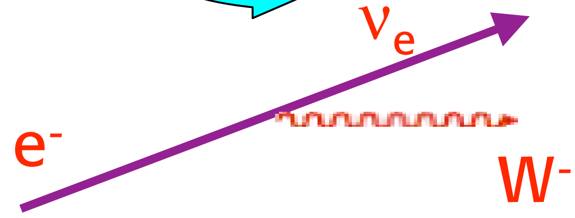
La Thuile, 3 March ‘09

Models of Neutrino Masses and Mixings

G. Altarelli
Universita' di Roma Tre/CERN

ν Oscillations Imply Different ν Masses

ν_e: same weak isospin doublet as e^-



flavour　　　mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

U: mixing matrix

$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

$\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$

$\nu_\mu = -\sin\theta\,\nu_1 + \cos\theta\,\nu_2$

e.g 2 flav.

Stationary source:

Stodolsky

$\nu_{1,2}$: different mass, different x-dep:

$\nu_a(x) = e^{ip_a x}\,\nu_a$　　　$p_a^2 = E^2 - m_a^2$

$$P(\nu_e \texttt{<->} \nu_\mu) = |\langle \nu_\mu(L)|\,\nu_e\rangle|^2 = \sin^2(2\theta)\cdot\sin^2(\Delta m^2 L/4E)$$

At a distance L, ν_μ from μ^- decay can produce e^- via charged weak interact's

Solid evidence for solar and atmosph. ν oscillations

Δm^2 values fixed:

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$,

$\Delta m^2_{sol} \sim 8\ 10^{-5}\ eV^2$

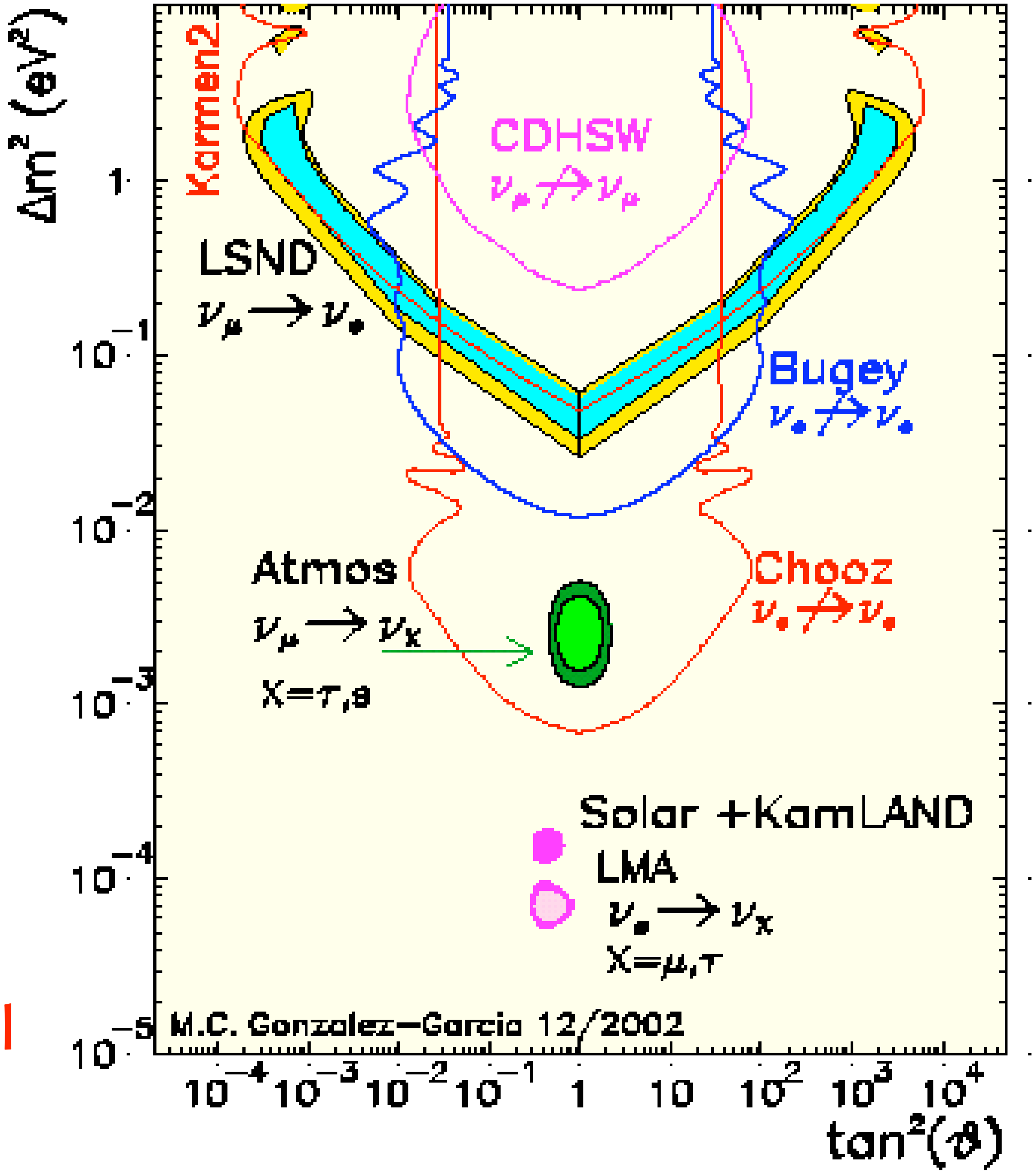
Miniboone has not confirmed LSND

mixing angles:

θ_{12} (solar) large

θ_{23} (atm) large,~ maximal

θ_{13} (CHOOZ) small



ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$; $\Delta m^2_{sun} \sim 8\ 10^{-5}\ eV^2$

- Direct limits

$m_{"\nu e"} < 2.2$ eV

End-point tritium β decay (Mainz, Troitsk)

$m_{"\nu\mu"} < 170$ KeV

$m_{"\nu\tau"} < 18.2$ MeV

$m_{ee} = |\Sigma\, U_{ei}^2\, m_i|$

- $0\nu\beta\beta$ $m_{ee} < 0.3 - 0.7 - ?$ eV (nucl. matrix elmnts)

Evidence of signal? Klapdor-Kleingrothaus

- Cosmology $\Omega_\nu\, h^2 \sim \Sigma_i m_i\ /94\text{eV}$ $(h^2 \sim 1/2)$

$\Sigma_i m_i < 0.17\text{-}0.68\text{-}2.1$ eV (dep. on data&priors)

Any ν mass $< 0.06\text{-}0.23\text{-}0.7$eV

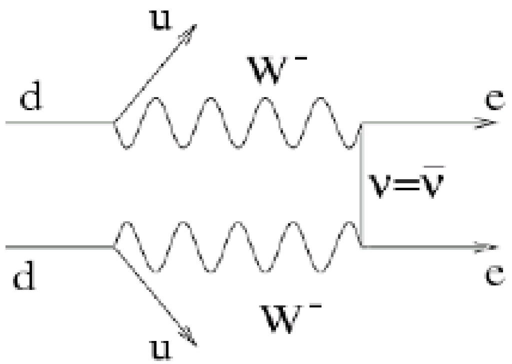
WMAP, SDSS, 2dFGRS, Ly-α

$0\nu\beta\beta$ experiments

$$\langle m_\nu \rangle^2 = \frac{1}{G(Q,Z)\, |M_{nucl}|^2\, \tau}$$

phase space

matrix elmnt
large uncrtnts



Pavan

Experiment	Isotope	$\tau_{1/2}^{0\nu}$ > [y]	range $\langle m_\nu \rangle$ [eV]
Heidelberg Moscow 2001	^{76}Ge	$1.9\ 10^{25}$	0.3-2.5
IGEX 2002	^{76}Ge	$1.57\ 10^{25}$	0.3-2.5
Cuoricino 2005	^{130}Te	$2\ 10^{24}$	0.3-0.7
NEMO 2005	^{100}Mo	$4.6\ 10^{23}$	0.6-1.0

claimed evidence only by a part of the collaboration

started in 2003

$$m_{ee} = \langle m_\nu \rangle = |\Sigma\, U_{ej}^2\, m_j\, e^{i\alpha j}|$$

Detecting $0\nu\beta\beta$ would prove L non conservation

Future: a factor ~ 10 improvement in next decade

$0\nu\beta\beta$ Decay Measurements

upper limit

Survey of some past and present experiments

isotope	experiment	latest result	$Q_{\beta\beta}$ [keV]	i.a. nat.	i.a. enrich.	exposure [kg×y]	technique	material	$\tau_{1/2}^{0\nu}$ [10^{23} y]	$\langle m_\nu \rangle$ [eV]
^{48}Ca	Elegant VI	2004[11]	4271	0.19	–	4.2	scintillator	CaF_2	0.14	7.2÷44.70
^{76}Ge	Heidelberg/Moscow	2004[17]	2039	7.8	87	71.7	ionization	Ge	120.0	0.44
^{82}Se	NEMO-3	2007[22]	2995	9.2	97	1.8	tracking	Se	1.2	1.60÷4.50
^{100}Mo	NEMO-3	2007[22]	3034	9.6	95÷99	13.1	tracking	Mo	5.8	0.60÷2.40
^{116}Cd	Solotvina	2003[12]	2805	7.5	83	0.5	scintillator	$CdWO_4$	1.7	1.70
^{130}Te	Cuoricino	2007[20]	2529	33.8	–	11.8	bolometer	TeO_2	30.0	0.16÷0.84
^{136}Xe	DAMA	2002[23]	2476	8.9	69	4.5	scintillator	Xe	12.0	1.10÷2.90
^{150}Nd	Irvine TPC	1997[14]	3367	5.6	91	0.01	tracking	Nd_2O_3	0.012	3.00
^{160}Gd	Solotvina	2001[13]	1791	21.8	–	1.0	scintillator	Gd_2SiO_5	0.013	26.00

A. Nucciotti arXiv:0707.2216 [nucl-ex]

$$0.16 < m_{\beta\beta}/\mathrm{eV} < 0.52 \qquad \text{(HM claim)},$$
$$0 \leq m_{\beta\beta}/\mathrm{eV} < 0.23 \qquad \text{(Cuoricino, “favorable” NME)},$$
$$0 \leq m_{\beta\beta}/\mathrm{eV} < 0.85 \qquad \text{(Cuoricino, “unfavorable” NME)}$$

Arnaboldi et al

The Heidelberg-Moscow claim not disproved by Cuoricino depending on nuclear matrix elements

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Bounds from cosmology

By itself CMB is only mildly sensitive to $\Sigma = \Sigma_i m_i$

Only in combination with LSS the limit becomes stronger.

And even stronger by adding the Lyman alpha forest data (but some tension among the data).

Fogli et al '08

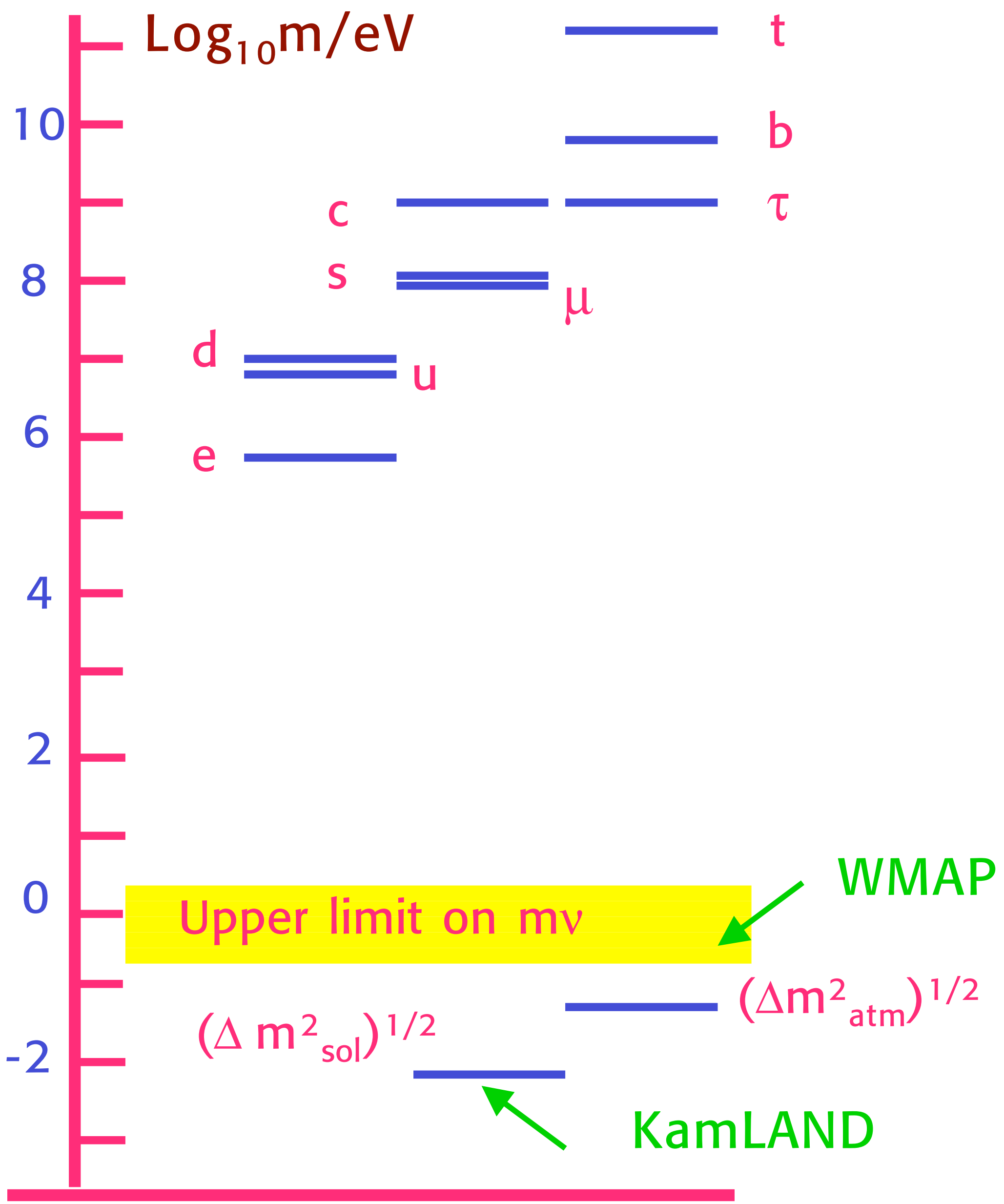
Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

CMB Cosmic Microwave Background: WMAP+ ACBAR+......

LSS Large Scale Structure (2dFGRS, SDSS)

HST +SN-Ia Hubble Space Tel. [h=0.72(7)]+ SuperNovae

BAO Baryonic Acoustic Oscillation (SDSS)



Neutrino masses are really special!

$m_t/(\Delta m^2_{atm})^{1/2} \sim 10^{12}$

Massless ν's?

- no ν_R
- L conserved

Small ν masses?

- ν_R very heavy
- L not conserved

A very natural and appealing explanation:

ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale M ~ M_{GUT}

$$m_\nu \sim \frac{m^2}{M}$$

m: ≤ m_t ~ v ~ 200 GeV
M: scale of L non cons.

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV
m ~ v ~ 200 GeV

M ~ 10^{15} GeV

Neutrino masses are a probe of physics at M_{GUT} !

Baryogenesis by decay of heavy Majorana ν's

BG via Leptogenesis near the GUT scale

$T \sim 10^{12\pm3}$ GeV (after inflation)

Buchmuller,Yanagida, Plumacher, Ellis, Lola, Giudice et al, Fujii et al ….

Only survives if Δ(B-L) is not zero
(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R ($M \sim 10^{12}$ GeV)

L non conserv. in ν_R out-of-equilibrium decay:
B-L excess survives at T_{ew} and gives the obs. B asymmetry.

Quantitative studies confirm that the range of m_i from ν oscill's is compatible with BG via (thermal) LG

In particular the bound was derived for hierarchy

$m_i < 10^{-1}$ eV

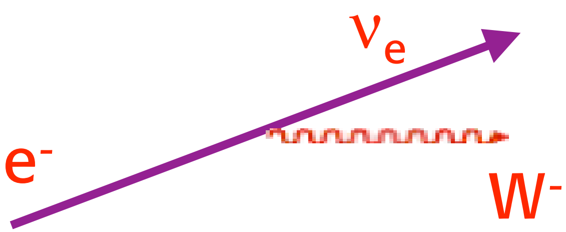
Buchmuller, Di Bari, Plumacher; Giudice et al; Pilaftsis et al; Hambye et al

Can be relaxed for degenerate neutrinos
So fully compatible with oscill'n data!!

3-ν Models

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U^{+}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

flavour mass



$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

δ: CP violation

$$U = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix} \sim$$

s = solar: large

CHOOZ: $|s_{13}| <\sim 0.2$

$$\sim \begin{pmatrix}c_{13}\,c_{12} & c_{13}\,s_{12} & s_{13}e^{-i\delta}\\ \dots & \dots & c_{13}\,s_{23}\\ \dots & \dots & c_{13}\,c_{23}\end{pmatrix}$$

atm.: ~ max

(some signs are conventional)

In general: $U = U^{+}{}_{e}U_{\nu}$

$$m_\nu \sim U^* \begin{bmatrix} e^{i\alpha_1}m_1 & 0 & 0 \\ 0 & e^{i\alpha_2}m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} U^+$$

$L^T m_\nu L$

In general 9 parameters:
3 masses, 3 angles,
3 phases

For $s_{13} \sim 0$:

$0\nu\beta\beta$

$$m_\nu \sim \begin{bmatrix} m_1c^2+m_2s^2 & (m_1-m_2)cs/\sqrt{2} & (m_1-m_2)cs/\sqrt{2} \\ \ldots & (m_1s^2+m_2c^2+m_3)/2 & (m_1s^2+m_2c^2-m_3)/2 \\ \ldots & \ldots & (m_1s^2+m_2c^2+m_3)/2 \end{bmatrix}$$

Note:
- m_ν is symmetric
- phases included in m_i

Relation between masses and frequencies:

$P(\nu_e \leftrightarrow \nu_\mu) = P(\nu_e \leftrightarrow \nu_\tau) = 1/2 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$P(\nu_\mu \leftrightarrow \nu_\tau) = \sin^2 \Delta_{atm} - 1/4 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$$\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L \quad ; \quad \Delta_{atm} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

In our def.: $\Delta_{sun} > 0$, $\Delta_{atm} >$ or < 0

Defining:

$$\Delta m^2_{atm} = m_3^2 - m_2^2 > \text{or} < 0$$

$$\Delta m^2_{sol} = m_2^2 - m_1^2 > 0$$

one has:

$$m_3^2 = \overline{m^2} + \frac{2}{3}\Delta m^2_{atm} + \frac{1}{3}\Delta m^2_{sol}$$

$$m_2^2 = \overline{m^2} - \frac{1}{3}\Delta m^2_{atm} + \frac{1}{3}\Delta m^2_{sol}$$

$$m_1^2 = \overline{m^2} - \frac{1}{3}\Delta m^2_{atm} - \frac{2}{3}\Delta m^2_{sol}$$

and

$$\overline{m^2} >> \left|\Delta m^2_{atm}\right| > \Delta m^2_{sol} \qquad \text{degenerate}$$

$$\Delta m^2_{atm} < 0 \qquad \text{inverse hierarchy}$$

$$\Delta m^2_{atm} > 0 \qquad \text{normal hierarchy}$$

Neutrino oscillation parameters

- 2 distinct frequencies
- 2 large angles, 1 small

parameter	best fit	2σ	3σ
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	$7.65^{+0.23}_{-0.20}$	7.25–8.11	7.05–8.34
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}\mathrm{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.27–0.35	0.25–0.37
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	≤ 0.040	≤ 0.056

Schwetz et al '08

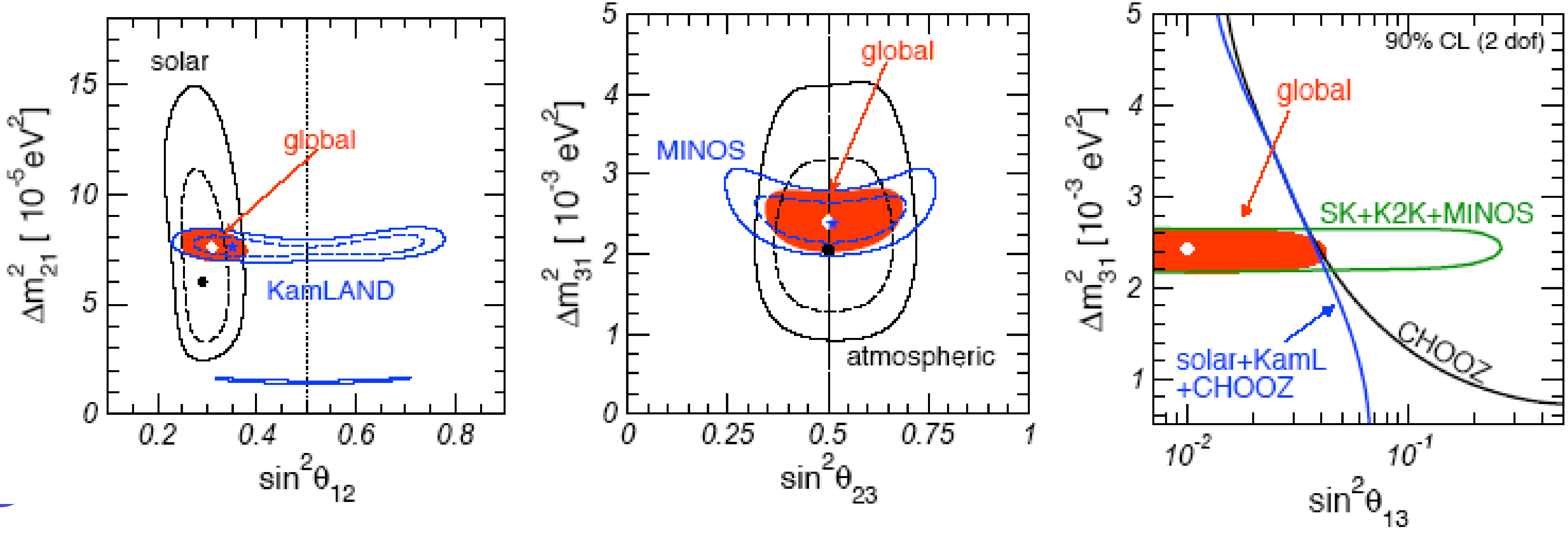
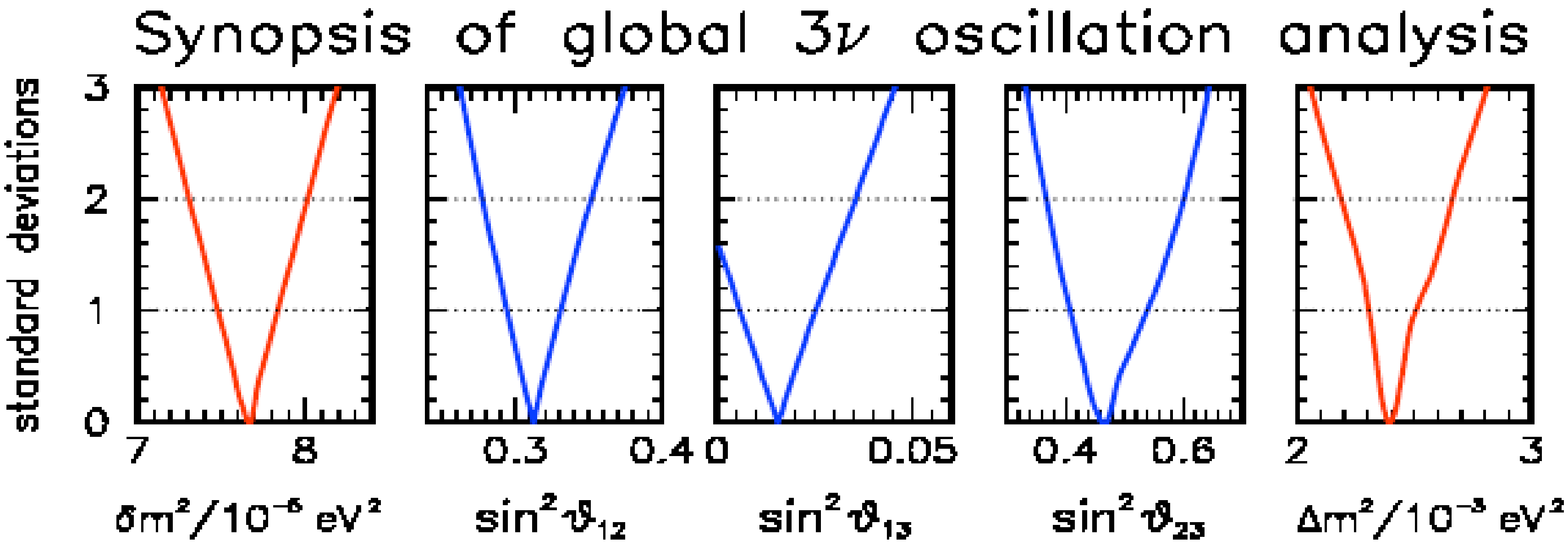


Table 1: Global 3ν oscillation analysis (2008): best-fit values and allowed n_σ ranges, from Ref. [4].

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.67	0.312	0.016	0.466	2.39
1σ range	7.48 – 7.83	0.294 – 0.331	0.006 – 0.026	0.408 – 0.539	2.31 – 2.50
2σ range	7.31 – 8.01	0.278 – 0.352	< 0.036	0.366 – 0.602	2.19 – 2.66
3σ range	7.14 – 8.19	0.263 – 0.375	< 0.046	0.331 – 0.644	2.06 – 2.81

Fogli et al '08

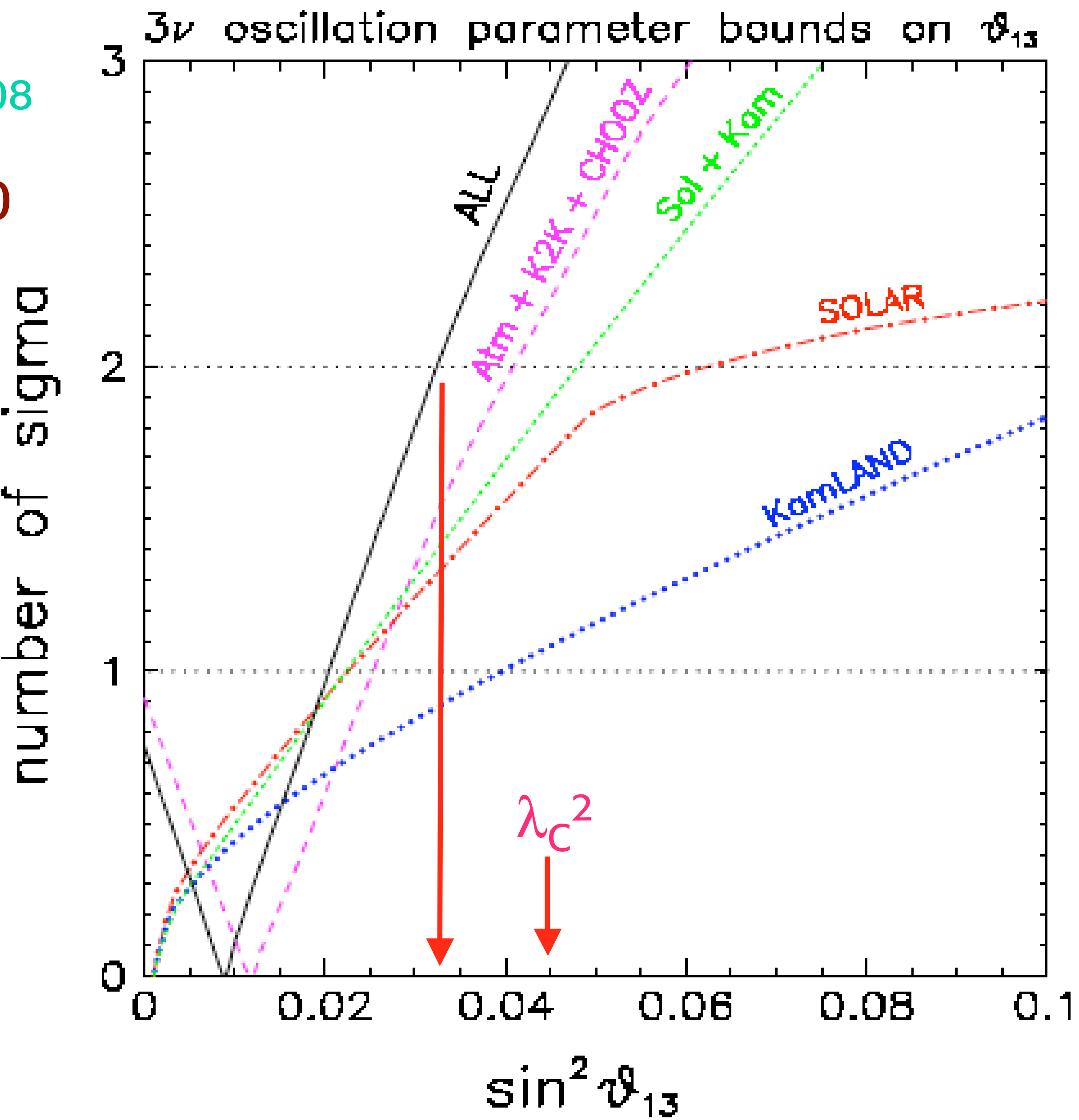


θ_{13} bounds

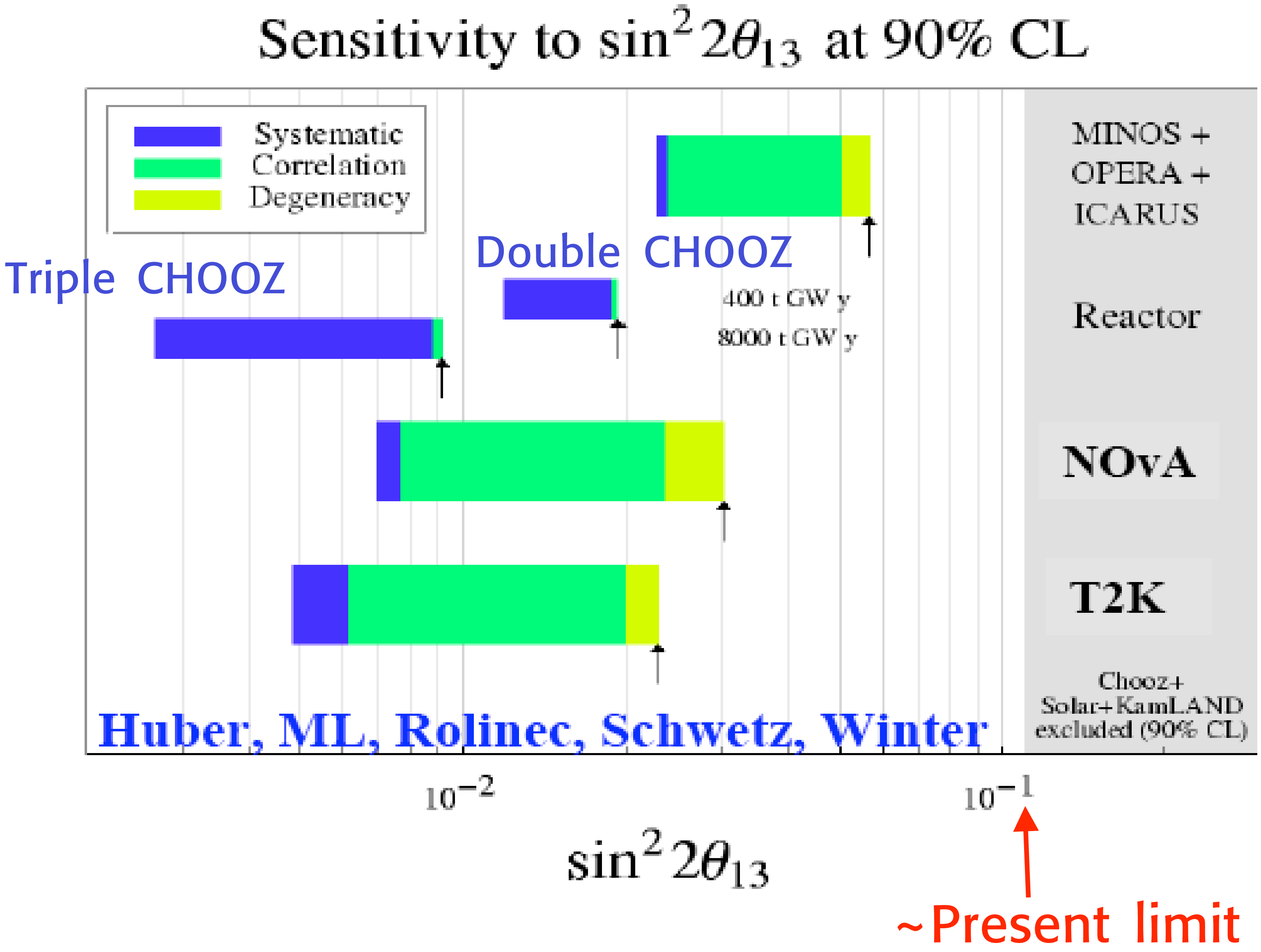
Fogli et al '08

$\sin^2\theta_{13}=0.016\pm0.010$

The 95% upper bound on $\sin\theta_{13}$ is close to $\lambda_C = \sin\theta_C$



Measuring θ_{13} is crucial for future ν-oscill's experiments (eg CP violation)



starts in '09!

$0\nu\beta\beta$ would prove that L is not conserved and ν's are Majorana
Also can tell degenerate, inverted or normal hierarchy

$$|m_{ee}|=c_{13}^2\,[m_1 c_{12}^2+e^{i\alpha}m_2 s_{12}^2]+m_3 e^{i\beta}s_{13}^2$$

Degenerate: $\sim|m||c_{12}^2+e^{i\alpha}s_{12}^2|\sim|m|(0.3\text{-}1)$

$|m_{ee}|\sim|m|\,(0.3\text{ -}1)\leq 0.23\text{-}1$ eV

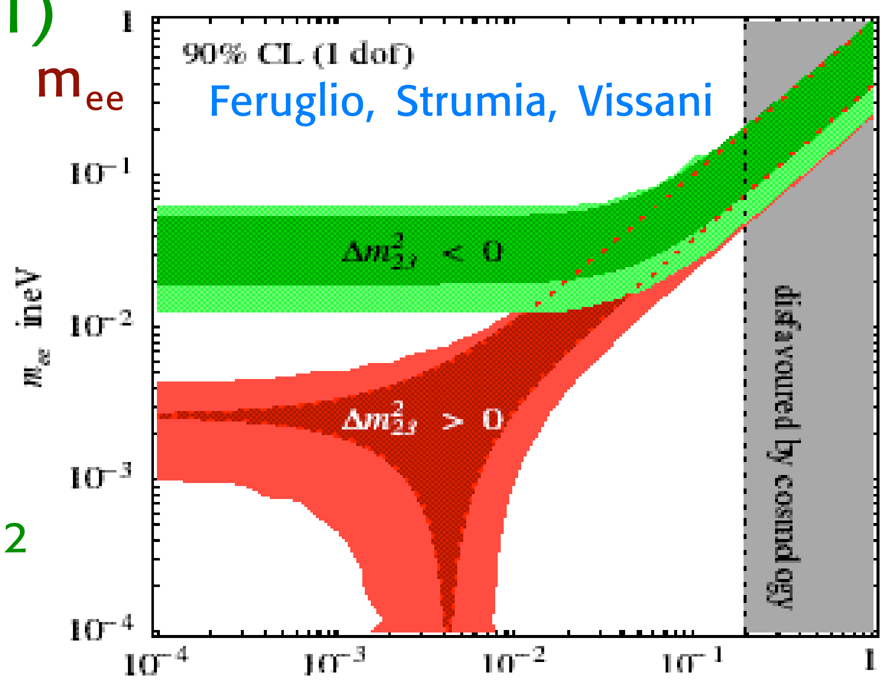
IH: $\sim(\Delta m^2_{atm})^{1/2}|c_{12}^2+e^{i\alpha}s_{12}^2|$

$|m_{ee}|\sim(1.6\text{-}5)\;10^{-2}$ eV

NH: $\sim(\Delta m^2_{sol})^{1/2}s_{12}^2+(\Delta m^2_{atm})^{1/2}e^{i\beta}s_{13}^2$

$|m_{ee}|\sim$ (few) 10^{-3} eV

Full dependence on min m_ν



Present exp. limit: $m_{ee}< 0.3\text{-}0.5$ eV
(and a hint of signal????? Klapdor Kleingrothaus)

General remarks

• After KamLAND, SNO and WMAP.... not too much hierarchy is found in ν masses:

$$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$$

Only a few years ago could be as small as 10^{-8}!

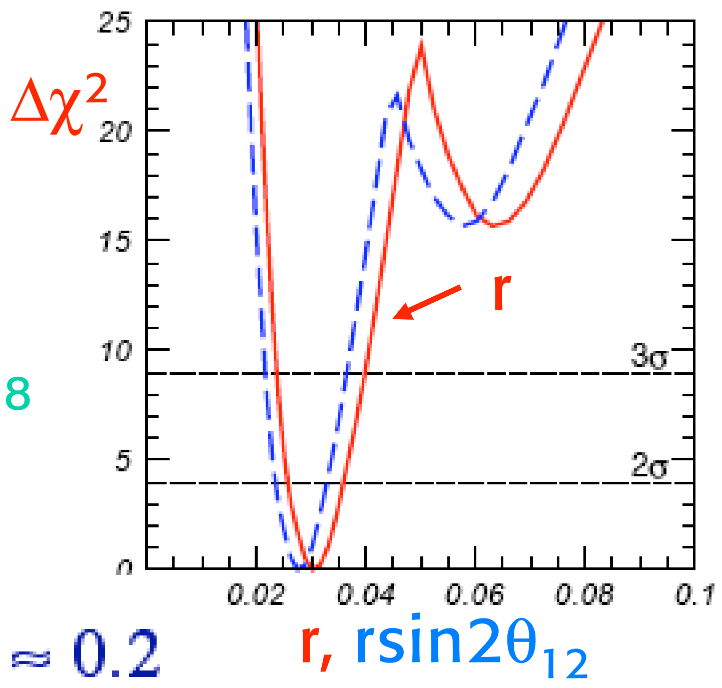
Precisely at 3σ: $0.025 < r < 0.039$

Schwetz et al '08

or

$m_{heaviest} < 0.2$ - 0.7 eV

$m_{next} > \sim 8\ 10^{-3}$ eV



For a hierarchical spectrum: $\dfrac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to $\lambda_C = \sin\theta_C$: $\lambda_C \approx 0.22$ or $\sqrt{\dfrac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν

(small powers of λ_C) $\longrightarrow$ e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

  2-σ interval $0.37 < \sin^2\theta_{23} < 0.60$ Fogli et al '08

  Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small

  probably accessible to exp.

  Very small θ_{13} theoretically hard

For some time people considered limiting models with $\theta_{13}= 0$ and θ_{23} maximal and θ_{12} generic

The most general mass matrix for $\theta_{13}= 0$ and θ_{23} maximal is given by
(after ch. lepton diagonalization!!!):

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

Neglecting Majorana phases it depends on 4 real parameters (3 mass eigenvalues and 1 mixing angle: θ_{12})

Inspired models based on $\mu-\tau$ symmetry
Grimus, Lavoura..., Ma,.... Mohapatra, Nasri, Hai-Bo Yu ....

Actually, at present, since KamLAND, the most accurately known angle is θ_{12}

G.L.Fogli et al'08

At $\sim 1\sigma$: $\boxed{\sin^2\theta_{12} = 0.294\text{-}0.331}$

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Harrison, Perkins, Scott '02

$\oplus$ Some additional ingredient other than μ–τ symmetry needed!

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Comparison with experiment:

At 1σ: G.L.Fogli et al'08

$\sin^2\theta_{12} = 1/3$: 0.29-0.33

$\sin^2\theta_{23} = 1/2$: 0.41-0.54

$\sin^2\theta_{13} = 0$: $< \sim 0.02$

The HPS mixing is clearly a very good approx. to the data!

Also called:
Tri-Bimaximal mixing

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

A lot of model building has been devoted to TB mixing

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Tribimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix} \longrightarrow m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$m_1 = x - y$
$m_2 = x + 2y$
$m_3 = x - y + 2v$

The 3 remaining parameters are the mass eigenvalues

Tribimaximal Mixing

A simple mixing matrix compatible with all present data

$$U=\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \to \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \to \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \to \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

Compare with quark mixings $\lambda_C \sim (m_d/m_s)^{1/2}$

- For the HPS mixing matrix all mixing angles are fixed to particularly symmetric values

Sparked interest in constructing models that can naturally produce this highly ordered structure

Models based on the A4 discrete symmetry (even permutations of 1234) offer a minimal solution

Ma...;
GA, Feruglio, hep-ph/0504165, hep-ph/0512103
GA, Feruglio, Lin hep-ph/0610165
GA, Feruglio, Hagedorn, 0802.0090 [hep-ph]
Y. Lin, 0804.2867 [hep-ph]; Csaki et al' 0806.0356.......

Larger finite groups: T', $\Delta(27)$, S4

Feruglio et al;
Chen, Mahanthappa;
Frampton, Kephart; Lam;
Bazzocchi et al ....

Alternative models based on $SU(3)_F$ or $SO(3)_F$ or their finite subgroups

Verzielas, G. Ross King ....
....

A4

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

A4 transformations can be written in terms of S and T as:

$$1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2$$

with: $S^2 = T^3 = (ST)^3 = 1$ [$(TS)^3 = 1$ also follows]

An element is abcd which means 1234 --> abcd

C_1: $1 = 1234$
C_2: $T = 2314$ $ST = 4132$ $TS = 3241$ $STS = 1423$
C_3: $T^2 = 3124$ $ST^2 = 4213$ $T^2S = 2431$ $TST = 1342$
C_4: $S = 4321$ $T^2ST = 3412$ $TST^2 = 2143$

x, x′ in same class if

C_1, C_2, C_3, C_4 are equivalence classes [$x' \sim gxg^{-1}$] g: group element

A4 has 4 inequivalent irreducible representations:
a triplet and 3 different singlets

$$3,\ 1,\ 1',\ 1''$$

(promising for 3 generations!)

Note:

as many representations as equivalence classes

$\sum d_i^2 = 12$ 9+1+1+1=12

Note: many models tried S3
S3 has no triplets but only 2 , 1, 1′
A4 is better in the lepton sector

Mohapatra, Nasri, Yu
Koide
Kubo et al
Kaneko et al
Caravaglios et al
Morisi
Picariello......

Three singlet inequivalent represent'ns:

$$\omega = \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\omega^3 = 1$$

$$1 + \omega + \omega^2 = 0$$

$$\omega^2 = \omega^*$$

Recall:
$S^2 = T^3 = (ST)^3 = 1$

$$\begin{cases} 1: \; S=1,\ T=1 \\ 1': \; S=1,\ T=\omega \\ 1'': \; S=1,\ T=\omega^2 \end{cases}$$

The only irreducible 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

Cabibbo '78

A4 has only 4 irreducible inequivalent represt'ns: 1,1',1'',3

Table of Multiplication:
$1' \times 1' = 1''$; $1'' \times 1'' = 1'$; $1' \times 1'' = 1$
$3 \times 3 = 1 + 1' + 1'' + 3 + 3$

A4 is well fit for 3 families!

Ch. leptons $l \sim 3$

$e^c, \mu^c, \tau^c \sim 1, 1'', 1'$

In the S-diag basis consider 3: $(a_1,a_2,a_3) \xrightarrow{S} (a_1,-a_2,-a_3)$

$(a_1,a_2,a_3) \xrightarrow{T} (a_2,a_3,a_1)$

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 \times 3_2$:

$$1 = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$1' = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3$$

$$1" = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3$$

$$3 \sim (a_2 b_3, a_3 b_1, a_1 b_2)$$

$$3 \sim (a_3 b_2, a_1 b_3, a_2 b_1)$$

e.g. $1" = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3 \xrightarrow{T} a_2 b_2 + \omega a_3 b_3 + \omega^2 a_1 b_1 =$
$= \omega^2 [a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3]$

(under S, 1" is invariant)

Under A4 the most common classification is:

lepton doublets $l \sim 3$

$e^c, \mu^c, \tau^c \sim 1, 1'', 1'$ respectively

A4 breaking gauge singlet flavons $\phi_S, \phi_T, \xi, (\xi') \sim 3, 3, 1, (1)$

For SUSY version: driving fields $\phi'_S, \phi'_T, \xi_0 \sim 3, 3, 1$

with the alignment:

!!!

$$\boxed{\begin{gathered}\langle\varphi_T\rangle = (v_T, 0, 0)\\ \langle\varphi_S\rangle = (v_S, v_S, v_S)\\ \langle\xi\rangle = u \quad , \quad \langle\tilde{\xi}\rangle = 0\end{gathered}}$$

In a serious model the alignment is a consequence of the symmetries

In all versions there are additional symmetries:

e.g. a broken $U(1)_F$ symmetry and/or discrete symmetries Z_n to ensure hierarchy of charged lepton masses and to restrict allowed couplings

Structure of the model (a 4-dim SUSY version)

GA, Feruglio, hep-ph/0512103

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + (x_a \xi + \tilde{x}_a \tilde{\xi})(ll) + x_b(\varphi_S ll) + h.c. + \ldots$$

shorthand: Higgs and cut-off scale Λ omitted, e.g.:

$$y_e e^c(\varphi l) \sim y_e e^c(\varphi l) h_d/\Lambda, \qquad x_a \xi(ll) \sim x_a \xi(l h_u l h_u)/\Lambda^2$$

In T-diag basis:

with this alignment:

$$\boxed{\begin{aligned} \langle\varphi_T\rangle &= (v_T, 0, 0) \\ \langle\varphi_S\rangle &= (v_S, v_S, v_S) \\ \langle\xi\rangle = u \quad &, \quad \langle\tilde{\xi}\rangle = 0 \end{aligned}}$$

Ch. leptons are diagonal

$$m_l = v_T \frac{v_d}{\Lambda}\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

ν's are tri-bimaximal

$$m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix}$$

$$a \equiv x_a \frac{u}{\Lambda} \qquad b \equiv x_b \frac{v_T}{\Lambda}$$

recall:

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

So, at LO TB mixing is exact

The only fine-tuning needed is to account for $r \sim 1/30$
[In most A4 models $r \sim 1$ would be expected as l, $\nu^c \sim 3$]

When NLO corrections are included from operators of higher dimension in the superpotential each mixing angle receives corrections of the same order $\delta\theta_{ij} \sim o(VEV/\Lambda)$

As the maximum allowed corrections to θ_{12} (and also to θ_{23}) are $o(\lambda_C^2)$, we need $VEV/\Lambda \sim o(\lambda_C^2)$ and we expect:

$\theta_{13} \sim o(\lambda_C^2)$ measurable in next run of exp's

(T2K starts at the end of '09)

- Many versions of A4 models exist by now

Why A4 works?

TB mixing corresponds to m
in the basis where
charged leptons are diagonal

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

m is the most general matrix invariant under
SmS = m and $A_{23}mA_{23}$= m with:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3
symmetry

Invariance under S can be made automatic in A4 while
invariance under A_{23} happens if 1′ and 1″ flavons are absent.

Charged lepton masses are a generic diagonal matrix, invariant under T
(or ηT with η a phase):

$$T^{\dagger} m_l T = m_l$$

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

The aligment occurs because is based on A4 group theory:

ϕ_T breaks A4 down to G_T
ϕ_S breaks A4 down to G_S
(G_T, G_S: subgroups generated by T, S)

Note that for TB mixing in A4 it is important that no flavons transforming as $1'$ and $1''$ exist

Recently Lam claimed that for "a natural" TB model the smallest group is S4 (instead A4 is a subgroup of S4)

This is because he calls "natural" a model only if all possible flavons are introduced

We do not accept this criterium:

In physics we call natural a model if the lagrangian is the most general given the symmetry and the representations of the fields
(for example the SM is natural even if only Higgs doublets are present)

Recent directions of research:

- Different (larger) finite groups

Ma;
Kobayashi et al;
Luhn, Nasri, Ramond [$\Delta(3n^2)$];
.....

- Trying to improve the quark mixings

Carr, Frampton
Feruglio et al
Frampton, Kephart.....

- Construct GUT models with approximate tribimaximal mixing

it is indeed possible, also for A4!
GA, Feruglio, Hagedorn 0802.0090

Ma, Sawanaka, Tanimoto; Ma;
Morisi, Picarello, Torrente Lujan; Bazzocchi et al;
de Madeiros Verzielas, King, Ross [$\Delta(27)$];
King, Malinsky [$SU(4)_C x SU(2)_L x SU(2)_R$]; Antusch et al;
Chen, MahanthappaBazzocchi et al [$\Delta(27)$]; .....

Or agreement with TB mixing could be accidental

If θ_{13} is found near its present bound (e.g $o(\lambda_C)$) this would hint that TB is accidental and bimaximal mixing (BM) could be a better first approximation

There is an intriguing empirical relation:

$$\theta_{12} + \theta_C = (47.0 \pm 1.7)^o \sim \pi/4$$

Raidal'04

Suggests bimaximal mixing in 1st approximation, corrected by charged lepton diagonalization.

Recall that

$$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

$\lambda_C = \sin\theta_C$

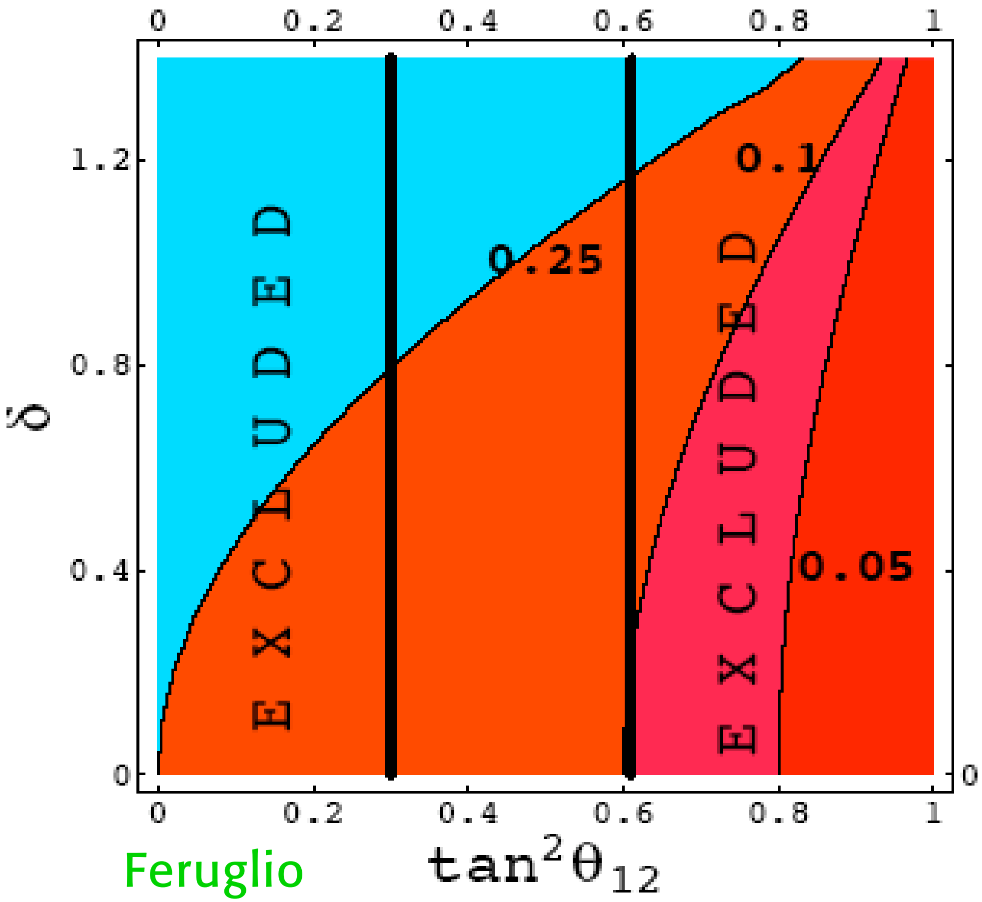
While $\theta_{12} + o(\theta_C) \sim \pi/4$ is easy to realize, exactly $\theta_{12} + \theta_C \sim \pi/4$ is more difficult: no compelling model

Minakata, Smirnov'04

Suggests that deviations from BiMaximal mixing arise from charged lepton diagonalisation (BM: $\theta_{12}= \theta_{23} =\pi/4$ $\theta_{12}=0$)

For the corrections from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1- \tan^2\theta_{12})/4\cos\delta \sim 0.15$

GA, Feruglio, Masina
Frampton et al
King
Antusch et al........



Feruglio

$$\bar{U}_{12} = -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s^e_{12}e^{-i\alpha_2} + s^e_{13}e^{i\delta_e}}{2}$$

$$\bar{U}_{13} = \frac{s^e_{12}e^{-i\alpha_2} - s^e_{13}e^{i\delta_e}}{\sqrt{2}}$$

$$\bar{U}_{23} = -e^{-i\alpha_2}\frac{1 + s^e_{23}e^{i\alpha_2}}{\sqrt{2}}$$

Corr.'s from s^e_{12}, s^e_{13} to U_{12} and U_{13} are of first order (2nd order to U_{23})

Here we construct a model where BM mixing holds in 1st approximation and is then corrected by terms o(λ_C)

Revisiting Bimaximal Neutrino Mixing in a Model with S_4 Discrete Symmetry

Guido Altarelli [1]

Dipartimento di Fisica 'E. Amaldi', Università di Roma Tre
INFN, Sezione di Roma Tre, I-00146 Rome, Italy

and

CERN, Department of Physics, Theory Division
CH-1211 Geneva 23, Switzerland

Ferruccio Feruglio [2] and Luca Merlo [3]

Dipartimento di Fisica 'G. Galilei', Università di Padova
INFN, Sezione di Padova, Via Marzolo 8, I-35131 Padua, Italy

soon to appear on the web

BM mixing

$\theta_{12} = \theta_{23} = \pi/4,\ \theta_{13} = 0$

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

By adding $\sin^2\theta_{12} \sim 1/2$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Bimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix} \longrightarrow m_{\nu BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$$m_1 = x + \sqrt{2}y$$

$$m_2 = x - \sqrt{2}y$$

$$m_3 = 2z - x$$

BM corresponds to $\tan^2\theta_{12}=1$
while exp.: $\tan^2\theta_{12}= 0.45 \pm 0.04$
so a large correction is needed

The 3 remaining parameters
are the mass eigenvalues

Bimaximal Mixing

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$

$$m_{\nu BM} = \left[\frac{m_3}{2} M_3 + \frac{m_2}{4} M_2 + \frac{m_1}{4} M_1\right]$$

$$M_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \; M_2 = \begin{pmatrix} 2 & -\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & 1 & 1 \\ -\sqrt{2} & 1 & 1 \end{pmatrix}, \; M_1 = \begin{pmatrix} 2 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & 1 & 1 \\ \sqrt{2} & 1 & 1 \end{pmatrix}$$

Eigenvectors: $(\sqrt{2}, 1, 1)/2, \; (-\sqrt{2}, 1, 1)/2 \; , \; (0, 1, -1)/\sqrt{2}$

S4: Group of permutations of 4 objects (24 transformations)

Irreducible representations: 1, 1', 2, 3, 3'

$$T^4=S^2=(ST)^3=(TS)^3=1$$

1 $\quad T = 1 \qquad S = 1$

2 $\quad T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad S = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

3 $\quad T = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{pmatrix} \qquad S = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

1 <-> 1' and 3<-> 3' by changing S, T <-> -S, -T

BM mixing corresponds to $m = m_{\nu BM}$
in the basis where
charged leptons are diagonal

$$m_{\nu BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

m is the most general matrix invariant under
$SmS = m$ and $A_{23} m A_{23} = m$ with:

$$S = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3 symmetry

Invariance under S can be made automatic in S4 while
invariance under A_{23} happens if the flavon content is suitable

	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	θ	φ_l	χ_l	ψ_l^0	χ_l^0	ξ_ν	φ_ν	ξ_ν^0	φ_ν^0
S_4	3	1	$1'$	1	3	1	1	3	$3'$	2	$3'$	1	3	1	3
Z_4	1	-1	-i	-i	1	1	1	i	i	-1	-1	1	1	1	1
$U(1)_{FN}$	0	2	1	0	0	0	-1	0	0	0	0	0	0	0	0
$U(1)_R$	1	1	1	1	1	1	0	0	0	2	2	0	0	2	2

$$\begin{aligned} w_l \; = \; & \frac{y_e^{(1)}}{\Lambda^2}\frac{\theta^2}{\Lambda^2}e^c(l\varphi_l\varphi_l) + \frac{y_e^{(2)}}{\Lambda^2}\frac{\theta^2}{\Lambda^2}e^c(l\chi_l\chi_l) + \frac{y_e^{(3)}}{\Lambda^2}\frac{\theta^2}{\Lambda^2}e^c(l\varphi_l\chi_l) + \\ + \; & \frac{y_\mu}{\Lambda}\frac{\theta}{\Lambda}\mu^c(l\chi_l)' + \frac{y_\tau}{\Lambda}\tau^c(l\varphi_l) + \ldots \end{aligned}$$

$$w_\nu \; = \; y(\nu^c l) + M\Lambda(\nu^c\nu^c) + a(\nu^c\nu^c\xi_\nu) + b(\nu^c\nu^c\varphi_\nu) + \ldots$$ see-saw

$$\frac{\langle\xi_\nu\rangle}{\Lambda} = E \qquad \frac{\langle\varphi_\nu\rangle}{\Lambda} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} C \qquad \frac{\langle\varphi_l\rangle}{\Lambda} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} A \qquad \frac{\langle\chi_l\rangle}{\Lambda} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} B$$

In this model BM mixing is exact at LO

For the special flavon content chosen, only θ_{12} and θ_{13} are corrected from the charged lepton sector by terms of $o(\lambda_C)$ (large correction!) while θ_{23} gets smaller corrections (great!) [for a generic flavon content also $\delta\theta_{23} \sim o(\lambda_C)$]

An experimental indication for this model would be that θ_{13} is found near its present bound at T2K

Conclusion

The observed pattern of neutrino masses can be accommodated

Quark and lepton mixings can be described together and GUT schemes are also possible

But no compelling illumination about the dynamics of flavour has emerged so far.