Quantum simulation of lattice gauge theories in AMO systems: challenges and perspectives

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arxiv.1507.06788

Joint work with **C. Laflamme, P. Zoller** (Innsbruck), **W. Evans, U.-J. Wiese** (Bern), **W. Bietenholz, U. Gerber, H. Megia-Diaz** (Mexico City)

Related works in collaboration with **E. Rico** (Bilbao), **S. Montangero's group** (Ulm), **U.-J. Wiese's group** (Bern)

How can quantum info and HEP interact?

Development of novel, quantum-information-based **numerical methods** (sign-

See talk by Pietro on Saturday

Rico, Pichler, MD et al. PRL 2014
Buyens et al., PRL 2014
Tagliacozzo et al., 2014
Silvi et al., 2014

Use quantum Info-related concepts in the context of gauge theories

(topological) entanglement entropies, characterization of phase transitions, ….

Design of **quantum simulation platforms**

What is quantum simulation

Feynman's lecture '*Simulating Physics with Computers'*

"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

R. P. Feynman, Int. J. Theor. Phys. (1982).

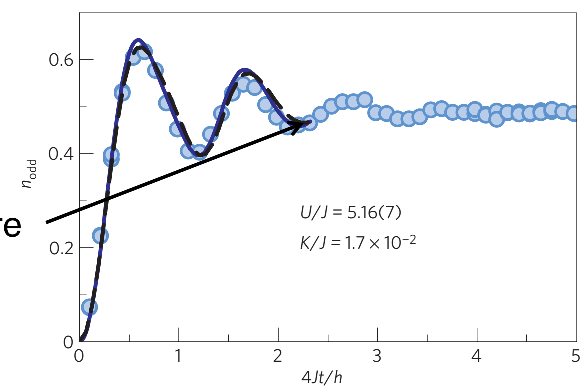
$$\dim \mathcal{H} \propto e^N!$$

Physics at equilibrium (sign problem for fermions, entanglement,...)

Time-evolution of many-particle quantum systems

quench

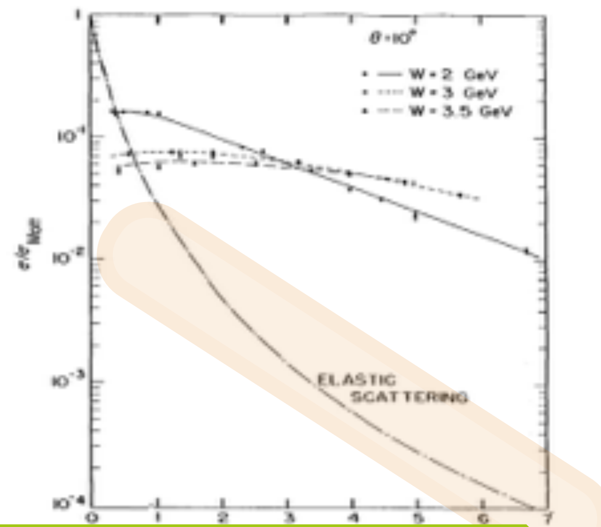
DMRG (theory)
Breaks down here



S. Trotzky et al.,
Nature Physics, 2232 (2012)

Today's dish: *Asymptotic freedom* and the *continuum limit*

1968- SLAC-MIT experiment on deep inelastic scattering cross sections [electron/nucleon]



$$g^2(k) = \frac{g^2}{\quad}$$

1974: *Politze* … QCD!

***SU(N)* quantum magnets** and
***CP(N-1)* quantum field theories**
with **alkaline-earth atoms**

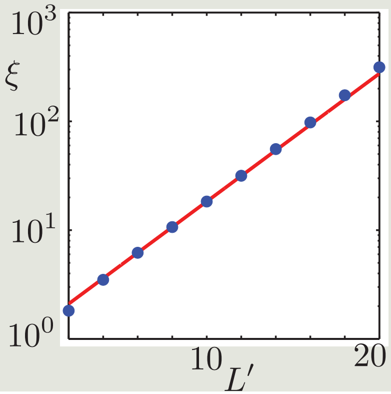
with synthetic systems?
Disclaimer: using toy models

Outline

Why shall we study **gauge theories** in synthetic system: **some motivations**

From lattice to field theories: ***the continuum limit***

Observe **asymptotic freedom** in cold atom systems using **alkaline-earth atoms** exploiting the **continuum limit**

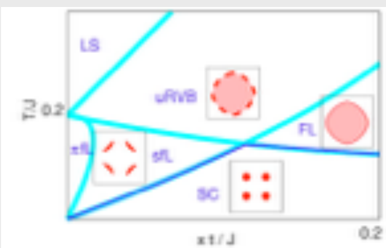


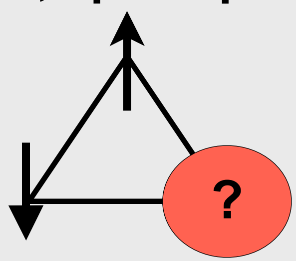
Gorshkov et al., Nat Phys. 2010; Cazalilla et al., NJP 2009.

Gauge theories are very interesting beasts…

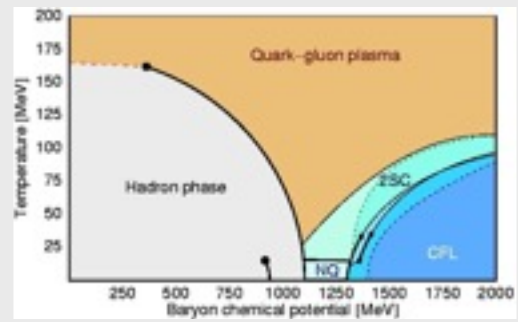
Ubiquitous theoretical framework

As emergent theories in condensed matter: HTc superconductors, **frustrated systems, spin liquids**





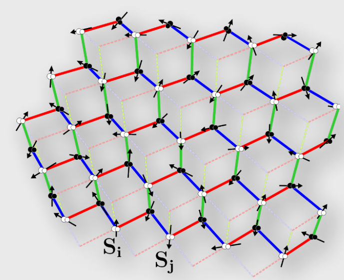
As fundamental description in particle physics: **standard model**



UniFrankfurt website

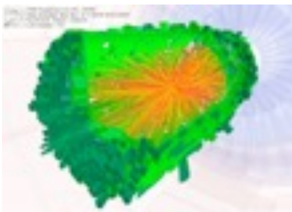
Phase diagram of QCD

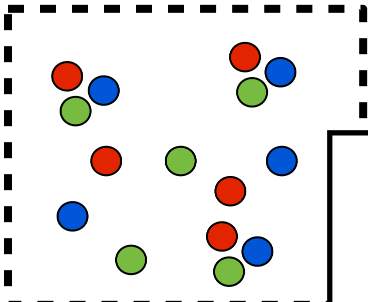
As resources in topological quantum computing: **Toric code**



Lee, Nagaosa and Wen, RMP 2006; *Introduction to Frustrated magnetism*, Springer 2011; Focus issue on quantum spin liquids, NJP2013. Montvay and Münster, *Quantum fields on a Lattice*

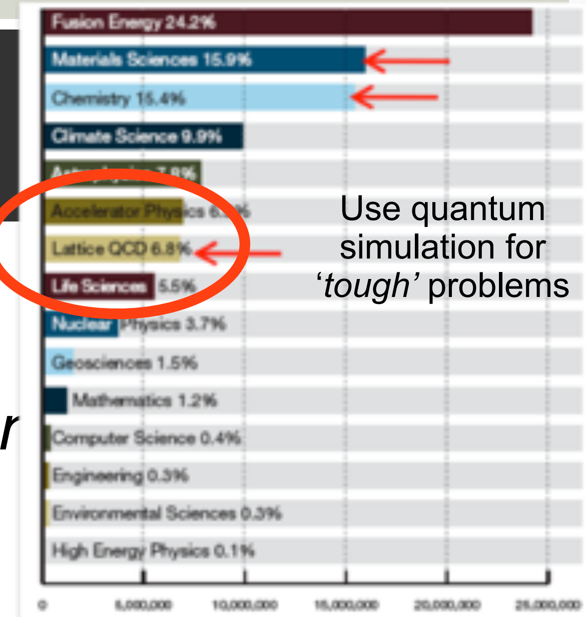
…Many open questions





Nucleons 'break' at high T

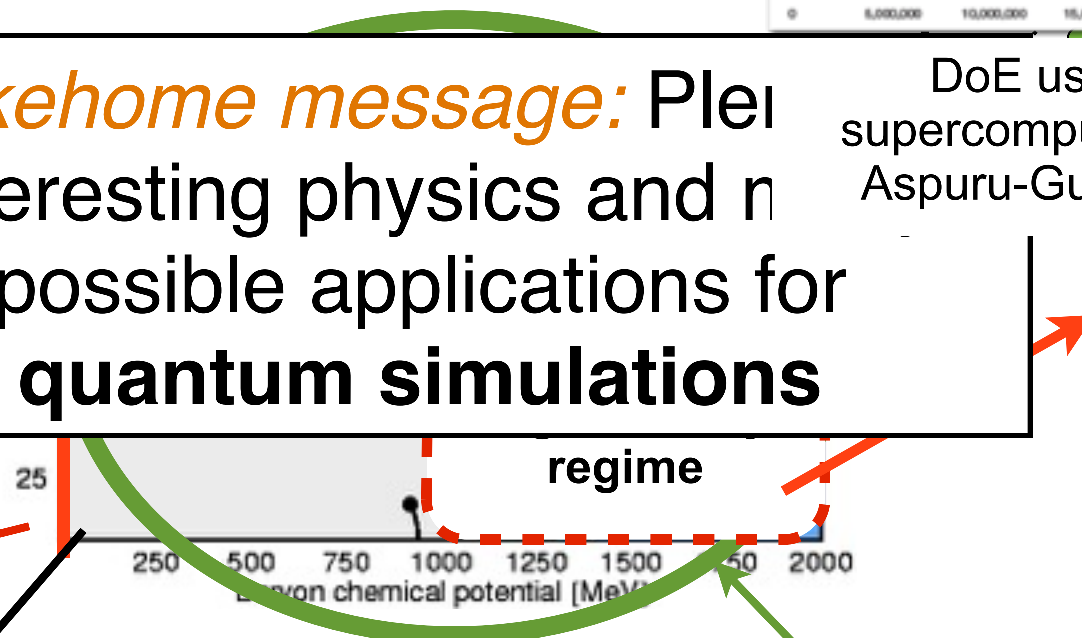
***Why** quantum simulatior*



Use quantum simulation for *'tough'* problems

DoE usage of supercomputers (from Aspuru-Guzik's talk)

Takehome message: Plei interesting physics and n possible applications for **quantum simulations**



Unbiased numerical results

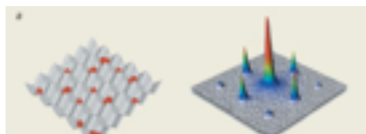
'normal' conditions

Sign problem! Unaccessible with **classical simulators**

…however, not yet realized in the panorama of synthetic systems…

Hubbard models of various sorts

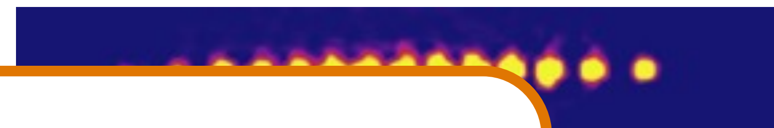
$$H = -t \sum_{<i,j>} a_i^\dagger a_j + U n_i + \ldots$$



Atoms, ions,
circuit QED
quantum dots

Non-local interactions

$$H = -t \sum_{i,j} \frac{S_i^z S_j^z}{|i-j|^\alpha} + h S_i^x + \ldots$$

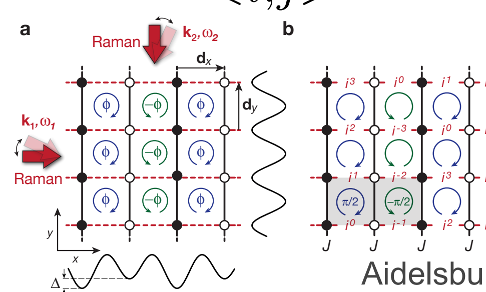


gnetic Atoms,…

However: only **global symmetries** (magnetization, spin-filp, particle number,…)! **No gauge symmetry…**

ields

$$H = -t \sum_{<i,j>} a_i^\dagger a_j e^{i\varphi_{ij}} + \ldots$$



Atoms,
circuit QED,
…

Aidelsburger et al. PRL2011

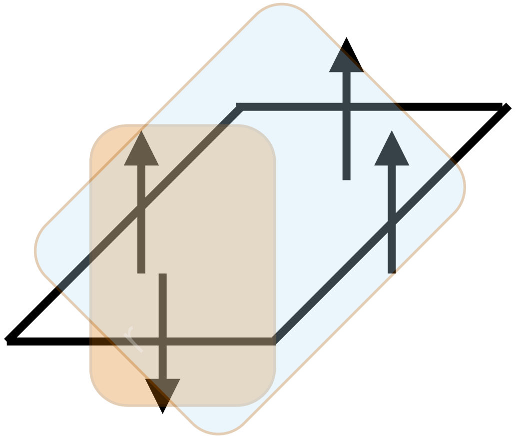
Blatt, Ross, Nat. Phys. 2012; Bloch, Dalibard, Nascimbene, Nat. Phys. 2012; Cirac and Zoller, Nat. Phys. 2012

The reason being that ...

It is very hard to get a **gauge symmetry** in synthetic systems!!!

From a Hamiltonian perspective, a **gauge symmetry** is nothing but a set of **local constraints**

Example: spin-flip in Ising models



$$H = \sum_{<i,j>} S_i^z S_j^z + S_i^x$$

$$P = \prod_i S_i^x$$

Conserved!

$$P_{12} = S_1^x S_2^x$$

Not Conserved!

No local conserved quantities

=

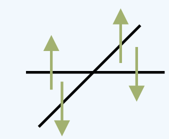
Absence of gauge symmetry

An example of a constrained system: Ising lattice gauge theory - aka the toric code

$$H = \sum_{\square} \prod_{j \in \square} \sigma_j^x$$

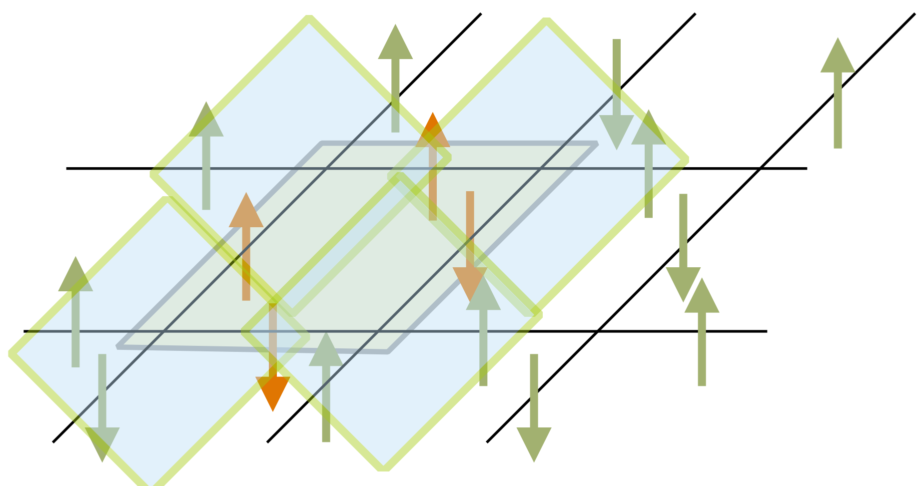
Hamiltonian

$$G_+ = \prod_{j \in +} \sigma_j^z$$



Constraints

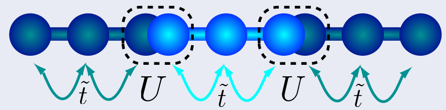
Flip all four spins around ***a plaquette.*** Since two spins the vertex are flipped, the parity is conserved



Wegner, 1971; Kitaev, 2003; Dissipative dynamics: Barreiro, Mueller et al, Nature 2011

Where we stand now - synthetic lattice gauge theories

Ultracold atoms

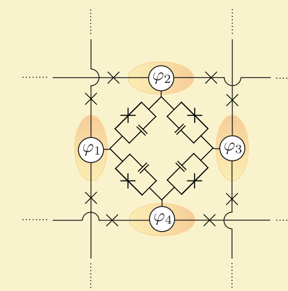


Tewari et al. (PRL2006);
Kapit and Mueller, PRA 2010
Banerjee, MD et al. PRL 2012, PRL 2013
Zohar et al, PRL 2012, PRL 2013, PRA2013
Notarnicola et al., Meurice et al., arxiv.2015;

$$U(1), \mathbb{Z}_N, SU(N), U(N), SO(3), \ldots$$

Circuit QED

Marcos et al., PRL 2013,
Ann. Phys. 2014
Mezzacapo et al.,
arxiv2015

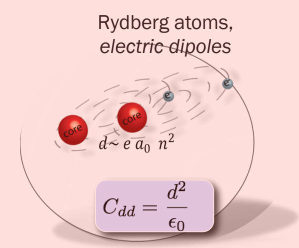


Digital approaches

Byrnes and Yamamoto, PRA 2006.
Weimer et al., Nat. Phys. 2010.
Tagliacozzo et al., Ann. Phys. 2012,
Tagliacozzo et al., Nat. Comm. 2013

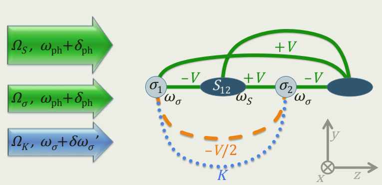
Rydberg atoms

A. Glätzle, MD et al., PRX
2014; arXiv2014



Trapped ions

Hauke, MD et al., PRX
2013; Nath, MD et al.
arxiv.1504.01474



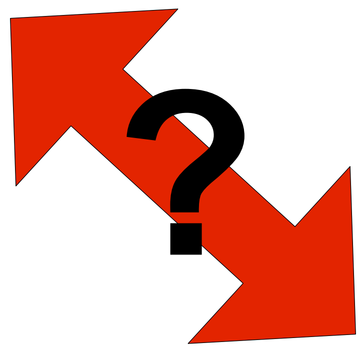
Quantum Zeno dynamics

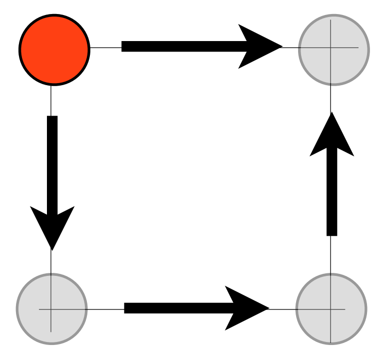
Stannigel et al., PRL 2014

How to connect lattice models to field theories?

High Energy - *field theories*

$$\mathcal{L} = \frac{1}{2g^2}\mathtt{Tr}F_{\mu\nu}F_{\mu\nu} + \ldots$$



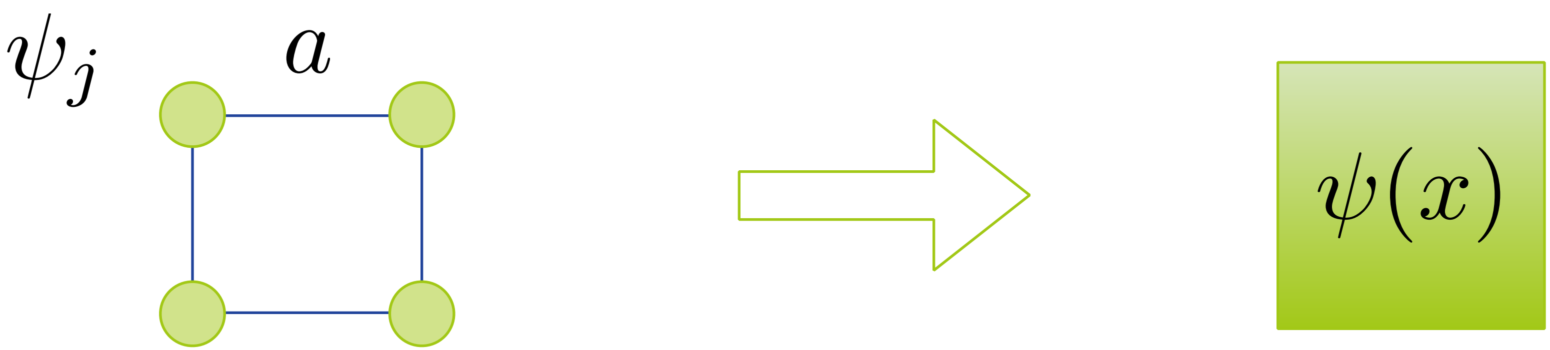


Fundamental question: the continuum limit

$$H = \sum_{i,j} \psi_i^\dagger U_{ij} \psi_j + \dots \qquad H = \int dx\, \tilde{\psi}^\dagger(x)(D_x)\psi(x') + \dots$$

In Wilson's lattice field theory, usually achieved by taking the limit

$$a \to 0$$



The dependence on *a* is present in the Hamiltonian/Lagrangian couplings in a controlled way

Open question: the continuum limit

Open question until now: *can we approach the* ***continuum limit*** *in* ***synthetic quantum systems***?

(1) *Orthodox way:* engineer **Wilson's lattice gauge theories!**

Wilson's theories

(2) *Unorthodox way*: **dimensional reduction** and **asymptotic freedom**

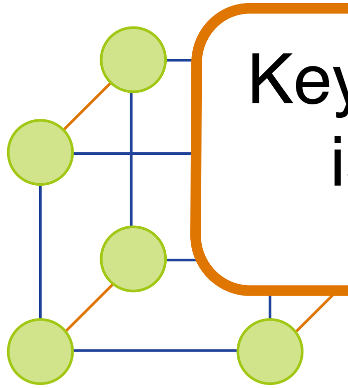
This talk

The continuum limit: the *unorthodox* way

$$H = \sum_{i,j} \psi_i^\dagger U_{ij} \psi_j + \ldots \qquad\qquad H = \int dx\, \tilde{\psi}^\dagger(x)(D_x)\psi(x') + \ldots$$

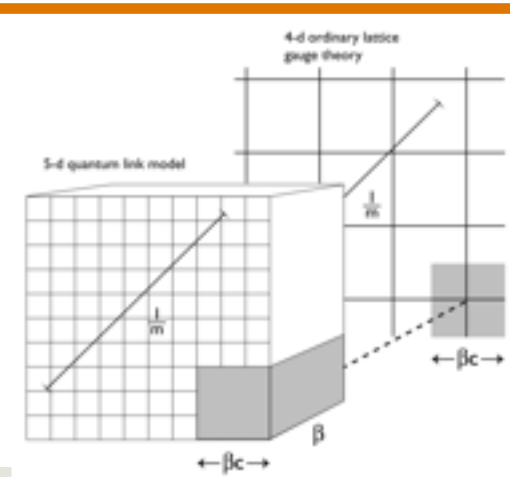
Take a theory in one dimension more, and extend this additional dimension until a correlation lengths is much larger than its width

ψ_j



a

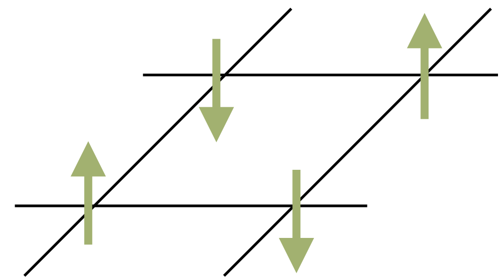
Key point: the **continuum limit** is achieved ***on the lattice***



A simple example of dimensional reduction

Intuition: Neel states in 2D antiferromagnets

$$H = -J \sum_{<i,j>} \vec{S}_i \vec{S}_j$$



At T=0, Neel order is stable, and described by an O(2) order parameter

Full-rotation -> rotation around one axis

$$SO(3)/SO(2) \rightarrow S^2$$

We get a field theory in 1D as (pseudo-)Goldstone bosons via ***spontaneous symmetry breaking***

B.B. Beard,et al., Phys. Rev. Lett. 94, 010603 (2005); N. Read and S. Sachdev, Phys. Rev. Lett. 62, 1694 (1989).

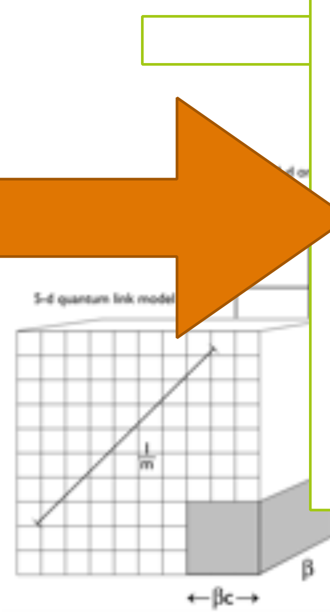
The continuum limit: the *unorthodox* way

$$H = \sum_{i,j} \psi_i^\dagger U_{ij} \psi_j + \dots \qquad\qquad H = \int dx\, \tilde{\psi}^\dagger(x)(D_x)\psi(x') + \dots$$

Take a theory in one dimension more, and extend this additional dimension until a correlation lengths is much larger than its width

SU(N) Heisenberg antiferromagnets

Widely investigated in condensed matter physics
Realizable with **alkaline-earth-like atoms**!



(1+1)d CP(N-1) quantum field theories

Toy models for QCD:
-asymptotically free
-confinement
- role of theta angles

Why CP(N-1) models?

$$S[P]=\int_0^\beta dct\int_0^L dx\,\frac{1}{g^2}\mathrm{Tr}[\partial_x P\partial_x P+\frac{1}{c^2}\partial_t P\partial_t P]-i\theta Q[P],$$

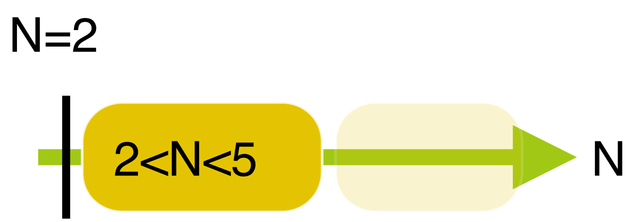
CP(N-1) fields: (N-1) x (N-1) matrices

$$Tr\,P=1,\,P^2=$$

Topological charge

$$\Pi_2[CP(N-1)]=\mathbb{Z}$$

Regime of interest

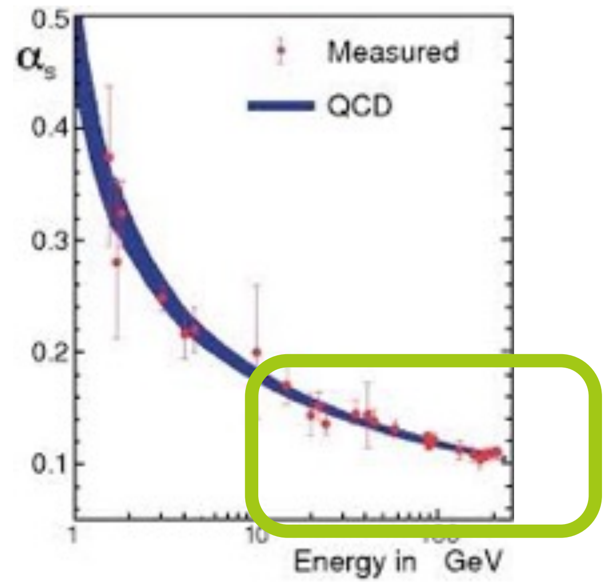


Display:

- ***asymptotic freedom***
- potentially rich ***bound***
- many-body effects due
- unexplored quench dyn

A.D'Adda, M. Luescher, and P. Di Vecchia, Nucl. Phys. B 146, 63 (1978).

Asymptotic freedom: what is that?



Interactions between particles **gets weaker and weaker** at **short distances** [= high energies = high momenta]

Formally, identified with the first coefficient of the beta function being negative

$$\beta(g) = \beta_0 * g^3$$

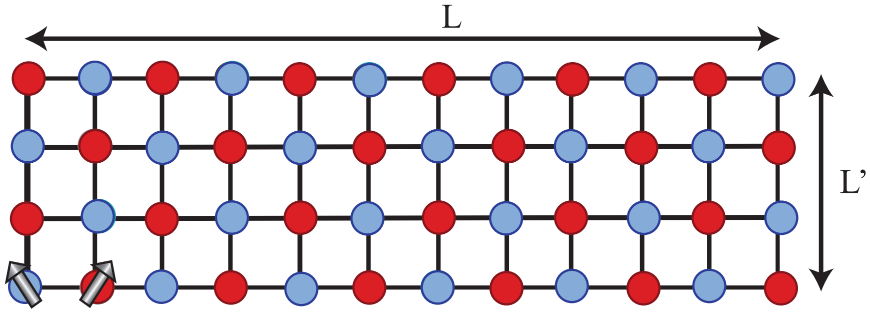
$$g^2(k) = \frac{g^2}{1 + \beta_0 g^2 \log[k^2/M^2]}$$

1974: *Politzer, Wilczek and Gross*: asymptotic freedom is fundamental in QCD!

See, e.g., Peskin's book for ways of calculating Beta in non-Abelian gauge theories

Dimensional reduction in CP(N-1) models: microscopics

Dimensional reduction from a 2+1d SU(N) antiferromagnet



$$H = -J \sum_{a=1}^{N^2-1} \sum_{\langle xy \rangle} T_x^a \, T_y^{a*}$$

$$[T_x^a, T_y^b] = \mathrm{i}\delta_{xy} f_{abc} T_x^c$$

SU(N) Heisenberg antiferromagnets extensively studied in condensed matter

SU(N) spi **s**

- dime ids
- exotic criti cality

Affleck, Nucl.Phys. B 1986

See, e.g., Read and Sachdev, NuclPhys. B 1989; for a review, Kaul et al., Annu. Rev. Con. Mat. Phys. 4, 179 (2013)

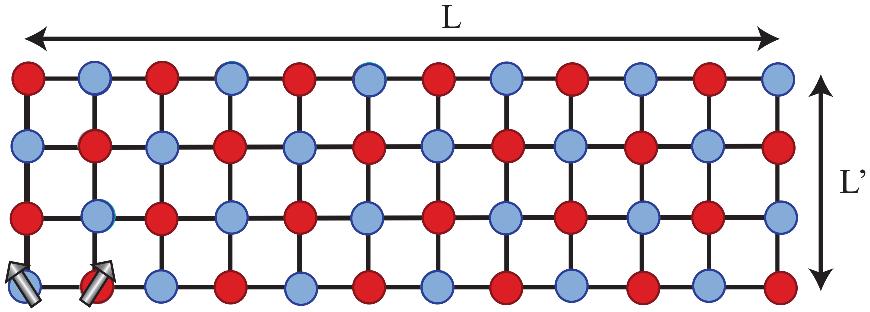
Key result: **spontaneous symmetry breaking** in the 2d case for N<5

Read and Sachdev, NuclPhys.B 1989;

NB: the two representations coincide for the N=2 case / Heisenberg model.

Dimensional reduction in CP(N-1) models

Dimensional reduction from a 2+1d SU(N) antiferromagnet



$$H = -J \sum_{a=1}^{N^2-1} \sum_{\langle xy \rangle} T_x^a \, T_y^{a*}$$

$$[T_x^a, T_y^b] = \mathrm{i}\delta_{xy} f_{abc} T_x^c$$

For L'>>1, Goldstone bosons live in a coset space

$$SU(N)/U(N-1) = \mathrm{CP}(N-1)$$

Which yields at low-energies a 1+1d theory:

$$S[P] = \int_0^\beta dct \int_0^L dx \, \frac{1}{g^2} \mathrm{Tr}[\partial_x P \partial_x P + \frac{1}{c^2} \partial_t P \partial_t P] - i\theta Q[P],$$

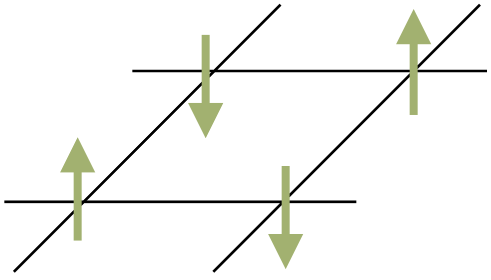
$$\theta = \pi n$$

B.B. Beard,et al., Phys. Rev. Lett. 94, 010603 (2005); N. Read and S. Sachdev, Phys. Rev. Lett. 62, 1694 (1989).

CP(1)

Intuition: Neel states in 2D antiferromagnets

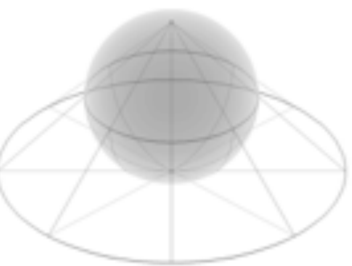
$$H = -J \sum_{<i,j>} \vec{S}_i \vec{S}_j$$



At T=0, Neel order is stable, and described by an O(2) order parameter

Full-rotation -> rotation around one axis

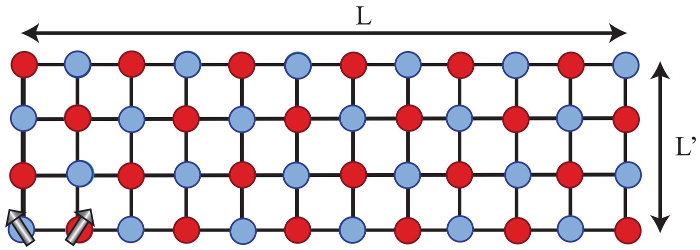
$$SO(3)/SO(2) \rightarrow S^2 = CP(1)$$



wikipedia.org

B.B. Beard,et al., Phys. Rev. Lett. 94, 010603 (2005); N. Read and S. Sachdev, Phys. Rev. Lett. 62, 1694 (1989).

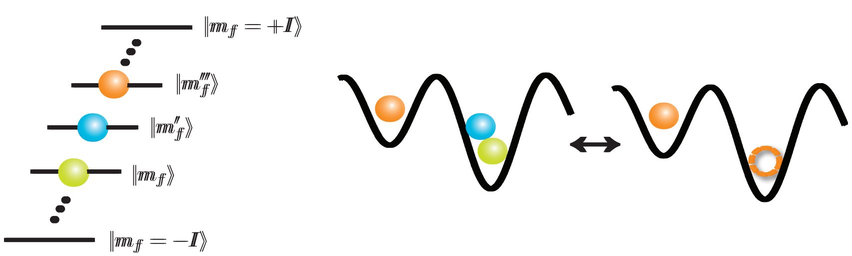
Implementing SU(N) AF magnets with alternate representations: N=3



$$H = -J \sum_{a=1}^{N^2-1} \sum_{\langle xy \rangle} T_x^a \, T_y^{a*}$$

$$[T_x^a, T_y^b] = \mathrm{i}\delta_{xy} f_{abc} T_x^c$$

Alkaline-earth atoms (Yt, Sr) in optical superlattices

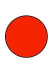


$$H = H_t + H_U$$

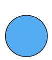
$$H_t = -\sum_m \sum_{\langle xy \rangle} t(\hat{c}^\dagger_{xm} \hat{c}_{ym} + \mathrm{h.c.})$$

$$H_U = \sum_x \frac{U}{2} n_x(n_x - 1) + V \sum_{x \in A} n_x,$$

Particle-hole transformation on blue sublattice

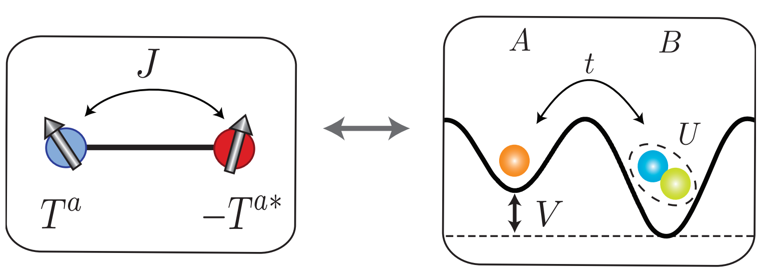


$$\hat{d}_{xm} = c_{x,m}$$



$$\hat{d}_{xm} = c^\dagger_{x,m}$$

Strong coupling limit: SU(N) spin model



Gorshkov et al., Nat Phys. 2010; Cazalilla et al., NJP 2009; for a review, see Cazalilla and Rey, Rep. Progr. Phys. 2014

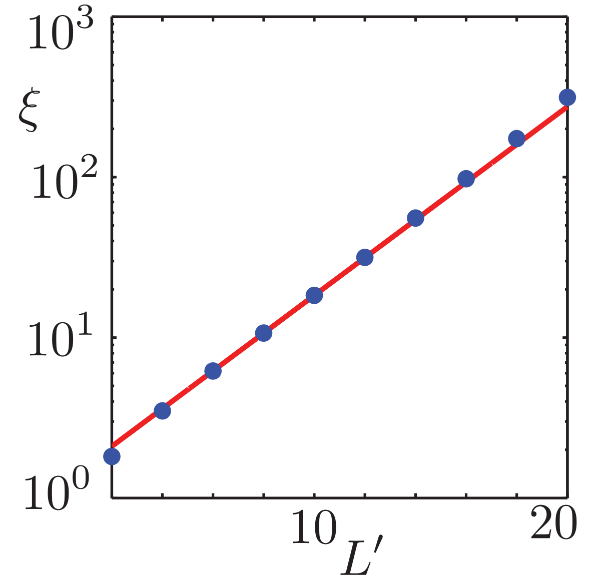
How to observe asymptotic freedom (N=3)?

Emergence of ***asymptotic freedom***

$$g^2(k) = \frac{g^2}{1 + \beta_0 g^2 \log[k^2/M^2]}$$

$$M \simeq 1/\xi \qquad g^2(k) \simeq 1/L'$$

$$\boxed{\xi \propto \exp(4\pi L' \rho_s/N) \gg L'}$$



QMC results using loop updates up to V=400 x 20

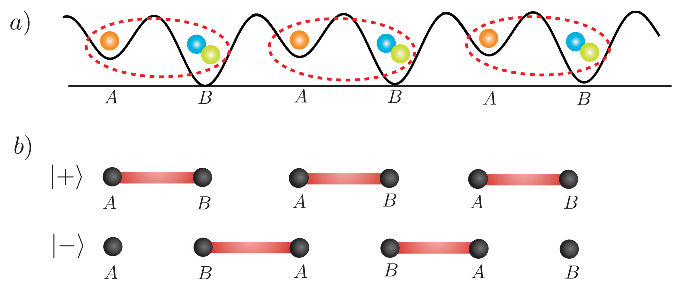
Can be measured by Bragg spectroscopy, microscope, noise correlations,…

B.B. Beard,et al., Phys. Rev. Lett. 94, 010603 (2005)

Theta angle and decay of false vacua

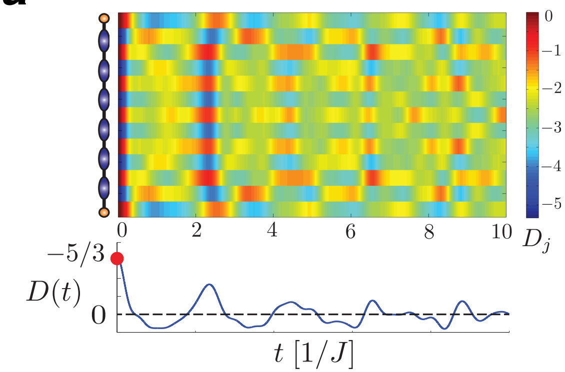
Effects of finite theta-angle: ***dimerized phases***

$$\theta = \pi e^{i\pi L'}$$



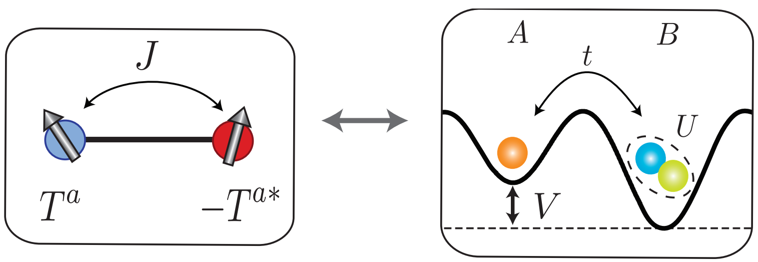
For odd number of chains, the system gets gapped and dimerization takes place

Quench dynamics: decay of ***false vacua***



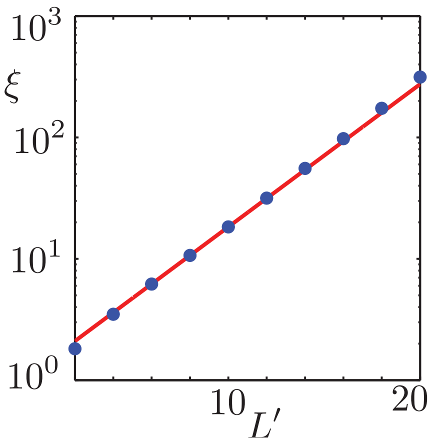
Conclusions

Synthetic systems are now approaching the level of control for realizing gauge theories



Synthetic quantum magnetism can be exploited to realize the **continuum limit** of **CP(N) models**

Asymptotic freedom of CP(N) can be observe using conventional probes



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Thank you

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C. Laflamme, P. Zoller (Innsbruck), **W. Evans, U.-J. Wiese** (Bern), **W. Bietenholz, U. Gerber, H. Megia-Diaz** (Mexico City), arxiv.1507.06788

Atoms U(1): Banerjee, MD, Mueller, Rico, Stebler, Wiese, Zoller, PRL ***109****, 175602;*
Atoms SU(N)/U(N): Banerjee, Boegli, MD, Rico, Stebler, Wiese, Zoller, PRL ***110****, 125303;*
Quantum Zeno: Stannigel, Hauke, Marcos, Hafezi, Diehl, MD, Zoller, PRL ***112****, 120406;*
Trapped Ions: Hauke, Marcos, MD, Zoller, PRX ***3****, 041018 (2013);*
TN approaches: Rico, Pichler, MD, Zoller, Montangero, PRL ***112****, 201601, arXiv.2015*
Rydbergs: Glätzle, MD, Nath, Rousochatzakis, Moessner, Zoller, PRX, ***4****, 041037.*