IQIS 2015 - 8th Italian Quantum Information Science Conference

Measuring statistical moments to detect a topological transition in a photonic quantum walk

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SLAM group

Physics of Structured Light And Matter, and of their interaction

Università degli Studi di Napoli Federico II

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Physics of Structured Light And Matter, and of their interaction

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+ Vittorio Cataudella & Giulio De Filippis

General picture

$$\hat{H} = \hat{H}(\delta)$$

physics depending some tunable parameter

The system shows two phases

$\delta < \delta^*$	$\delta > \delta^*$
Phase 1	Phase 2

General picture

$$\hat{H} = \hat{H}(\delta)$$

physics depending some tunable parameter

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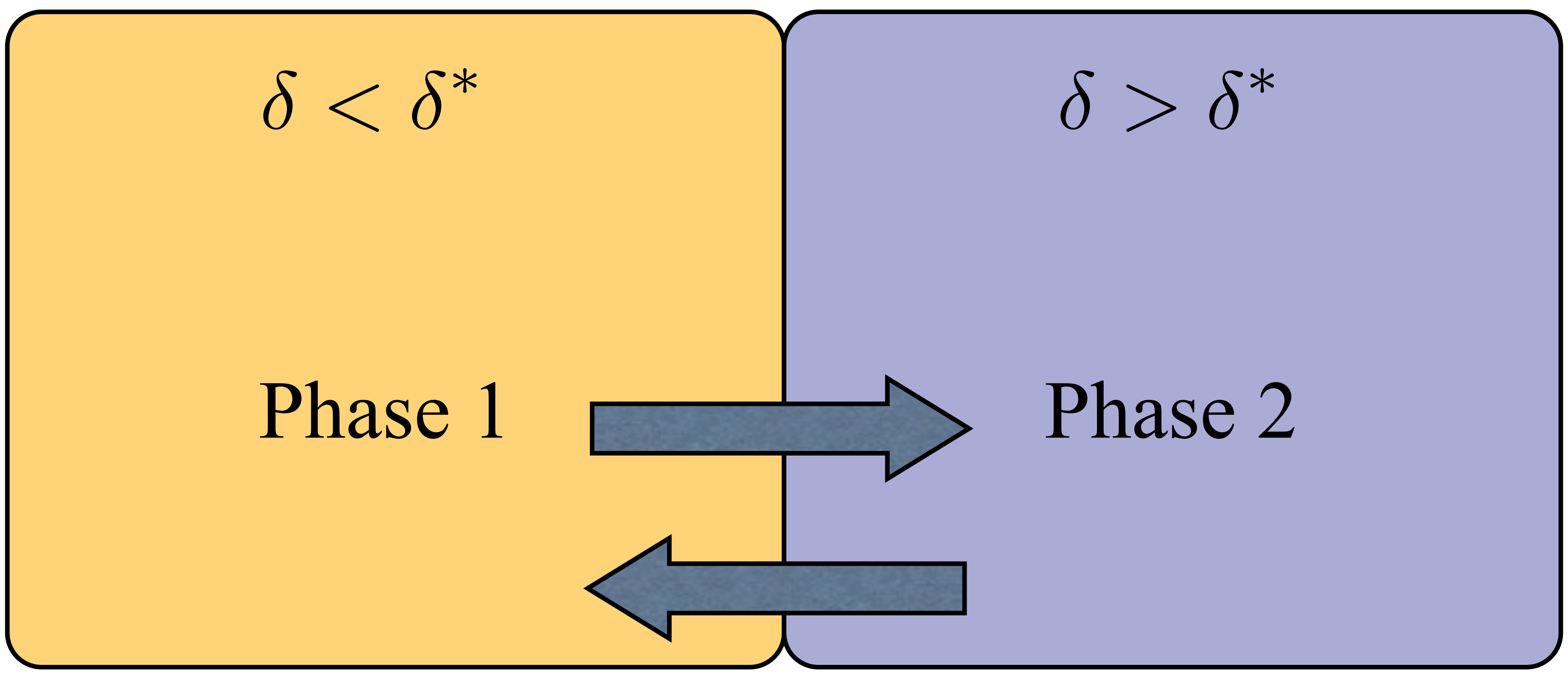
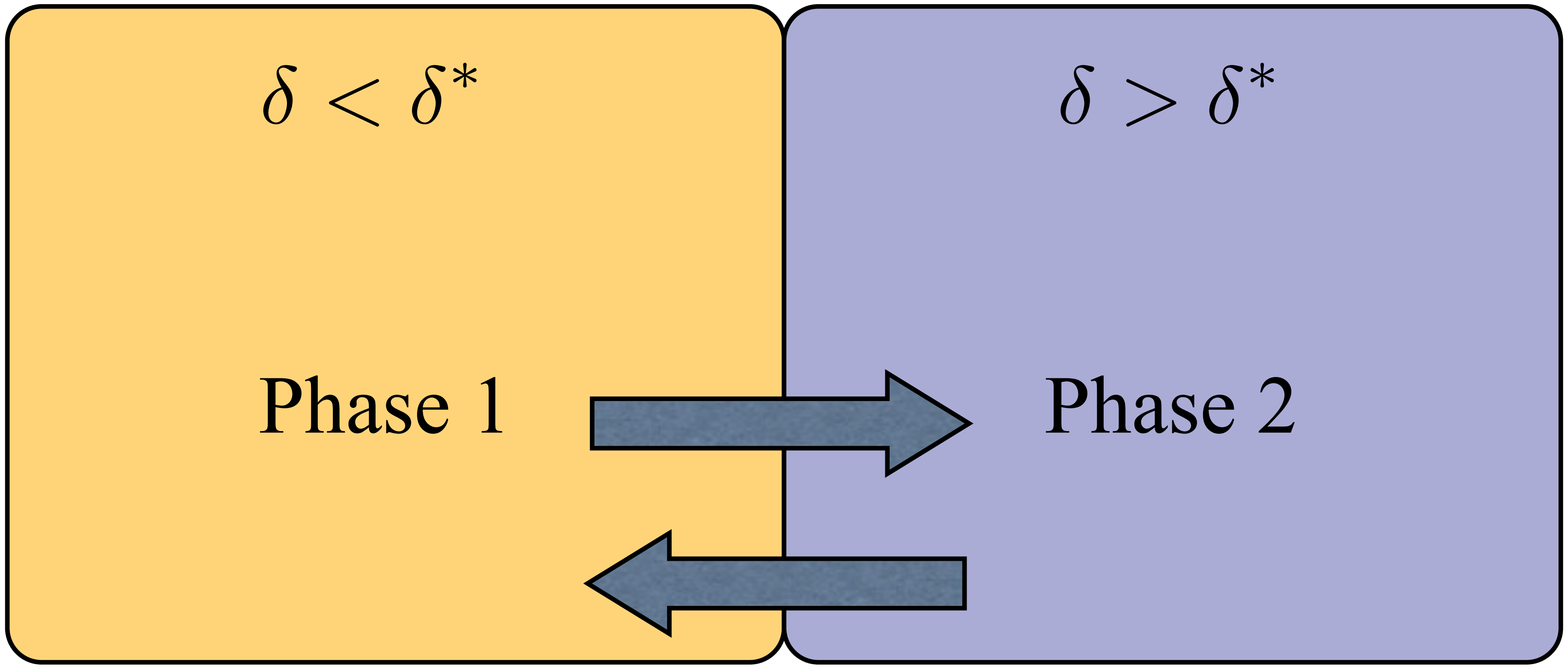


General picture



DETECTION OF THE PHASE TRANSITION

IN THE CONTEXT OF TOPOLOGICAL PHASES

Short Outline

- Quantum Walk at a glance
- QWs with twisted photons
- Topological phases in QWs
- Detection of the phase transition

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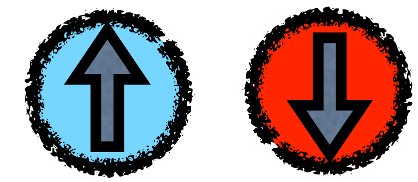
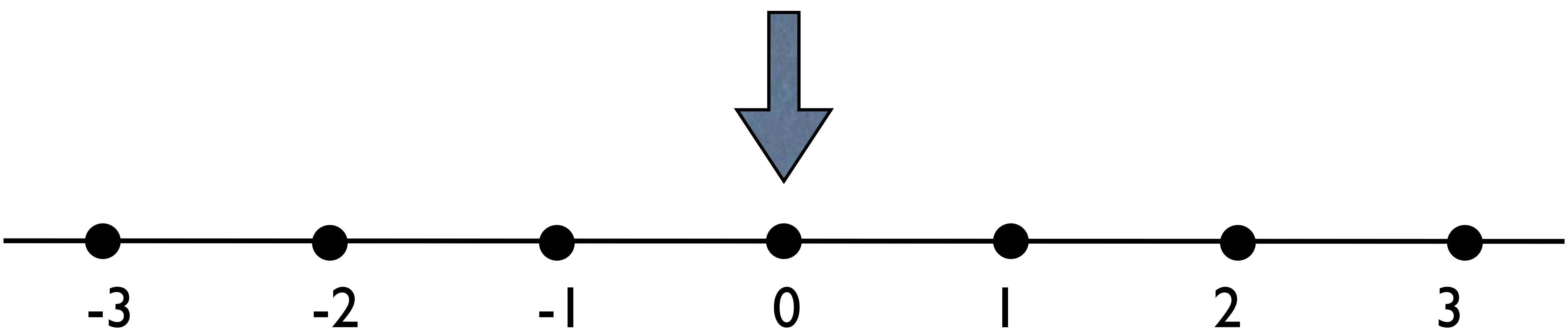
The classical random walk

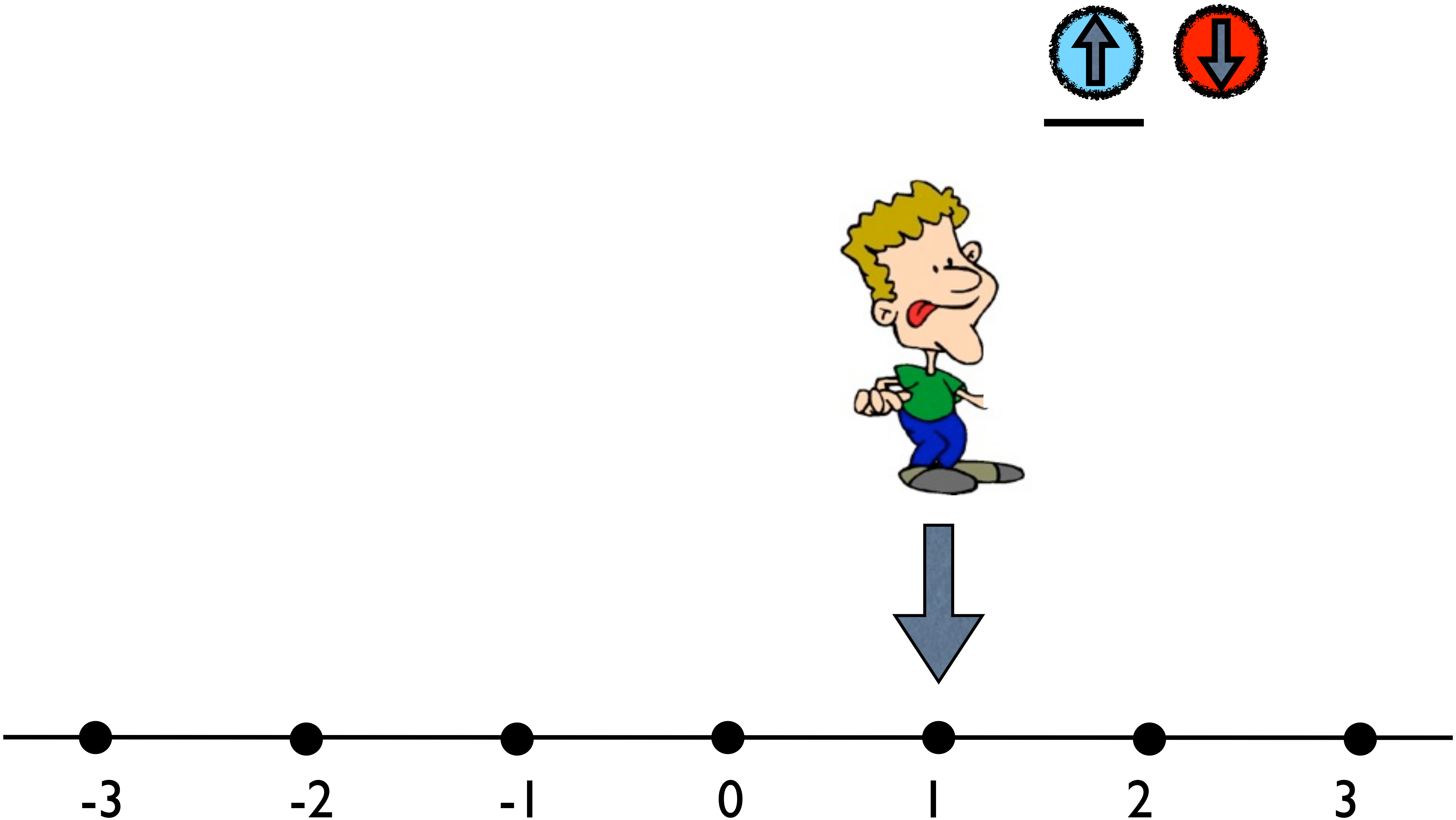
with a coin 

a walker 



on a lattice

The classical random walk



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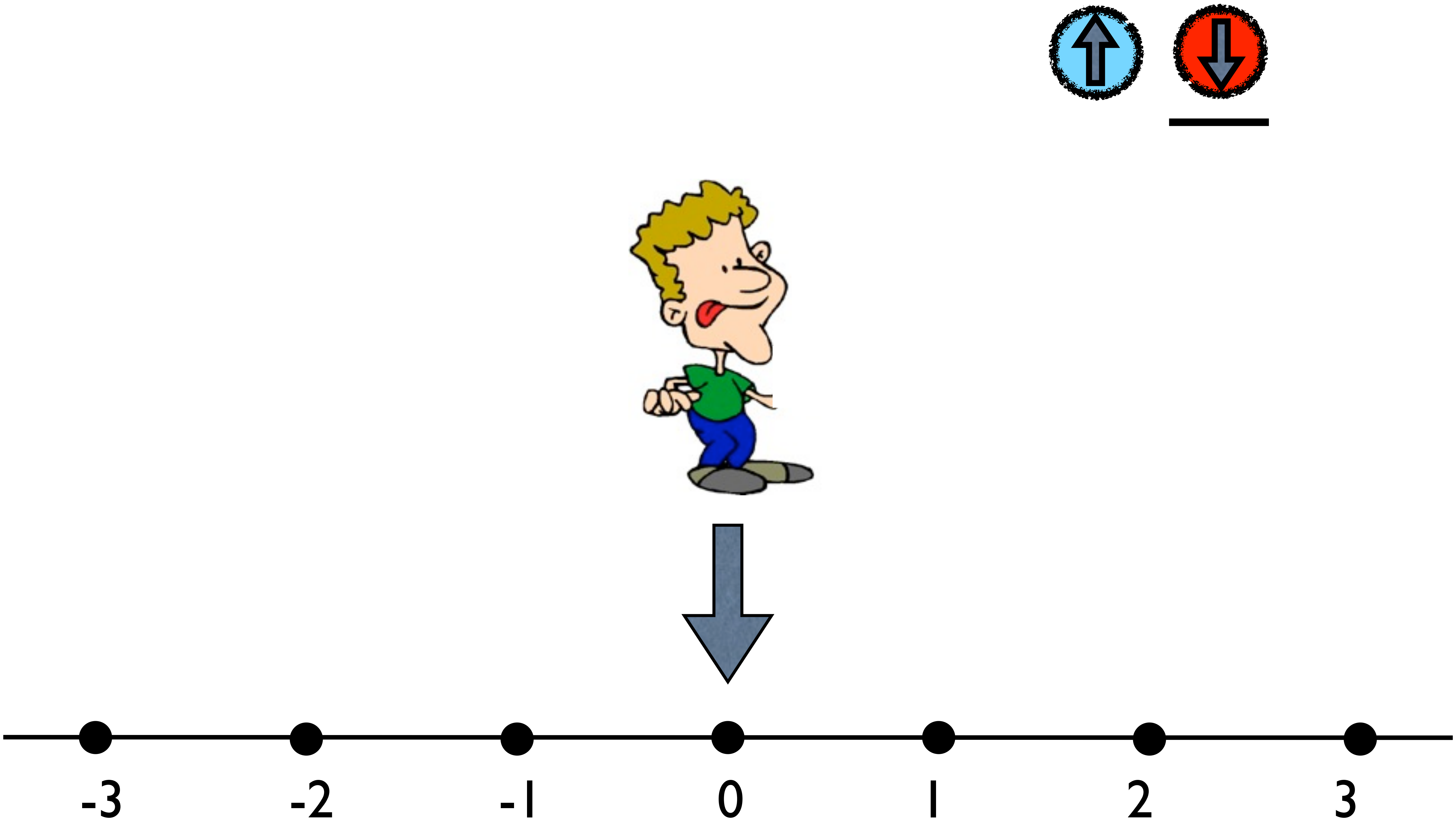
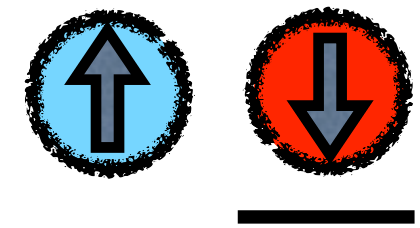
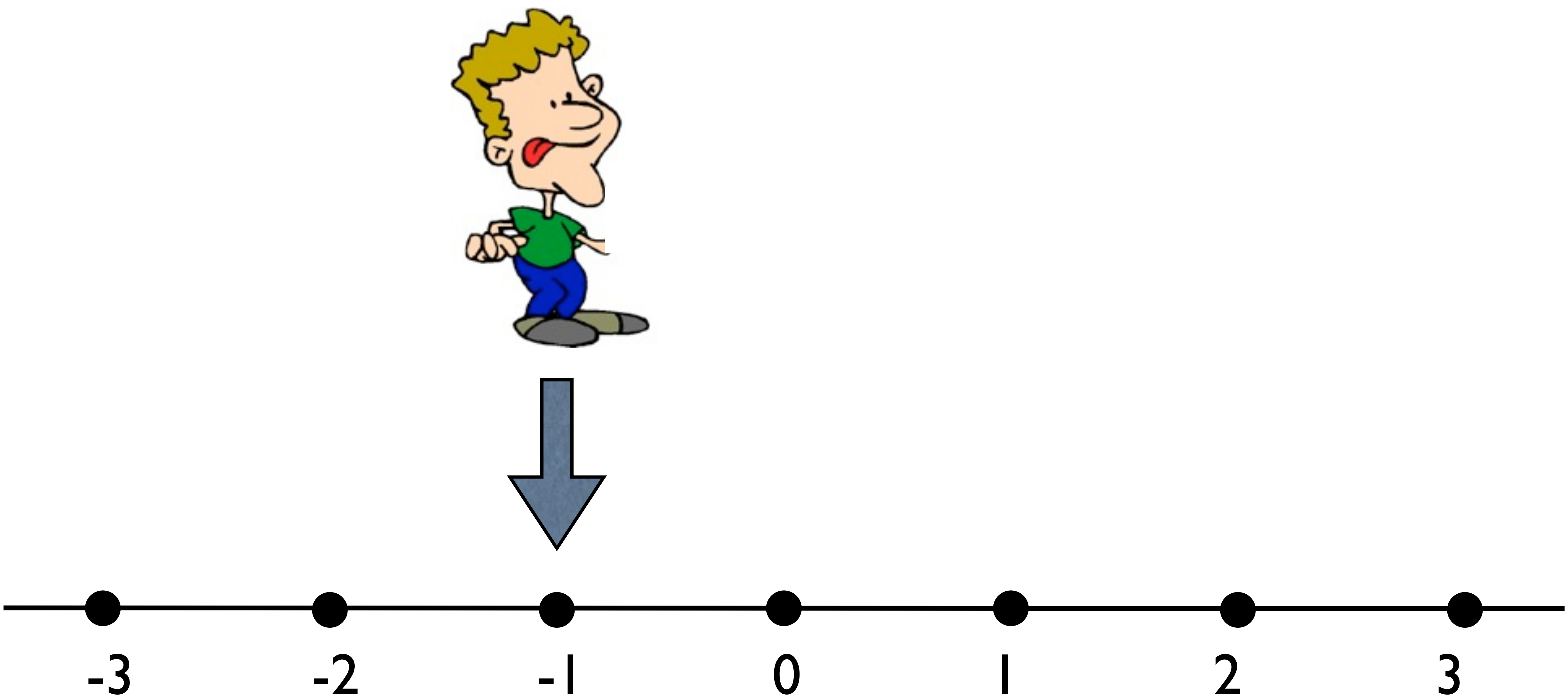


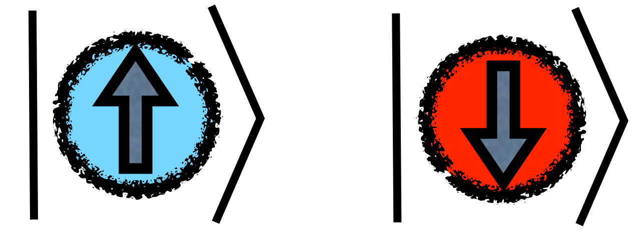
The classical random walk





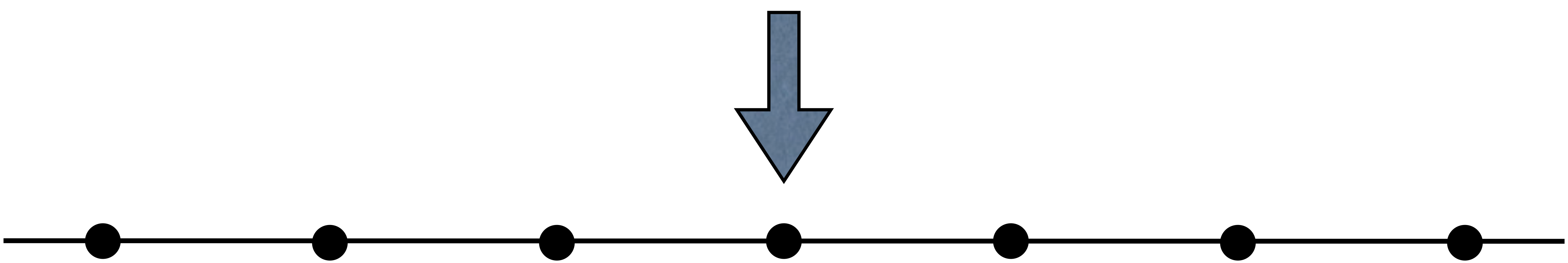
The quantum walk

with a quantum coin



2D Hilbert Space

a quantum walker



$|-3\rangle$ $|-2\rangle$ $|-1\rangle$ $|0\rangle$ $|1\rangle$ $|2\rangle$ $|3\rangle$

infinite D Hilbert Space

Quantum Walk Dynamics

...the simplest scenario...

$$\hat{S} = |\uparrow\rangle\langle\uparrow|\hat{T}_+ + |\downarrow\rangle\langle\downarrow|\hat{T}_- \qquad \hat{U}_c = 1/\sqrt{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

conditional shift *coin rotation*

$$\hat{U} = \hat{S} \cdot (\hat{U}_c \otimes \hat{1})$$

single step operator

state evolved after *n* steps

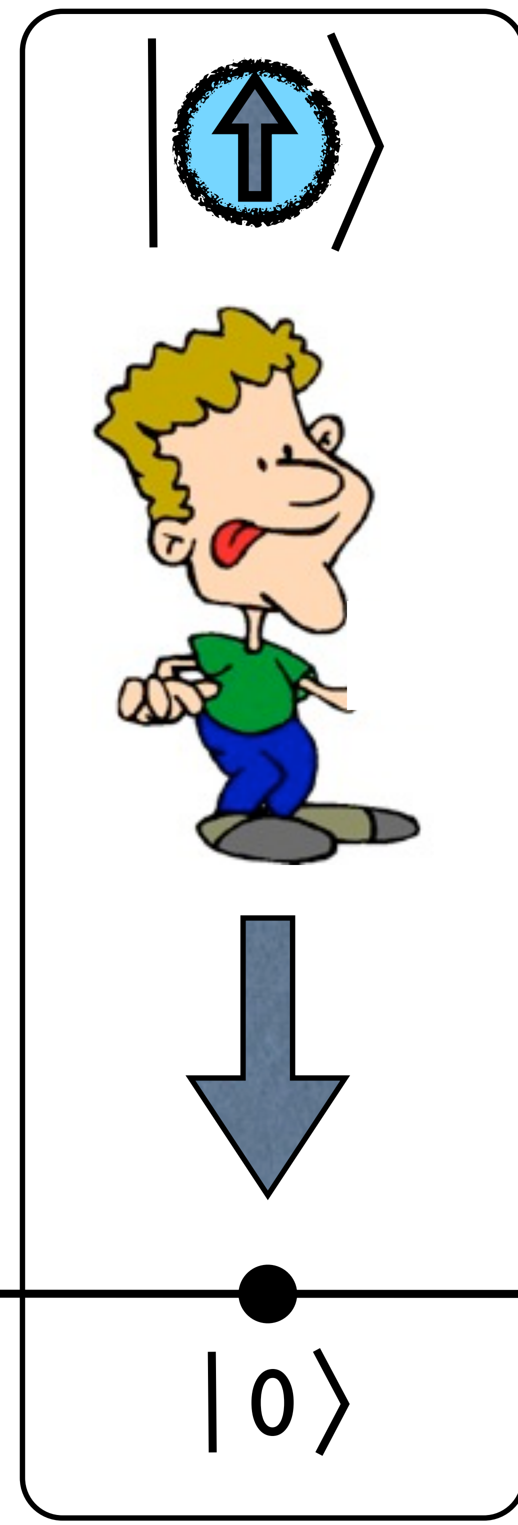
$$|\psi_f\rangle = \hat{U}^n|\psi_0\rangle$$

A cartoon quantum walk

let's start!

$\hat{S}\cdot$ $(\hat{U}_c \otimes \hat{1})$

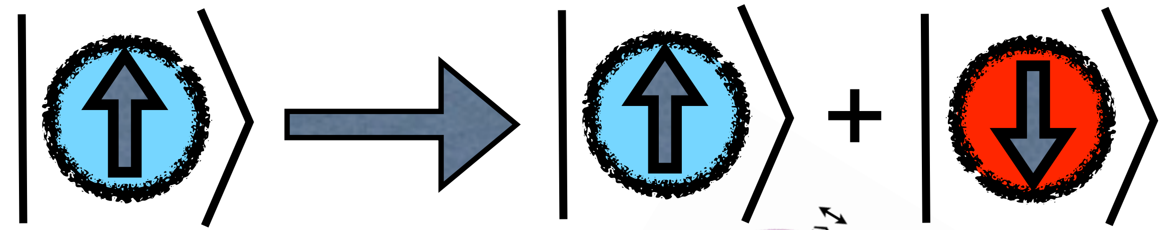
ex:
Hadamard



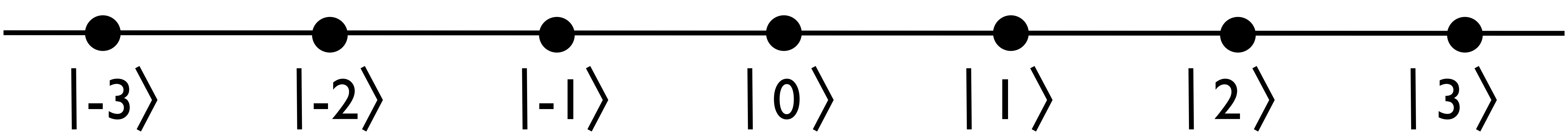
$= |\psi_0\rangle = |\uparrow\rangle \otimes |0\rangle$

$|-3\rangle$ $|-2\rangle$ $|-1\rangle$ $|0\rangle$ $|1\rangle$ $|2\rangle$ $|3\rangle$

A cartoon quantum walk



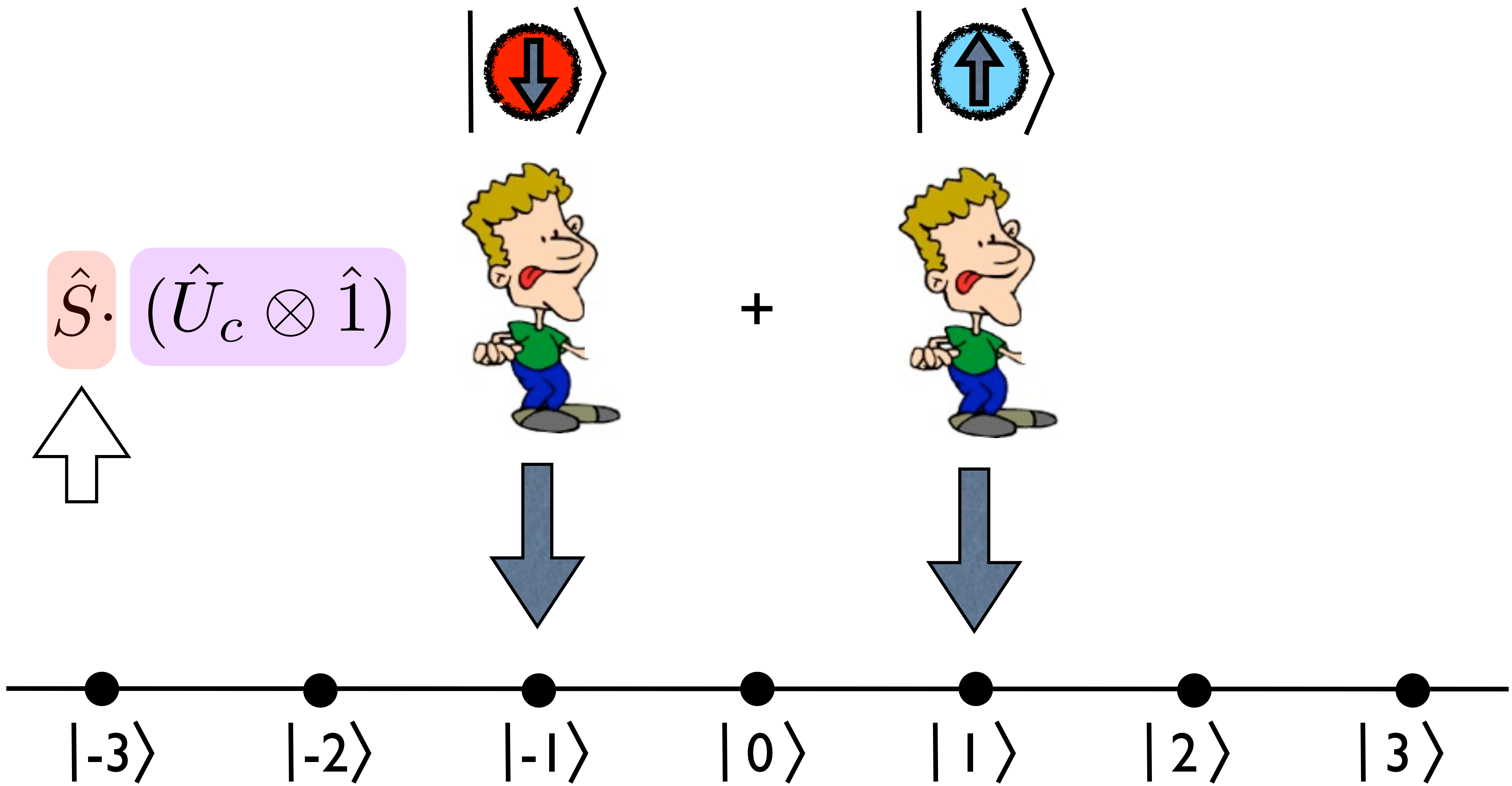
$\hat{S}\cdot\,(\hat{U}_c \otimes \hat{1})$



1st step

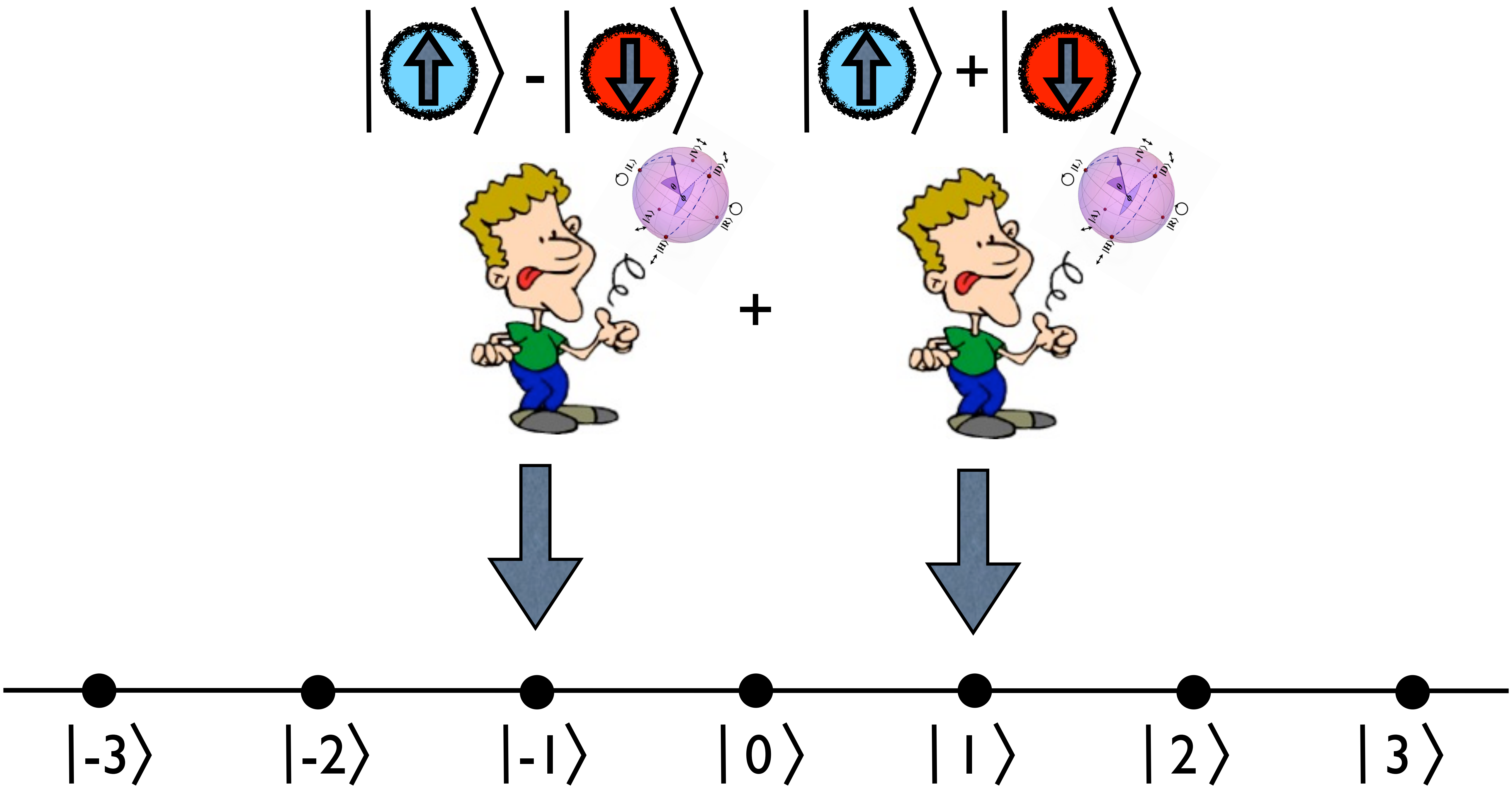
A cartoon quantum walk

$$\hat{S} \cdot (\hat{U}_c \otimes \hat{1})$$



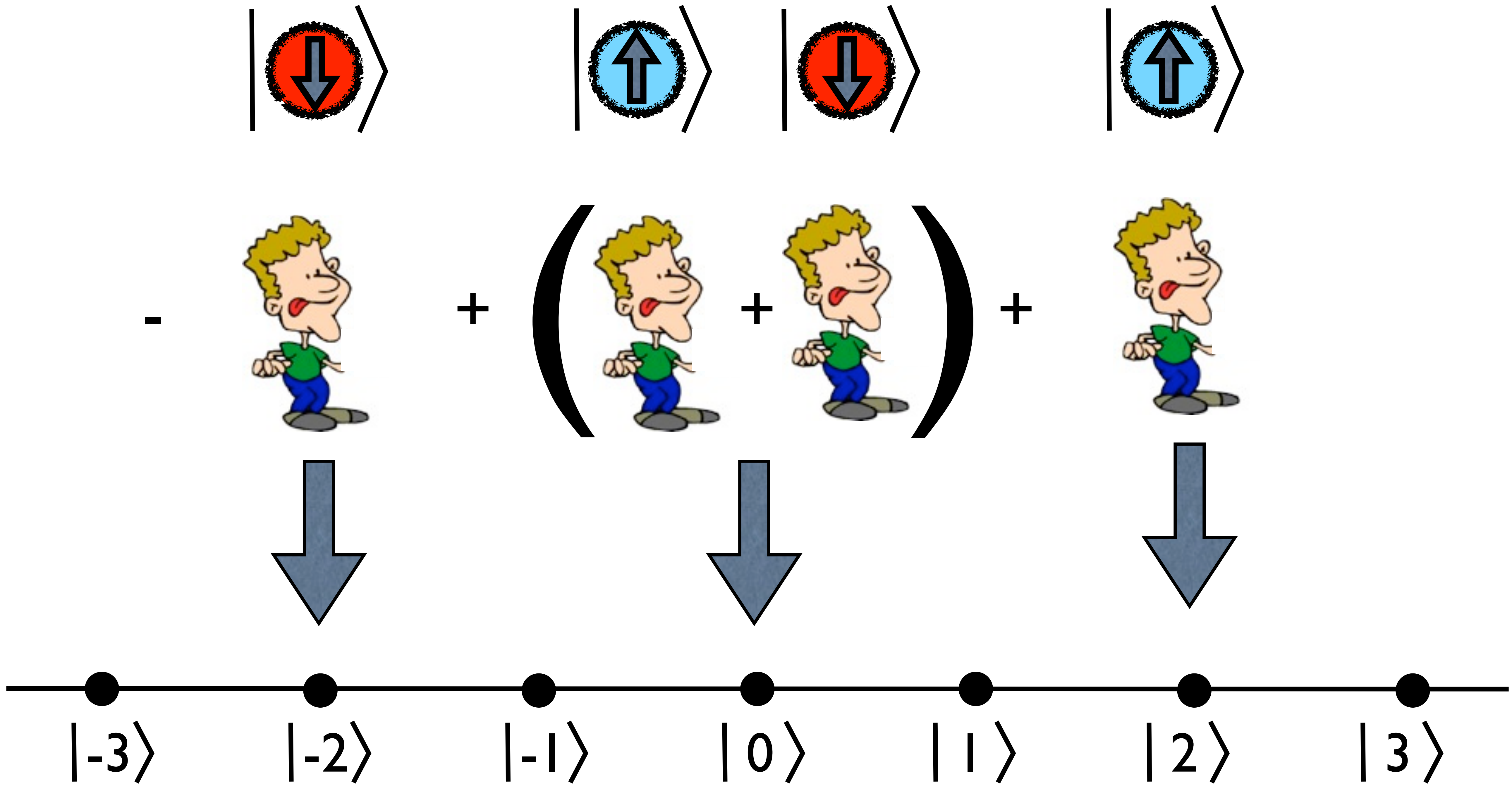
1st step

A cartoon quantum walk



2nd step

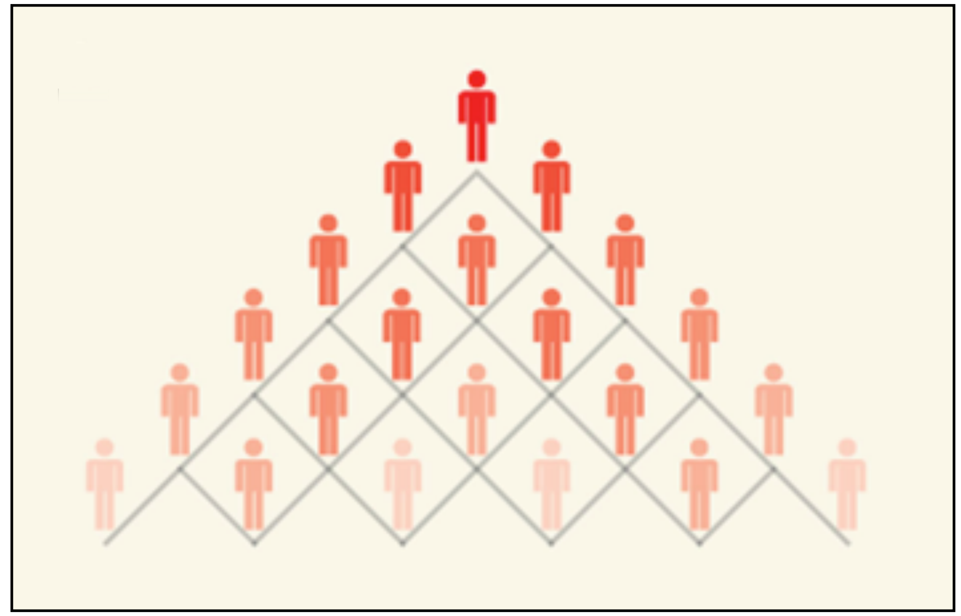
A cartoon quantum walk

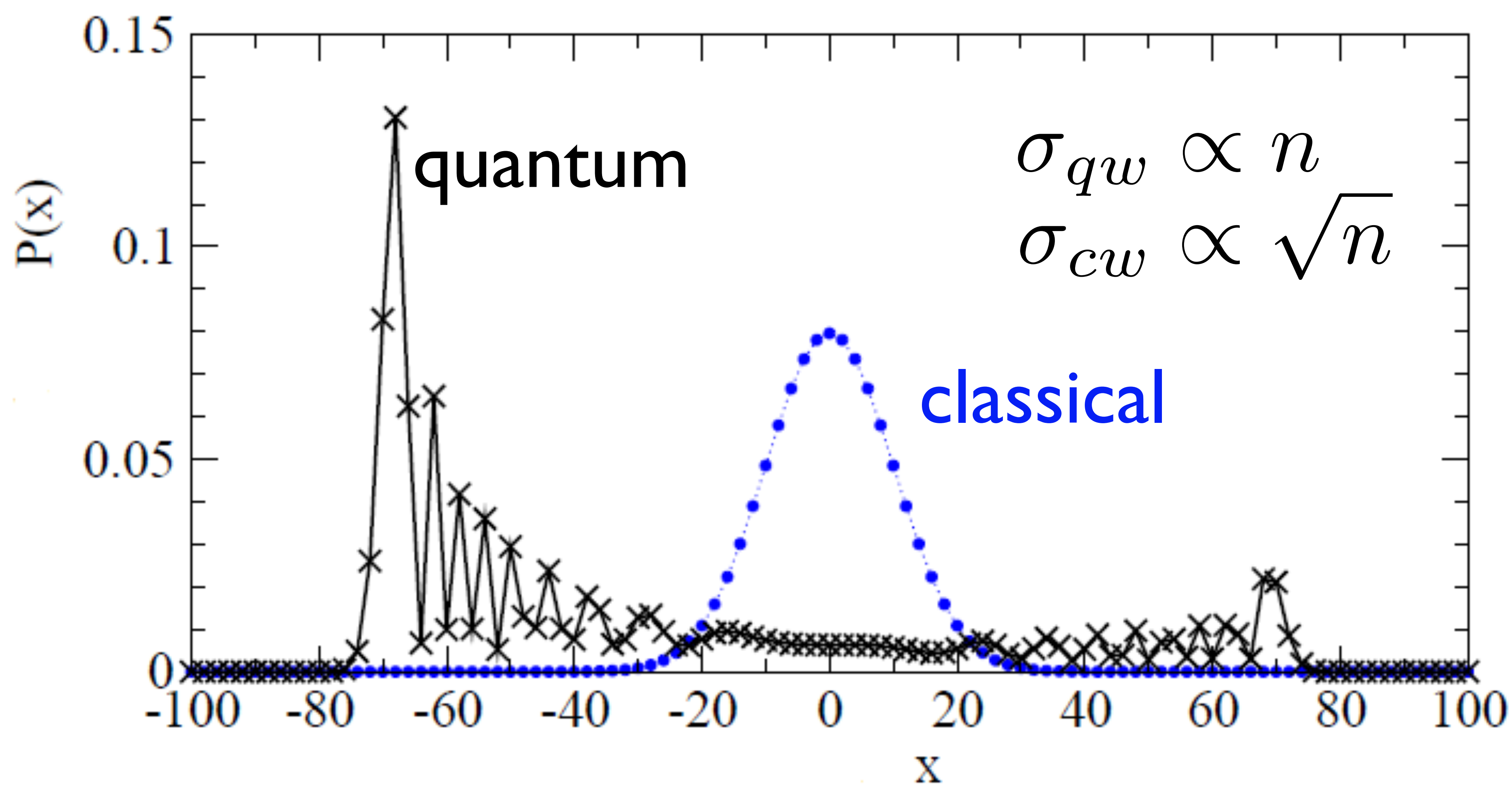


2nd step

Quantum vs Classical

Interference between different paths!





Why realizing QWs?

Quantum algorithms and universal quantum computation

Quantum simulations

transport phenomena

topological phases

...

S. E. Venegas-Andraca, *Quantum Information Processing*, **11**(5):1015–1106, 2012.

Short Outline

- Quantum Walk at a glance
- QWs with twisted photons
- Topological phases in QWs
- Detection of the phase transition

Needed resources for QW simulation

a qu-*dit*

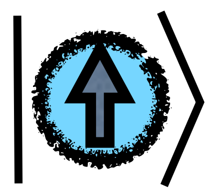
a qu-*bit*

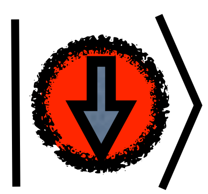
$\hat{U}_c$ **the coin rotation**

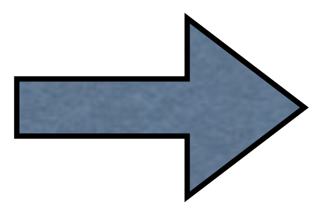
$\hat{S}$ **the conditional shift**

The coin

Coin States

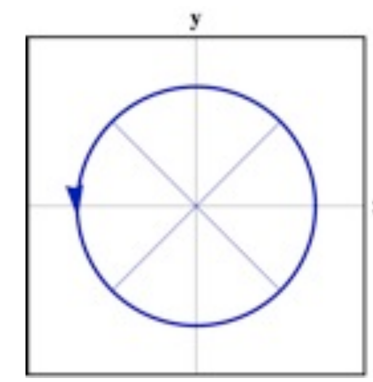




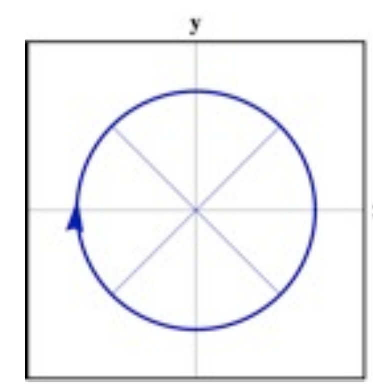


Spin Angular Momentum (Polarization)

$|L\rangle$

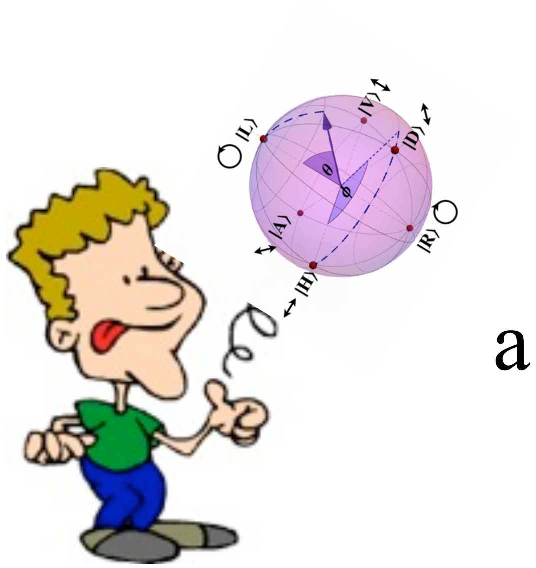


$|R\rangle$



<u>Left/Right Circular Polarization states</u>

The coin rotation



a suitable combination of half-wave plates (HWP) and quarter-wave plates (QWP)

specific case: Hadamard walk

$$|R\rangle \longrightarrow \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$

$$|L\rangle \longrightarrow \frac{|L\rangle - |R\rangle}{\sqrt{2}}$$

QWP @ 45°

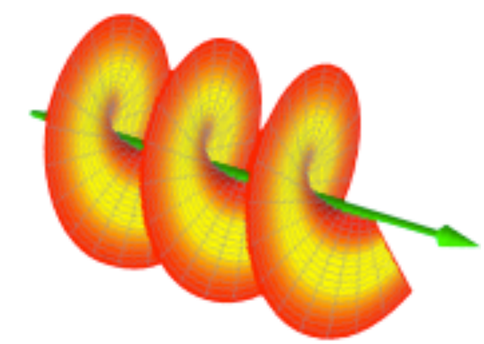
The walker

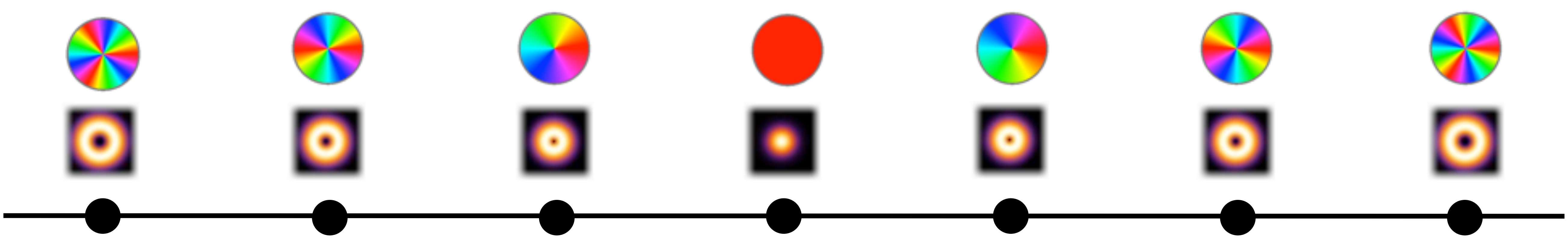
Walker States

Light
Orbital Angular Momentum
OAM

$|x\rangle$

$|m\rangle$

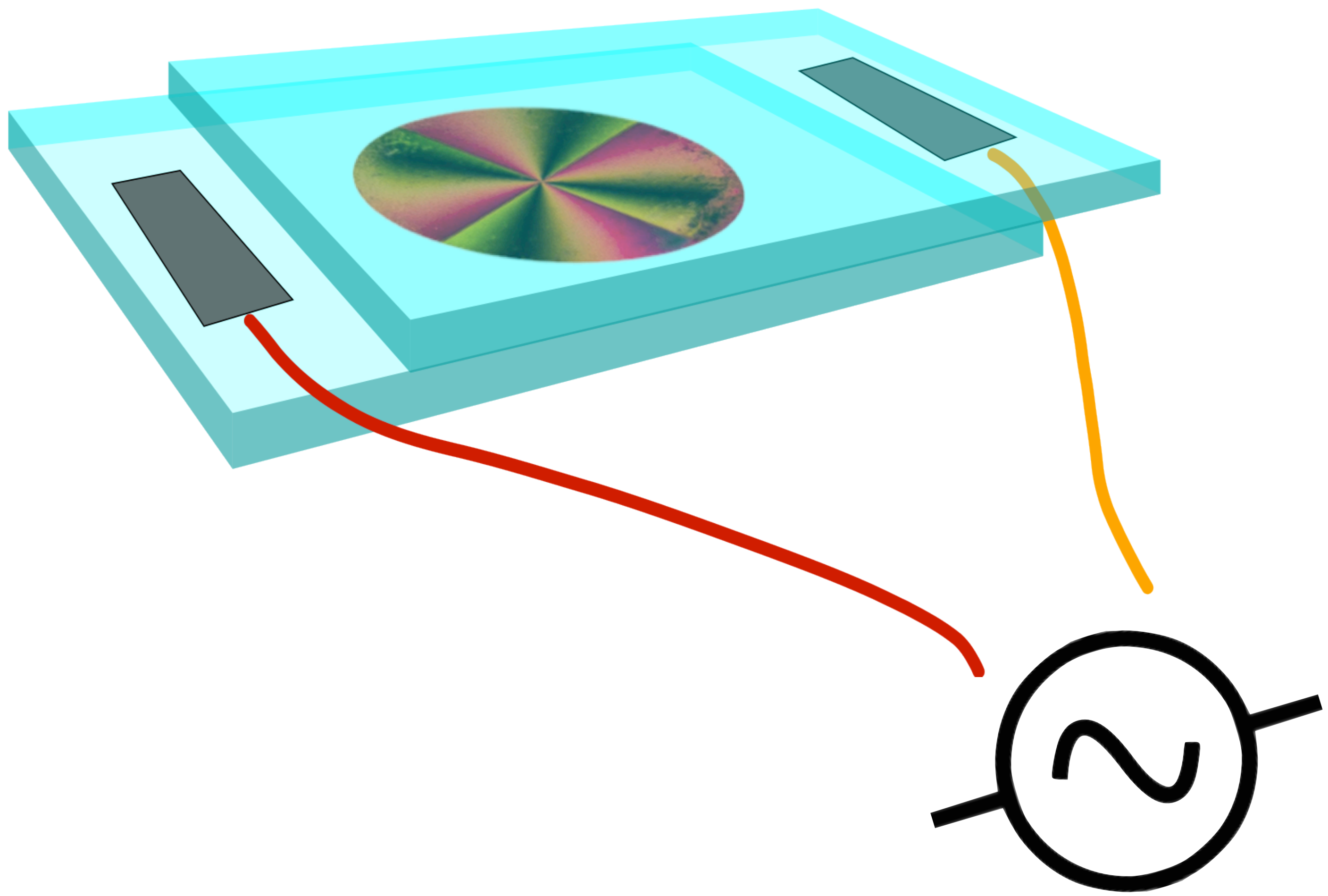




$|{-3}\rangle$ $|{-2}\rangle$ $|{-1}\rangle$ $|0\rangle$ $|1\rangle$ $|2\rangle$ $|3\rangle$

helical modes of single photons

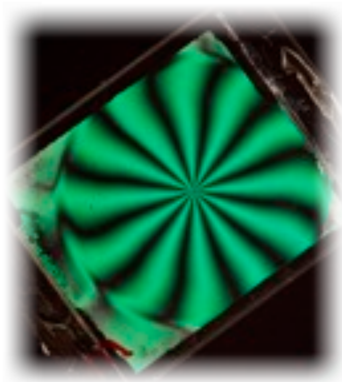
The conditional shift: the *q*-plate!



L. Marrucci, C. Manzo, D. Paparo, *Phys. Rev. Lett*. **96**, 163905 (2006)
Appl. Phys. Lett. **88**, 221102 (2006)

Engineering spin-orbit interaction: the q-plate

Engineering spin-orbit interaction: the *q*-plate



q - plate

The optic axis orientation in the plate is **patterned,** with a **central defect of topological charge *q***

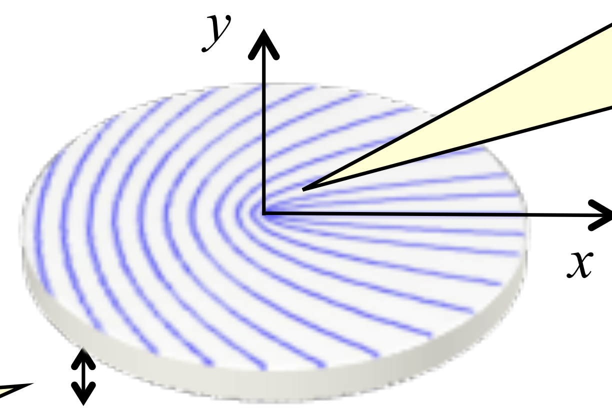


Plate thickness and birefringence chosen so as to have uniform **phase retardation** δ

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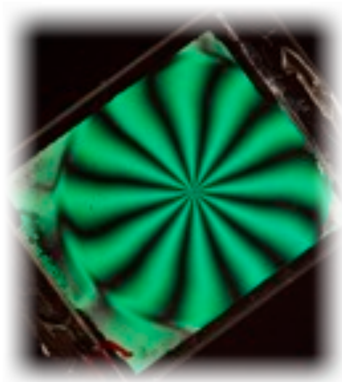

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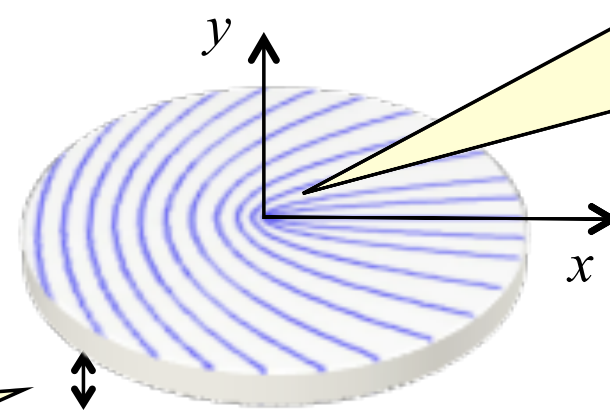

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General pattern:

$$\alpha(x,y)=\alpha(r,\varphi)=q\varphi+\alpha_0$$

with q integer or half-integer

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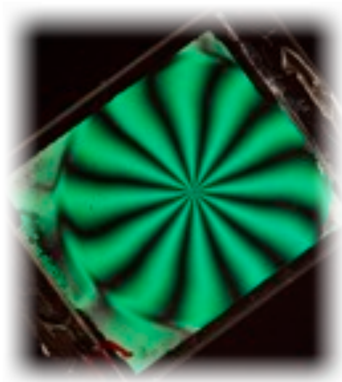

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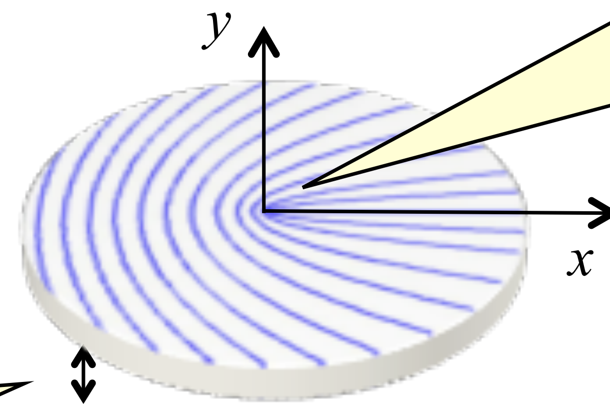
The optic axis orientation in the plate is **patterned,** with a **central defect of topological charge q**



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General pattern:

$$\alpha(x,y)=\alpha(r,\varphi)=q\varphi+\alpha_0$$

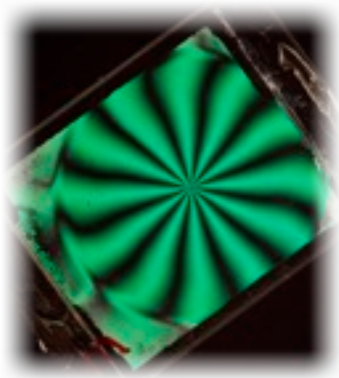
with q integer or half-integer

$$\delta=\pi$$

patterned half-wave plate with non uniform orientation

$$|L\rangle \longrightarrow e^{i\,2\alpha}|R\rangle$$

Engineering spin-orbit interaction: the *q*-plate



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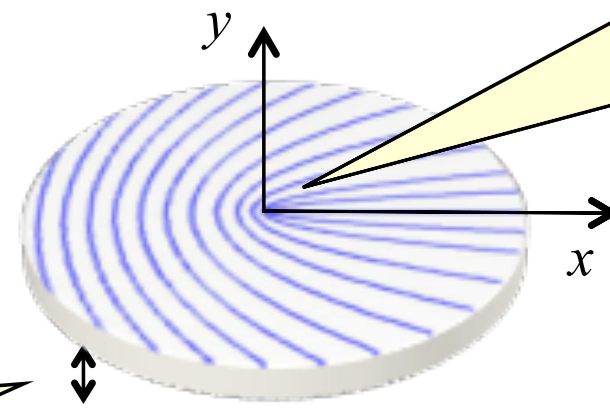

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General pattern:

$$\boxed{\alpha(x,y) = \alpha(r,\varphi) = q\varphi + \alpha_0}$$

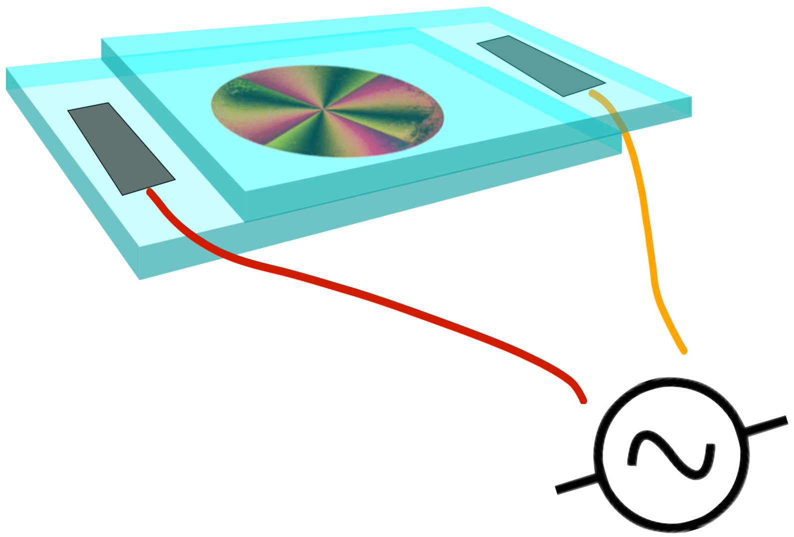
with q integer or half-integer

$$\delta = \pi$$

patterned half-wave plate with non uniform orientation

$$|L\rangle \longrightarrow e^{\boxed{i\,2\alpha}}|R\rangle$$

The conditional shift: the q-plate!



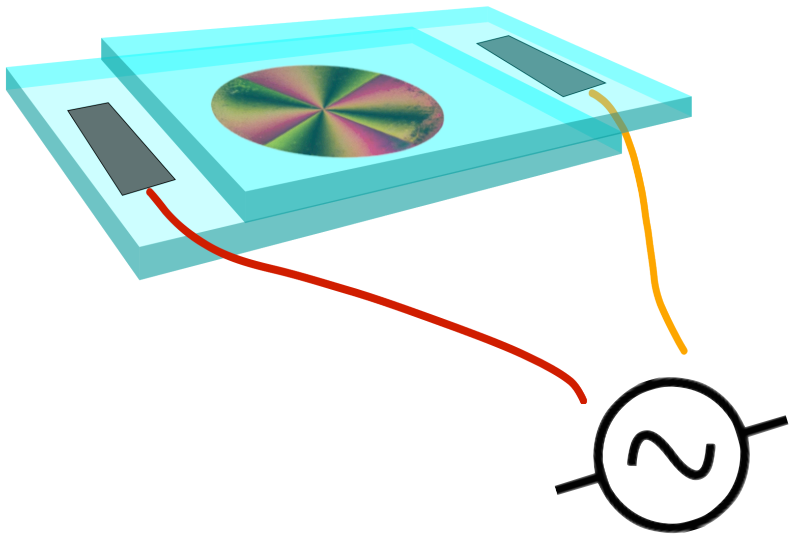
q is the topological charge

$\delta(\lambda, V)$ is the optical retardation

$$|L\rangle_\pi |m\rangle_o \longrightarrow \cos\frac{\delta}{2}|L\rangle_\pi|m\rangle_o + i\sin\frac{\delta}{2}|R\rangle_\pi|m+2q\rangle_o$$

$$|R\rangle_\pi |m\rangle_o \longrightarrow \cos\frac{\delta}{2}|R\rangle_\pi|m\rangle_o + i\sin\frac{\delta}{2}|L\rangle_\pi|m-2q\rangle_o$$

The conditional shift: the *q*-plate!



q is the topological charge

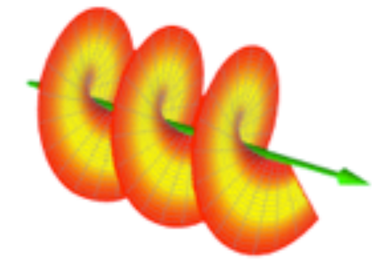
$\delta(\lambda, V)$ is the optical retardation

$$|L\rangle_\pi |m\rangle_o \longrightarrow \qquad \qquad |R\rangle_\pi |m+2q\rangle_o$$

$$\delta = \pi$$

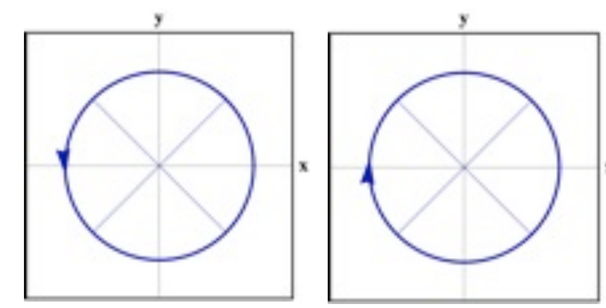
$$|R\rangle_\pi |m\rangle_o \longrightarrow \qquad \qquad |L\rangle_\pi |m-2q\rangle_o$$

Needed resources for QW simulation



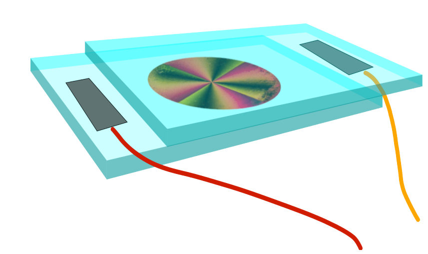
a qu-*dit*

a qu-*bit*



$\hat{U}_c$ **the coin rotation**

$\hat{S}$ **the conditional shift**



Quantum Walk Dynamics

...the simplest scenario...

$$\hat{S} = |\uparrow\rangle\langle\uparrow|\hat{T}_{+} + |\downarrow\rangle\langle\downarrow|\hat{T}_{-} \qquad \hat{U}_c = 1/\sqrt{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

conditional shift *coin rotation*

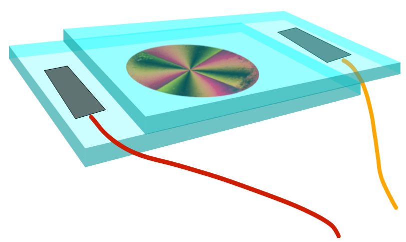
$$\hat{U} = \hat{S}\cdot(\hat{U}_c \otimes \hat{1})$$

single step operator

state evolved after *n* steps $$|\psi_f\rangle = \hat{U}^n|\psi_0\rangle$$

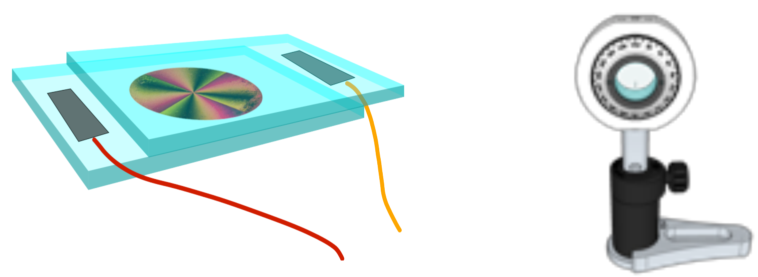
Quantum Walk Dynamics

...our implementation...



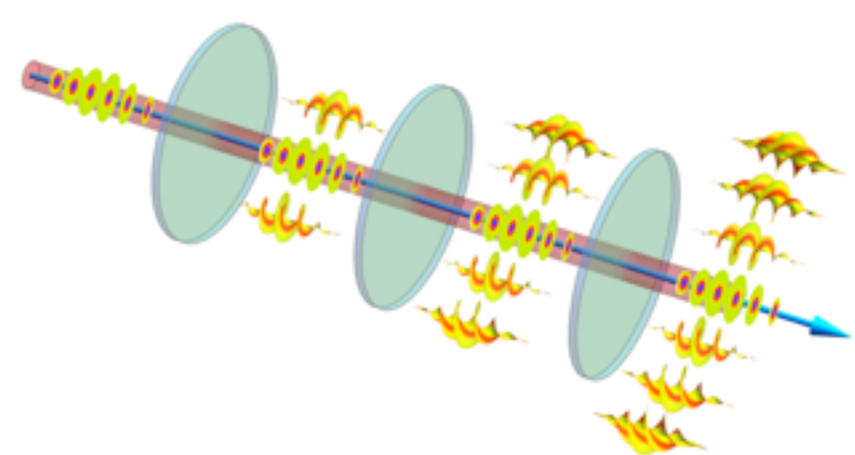
conditional shift

coin rotation

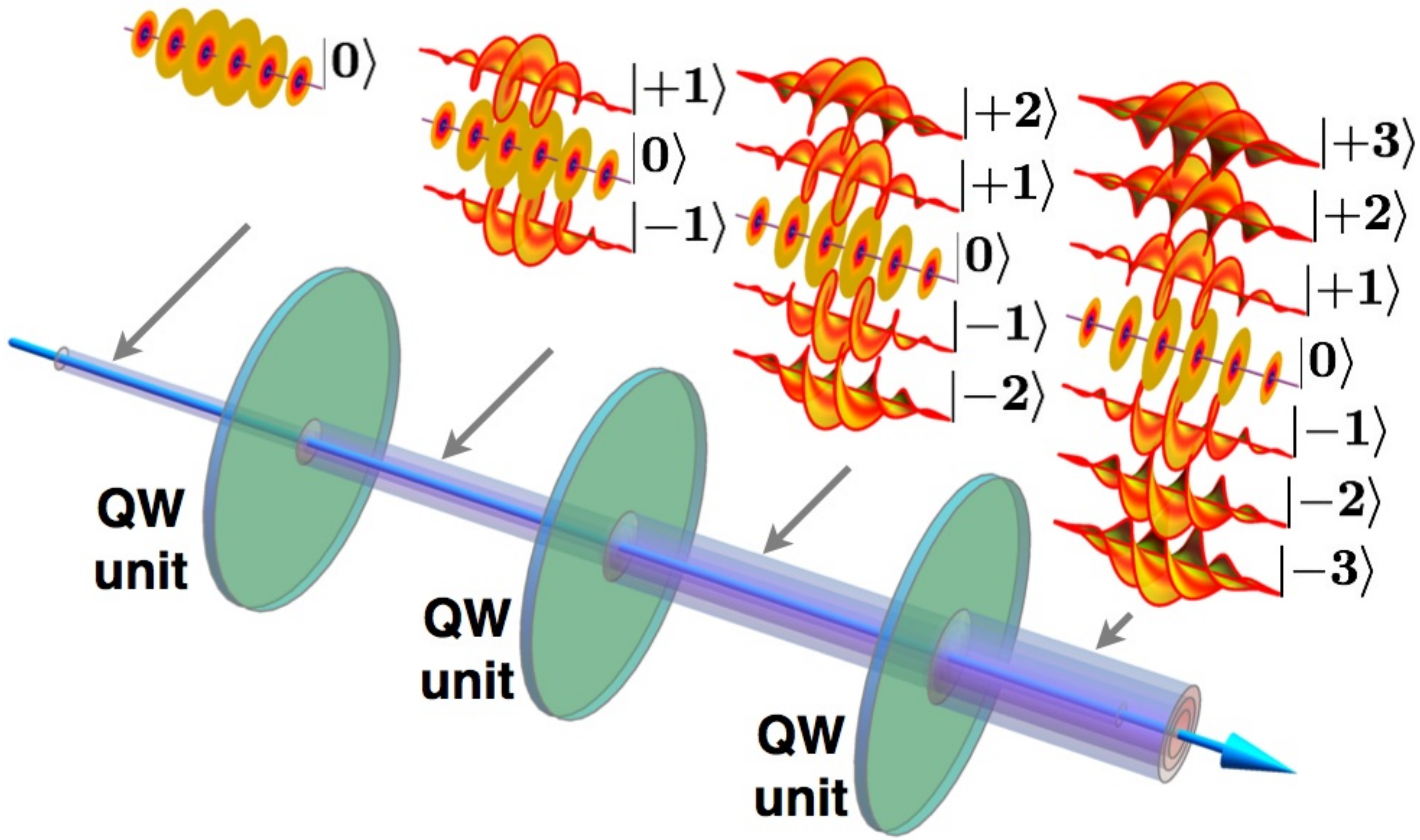


single step operator

state evolved after *n* steps



QW in the OAM space of light



a single light beam

interferometric stability is intrinsic

Cardano F. *et al.*, *Science Advances* **1**, e1500087 (2015)

Short Outline

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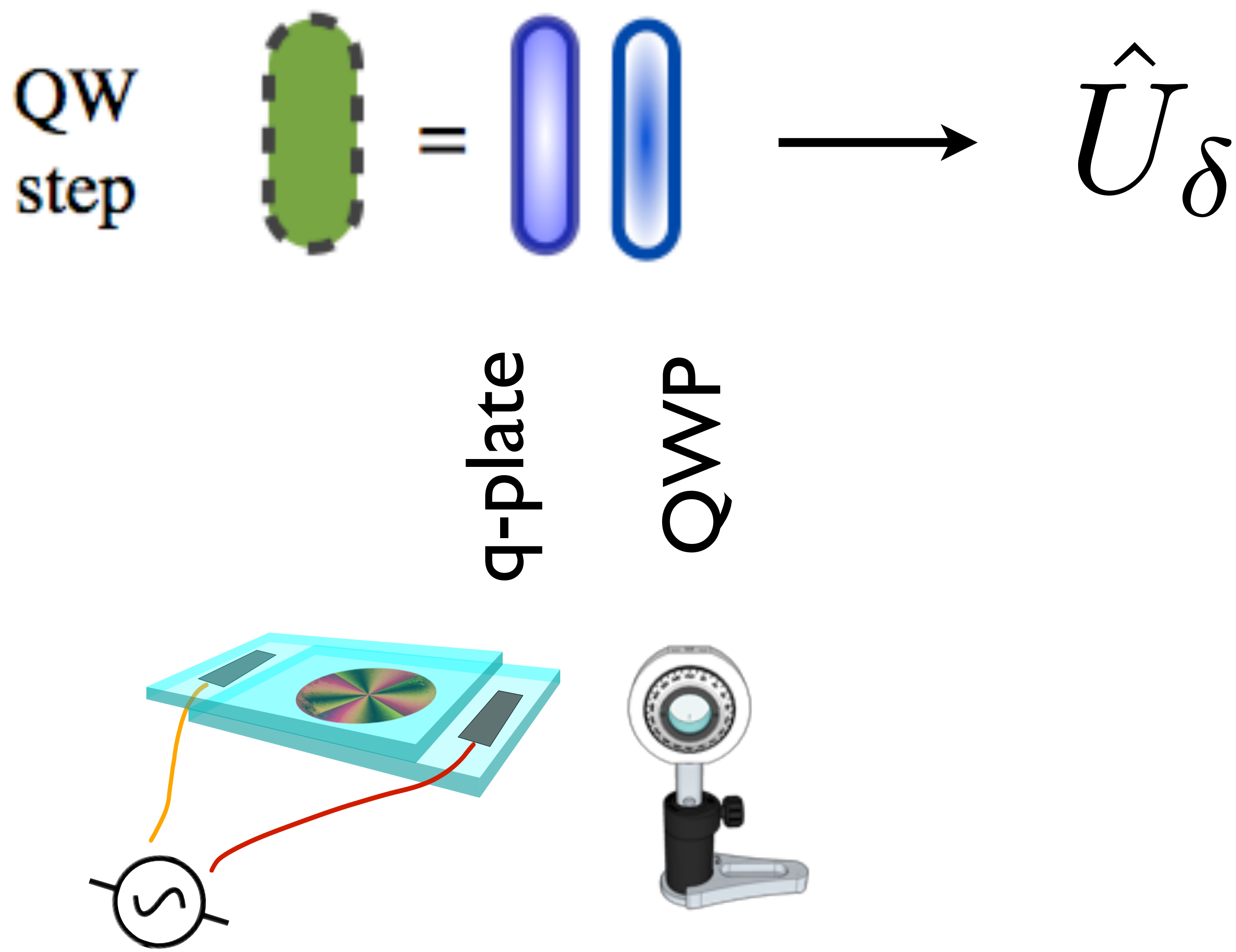
A strange feature...



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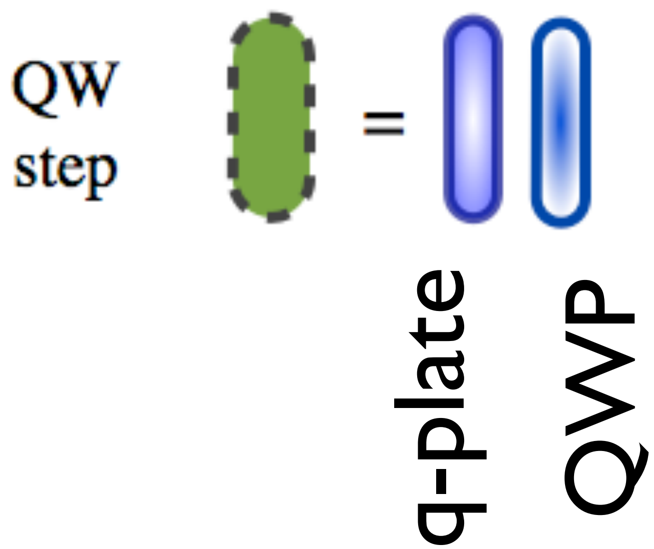
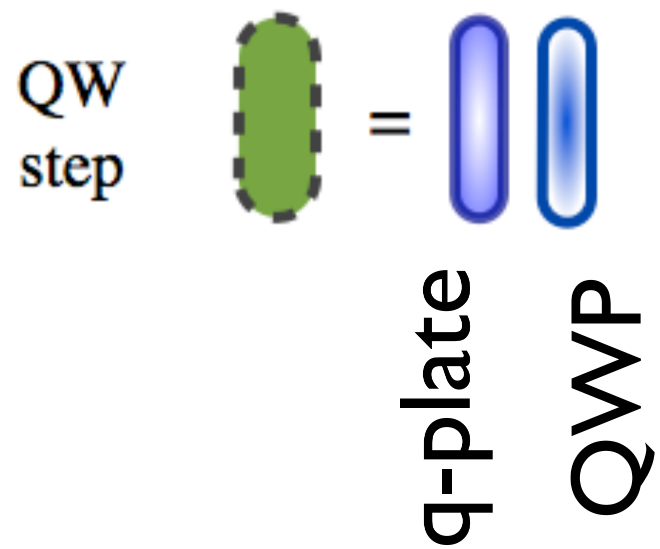


$$\longrightarrow \hat{U}_\delta$$

$$M_2 = \sum_m m^2 \, P(m)$$

$$P(m) = |\langle m|\psi_f\rangle|^2$$

A strange feature...



$\longrightarrow \hat{U}_\delta$

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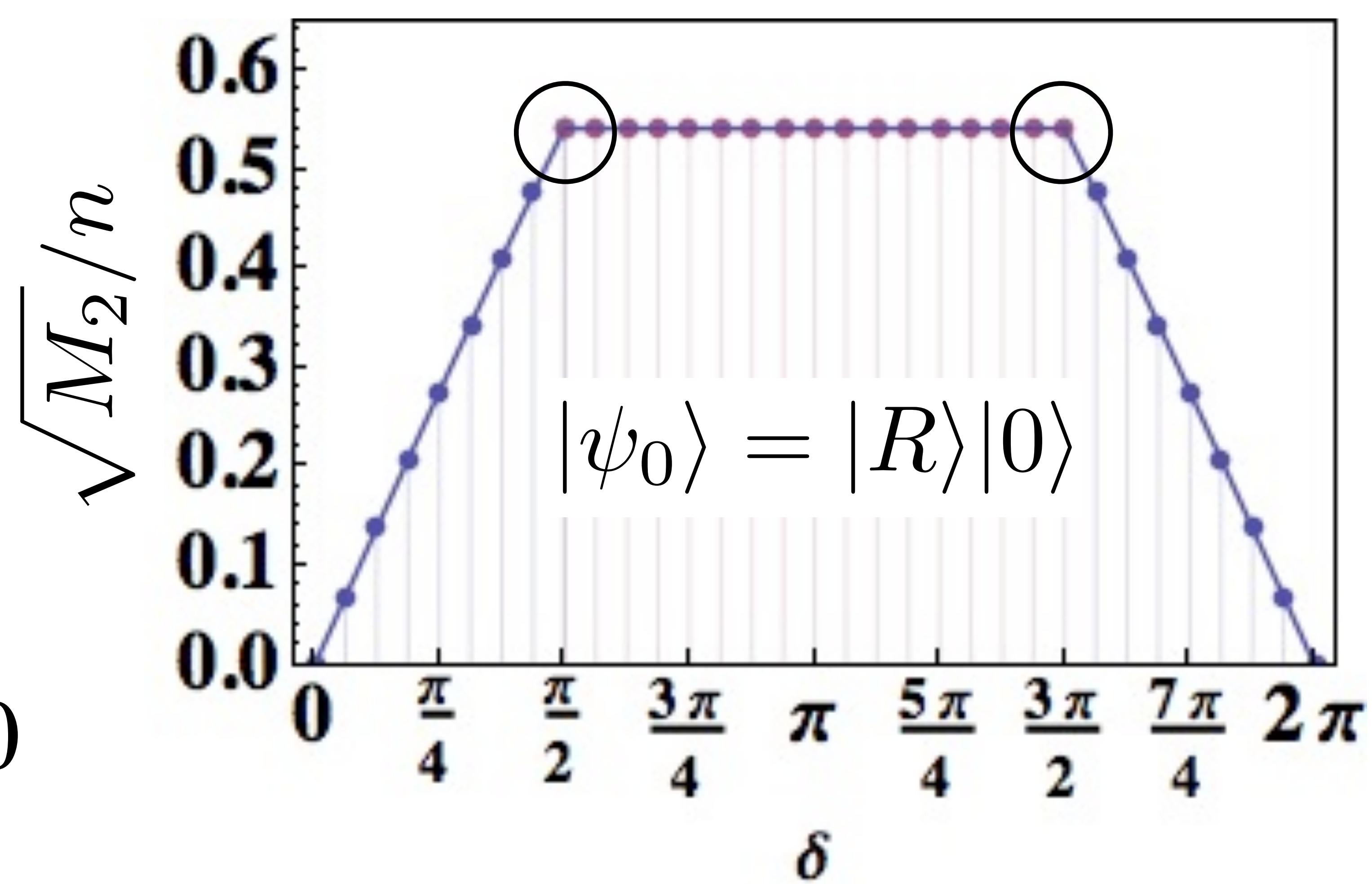


n = 50

why slope discontinuities?

QW Band Structure

looking for QW eigenstates and eigenvalues (quasi - energies)

$$\hat{U}|\psi\rangle = e^{-i\,E}\,|\psi\rangle$$

QW Band Structure

eigenstates

translation symmetry
$+$
lattice

quasi-momentum *k*

$$k \in \{-\pi, \pi\}$$

two possible states in each lattice site

band index *s*

$$s \in \{1, 2\}$$

$$|\psi\rangle = |k, s\rangle = \boxed{|k\rangle} \otimes \boxed{|\phi_s(k)\rangle}$$

walker **coin**

Coin Eigenstates

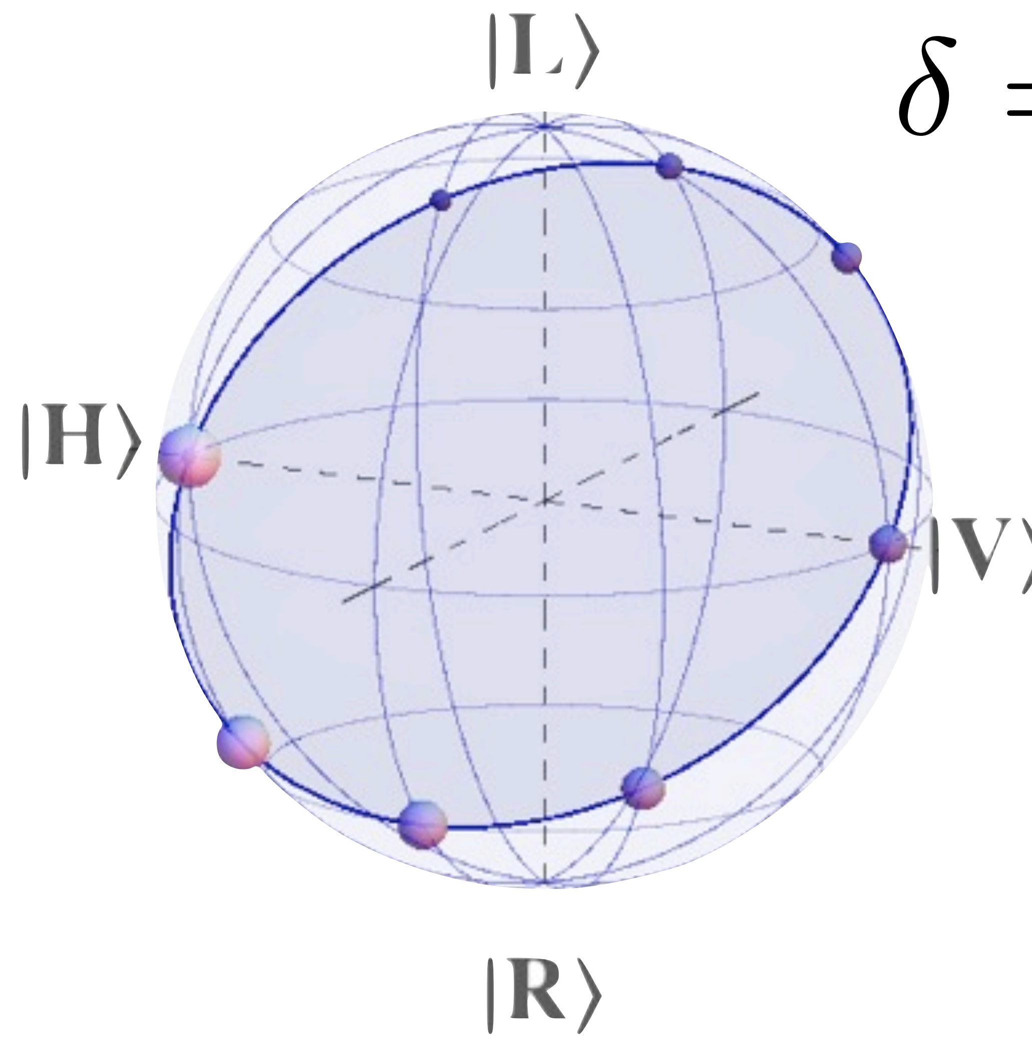
$$|k, s\rangle = |k\rangle \otimes \boxed{|\phi_s(k)\rangle}$$

coin

$\mathbf{n}(k)$

$-\pi$ π k

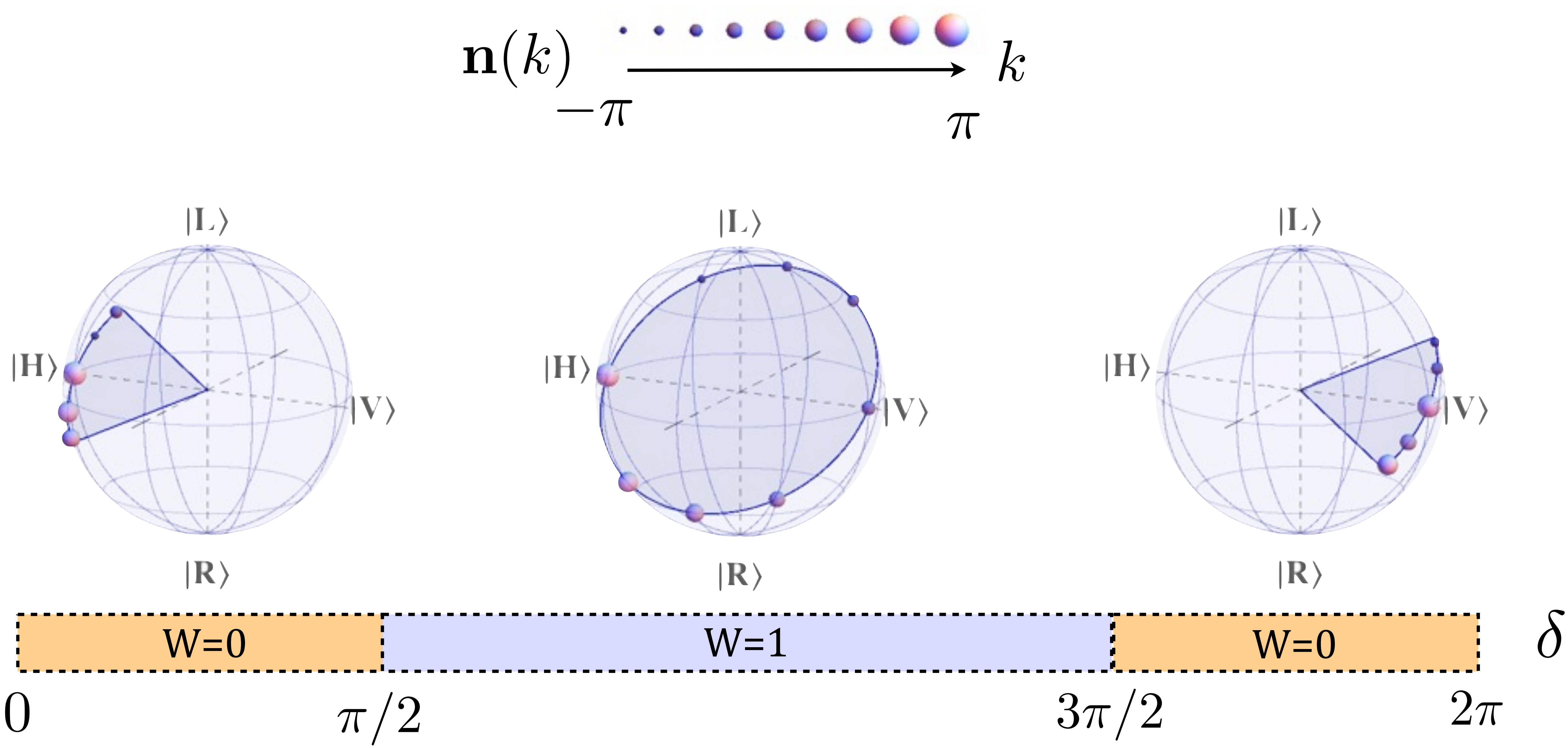
3D unit vector on the Poincaré sphere



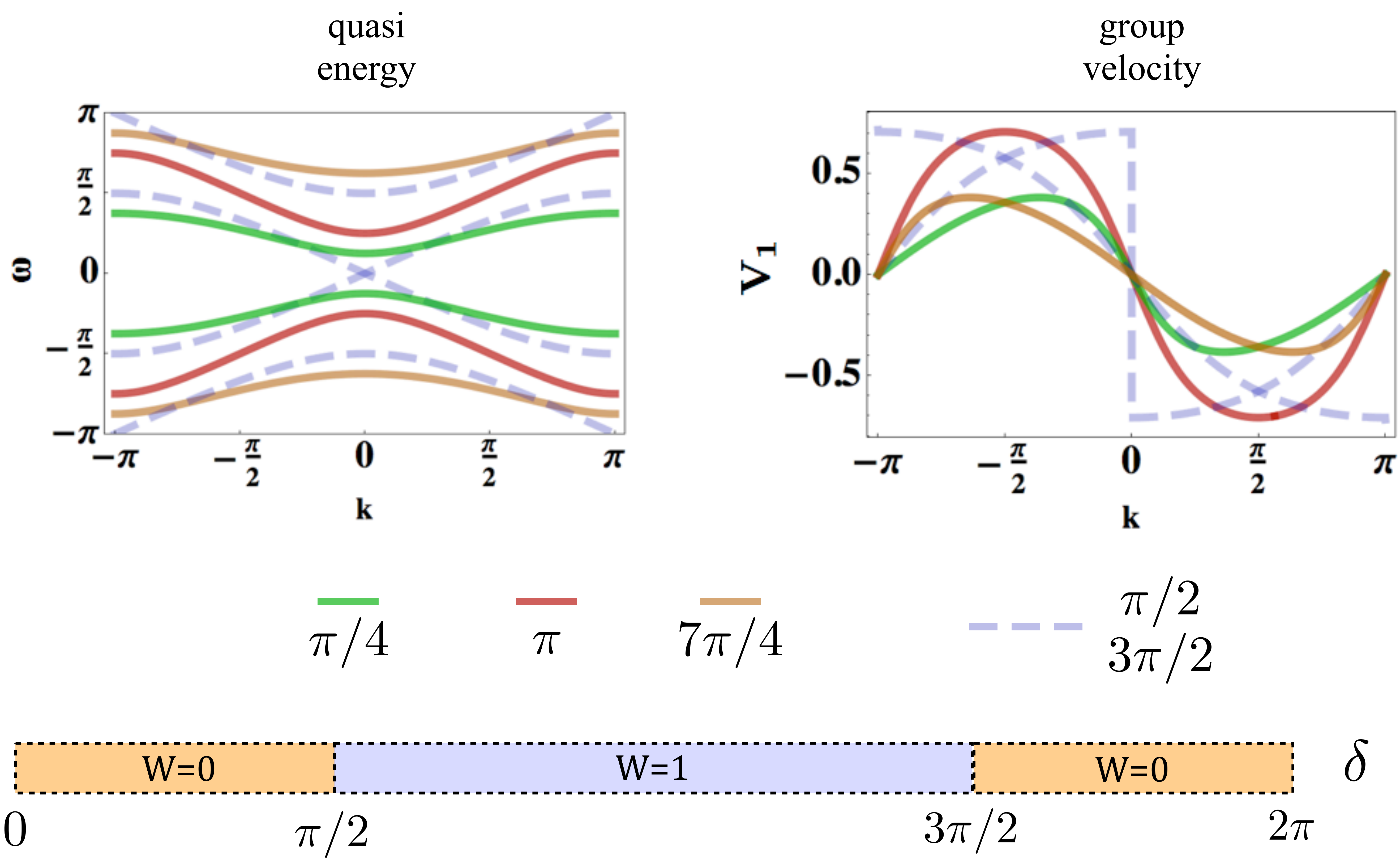
$\delta = \pi$

the winding number W of coin eigenstates is a topological invariant!

Distinct topological phases

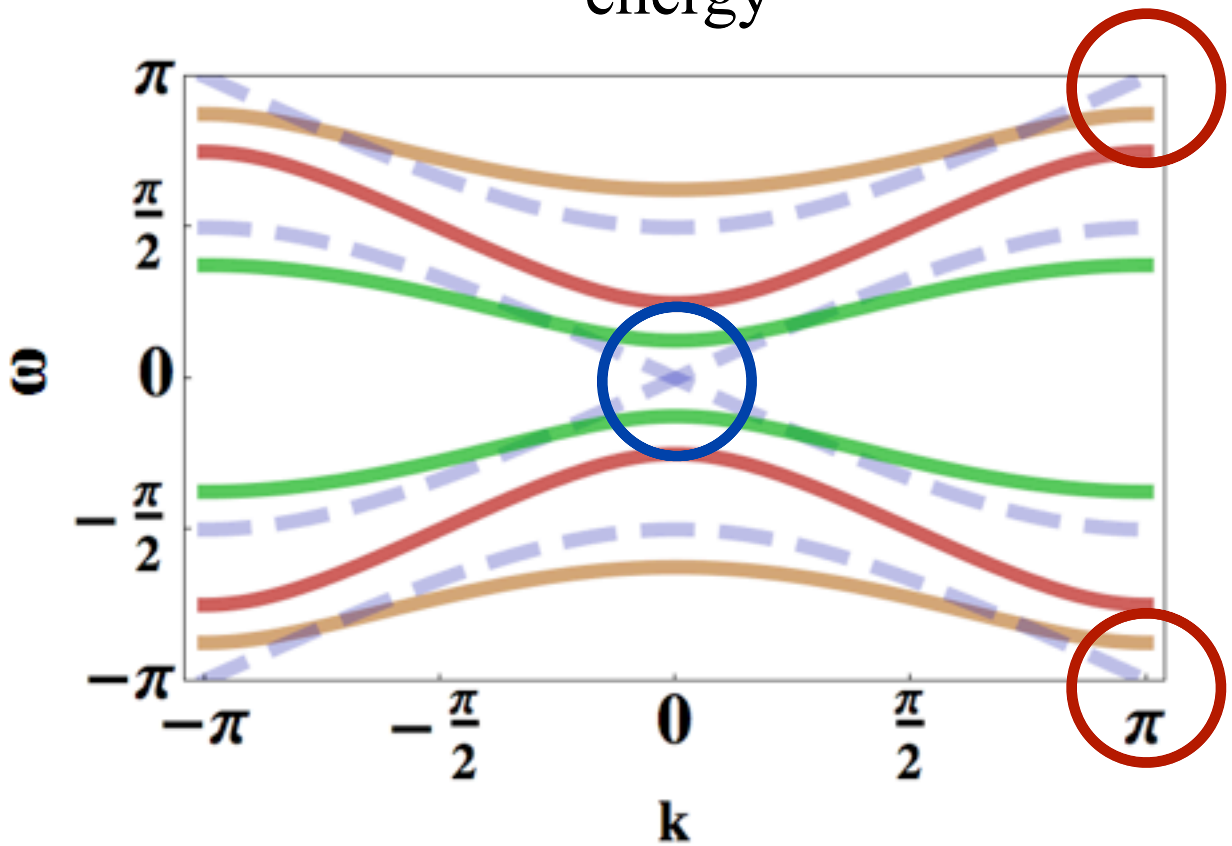


Distinct topological phases

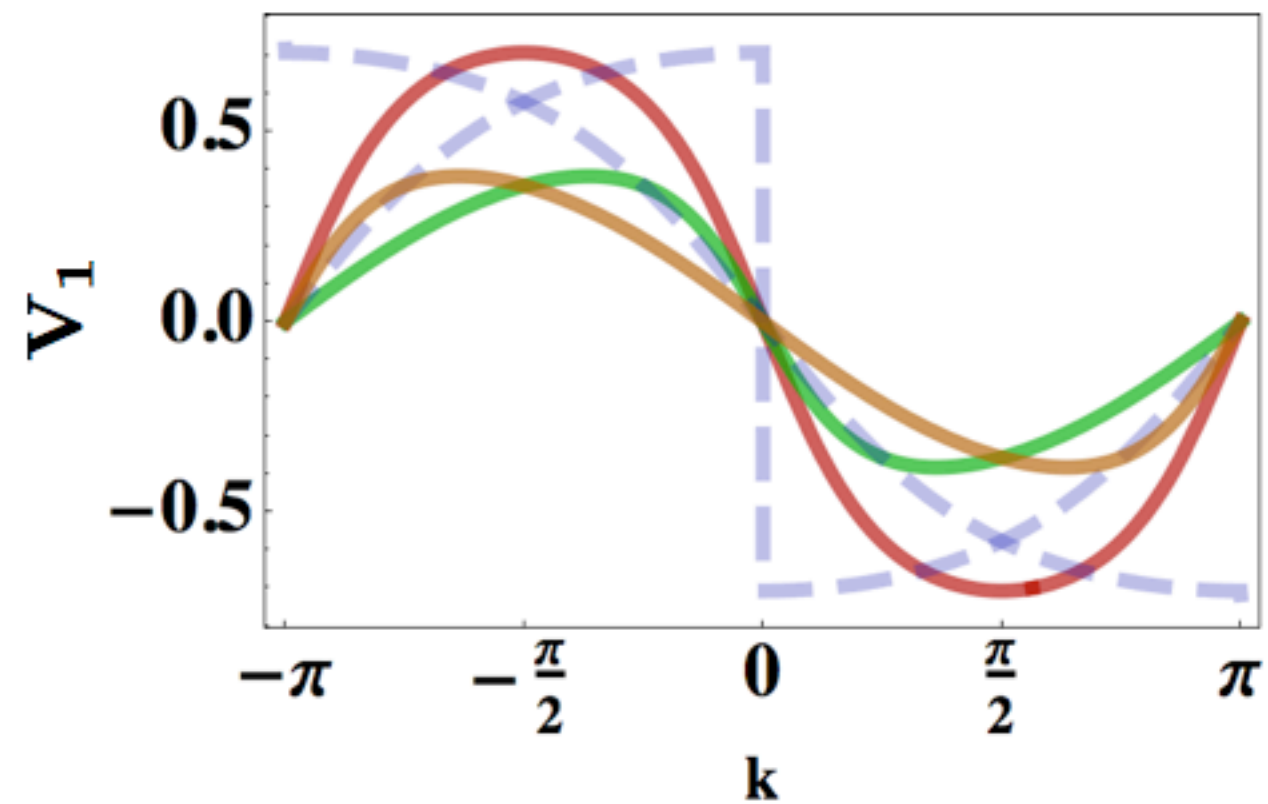


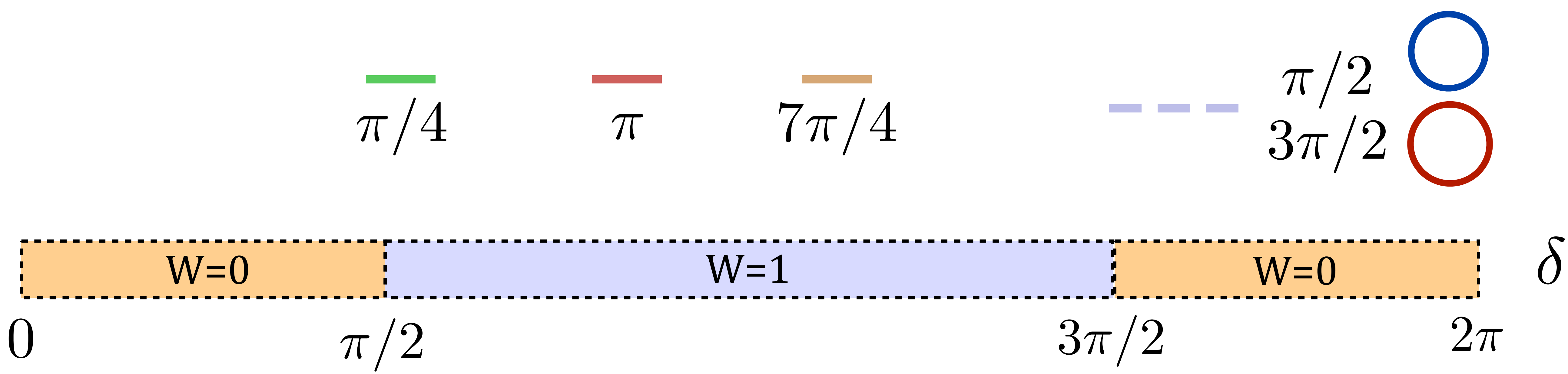
Distinct topological phases

quasi energy



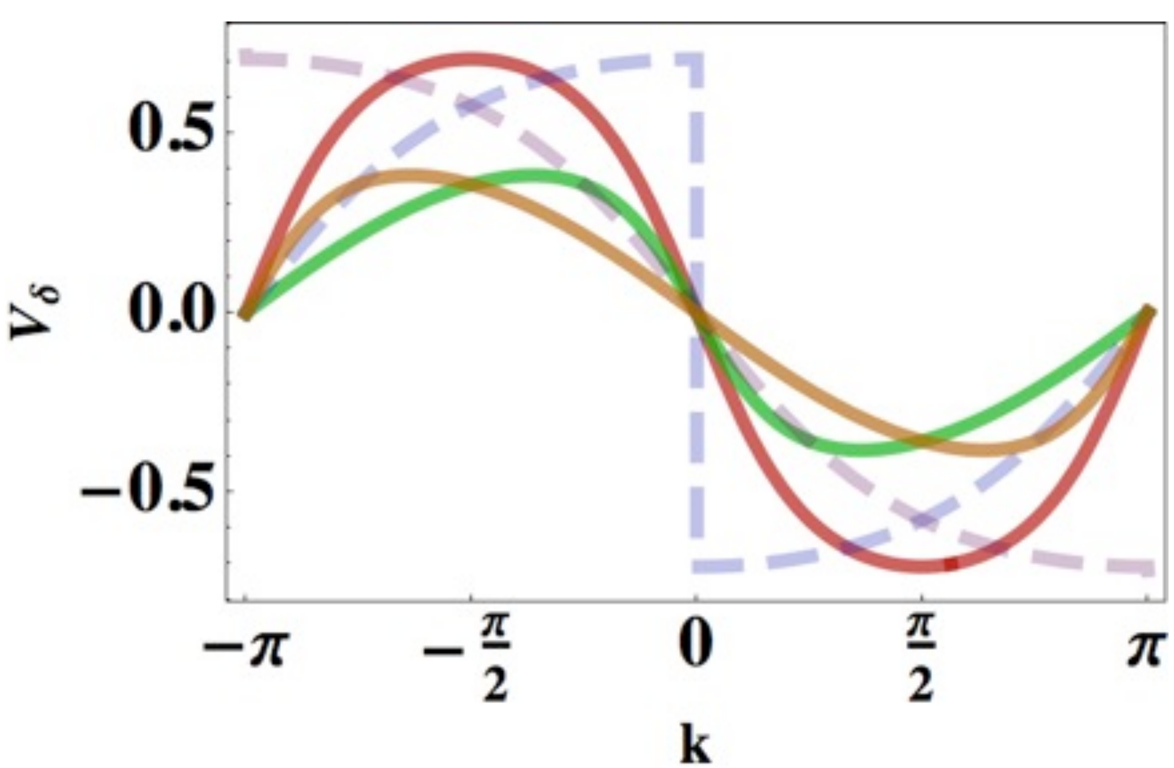
group velocity

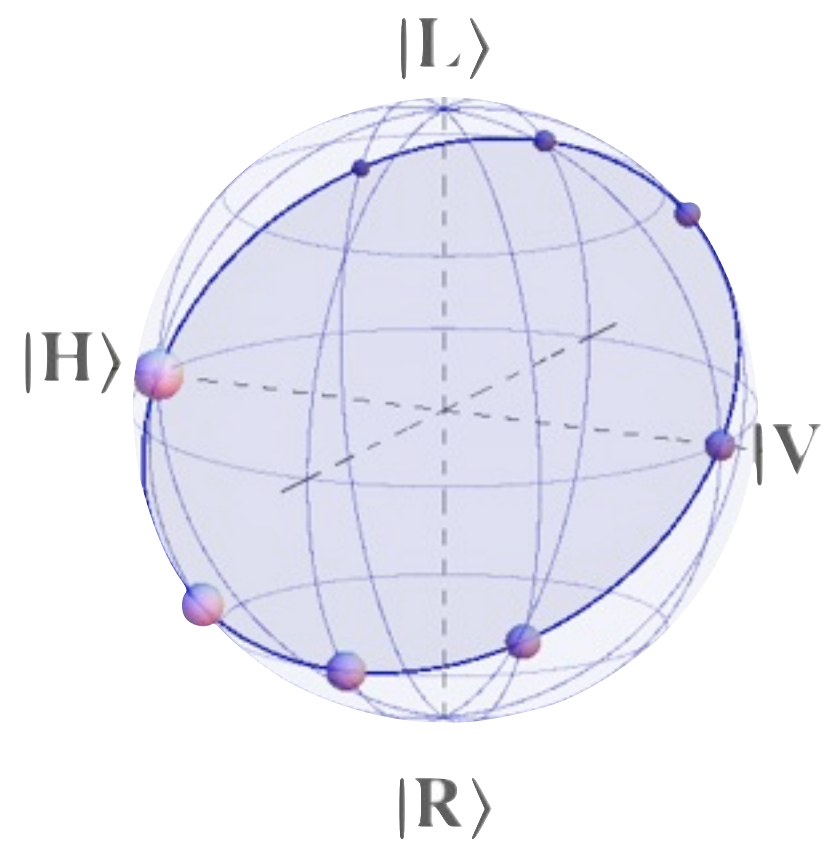




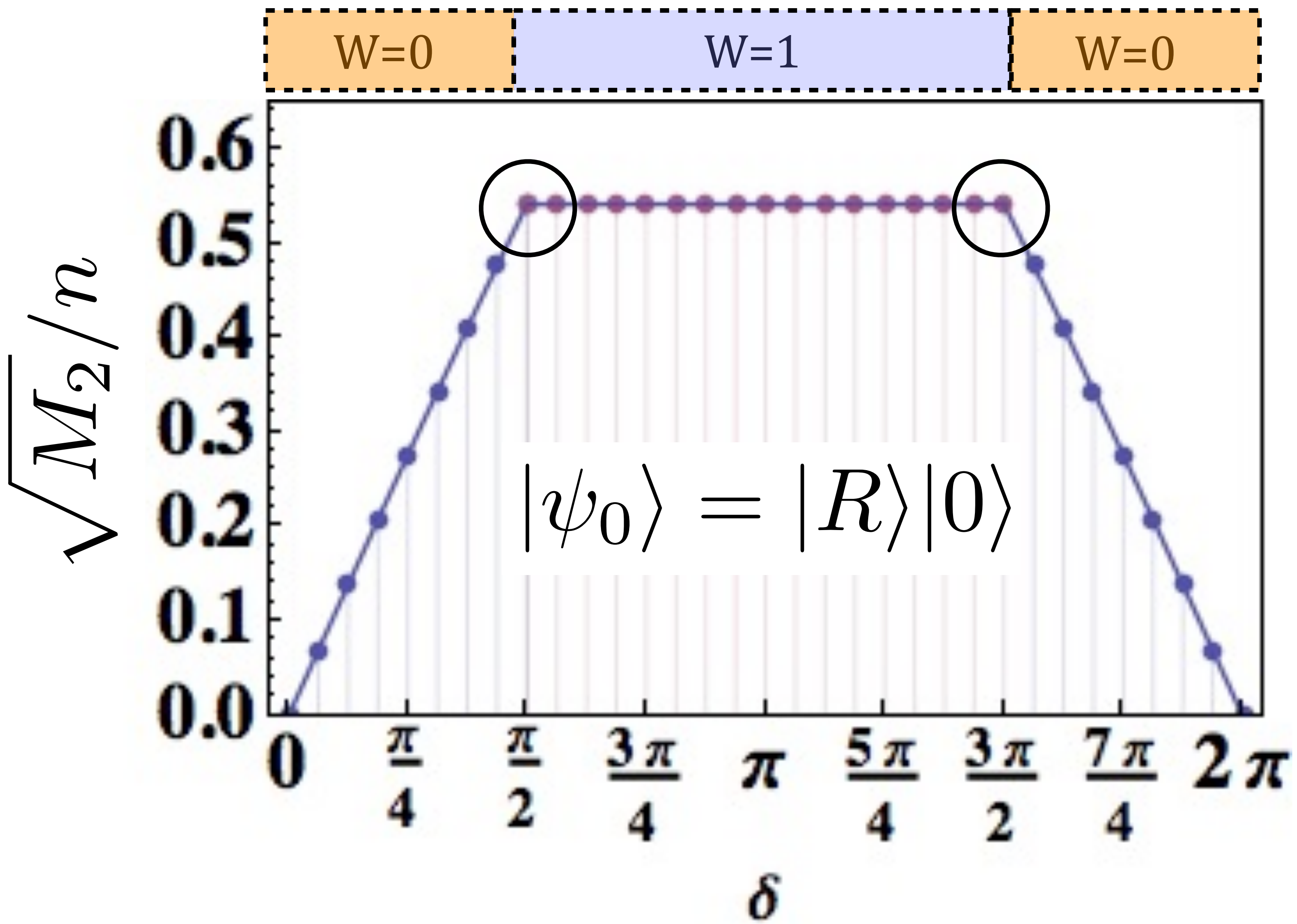
Dynamics and topology

$$V_\delta = \frac{dE_\delta}{dk} = n_y(k)$$





Distinct topological phases



$$V_\delta = \frac{dE_\delta}{dk} = n_y(k)$$

Short Outline

- Quantum Walk at a glance
- QWs with twisted photons
- Topological phases in QWs
- Detection of the phase transition

Moments are linked to topological features

$$|\psi_0\rangle = |\phi_0\rangle|0\rangle$$ localized input

Cardano F. *et al.*, Arxiv 1507.01785 (2015)

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$\downarrow$ asymptotic analysis (large n)

$$M_2/n^2 = L(\delta) + O(1/n^2)$$

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$$L(\delta) = \int_{-\pi}^{\pi} \frac{\mathrm{d}k}{2\pi}\,[V_\delta(k)]^2 = \int_{\pi}^{\pi} \frac{dk}{2\pi}\,[n_y(k)]^2$$

IMPORTANT: independent of the coin initial state $|\phi_0\rangle$

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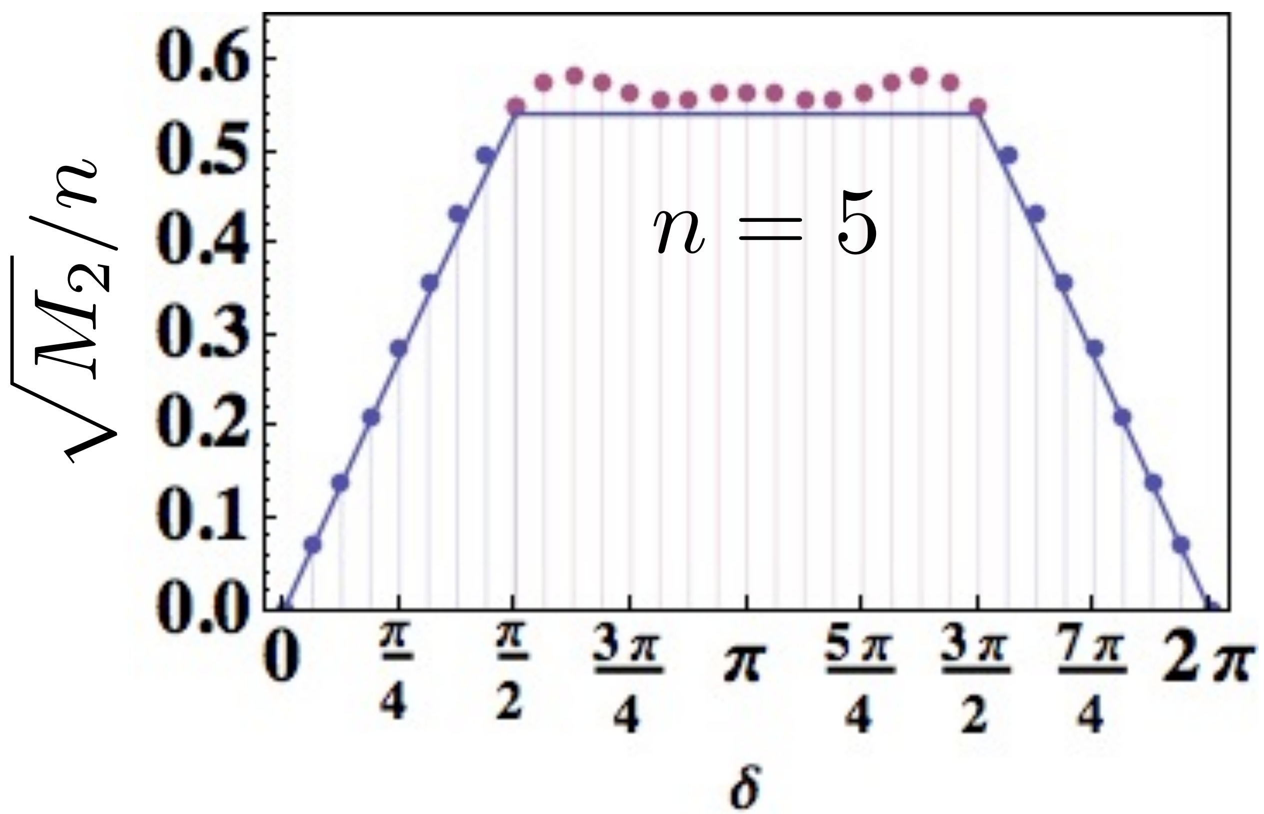
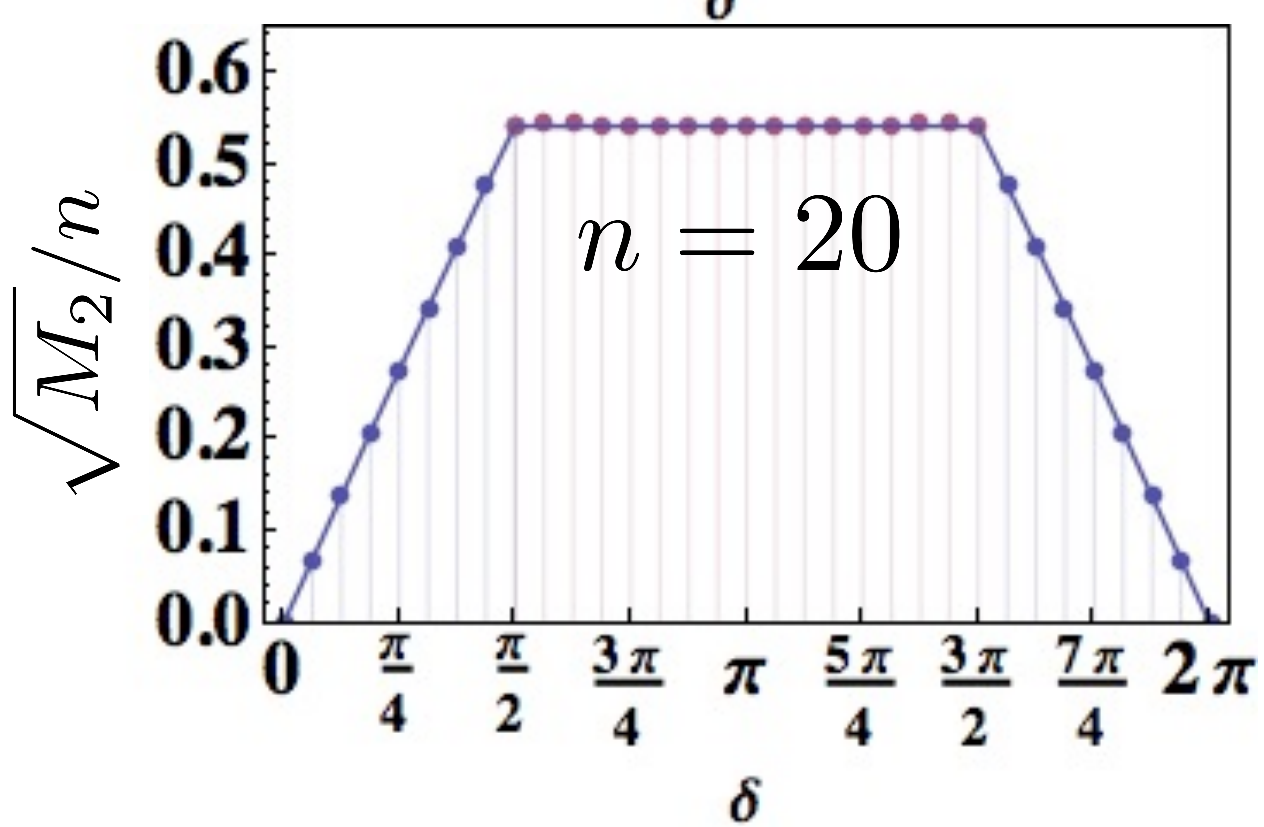
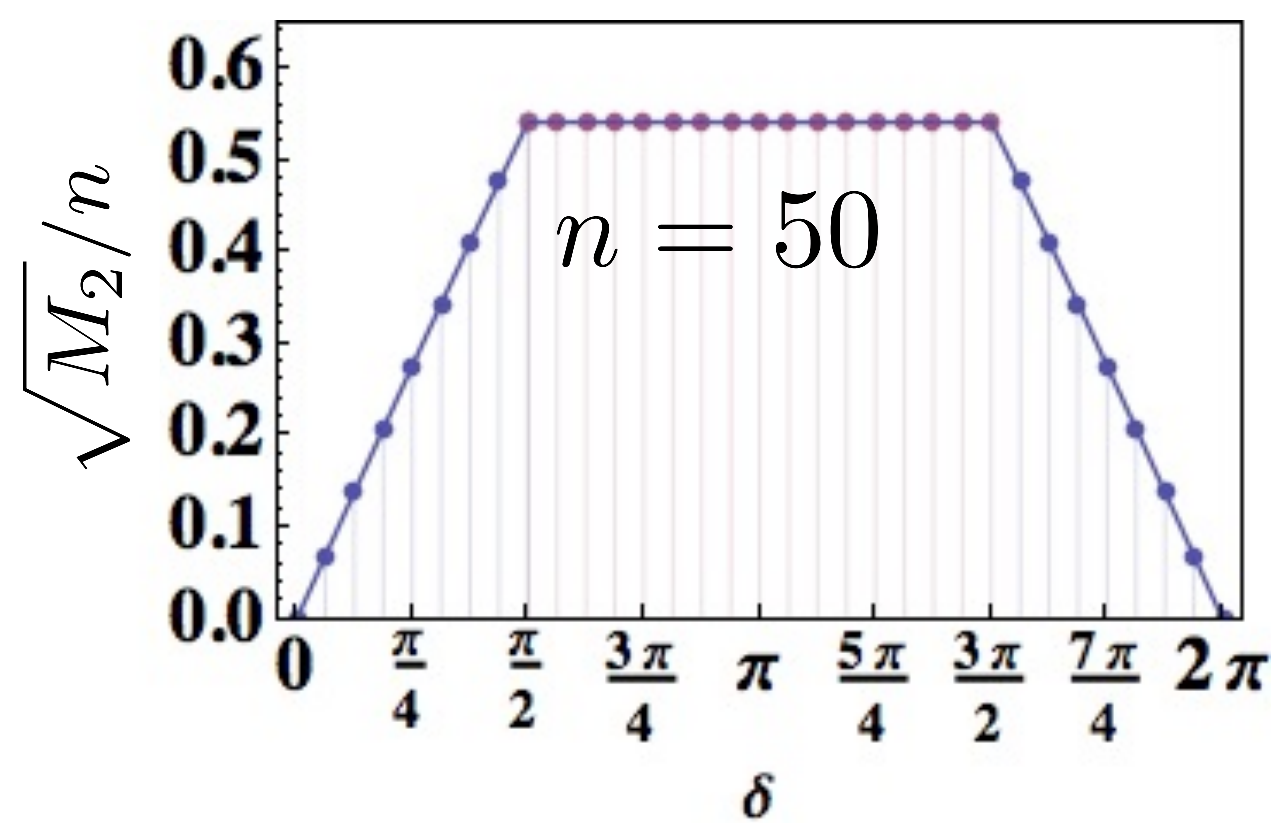
Statistical moments at the quantum transition



numerical simulations

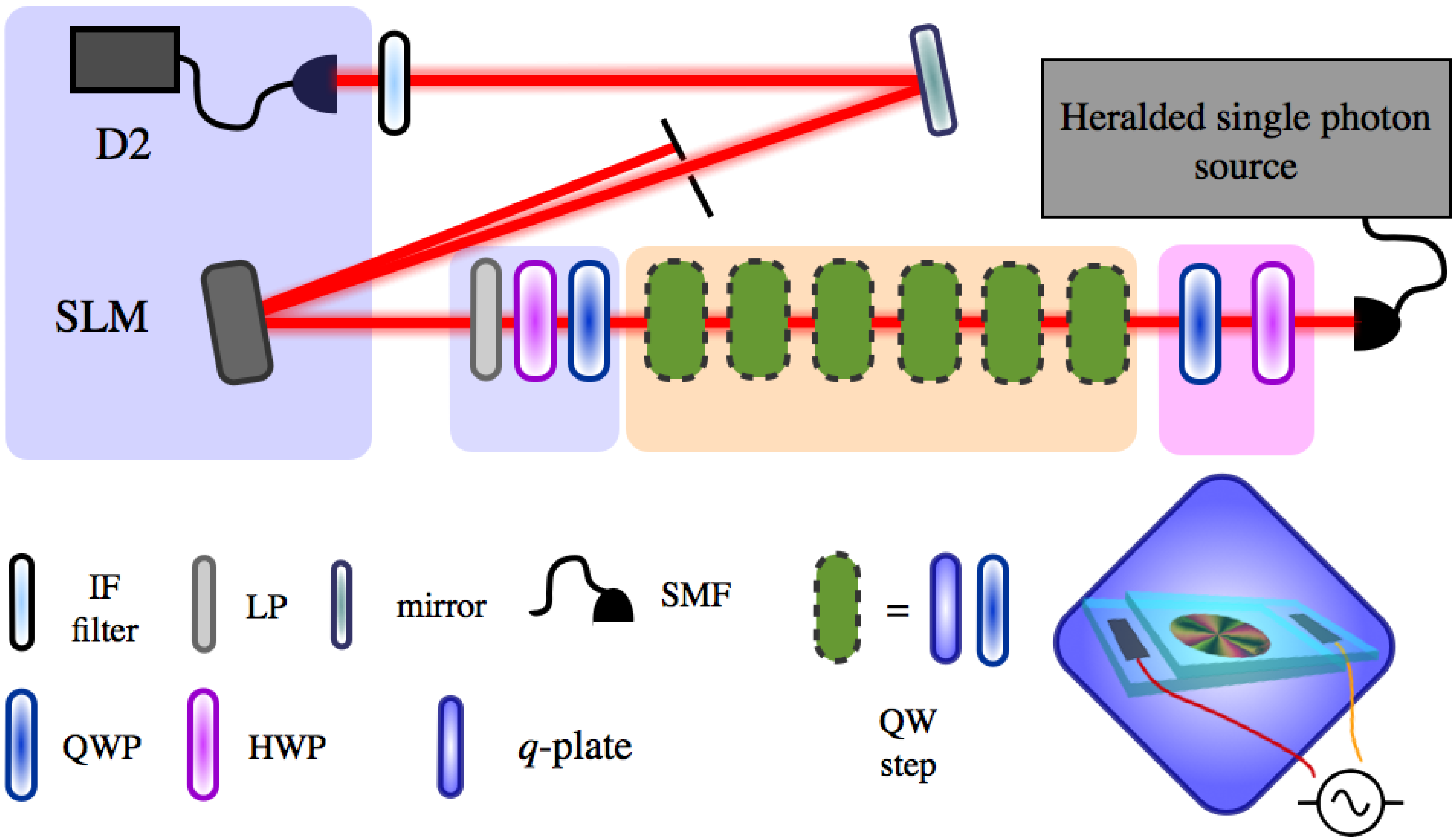
analytical expression for $L(\delta)$





Cardano F. *et al.*, Arxiv 1507.01785 (2015)

Experimental layout



control δ

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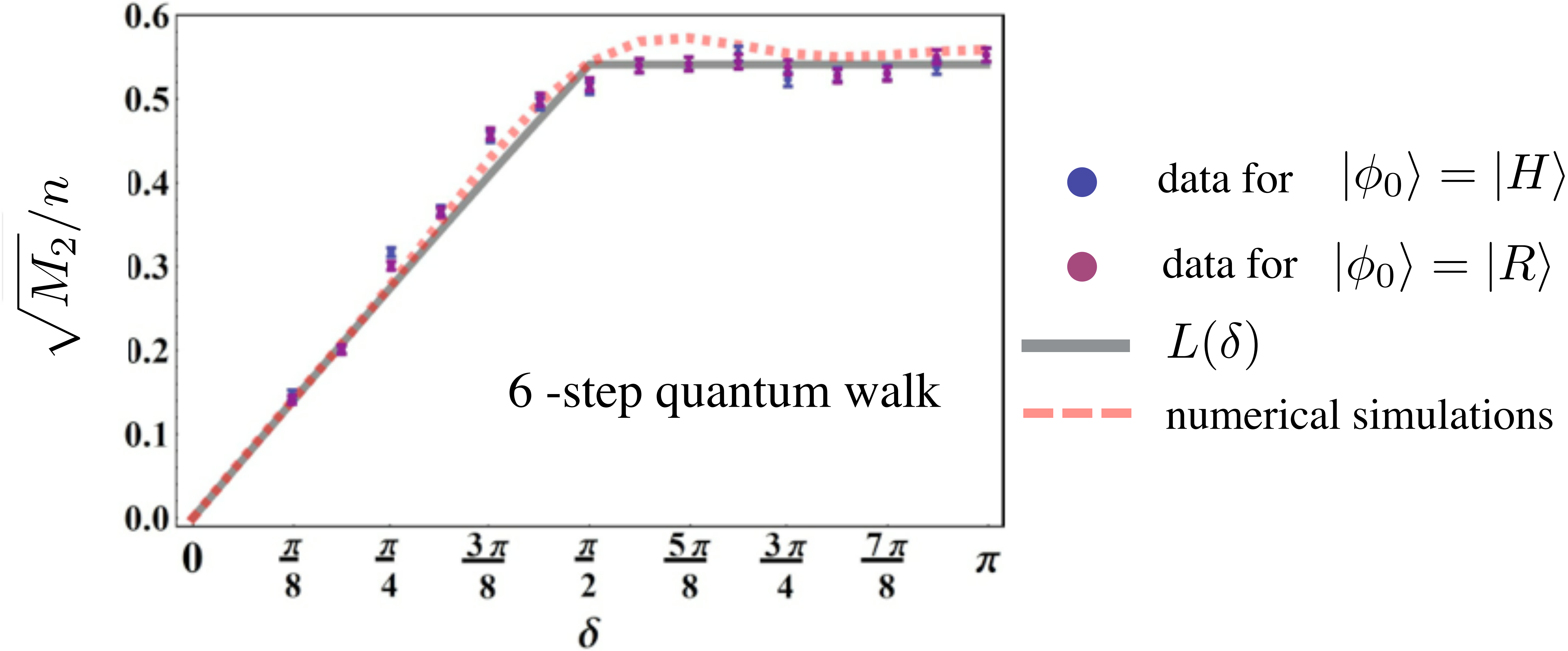
Experimental detection of the quantum transition



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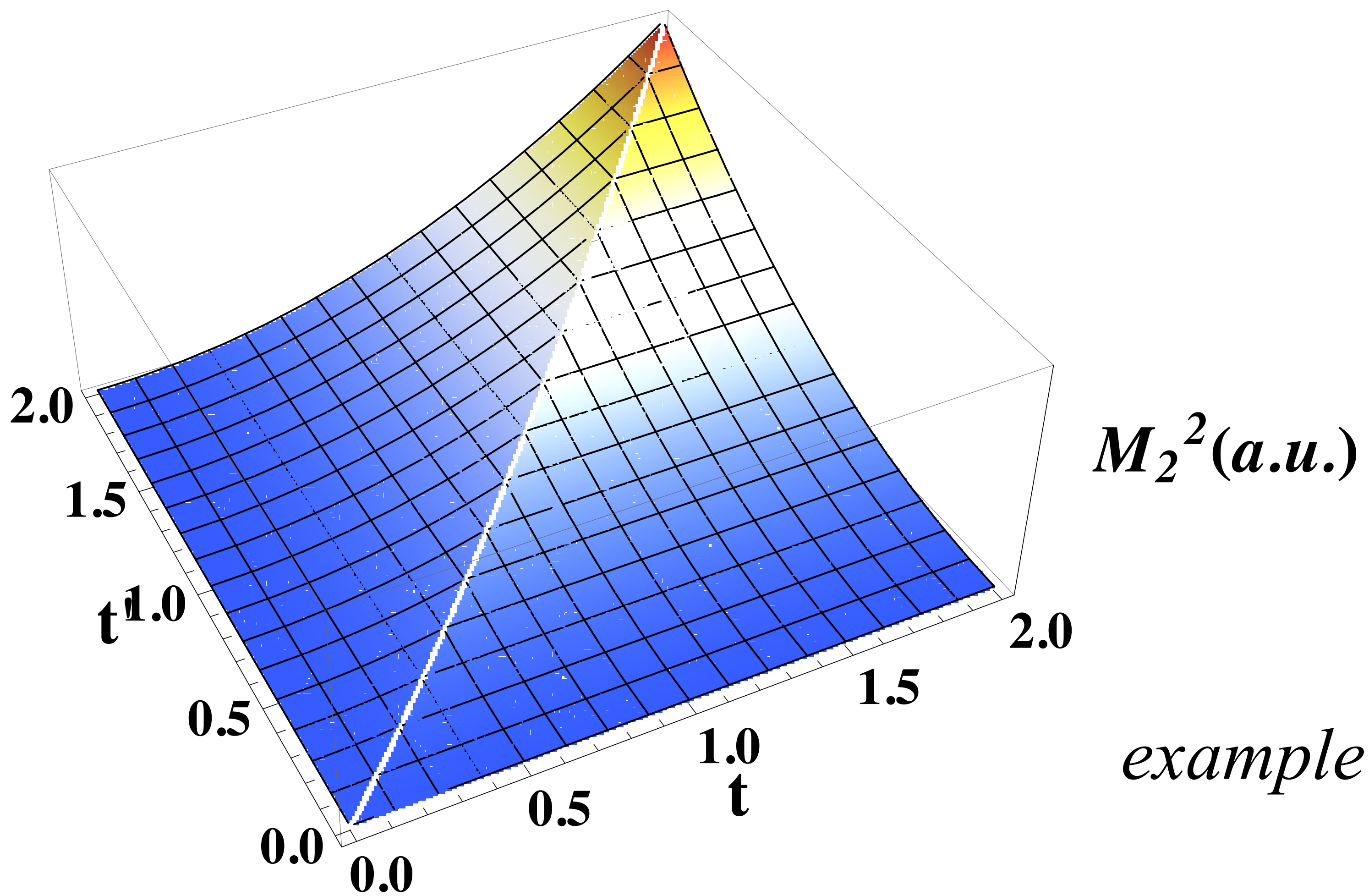
Is this result general?

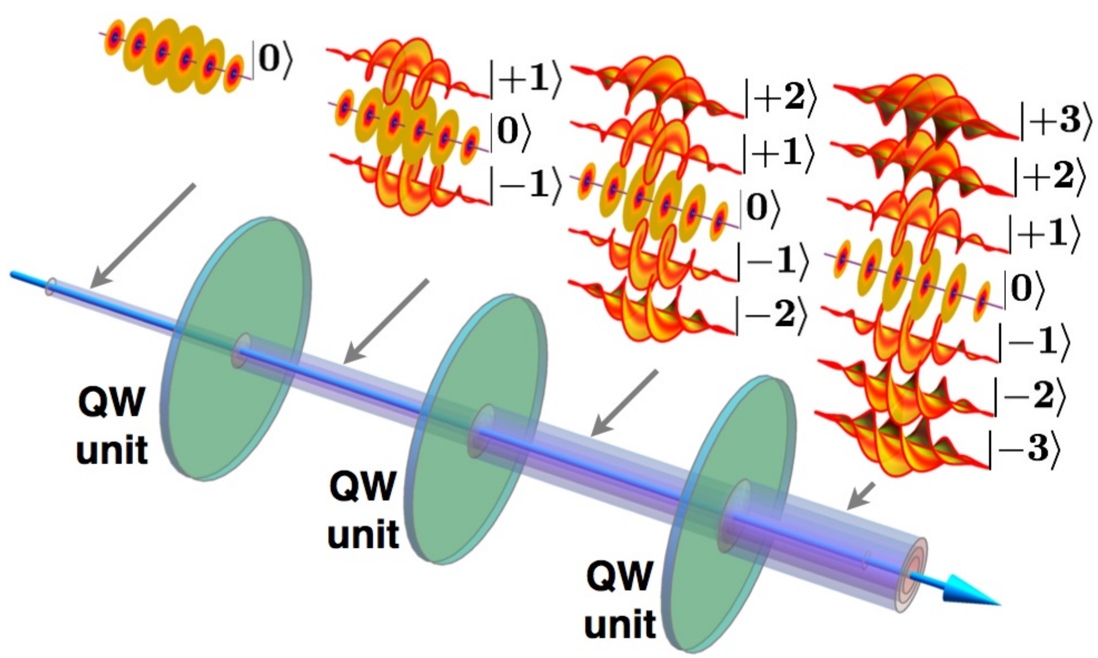
1D systems: no proof, but strong evidences



example: SSH model

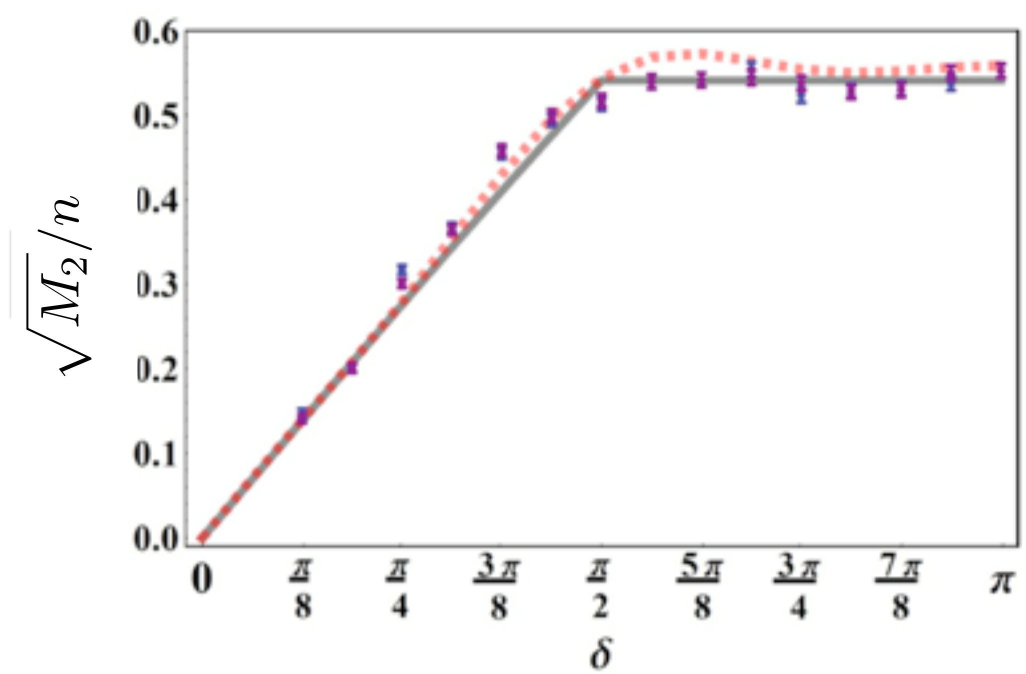
2D...work in progress!

Conclusions



Novel platform for QW simulations

Cardano F. *et al.*,
Science Advances **1**, e1500087 (2015)



Dynamical moments detect a topological quantum transition

Cardano F. *et al.*, Arxiv 1507.01785 (2015)

Thanks!