Quantum State Reconstruction and Control on Atom-Chips

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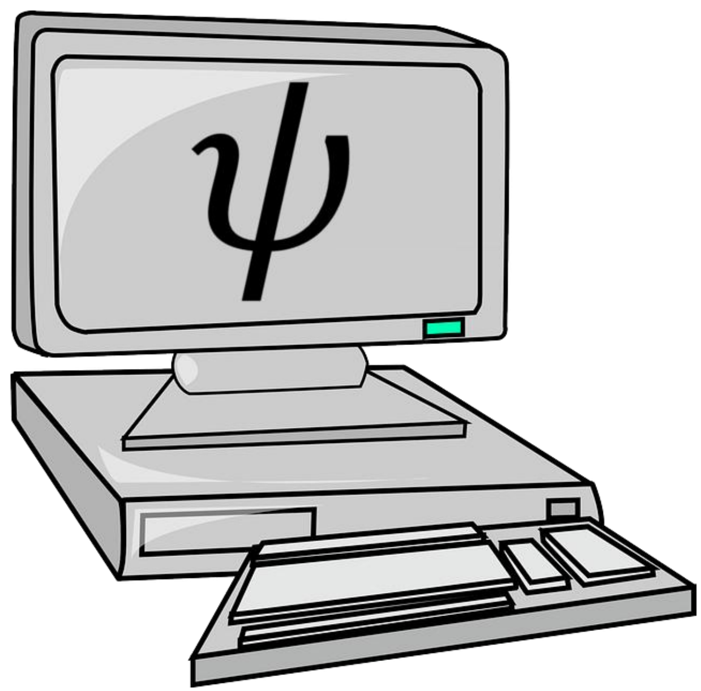
IQIS 2015

Goal

State Preparation

State Reconstruction



Outline

- AtomChip Experiment
- Model
- Quantum state reconstruction
- Quantum state preparation
- Conclusions

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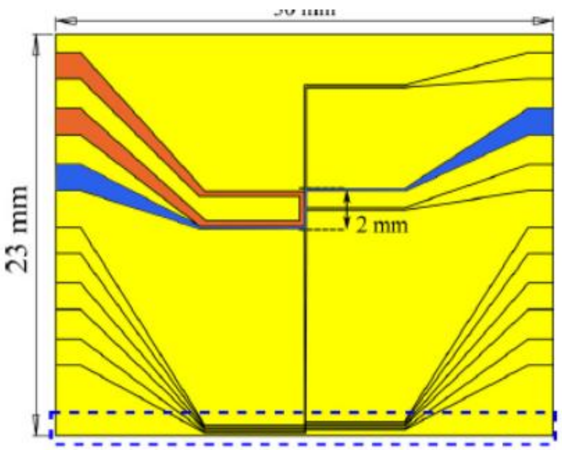
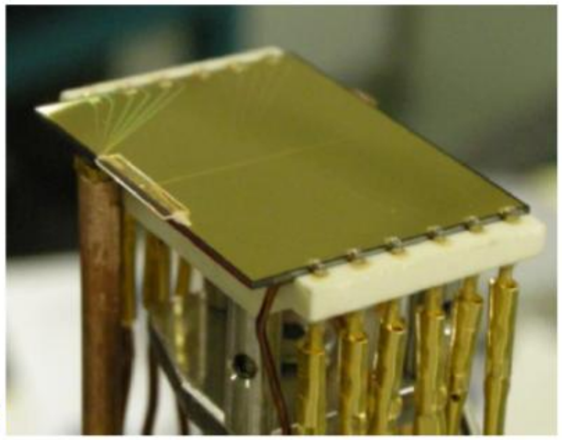
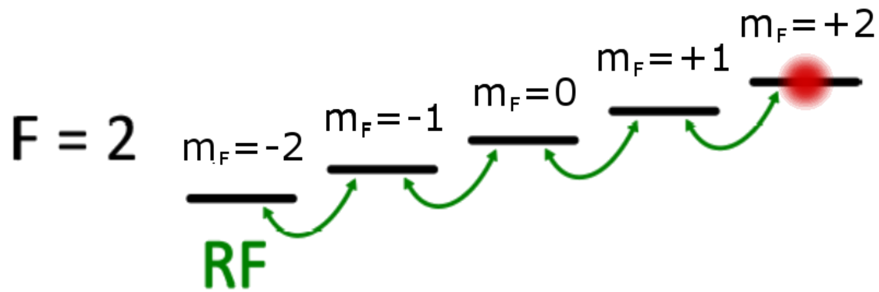
Atom-Chip Experiment



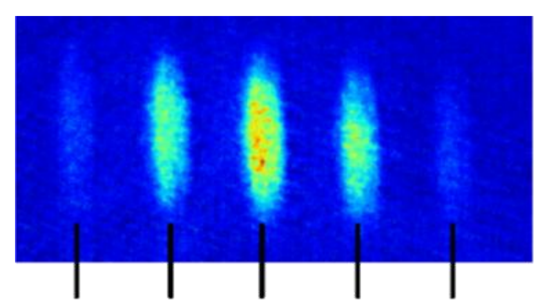


- Energy landscape in presence of magnetic field B



- ^{87}Rb BEC on atom-chip
- From zero to BEC in 8 sec
- Compact, easy-to-use and stable setup
- Integrated auxiliary conductors as RF antenna

- Stern-Gerlach discrimination



$m_F = -2$ $m_F = +2$

“Degenerate Quantum Gases Manipulation on Atom-chips”
I. Herrera, J. Petrovic, P.Lombardi, S. Bartalini and F.S. Cataliotti
Physica Scripta **T149**, 014002 (2012).

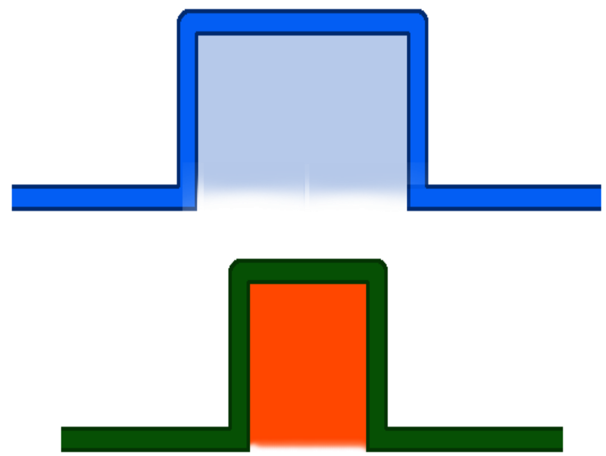
A multi-state interferometer on an atom chip
J. *Petrovic*, I. Herrera, P. Lombardi, F. Schaefer, F. S. Cataliotti
New Journal of Physics **15** (4), 043002 (2013)

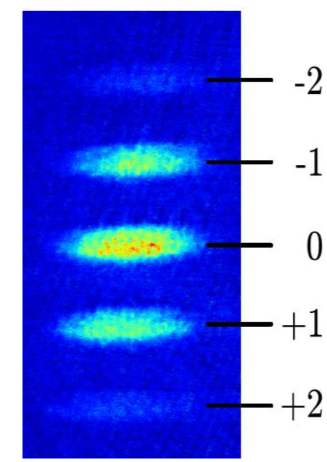
Atom-Chip Experiment

Time

Homogeneous magnetic field

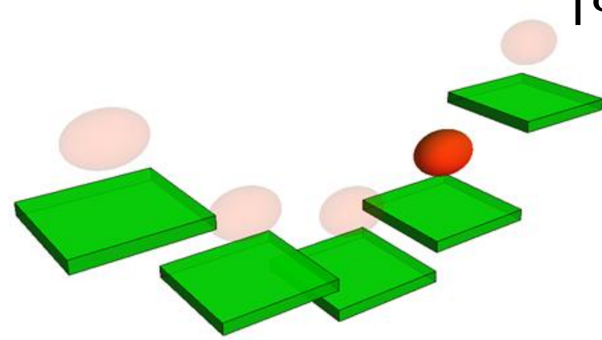
RF driving field





$$|\psi_{in} > = |m_F = 2 >$$

$$|\psi_{out} >= \sum_{i=-2}^{2} c_i|m_i > \rightarrow |c_i|^2$$



Model

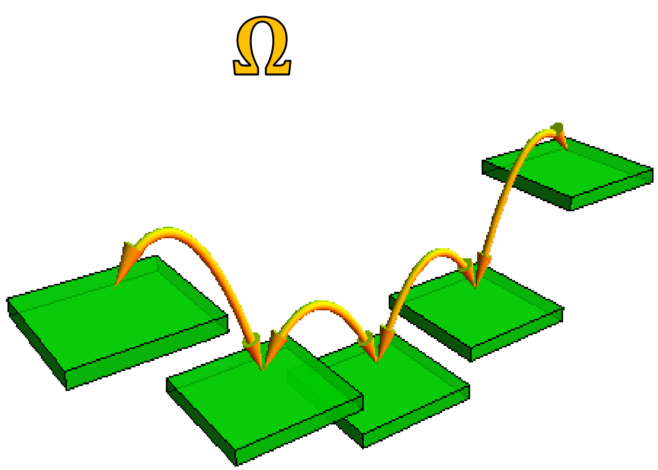
$$H_{RWA}(\alpha) = \hbar \begin{pmatrix} \omega_2(B) - 2\omega & \Omega & 0 & 0 & 0 \\ \Omega & \omega_1(B) - \omega & \sqrt{3/2}\,\Omega & 0 & 0 \\ 0 & \sqrt{3/2}\,\Omega & \omega_0(B) & \sqrt{3/2}\,\Omega & 0 \\ 0 & 0 & \sqrt{3/2}\,\Omega & \omega_{-1}(B) + \omega & \Omega \\ 0 & 0 & 0 & \Omega & \omega_{-2}(B) + 2\omega \end{pmatrix}$$

- Breit-Rabi formula

$$\omega_{mf}(B) = \alpha + \chi\, B + \eta \sqrt{1 + \gamma\, B + \ B^2}$$

- Set of parameter $\alpha = \{\Omega, B, \omega\}$

$$\omega_n(B) - n\,\omega$$



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Quantum State Reconstruction

Unknown state

$$\rho_{in} = \begin{pmatrix} \rho_{11} & \cdots & \rho_{1n} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & \rho_{nn} \end{pmatrix}$$

Output channels

$$\{M_i\}_{i=1\ldots m}$$

Measurements

$$f_i \sim Tr[\rho_{in} M_i]$$

Reconstruction (MLE)

$$\mathcal{L}(\rho_0) = \prod_i Tr[\rho_0 E_i]^{f_i}$$

Quantum State Reconstruction

Common problems of a standard reconstruction

- Size of the set of operators scale as n^2
- Different measurements are different experimental condition
- Computational time complexity growth exponentially

Solution?

Quantum State Reconstruction

Common problems of a standard reconstruction

- Size of the set of operators scale as n^2
- Different measurements are different experimental condition
- Computational time complexity growth exponentially

Solution?

Different strategy:

- Unknow state as initial point of a known evolution
- Use only one measurement operator
- Try to reproduce the experimental data

Quantum State Reconstruction

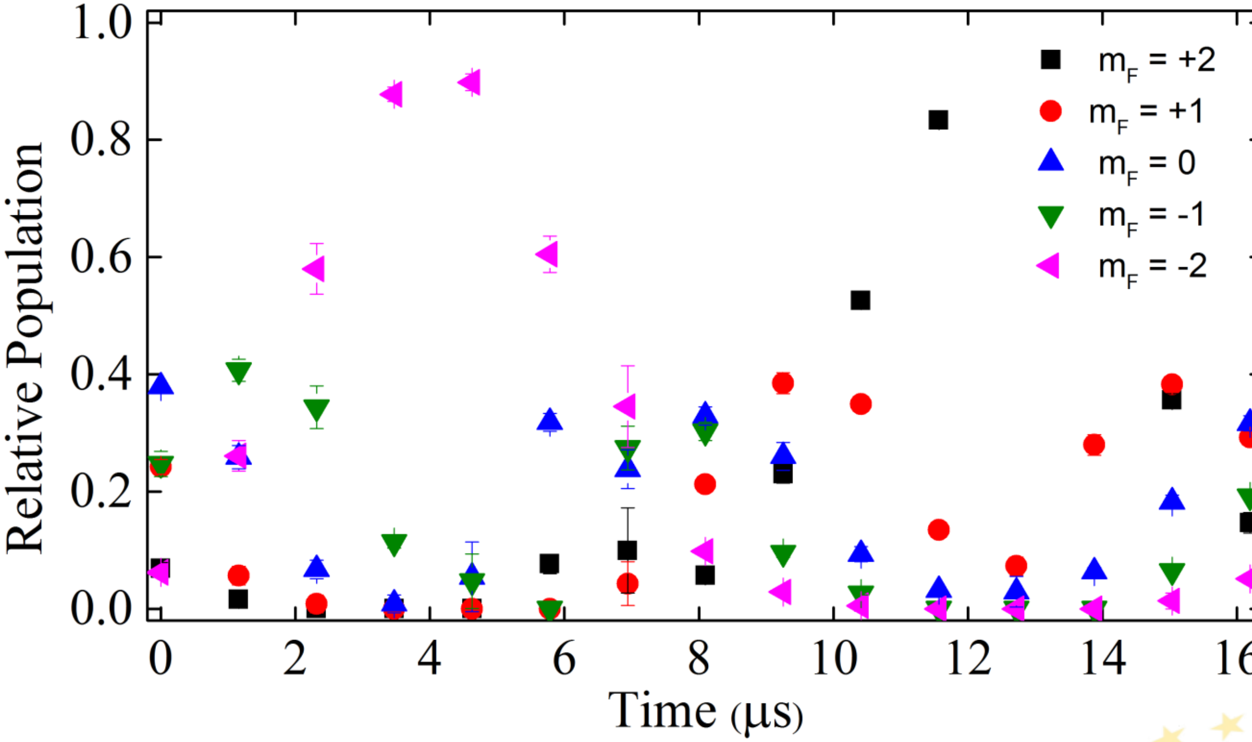
- Experimentally set the parameter $\{\Omega, B, \omega\}$ in H to defined values and evolve the unknow ρ_{in}
- Same Stern-Gerlach measurement at discrete times, in the time window T

$$p_{i,j} = Tr\left[\,\rho_{in}(t_j)\hat{a}_i\hat{a}_i^{+}\right],\ \sigma_{i,j}$$

$$p_{i,j} = Tr\left[\,\rho_{in}(t=0)A_i(t_j)\right]$$

$$A_i(t_j) = U^{+}\,(t_j)\hat{a}_i\hat{a}_i^{+}U(t_j)$$

$$\{M_i\}_{i=1\ldots n} \rightarrow A_i(t_j)$$



C. Lovecchio et al. arXiv:1504.01963, soon on NJP

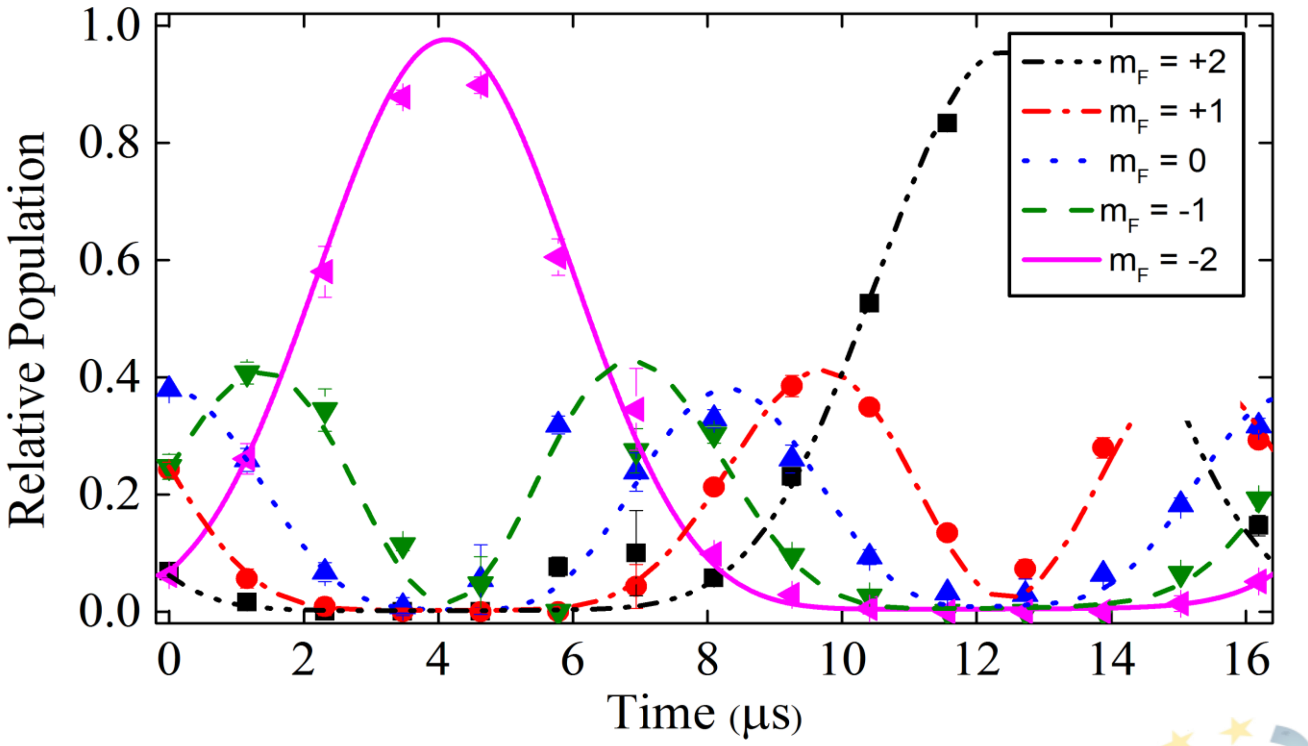
Quantum State Reconstruction

- Random initial guess $\rho_0 = \rho(t=0)$
- Simulate numerically the evoution set by H
  $$\frac{d}{dt}\rho(t) = -i[H,\rho(t)] + \mathcal{L}(\rho(t))$$
- Evaluate deviation between theoretical evolution and experimental data

$\bar{p}_{i,j}(\rho_0)$ theoretical prediction

$$\varepsilon(\rho_0) = \frac{1}{5}\sum_i \sqrt{\frac{\sum_j \omega_{i,j}|\bar{p}_{i,j}(\rho_0) - p_{i,j}|^2}{\sum_i \omega_{i,j}}}$$

$$\omega_{i,j} = 1/\sigma_{i,j}^2$$



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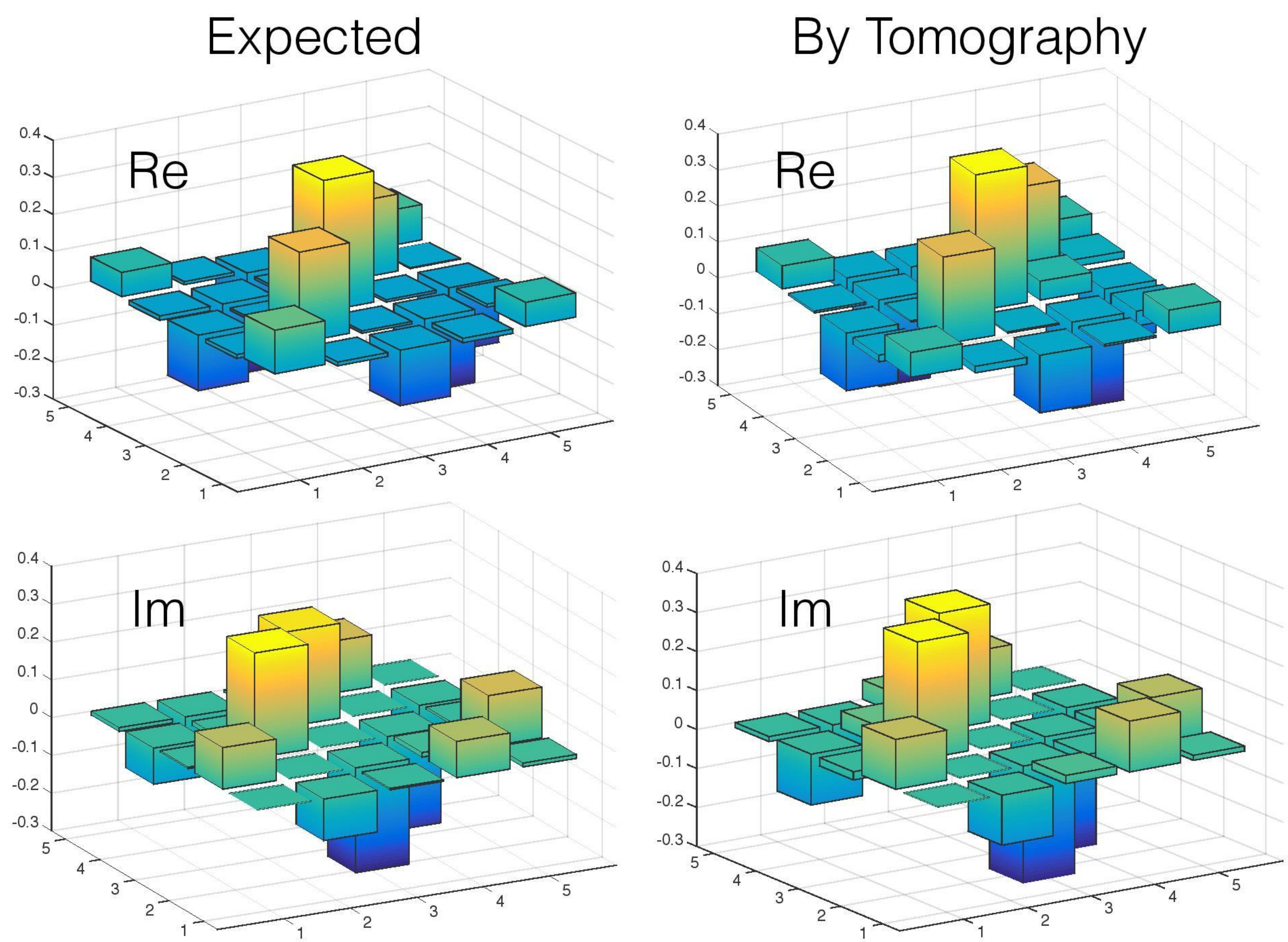
Quantum State Reconstruction

- $\varepsilon(\rho_0) < 3 \times 10^{-6}$
- Uhlmann Fidelity $\mathcal{F} > 0.95$



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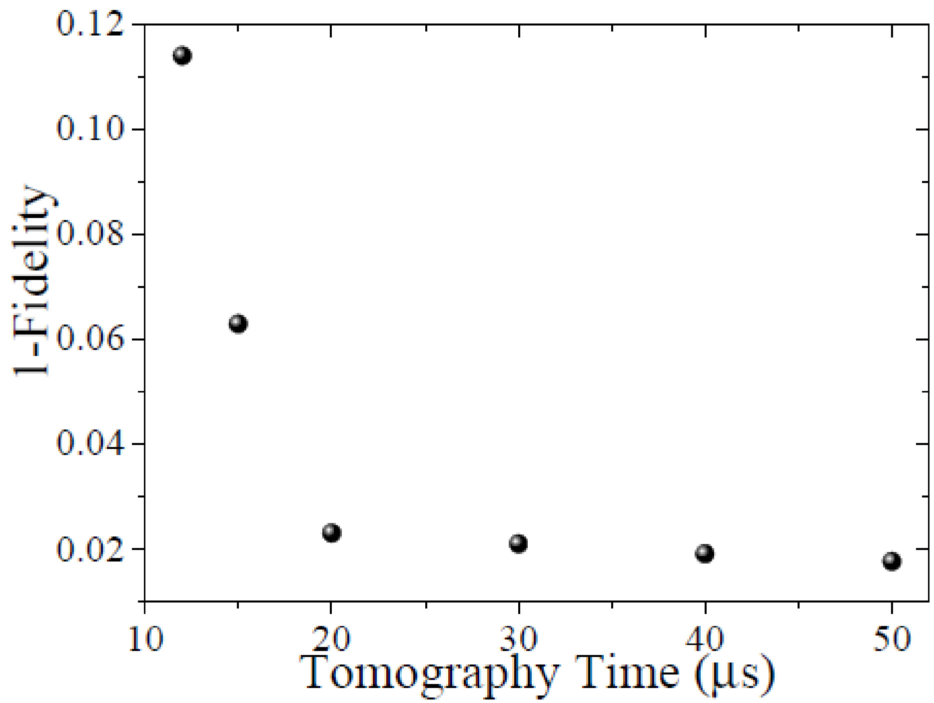
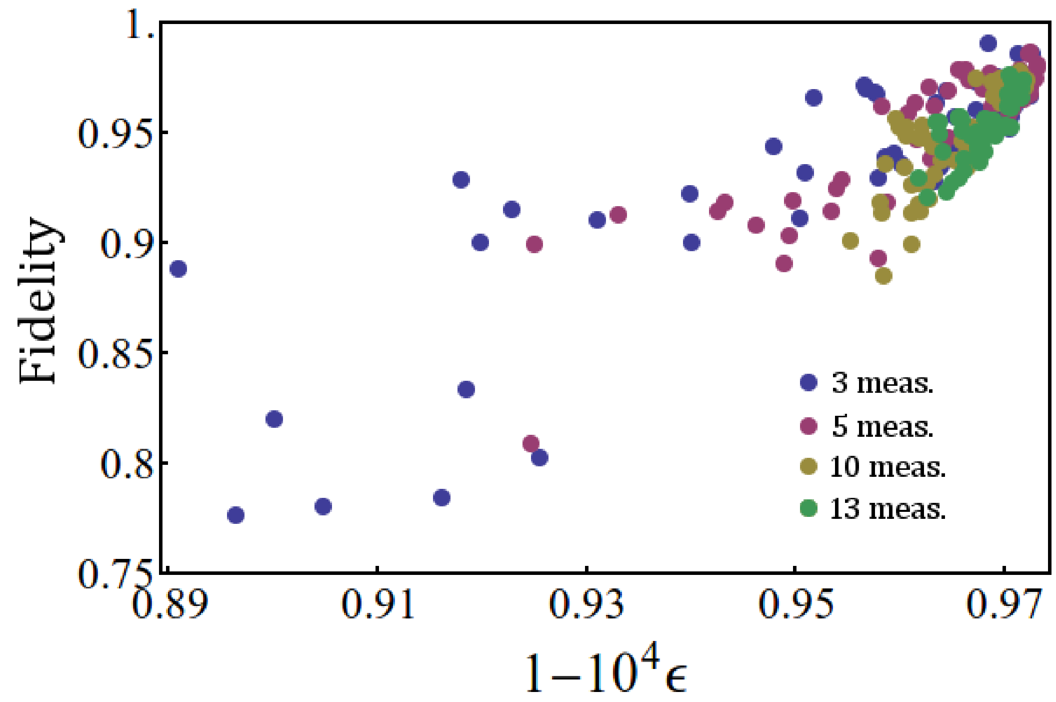
Quantum State Reconstruction





The Fidelity of the reconstructed state quickly converges to a maximum

At low Deviation correspond higher Fidelities

C. Lovecchio et al. arXiv:1504.01963, soon on NJP

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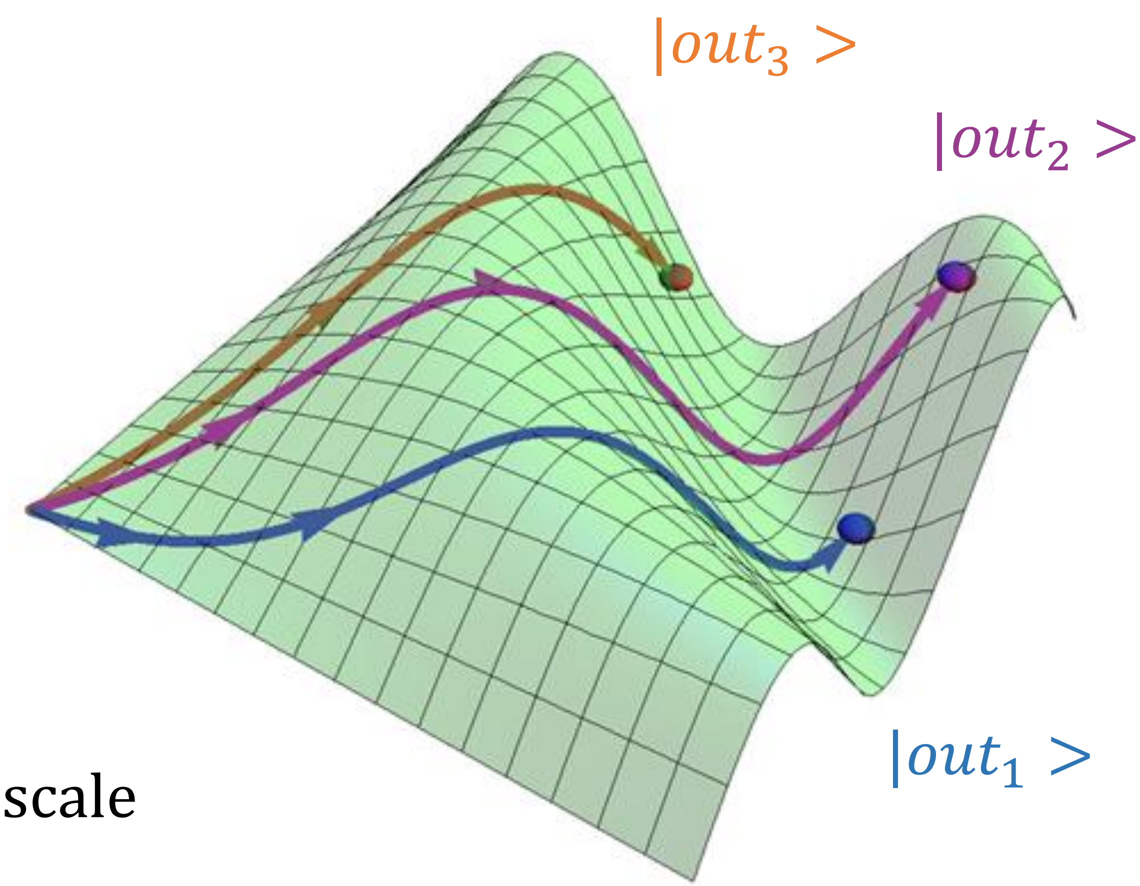
Quantum State Preparation

- $H(t, \alpha)$
  T_0, ε_0
- $\alpha \rightarrow \alpha_i$
  $T_s \leq T_0, \varepsilon_s \leq \varepsilon_0$
- $\alpha \rightarrow \alpha(t)$



Why?

- Arbitrary state preparation
- Faster than decoherence time scale

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Quantum State Preparation

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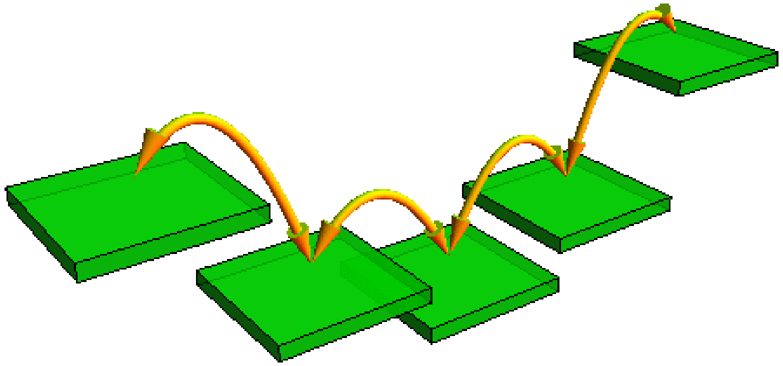
$\alpha = \{\Omega, B, \omega\} \rightarrow \omega = \omega(\mathrm{t})$

Ω

$\omega_n(B) - n\,\omega(\mathrm{t})$



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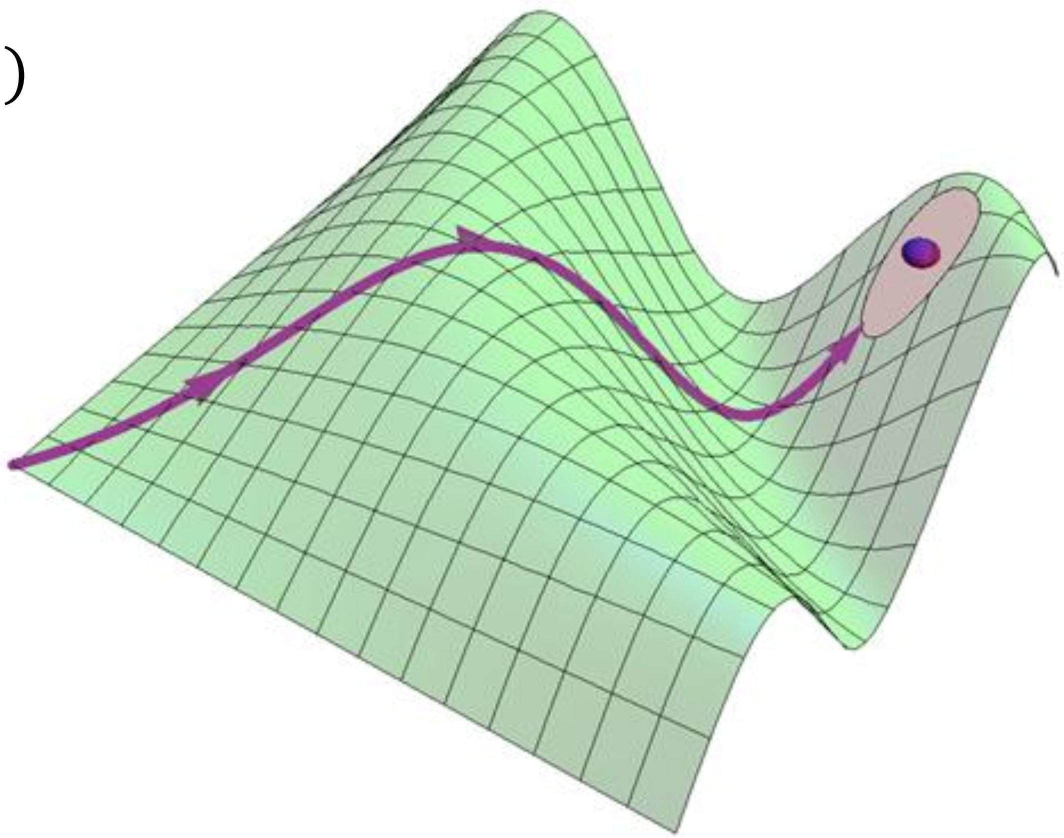
Quantum State Preparation

CRAB optimization

- $\varepsilon = \sum_i \frac{|p_i - b_i|}{2} \rightarrow \varepsilon_{T,}\ \varepsilon_E\ (\varepsilon \in [0,1])$
- $p_i = \rho_{ii}(T)$
- b_i target state population



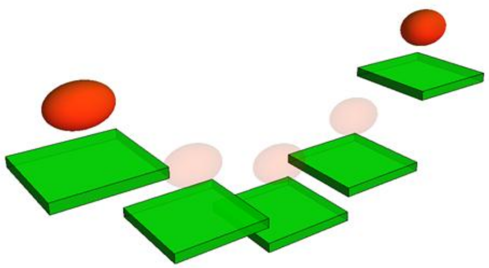
Experimental constraints

- $\omega\ (t) \in 2\pi\ [4150, 4600]$ kHz
- $B = 6.1794$ Gauss
- $\Omega = 2\ \pi\ 60$ kHz
- $T=100\ \mu s$

T. Caneva, T. Calarco, and S. Montangero, Phys. Rev. A 84, 022326 (2011).
P. Doria, T. Calarco, and S. Montangero, Phys. Rev. Lett. 106, 190501 (2011).

Quantum State Preparation

Target State	ρ_{11}	ρ_{22}	ρ_{33}	ρ_{44}	ρ_{55}
A	1/2	0	0	0	1/2
B	1/2	0	0	1/2	0
C	0	1/2	0	1/2	0
D	1/2	1/2	0	0	0
E	0	1/3	1/3	1/3	0
F	1/5	1/5	1/5	1/5	1/5
G	0	1	0	0	0
H	0	0	0	1	0
I	0	0	1	0	0



$$\rho_{11} \to m_F = +2, \dots, \rho_{55} \to m_F = -2$$

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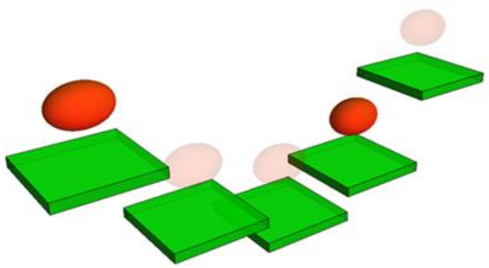

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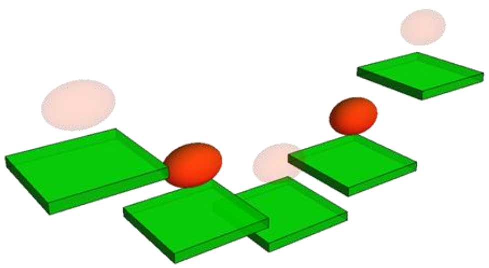

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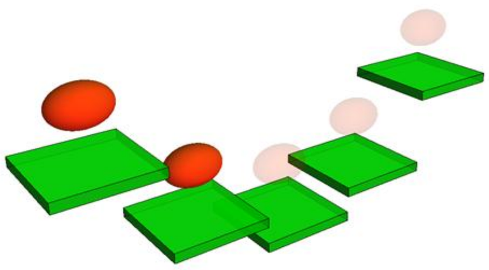

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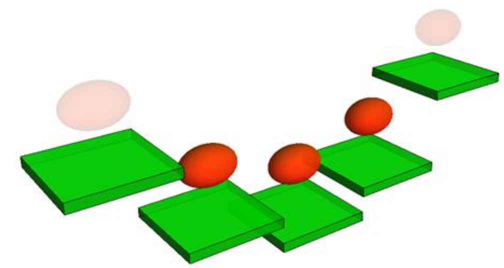

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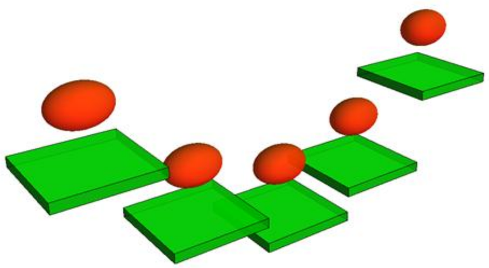

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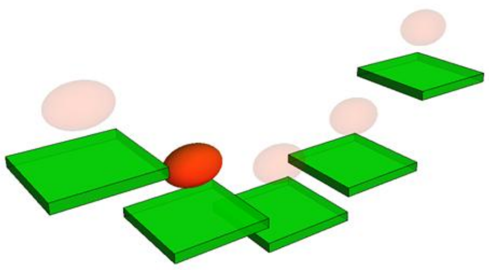

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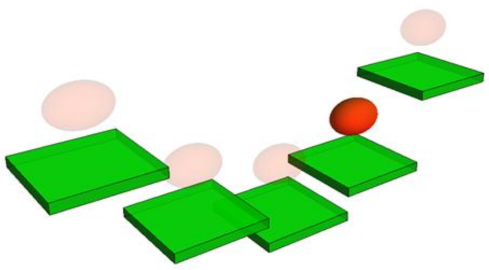

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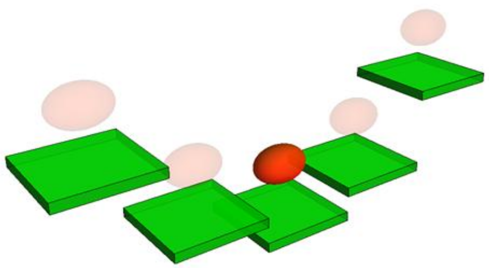

$$\rho_{11} \rightarrow m_F = +2, \dots, \rho_{55} \rightarrow m_F = -2$$

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Quantum State Preparation

Target State	ε_T	ε_E	$\mathcal{F}$
A	0,04(3)	???	0,71
B	0,04(2)	???	0,67
C	0,04(3)	???	0,11
D	0,03(2)	???	0,71
E	0,04(2)	???	0,02
F	0,02(1)	???	0,45
G	0,05(4)	???	0,15
H	0,04(3)	???	0,07
I	0,07(3)	???	0,15

$$\varepsilon = \sum_i \frac{|p_i - b_i|}{2}$$

- $\mathcal{F}(\rho_0, \rho_T) = Tr\sqrt{{\rho_0}^{1/2} \rho_T {\rho_0}^{1/2}}$ Uhlman fidelity

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Quantum State Preparation

Target State	ε_T	ε_E	$\mathcal{F}$
A	0,04(3)	0,07(1)	0,71
B	0,04(2)	0,02(1)	0,67
C	0,04(3)	0,04(1)	0,11
D	0,03(2)	0,02(1)	0,71
E	0,04(2)	0,03(1)	0,02
F	0,02(1)	0,03(1)	0,45
G	0,05(4)	0,04(1)	0,15
H	0,04(3)	0,03(1)	0,07
I	0,07(3)	0,07(1)	0,15

$$\varepsilon = \sum_i \frac{|p_i - b_i|}{2}$$

- $\mathcal{F}(\rho_0, \rho_T) = Tr\sqrt{{\rho_0}^{1/2}\rho_T{\rho_0}^{1/2}}$ Uhlman fidelity

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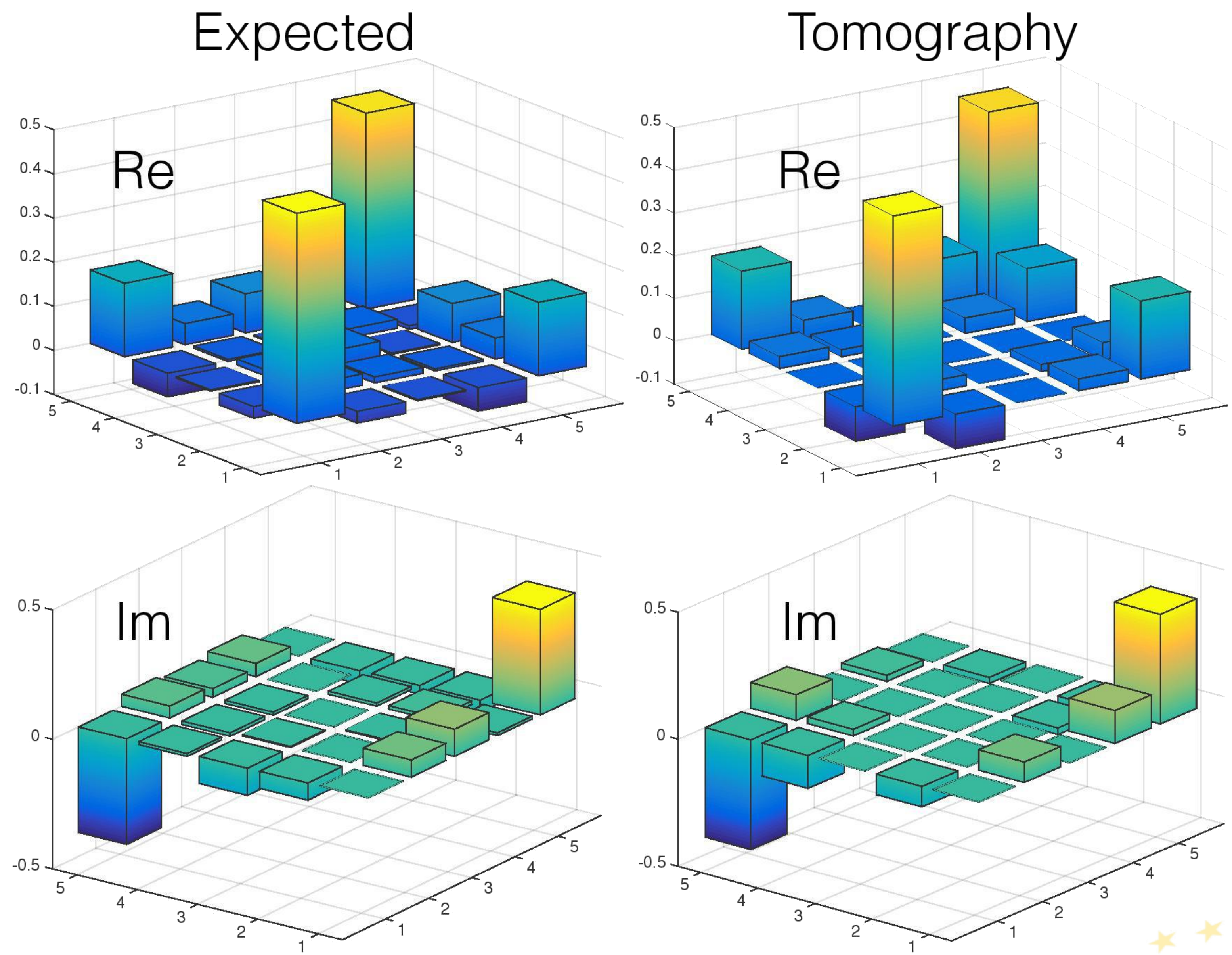
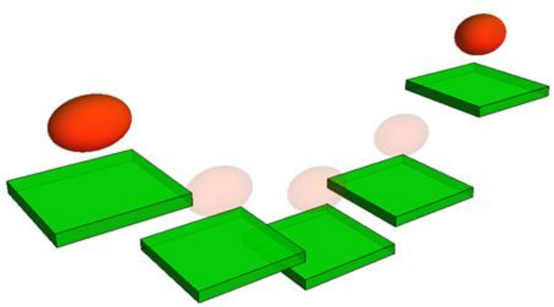
Quantum State Preparation

- State A

$\mathcal{F} = 0{,}93$





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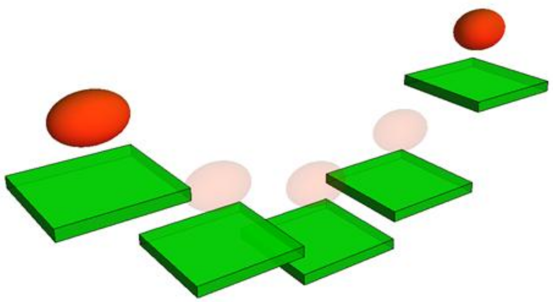
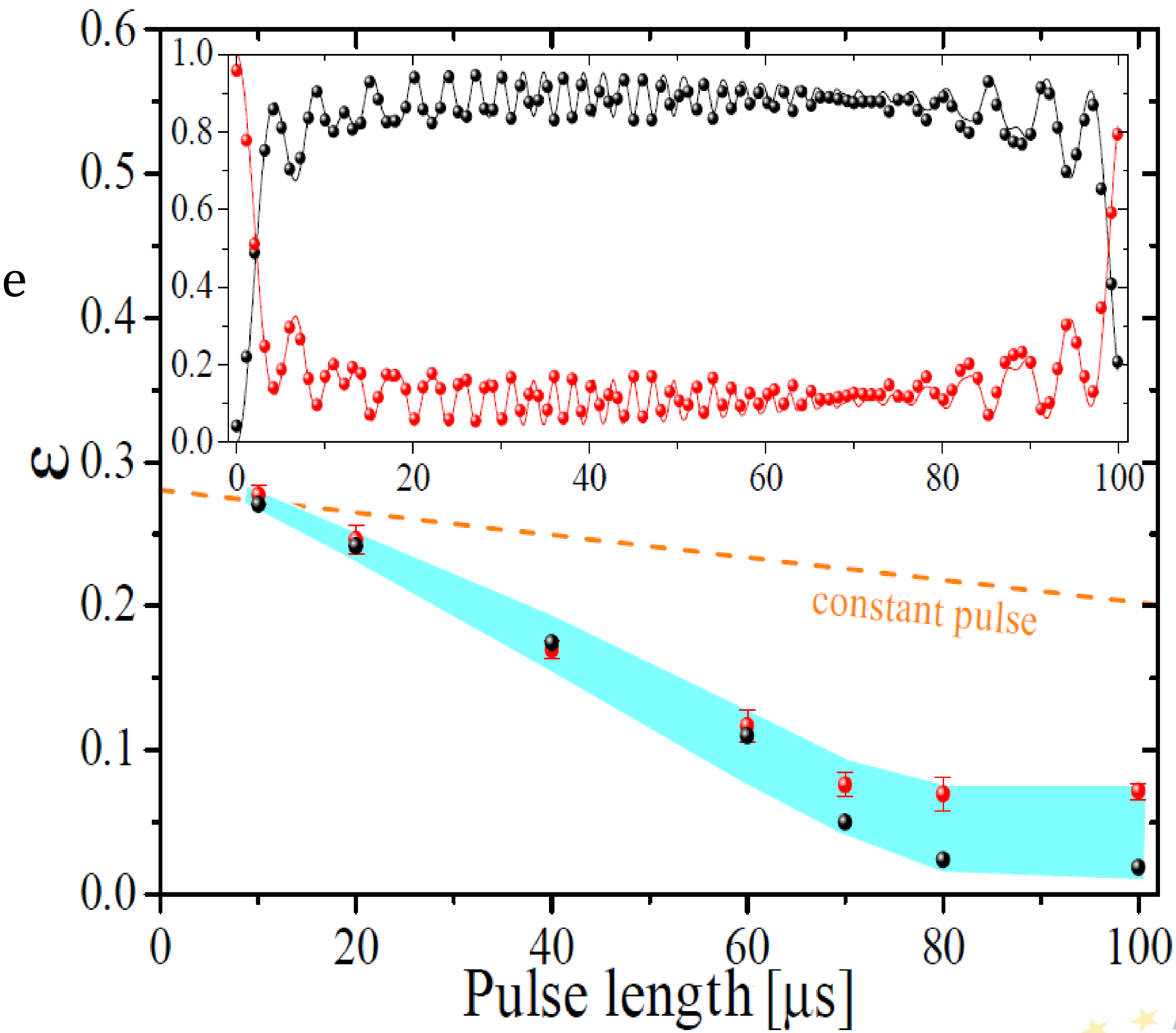
Quantum State Preparation

- State A
- Different optimized pulse length T
- Same constraints for all pulses





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Conclusion

- Good knowledge of quantum dynamics allows for easy full reconstruction of an unknown quantum state
- Optimal control strategy allow to reach any point of the Hilbert space of interest
- The error in the states preparation depends on the time length of the optimized evolution
- By repeated measurements it is also possible to control the size of the Hilbert space in which the system is allowed to evolve [F. Schaefer et al Nat. Comm. **5**:3194 (2014)]

Thank you

Filippo Caruso

Prof. Francesco
Saverio Cataliotti

Shahid Cherukattil

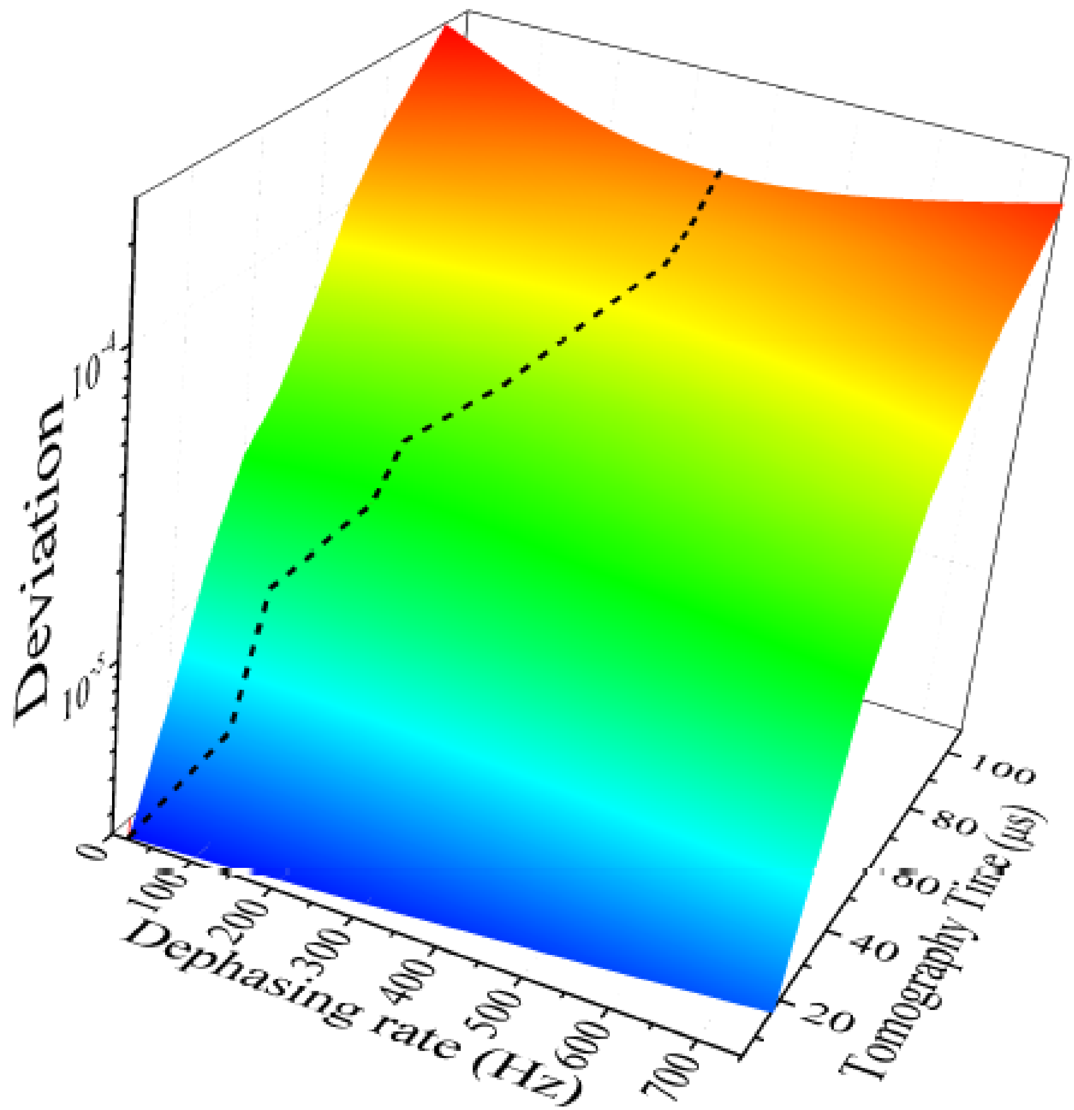
Murtaza Ali Khan

Augusto Smerzi

and
Florian Schäefer, Ivan Herrera
Simone Montagero, Tommaso
Calarco

Quantum State Reconstruction

At longer times dephasing from external noise dominates ultimately setting the maximum attainable fidelity



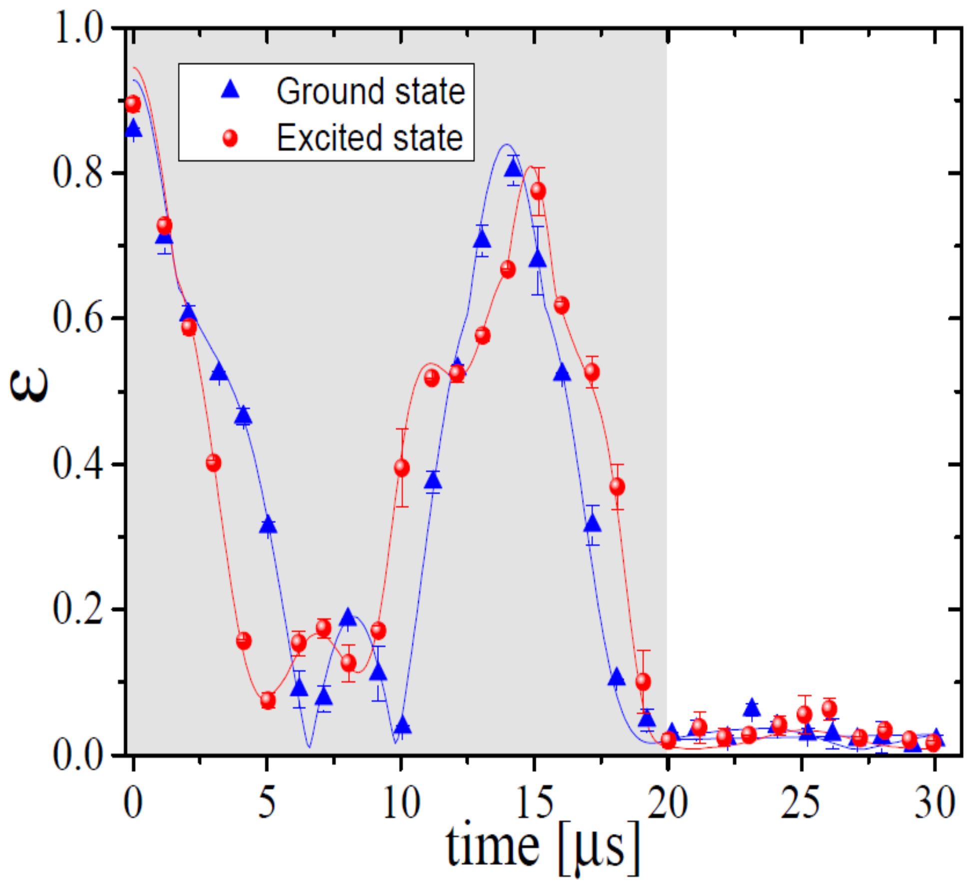
Quantum State Preparation

Excited and ground state preparation of the RF driven Hamiltonian

$$H(t,\alpha) = \hbar \begin{pmatrix} \omega_2 - 2\omega & \Omega & 0 & 0 & 0 \\ \Omega & \omega_1 - \omega & \sqrt{3/2}\,\Omega & 0 & 0 \\ 0 & \sqrt{3/2}\,\Omega & \omega_0 & \sqrt{3/2}\,\Omega & 0 \\ 0 & 0 & \sqrt{3/2}\,\Omega & \omega_{-1} + \omega & \Omega \\ 0 & 0 & 0 & \Omega & \omega_{-2} + 2\omega \end{pmatrix}$$

$$\omega_n = \omega_n(B)$$

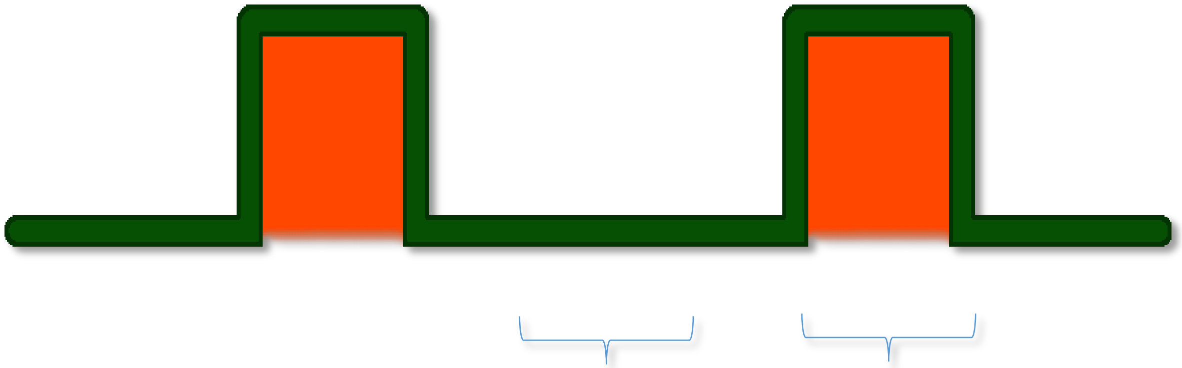
- $B =$ 6,1794 Gauss
- $\omega = 4{,}323$MHz



Application to Interferometry

RF driving field

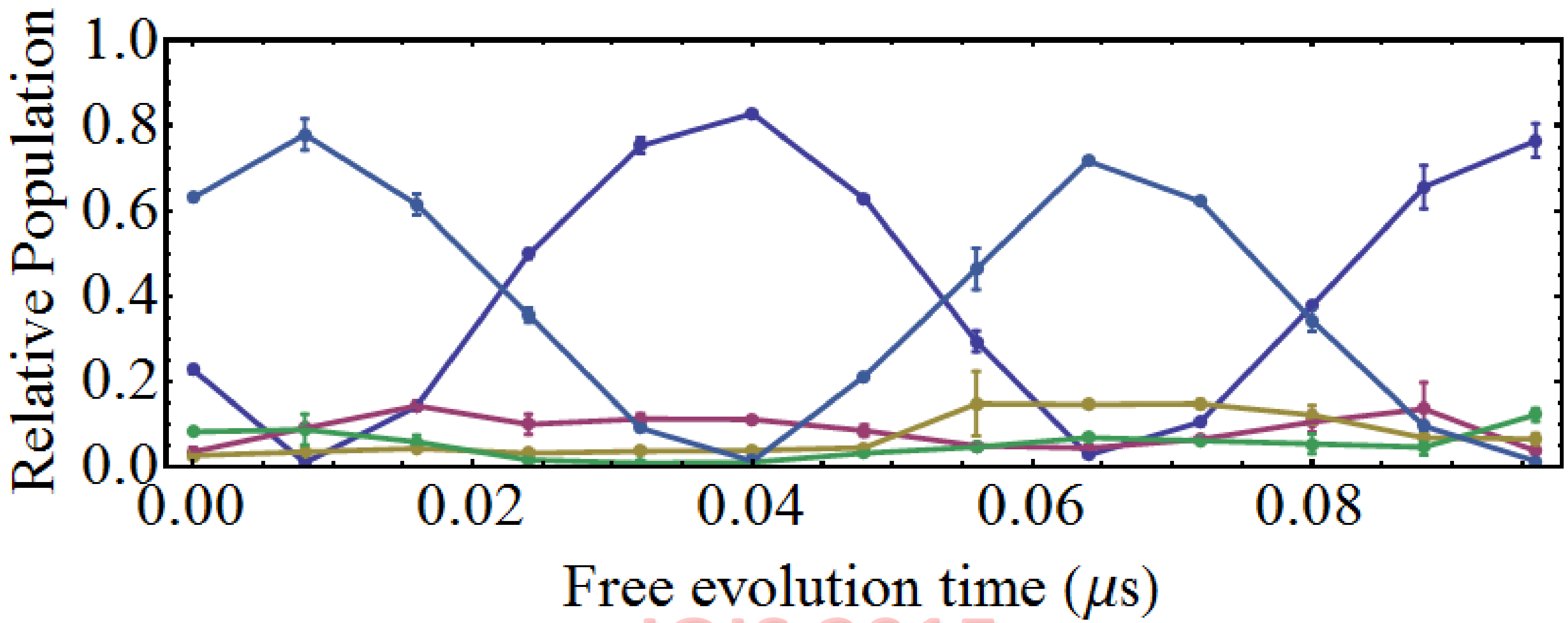
Time



T recombination

$$| \psi > = \sum_{m_F} a_{m_F}\, e^{i\, \Delta\phi_{m_F}} |F = 2, m_F >$$

$$\Delta\phi_{m_F} = \frac{\mu_B g_F F_z}{\hbar} B_z T$$



Application to Interferometry

- State preparation A, B, C, D
- $\Delta E(B) \cong n\, 2\pi\, 4.3$ MHz
- $\Delta\phi \propto n\, \Delta E(B)$
- In the best case $S \propto \Delta E(B)$
- In our case $S/A \propto \Delta E(B)$