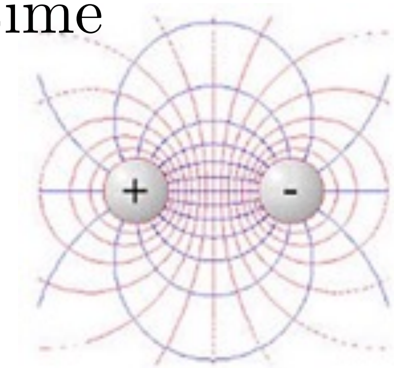
The fate of Lorentz covariance: from QCA to deformed relativity models

Final remarks and (many) open problems

Quantum Field Theory

Classical Field Theory

Spacetime



$$S = \frac{1}{2}\int_M \mathrm{Tr}[F \wedge \star F]$$

Symmetries

$+$

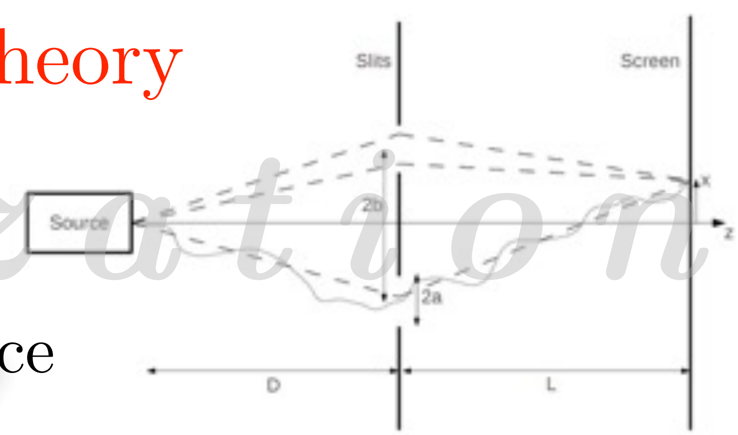
Quantum Theory

$$[\psi_a(x), \psi_b^\dagger(y)] = \delta(x-y)\delta_{ab}$$

Quantization

Operators

Fock space



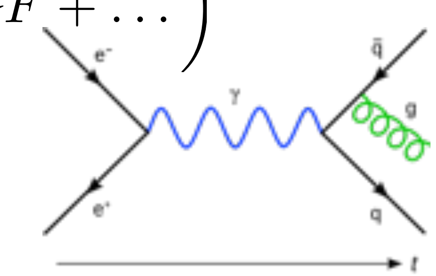
$=$

Quantum Field Theory

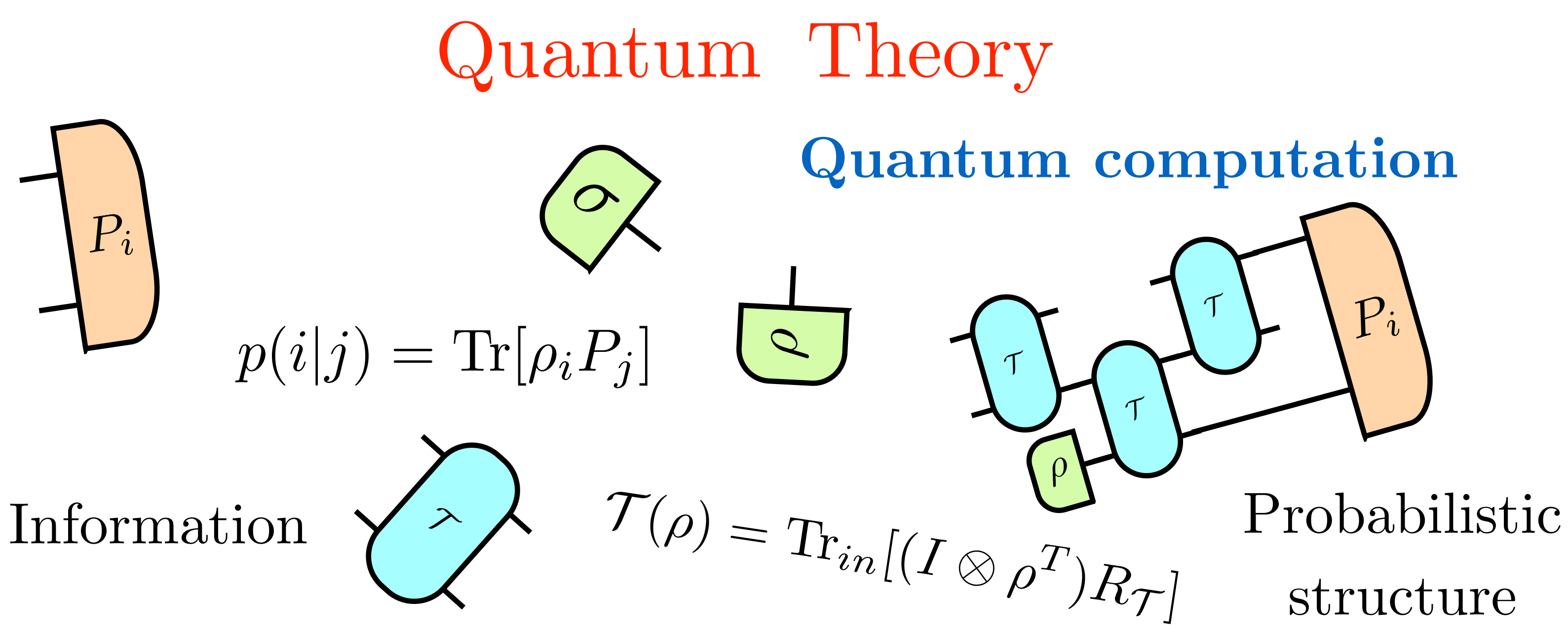
$$Z = \int \mathrm{d}\mu \, \exp\left(-\frac{i}{2}\int_M F \wedge \star F + \dots\right)$$

Anomalies

Divergences

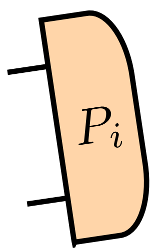


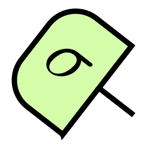
Quantum Field Theory



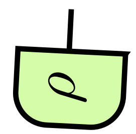
Quantum Field Theory

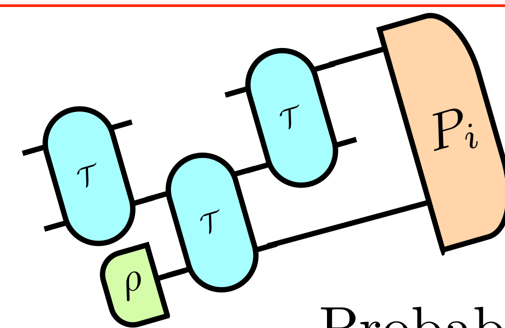
Quantum Theory





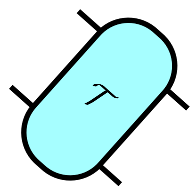
Quantum computation





$p(i|j) = \mathrm{Tr}[\rho_i P_j]$

Information



$\mathcal{T}(\rho) = \mathrm{Tr}_{in}[(I \otimes \rho^T) R_{\mathcal{T}}]$

Probabilistic structure

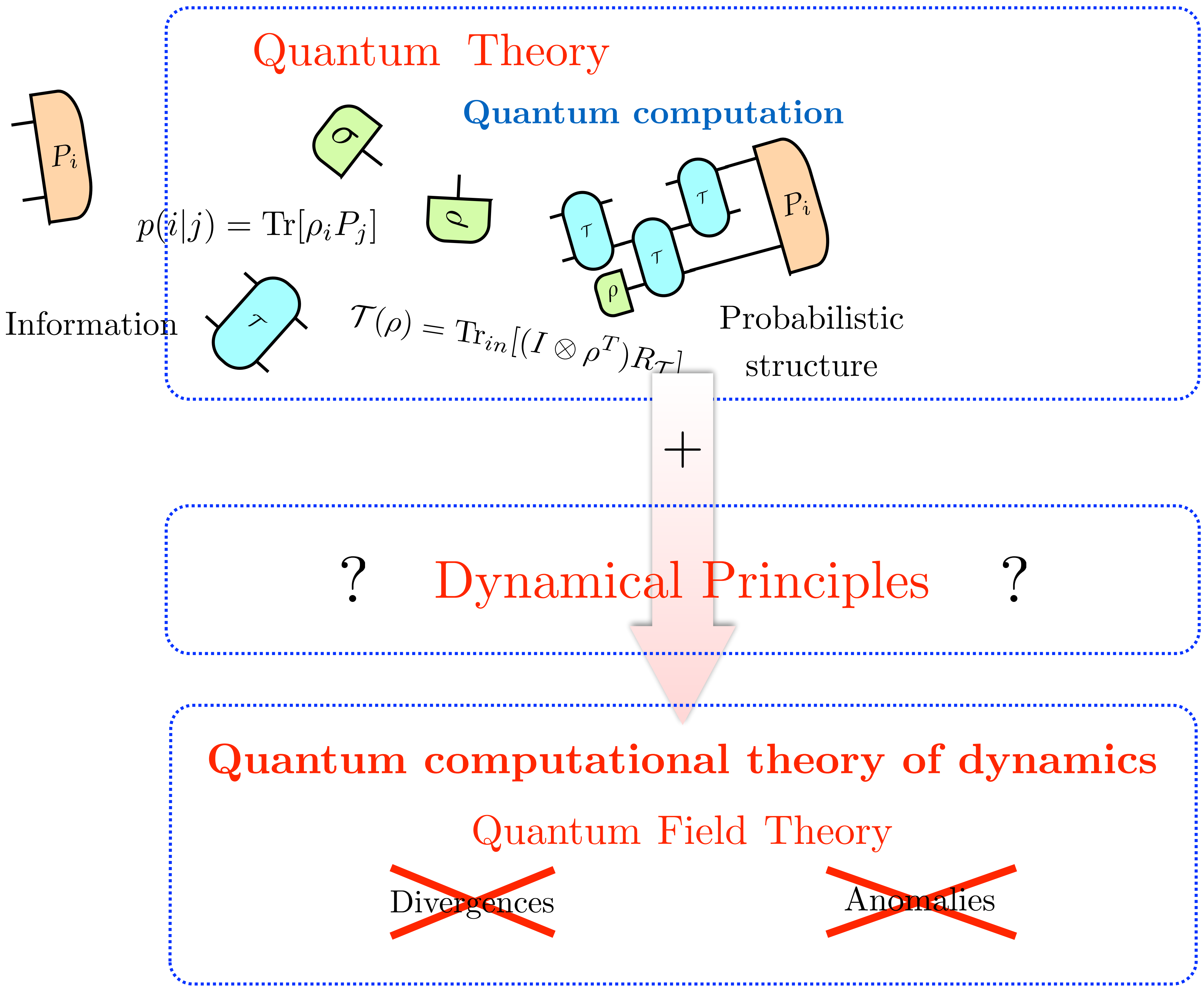
Simulating Physics with Computers

R. P. Feynman, Int. J. Theo. Phys. 21, 467 (1982)

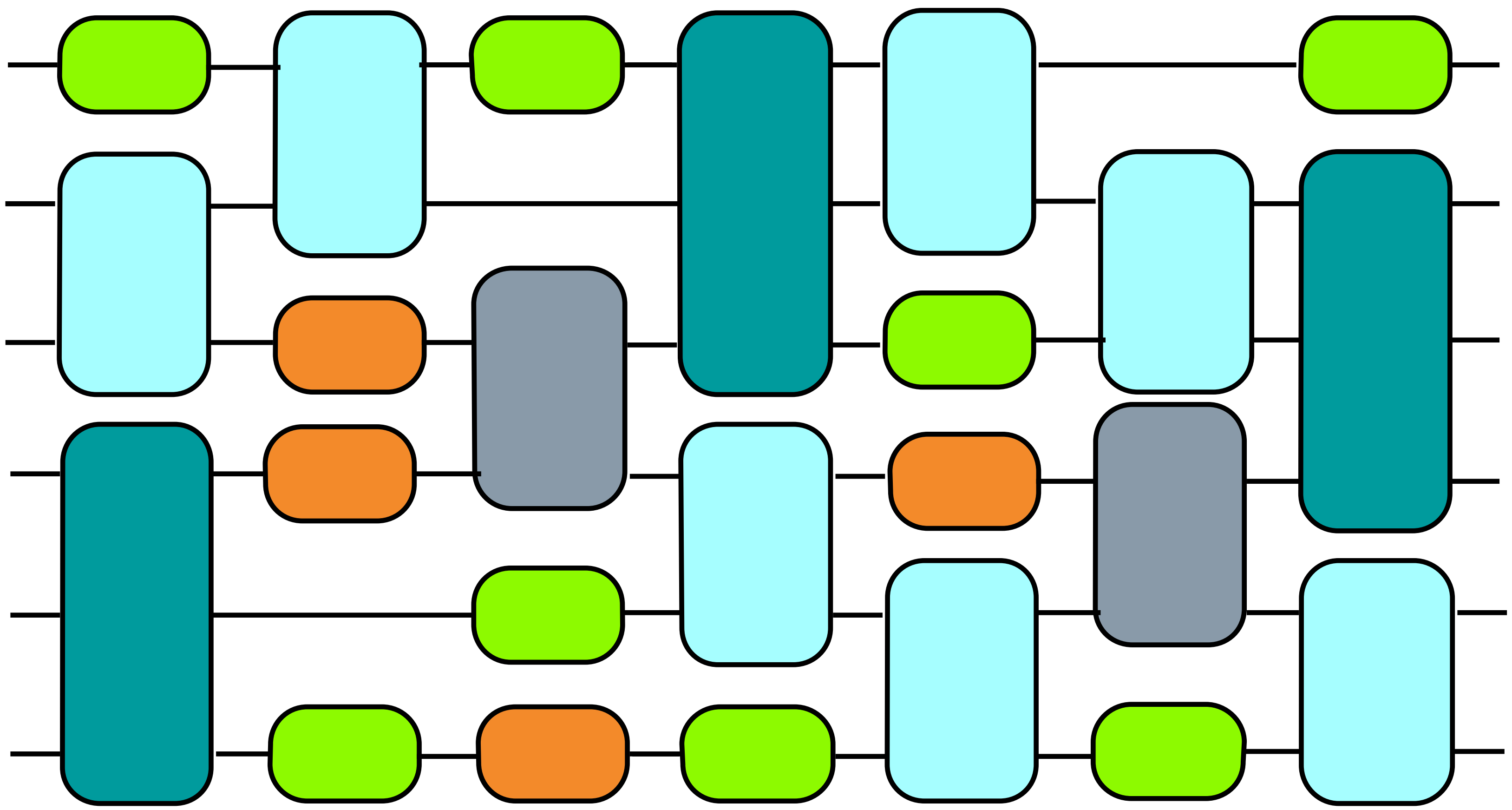
Can a Quantum Computer **exactly** simulate physical systems?

Replace physical laws with quantum algorithms

Quantum Field Theory



What kind of computer?



Quantum Circuit

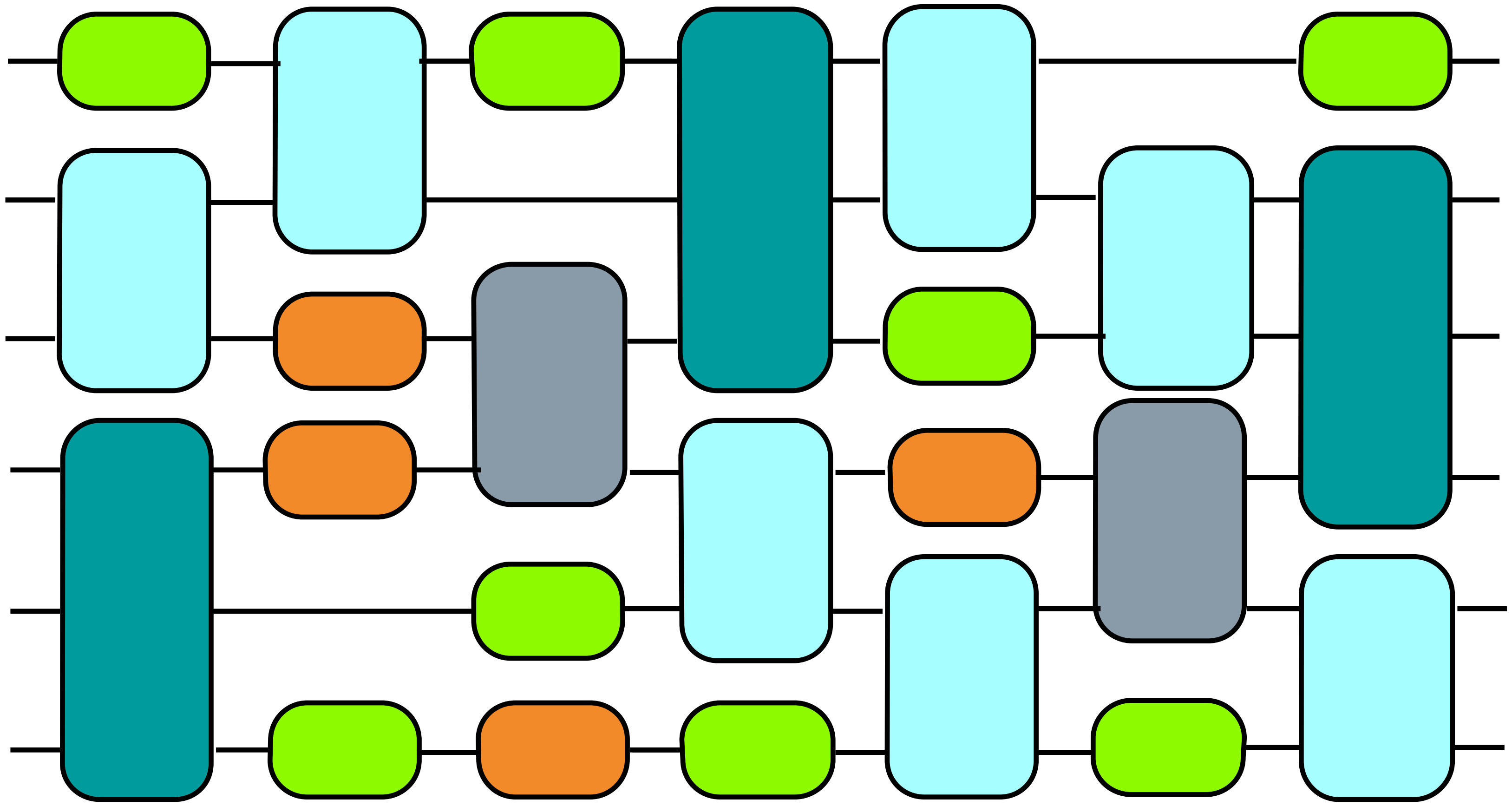
"[...] everything that happens in a finite volume of space and time would have to be exactly analyzable with a finite numbers of logical operations" R. Feynman

Each system interacts with a finite number of neighbors: **locality**

Reversible Quantum Computation: **unitary evolution**

Homogeneity, isotropy, ...

What kind of computer?



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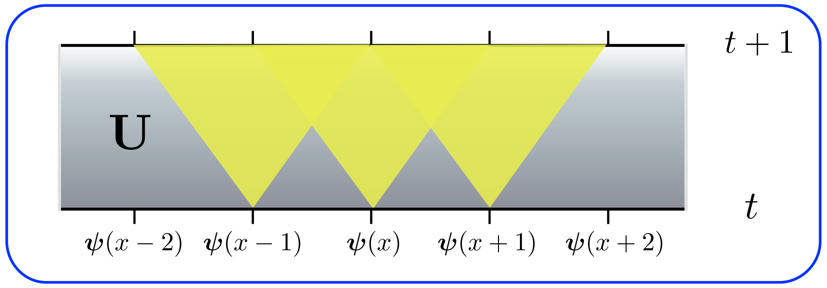

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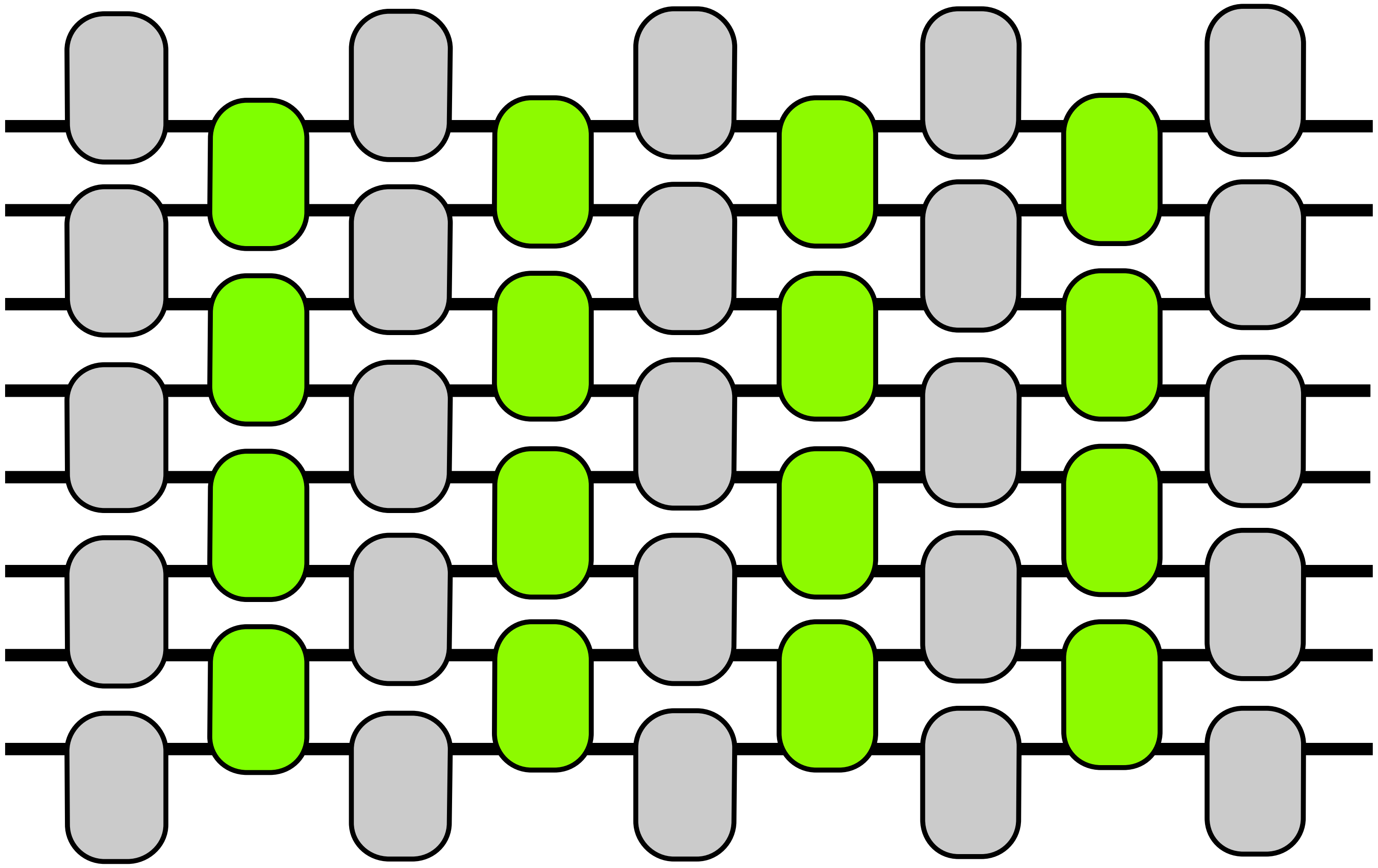

Quantum Circuit

↓

Quantum Cellular Automaton

B. Schumacher, R.F. Werner
e-print arXiv:0405174.

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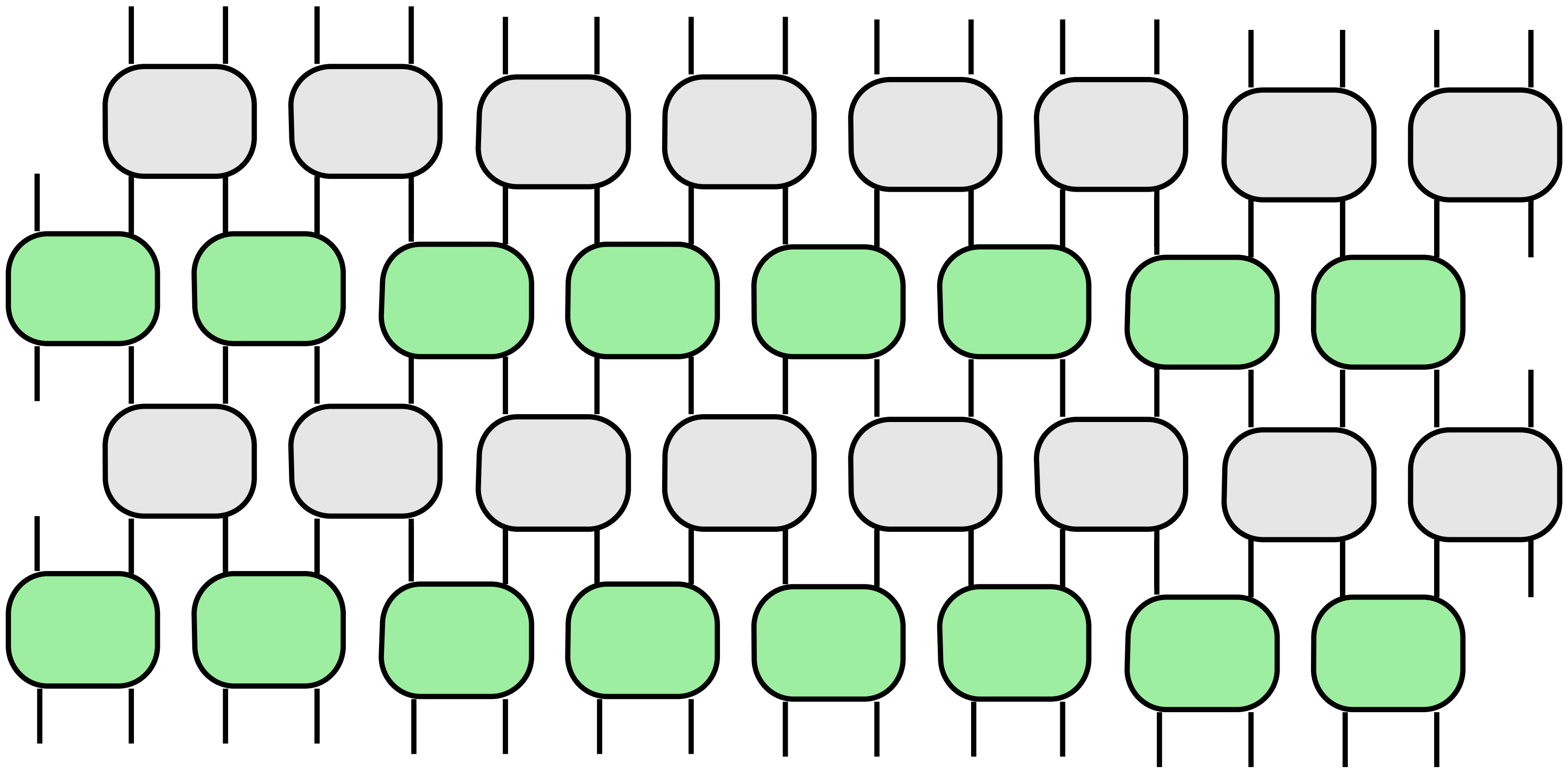
QCA for the Dirac field

(1+1)-dimensional case



Linearity

$$\psi_i(t+1) = \mathbf{U}_{i,j}\psi_j(t)$$

Quantum Walk

$$\psi(x,t+1) = \mathbf{U}\psi(x,t)$$

$$\mathbf{U} = \begin{pmatrix} nS & -im \\ -im & nS^\dagger \end{pmatrix} \xrightarrow{\text{Fourier}} \mathbf{U} = \int_{-\pi}^{\pi} \mathrm{d}k\ \mathbf{U}(k) \otimes |k\rangle\langle k|$$

band-limited

$$S\psi(x) = \psi(x+1)$$

$$n^2 + m^2 = 1,$$

$$0 \leqslant m \leqslant 1$$

bounded mass

$$\psi(x) = \begin{pmatrix} \psi^R(x) \\ \psi^L(x) \end{pmatrix}$$

$$\mathbf{U}(k) = \begin{pmatrix} ne^{ik} & -im \\ -im & ne^{-ik} \end{pmatrix}$$

AB, G. M. D'Ariano, A. Tosini, Ann. of Phys 354, 244 (2015).

AB, G. M. D'Ariano, A. Tosini, Phys. Rev. A 88, 032301 (2013).

Dirac QCA vs Dirac evolution

$$\mathbf{U}(k) = \exp(-i\mathbf{H}_A(k)) \qquad \mathbf{H}_A(k) \xrightarrow{m,k\to 0} \mathbf{H}_D(k) + O(m^2 k)$$

$$\mathbf{H}_D(k) = \begin{pmatrix} -k & m \\ m & k \end{pmatrix}$$

Dispersion relation

$$\cos^2(\omega_A) = (1-m^2)\cos^2(k)$$

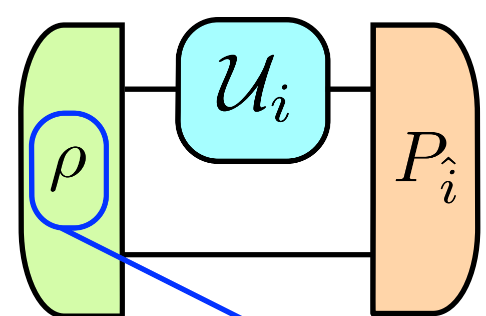
$$\omega_A \xrightarrow{m,k\to 0} \omega_D\left(1 - \frac{m^2}{6}\frac{k^2-m^2}{k^2+m^2}\right)$$

$$\omega_D^2 = k^2 + m^2$$

Discrimination between black boxes

$$—\boxed{\mathcal{U}_A}— \; = \; \exp(-iH_A t) \quad \text{Automaton}$$

$$—\boxed{\mathcal{U}_D}— \; = \; \exp(-iH_D t) \quad \text{Dirac}$$



$i = A, D$

$\rho \in \mathcal{S}_{\bar{k},\bar{N}}$ less than $\bar{N}$ particles
momentum smaller than $\bar{k}$

$$p_{err} = \frac{1}{2}\Big(p(A|D) + p(D|A)\Big) \geq \frac{1}{2}\left(1 - \frac{1}{6}m^2\overline{kN}t\right)$$

AB, G. M. D'Ariano, A. Tosini, Ann. of Phys 354, 244 (2015).
AB, G. M. D'Ariano, A. Tosini, Phys. Rev. A 88, 032301 (2013).

QCA and Lorentz transformations

Quantum Cellular Automata

$m, k \to 0$

Quantum Field Theory
Lorentz invariant equations

Relativity

The observer is the same!
Boosted observer?

1D Dirac automaton dispersion relation

$$\cos^2(\omega) = (1 - m^2)\cos^2(k)$$

classical kinematics
emergent from the automaton

non Lorentz invariant

Lorentz transformation

$$\begin{pmatrix} \omega' \\ k' \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} \omega \\ k \end{pmatrix}$$

$$\gamma := \frac{1}{\sqrt{1-\beta^2}}$$

Violations of Lorentz invariance
at ultra-relativistic scales

different transformation or privileged reference frame

Deformed relativity

A simple speculation from Quantum Gravity → In whose reference frame is the Planck energy the threshold for new phenomena?

Preserve relativity principle
Lorentz group

AND

invariant energy scale

⇓

Modify the action of Lorentz group

non-linear action in momentum space

$$L_\beta^D := \mathcal{D}^{-1} \circ L_\beta \circ \mathcal{D},$$

$$L_\beta = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix}$$

momentum space is more fundamental

$\mathcal{D}$ is a non-linear map

- $J_{\mathcal{D}}(0,0) = I$
- singular point ⇔ invariant energy
- invertible

which $\mathcal{D}$?

G. Amelino-Camelia, Physics Letters B 510, 255 (2001).
J. Magueijo, L. Smolin, Phys. Rev. Lett. 88, 190403 (2002).

Deformed relativity and QCA

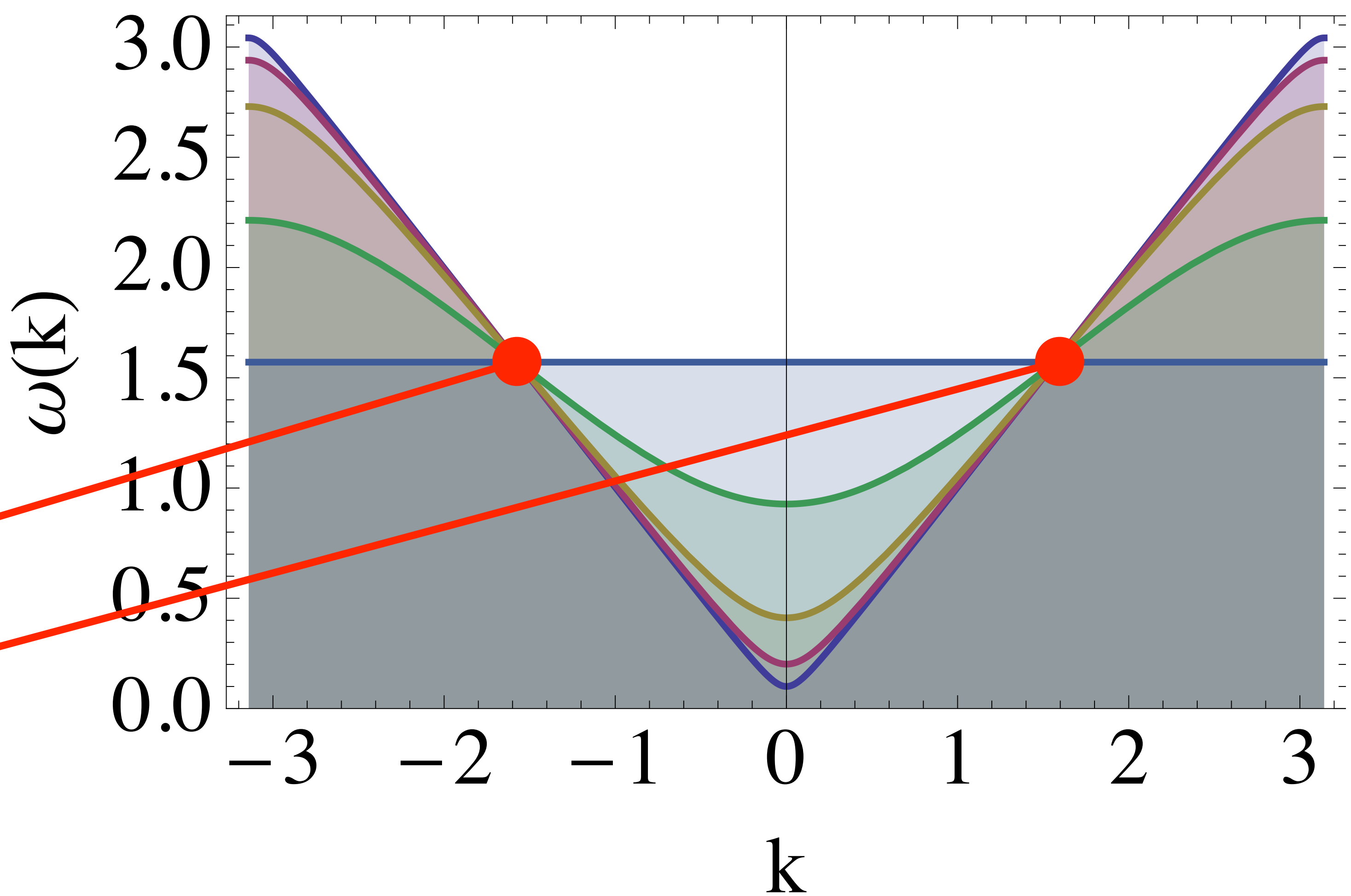
Automaton dispersion relation

$$\cos^2(\omega) = (1 - m^2)\cos^2(k) \;\Rightarrow\; \frac{\sin^2(\omega)}{\cos^2(k)} - \tan^2(k) = m^2$$

$$\tilde{\omega}^2 - \tilde{k}^2 = m^2$$

$$\mathcal{D}\begin{pmatrix}\omega \\ k\end{pmatrix} = \begin{pmatrix}\frac{\sin(\omega)}{\cos(k)} \\ \tan(k)\end{pmatrix}$$

$$-\frac{\pi}{2} \leqslant k \leqslant \frac{\pi}{2}$$



$$\omega_{inv} = \frac{\pi}{2}$$

A. Bibeau-Delisle, AB, G. M. D'Ariano, P. Perinotti, A. Tosini, EPL 109, 50003 (2015).

AB, G. M. D'Ariano, P. Perinotti, arXiv 1503.01017

Deformed relativity in position space

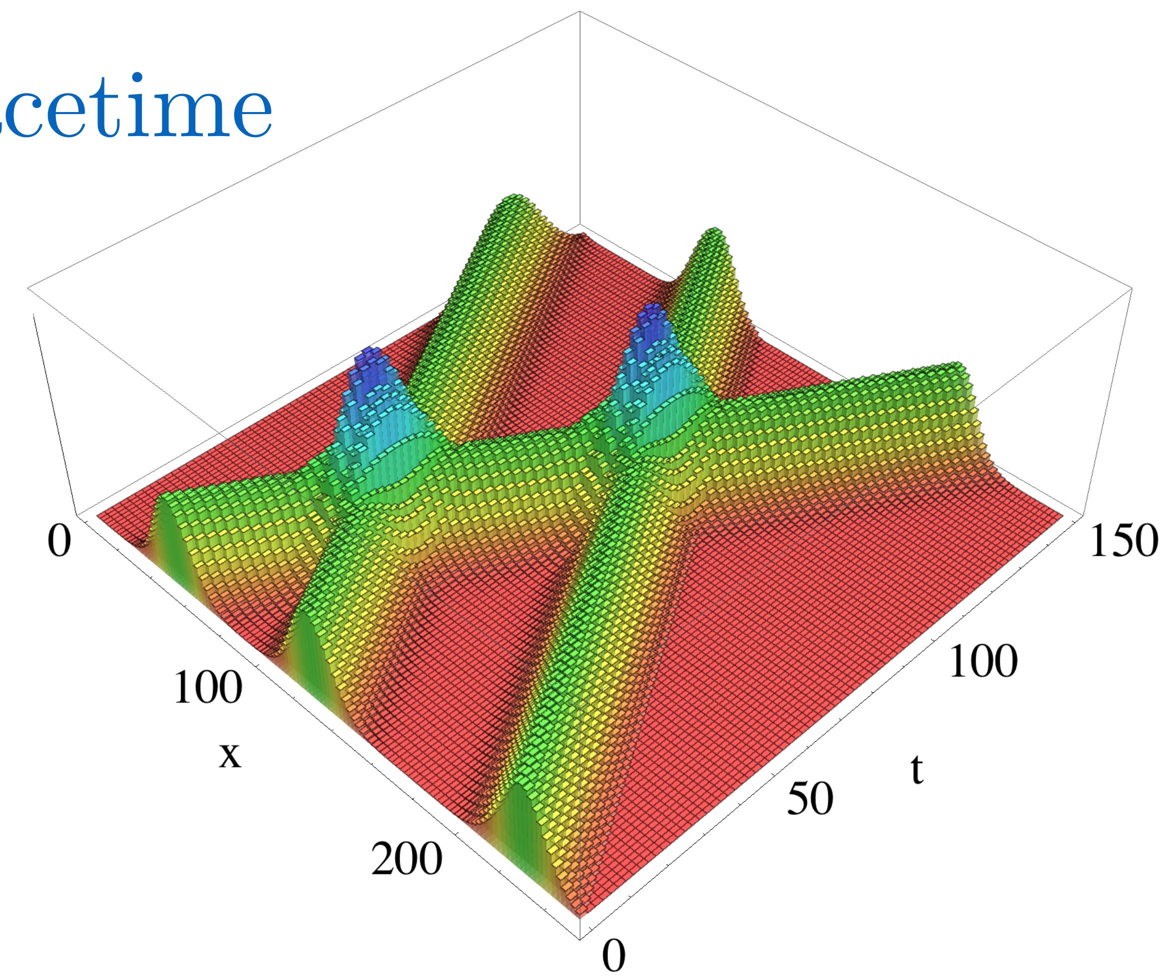
The model is defined in the momentum space → ? → Transformations in the position space?

Operational toy model of spacetime

coincidence of wavepackets

⇕

point in spacetime



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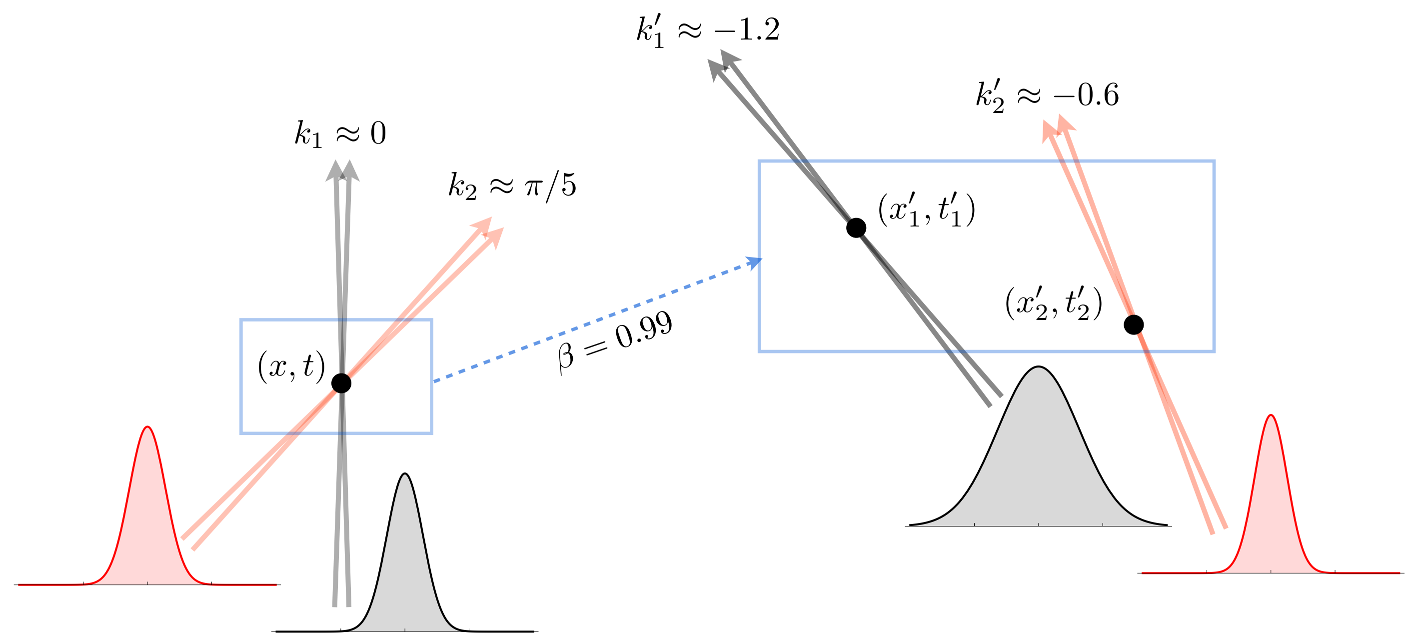


transformation of the coincidence points:

$$\begin{pmatrix} t' \\ x' \end{pmatrix} \approx \begin{pmatrix} -\partial_{\omega'} k & \partial_{k'} k \\ \partial_{\omega'} \omega & -\partial_{k'} \omega \end{pmatrix}_{k'=k'_0} \begin{pmatrix} t \\ x \end{pmatrix}$$

momentum-dependent spacetime

Relative Locality

A. Bibeau-Delisle, AB, G. M. D'Ariano, P. Perinotti, A. Tosini, EPL 109, 50003 (2015).

AB, G. M. D'Ariano, P. Perinotti, arXiv 1503.01017

Deformed relativity in position space

Relative locality ⇒ Observer-dependent spacetime

"before"

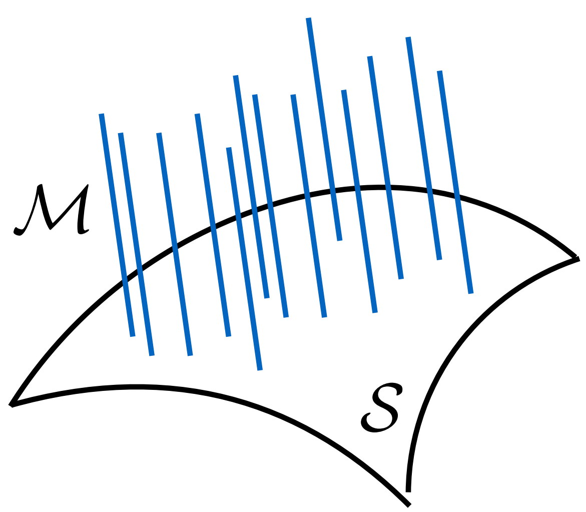
spacetime $\mathcal{S}$

"objective arena"

flat momentum space $\mathcal{M}$

phase space

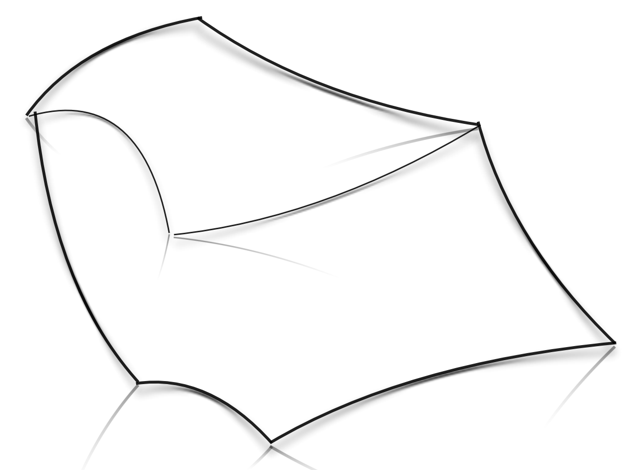
$\mathcal{P} = \mathcal{T}^*\mathcal{S}$



"after"

phase space

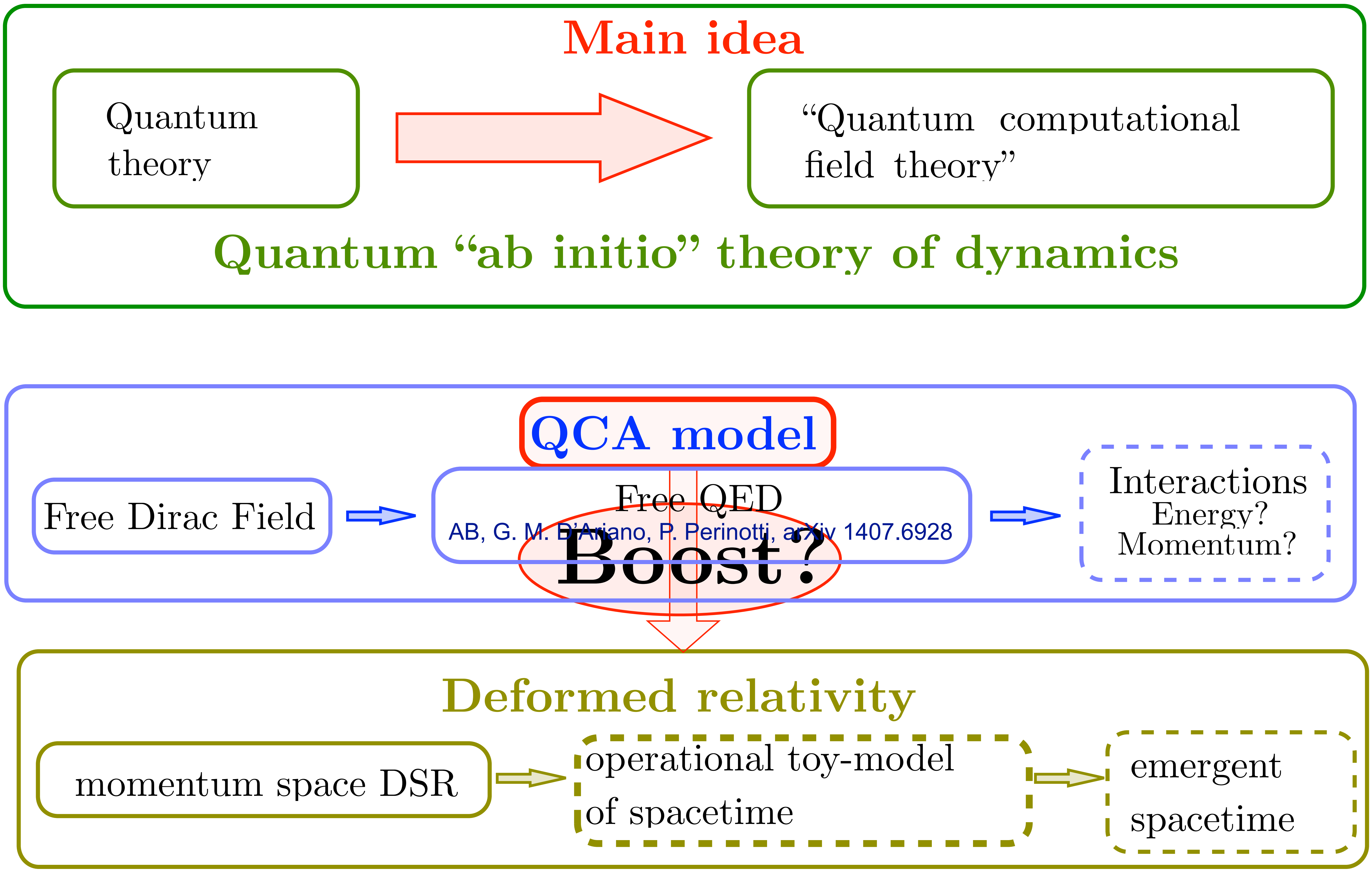
$\mathcal{P} \neq \mathcal{T}^*\mathcal{S}$



no canonical projection that gives a description of processes in spacetime

R. Schutzhold, W. G. Unruh, JETP Lett. 78, 431 (2003).
G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, L. Smolin, Phys. Rev. D 84, 084010 (2011).
A. Bibeau-Delisle, AB, G. M. D'Ariano, P. Perinotti, A. Tosini, EPL 109, 50003 (2015).

A final overlook



Thank you!