

Les Rencontres de Physique de la Vallee d'Aoste 2009

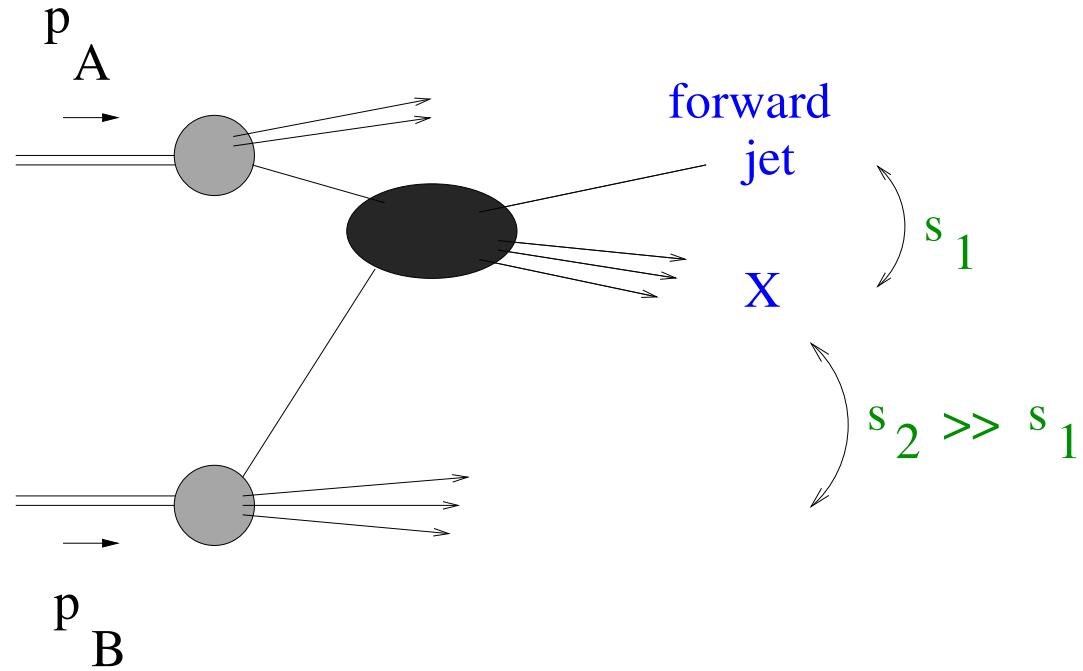
## High- $p_T$ physics in the forward region and QCD coherence effects

F. Hautmann (Oxford)

- Motivation — hard processes at forward rapidities at the LHC
- Theoretical issues on space-like parton showers and coherent gluon radiation
  - Applications in  $ep$  and  $p\bar{p}$  jet production

# INTRODUCTION

## High- $p_T$ production in the forward region at the LHC



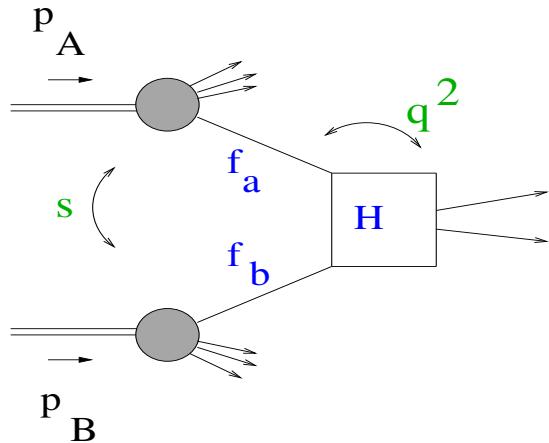
- ▷ experimental coverage of large rapidities
- ▷ phase space opening up for large  $\sqrt{s}$

$\Downarrow$

- events with **multiple** hard scales:  $q_1^2, \dots, q_n^2$
- potentially large corrections to all orders in  $\alpha_s$ ,  $\sim \ln^k(q_i^2/q_j^2)$

- asymmetric parton kinematics

$\Rightarrow$  parton distributions probed near kinem. boundaries  $x \rightarrow 0, 1 - x \rightarrow 0$



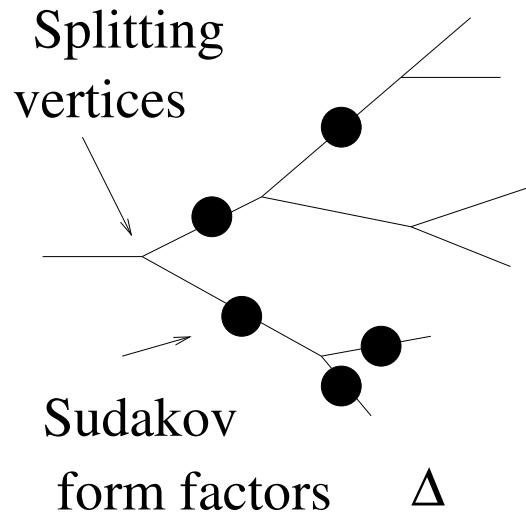
▷ coherence effects for multi-gluon radiation at  
small longitudinal momentum fractions  $x$ :

- not included in standard shower Monte Carlo generators
  - included partially in NLO multi-jet calculations
- present to all orders and enhanced by logs of  $\sqrt{s}/E_T$

# OUTLINE

- I. Parton branching and multi-gluon emission
- II. Coherence in space-like showers at high energies
- III. Issues on unintegrated parton distributions
- IV. Angular correlations and jet production data
- V. Summary and prospects for LHC final states

# I. PARTON SHOWER METHODS

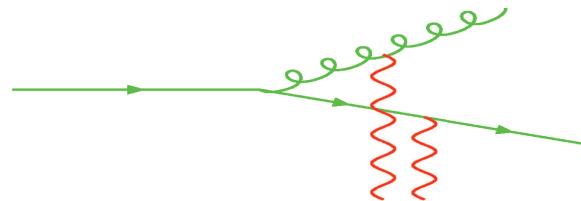


$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \alpha_S P(z) \Delta(q^2, q_0^2)$$

- based on dominance of collinear evolution of jets
- Factorization of QCD cross sections in collinear limit  
→ probabilistic (Markov) picture
- summation of logarithmically enhanced radiative contributions  
 $(\alpha_S \ln p_T/\Lambda)^n$
- soft gluon radiation by coherent branching [HERWIG, new PYTHIA]



▷ soft gluons radiated over long times  $\longrightarrow$  quantum interferences



↙ factorization in soft limit

$$|M_{n+1}^{a_1 \dots a_n a}(p_1, p_n, q)\rangle = \mathbf{J}^a |M_n^{a_1 \dots a_n}(p_1, p_n)\rangle , \quad \mathbf{J}^{a\mu} = \sum_i \mathbf{Q}_i^a \frac{p_i^\mu}{p_i \cdot q} , \quad \mathbf{Q} = \text{color charge}$$

interference terms ↓

$$d\sigma_{n+1} = d\sigma_n \frac{d^3 q}{(q^0)^3} \sum_{i,j} \mathbf{Q}_i \cdot \mathbf{Q}_j w_{ij} , \quad w_{ij} = \frac{(q^0)^2 p_i \cdot p_j}{(p_i \cdot q)(p_j \cdot q)}$$

— not positive definite, non-Markov..?

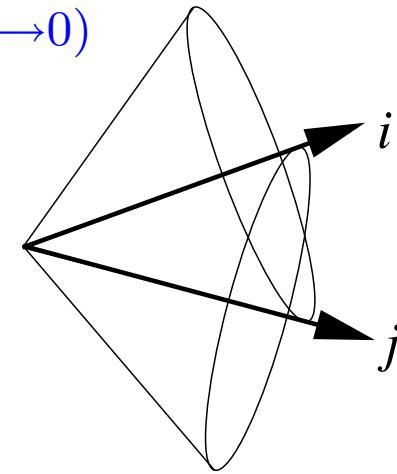
→ spoils probabilistic picture? NO, owing to soft-gluon coherence ↪

- single-emission: separate singularities along emitters' directions

$$\begin{aligned} \frac{(q^0)^2 p_i \cdot p_j}{(p_i \cdot q)(p_j \cdot q)} &\equiv \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} \\ &= \frac{1}{2} \left( \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} - \frac{1}{\zeta_{jq}} + \frac{1}{\zeta_{iq}} \right) + \frac{1}{2} \left( \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} - \frac{1}{\zeta_{iq}} + \frac{1}{\zeta_{jq}} \right) \end{aligned}$$

where  $\zeta_{nk} \equiv \frac{p_n \cdot p_k}{p_n^0 p_k^0} \simeq 1 - \cos \theta_{nk}$  ( $m \rightarrow 0$ )

→ by azimuthal average

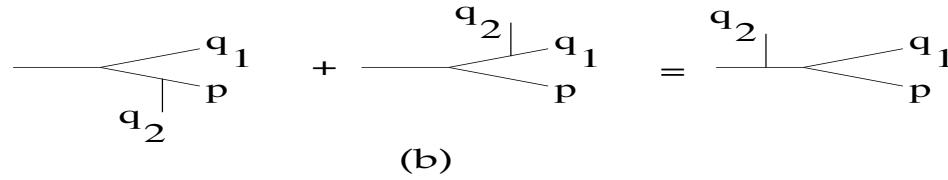
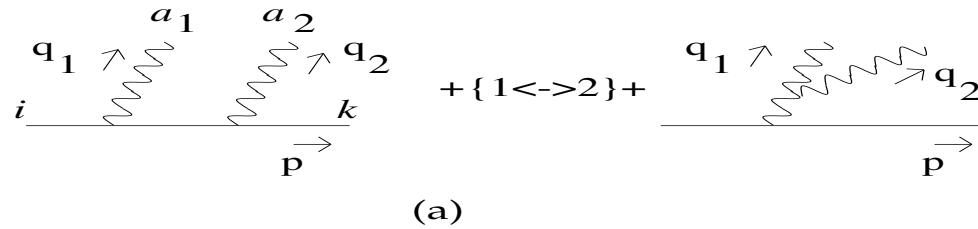


$$\langle \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} \rangle = \frac{1}{\zeta_{iq}} \Theta(\zeta_{ij} - \zeta_{iq}) + \frac{1}{\zeta_{jq}} \Theta(\zeta_{ij} - \zeta_{jq})$$

◇ large-angle emissions of soft gluons sum coherently outside angular-ordered cones

- multiple emission: ( $q_1, q_2$  with  $q_2^0 \ll q_1^0$ )

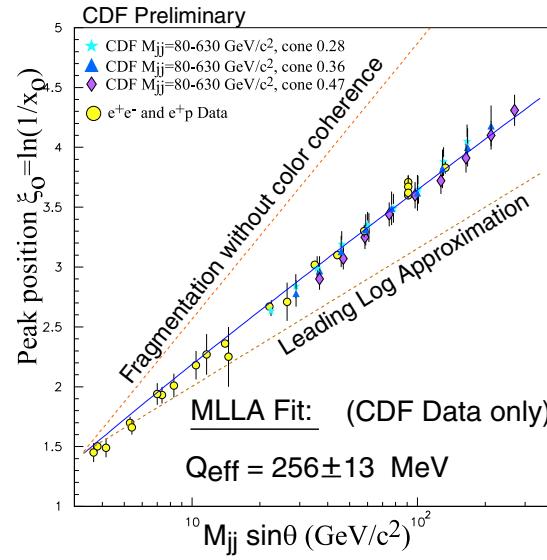
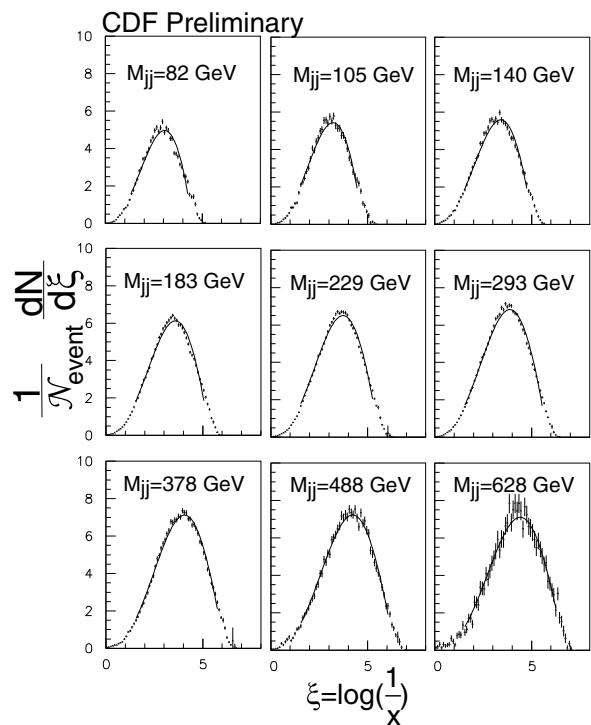
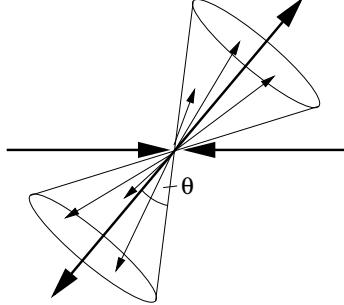
$$\mathbf{J}_1^{\mu a_1} = \mathbf{Q}_p^{a_1} \frac{p^\mu}{p \cdot q_1} \quad , \quad \mathbf{J}_2^{\mu a_2} = \mathbf{Q}_p^{a_2} \frac{p^\mu}{p \cdot q_2} + \mathbf{Q}_{q_1}^{a_2} \frac{q_1^\mu}{q_1 \cdot q_2}$$



$$\begin{aligned} \mathcal{M}_{ki}^{a_1 a_2} &= g_s^2 \langle a_1 | k | \mathbf{J}_2 \cdot \boldsymbol{\varepsilon}_2 | a' | i' \rangle \langle i' | \mathbf{J}_1 \cdot \boldsymbol{\varepsilon}_1 | i \rangle \\ &= g_s^2 \frac{p \cdot \boldsymbol{\varepsilon}_1}{p \cdot q_1} \left( \frac{p \cdot \boldsymbol{\varepsilon}_2}{p \cdot q_2} t^{a_2} t^{a_1} + \frac{q_1 \cdot \boldsymbol{\varepsilon}_2}{q_1 \cdot q_2} [t^{a_1}, t^{a_2}] \right)_{ki} \end{aligned}$$

- small angle: bremsstrahlung cones
- large angle ( $\theta_{pq_2} \gg \theta_{pq_1}$ ): sees total charge  $\mathbf{Q}_p + \mathbf{Q}_{q_1}$

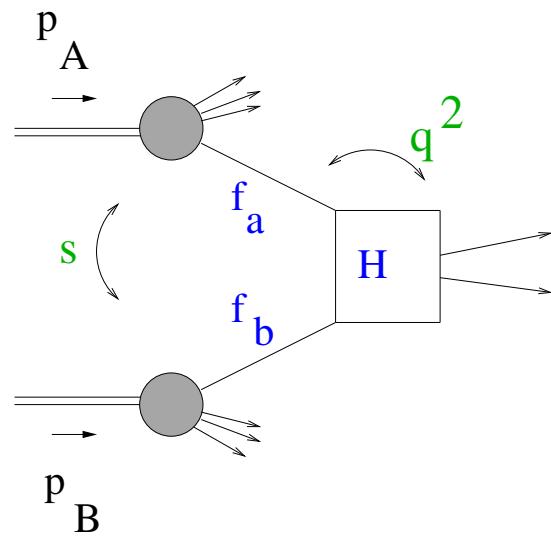
- Extensive collider data studies emphasize the phenomenological relevance of coherence effects. Example:  $p\bar{p}$  dijets



[B. Webber, CERN seminar, 2008]

## II. COHERENCE IN HIGH-ENERGY, SMALL-X PARTON SHOWERS

- Arguments used above rely on soft vector emission current from **external** legs → leading IR singularities
  - appropriate in single-scale hard processes

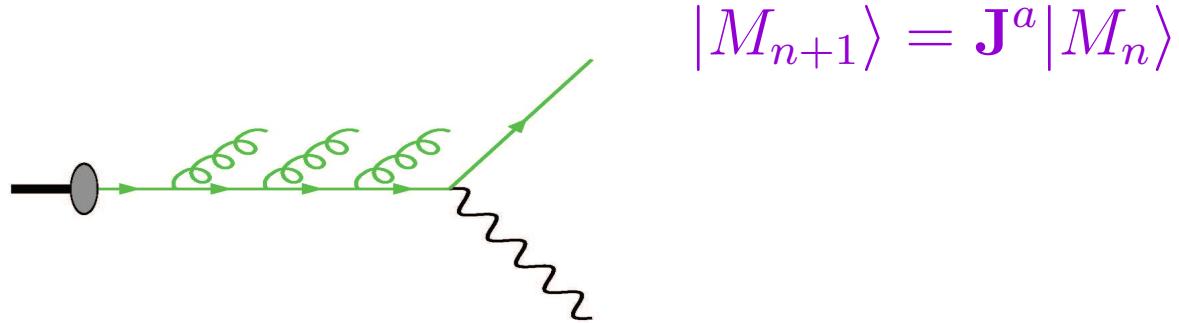


LHC forward hard processes are  
multi-scale:  $s = q_1^2 \gg \dots \gg q_n^2 \gg \Lambda^2$



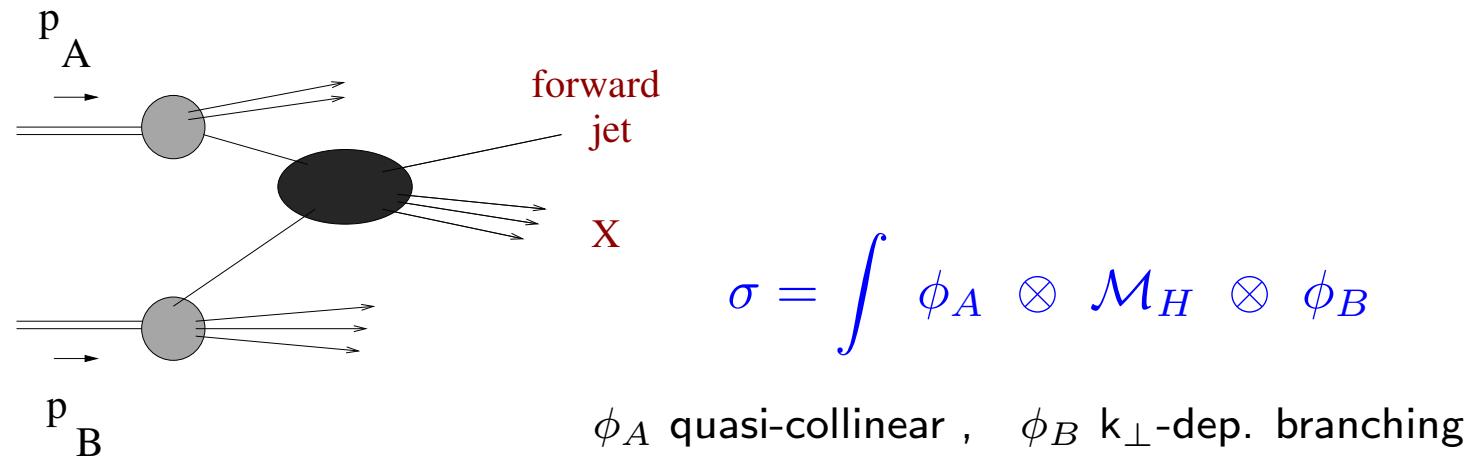
- initial-state branchings not collinearly-ordered potentially non-negligible
- emissions from internal legs become leading for  $x \ll 1$   
⇒ associated coherence effects

- ▷ internal-emission current also factorizable at high-energy



- ▷ BUT:
  - $\mathbf{J}$  depends on total transverse momentum transmitted  
⇒ matrix elements and pdf at fixed  $k_\perp$  (“unintegrated”)
  - virtual corrections not all in  $\Delta$  form factor  
⇒ modified branching probability  $P(z, k_\perp)$
- ◊ radiative corrections enhanced by  $\alpha_S^k \ln^m s/p_T^2$
- ◊ superleading logs cancel in fully inclusive quantities (space-like anomalous dimensions)

# STRUCTURE OF FORWARD JET CROSS SECTION



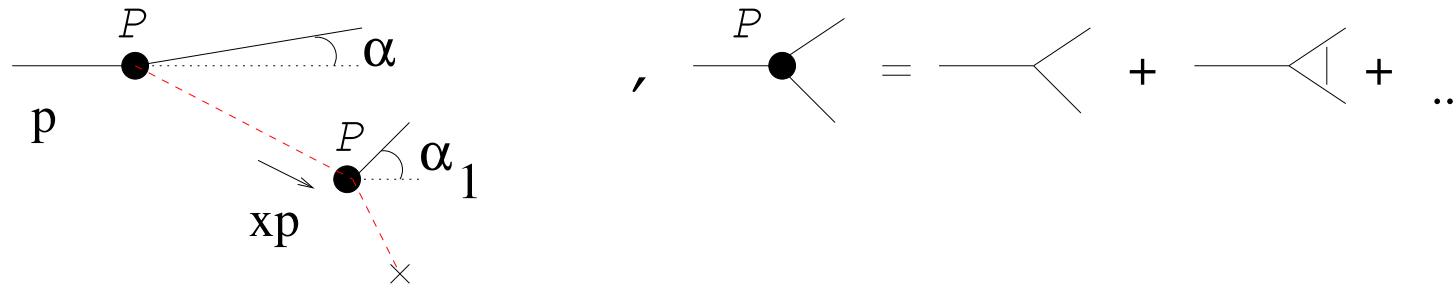
[Deak, Jung, Kutak & H, in progress]

- $\mathcal{M}_H$  from perturbative off-shell amplitudes  
( $\hookrightarrow$  q and g channels) [Fadin-Lipatov,  
Catani-Ciafaloni-H, ...]
- unintegrated distributions from branching equation + data fits  
 $\hookrightarrow$

# $K_\perp$ -DEPENDENT PARTON BRANCHING

- implement all-order summation of  $(\alpha_S \ln s/p_T^2) \oplus$  IR  $x \rightarrow 1$  behavior

$$\text{branching eq. : } \mathcal{A}(x, k_T, \mu) = \mathcal{A}_0(x, k_T, \mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(\mu - zq) \\ \times \underbrace{\Delta(\mu, zq)}_{\text{Sudakov}} \underbrace{\mathcal{P}(z, q, k_T)}_{\text{unintegr. splitting}} \mathcal{A}\left(\frac{x}{z}, k_T + (1-z)q, q\right)$$

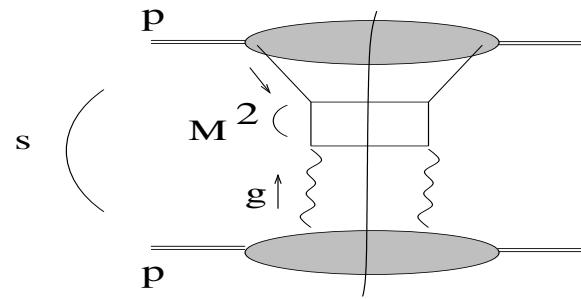


(left) Coherent radiation in the space-like parton shower for  $x \ll 1$ ;  
 (right) the unintegrated splitting function  $\mathcal{P}$ , including small- $x$  virtual corrections.

$$\alpha/x > \alpha_1 > \alpha \quad (\text{small } x \text{ coherence region})$$

### III. TOWARD PRECISE CHARACTERIZATIONS OF UNINTEGRATED PDF's

Example 1: Unintegrated (TMD) pdf from high energy factorization:



◊ single gluon polarization dominates  $s \gg M^2 \gg \Lambda_{\text{QCD}}^2$

    → gauge invariance rescued (despite gluon off-shell)

[Lipatov; Ciafaloni; Catani, H, ...]

◊ energy evolution equations / corrections down by  $1/\ln s$  rather than  $1/Q$

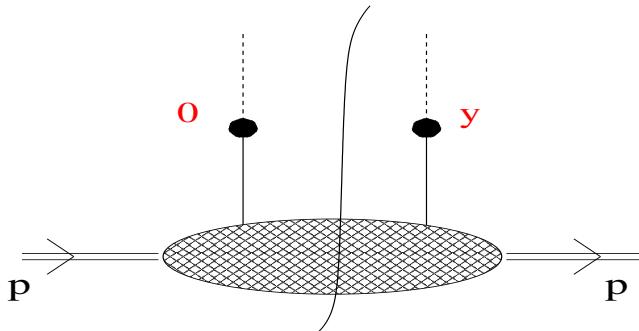
    → BFKL (+ its variants)

◊ Note: can match on to arbitrarily high  $k_\perp$  in the UV ⇒

- suitable for simulations of jet physics at the LHC
- well-defined summation of higher-order radiative corrections

◇ Gauge-invariant characterization over **whole** phase space is more difficult

Example 2: Generalize ordinary (lightcone) pdf to non-lightlike distances:



$$\mathbf{p} = (p^+, m^2/2p^+, \mathbf{o}_\perp)$$

$$\tilde{f}(y) = \langle P | \bar{\psi}(y) V_y^\dagger(n) \gamma^+ V_0(n) \psi(0) | P \rangle , \quad y = (0, y^-, y_\perp)$$

$$V_y(n) = \mathcal{P} \exp \left( ig_s \int_0^\infty d\tau \, n \cdot A(y + \tau n) \right) \quad \text{eikonal Wilson line in direction } n$$

- works at tree level [Mulders, 2002; Belitsky et al., 2003; ... ]
- subtler at level of radiative corrections [Collins; Hautmann; Cherednikov, ...]
  - ↪ endpoint  $x \rightarrow 1$  behavior ⇒ regularization method

- spectator interactions possibly non-decoupling (non-abelian Coulomb phase)
  - [Mulders, Bomhof; Collins, Qiu; Brodsky, ...]

## CUT-OFF REGULARIZATION

- ▷ cut-off in Monte-Carlo generators using u-pdf's

S. Höche, F. Krauss and T. Teubner, arXiv:0705.4577 (KMR)

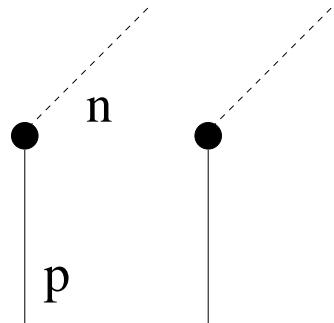
S. Jadach, W. Placzek, P. Stephens, K. Golec, W. Skrzypek, hep-ph/0703317 (CCFM)

LDCMC Lönnblad & Sjödahl, 2005; Gustafson, Lönnblad & Miu, 2002 (LDC)

CASCADE Jung, 2004, 2002; Jung and Salam, 2001 (CCFM)

SMALLX Marchesini & Webber, 1992 (CCFM)

- ▷ cut-off from gauge link in non-lightlike direction  $n$ :



$$\eta = (p \cdot n)^2 / n^2$$

Collins, Rogers & Stasto, arXiv:0708.2833

Ji, Ma & Yuan, 2005, 2006

earlier work from 80's and 90's

finite  $\eta \Rightarrow$  singularity is cut off at  $1 - x \gtrsim \sqrt{k_\perp/4\eta}$

\* Note: Subtractive regularization is possible alternative to cut-off [Collins & H, 2001]

## IV. APPLICATIONS TO JET FINAL STATES IN $p\bar{p}$ AND EP

- ▷ jet correlations provide sensitive probes of QCD multiple-radiation effects

Ex.: azimuthal  $\Delta\phi$  correlation (between two hardest jets)

- ▷ Tevatron  $\Delta\phi$  dominated by leading-order processes

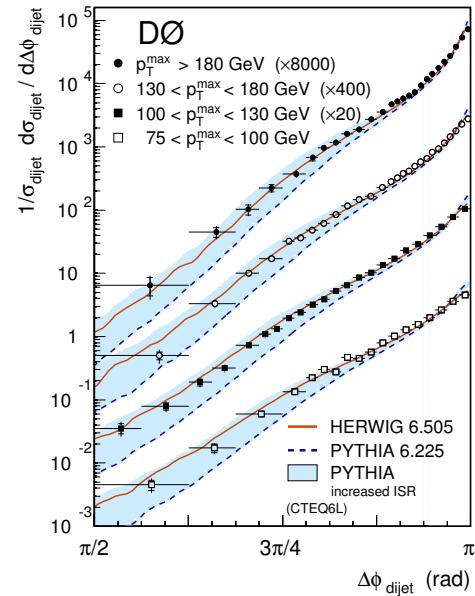
- distribution well described by HERWIG
- used for MC parameter tuning in PYTHIA

- ▷ HERA  $\Delta\phi$  not well described by standard MC

↪ see next

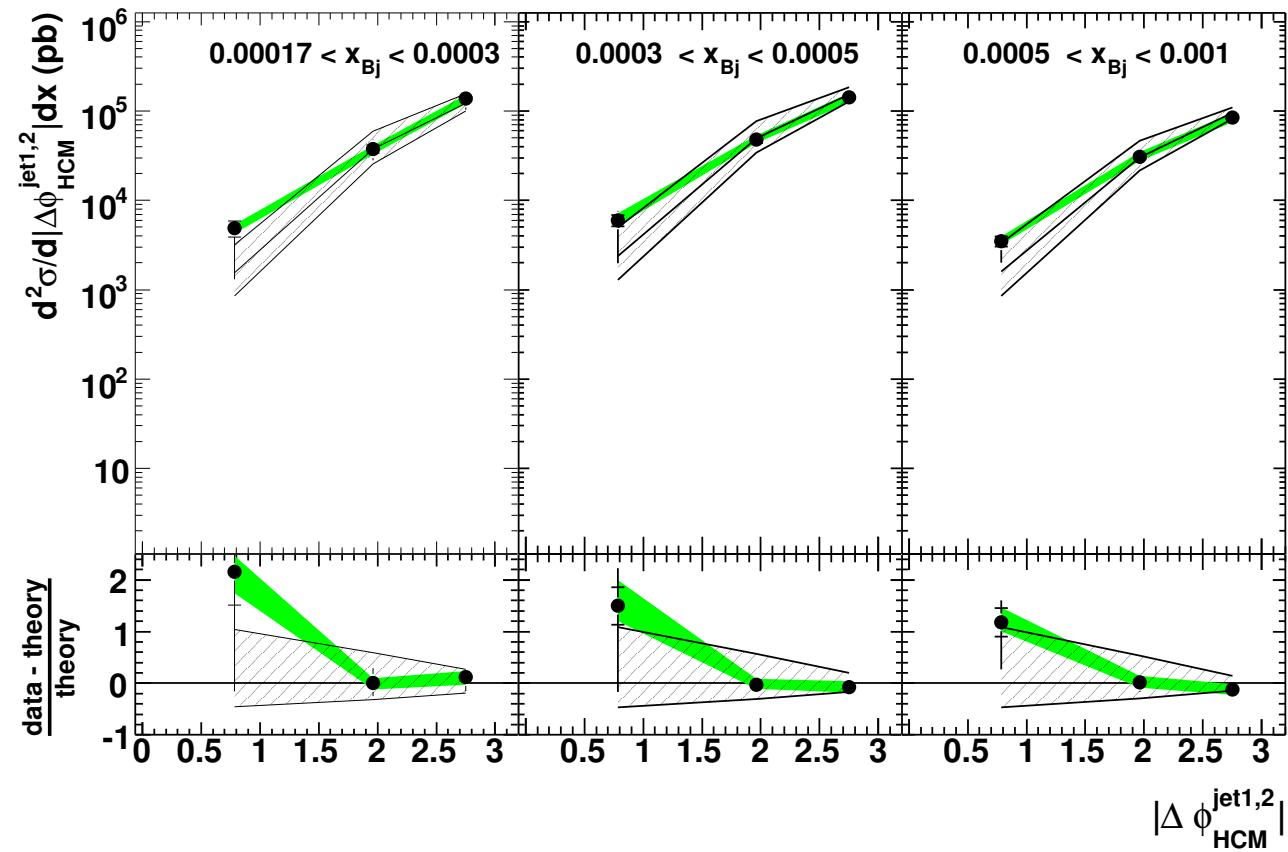
- ▷ accessible at the LHC relatively early

↪ probe coherence effects in high-energy spacelike showers



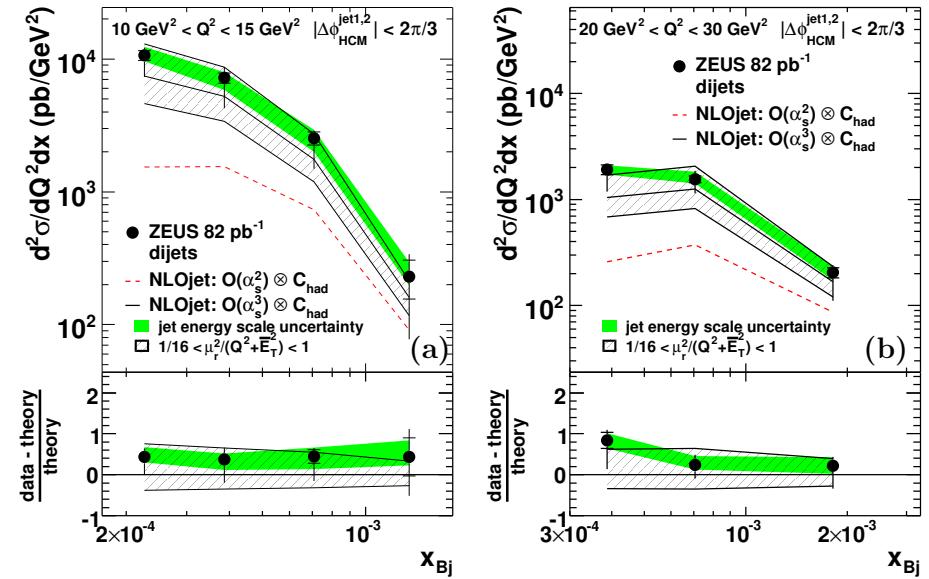
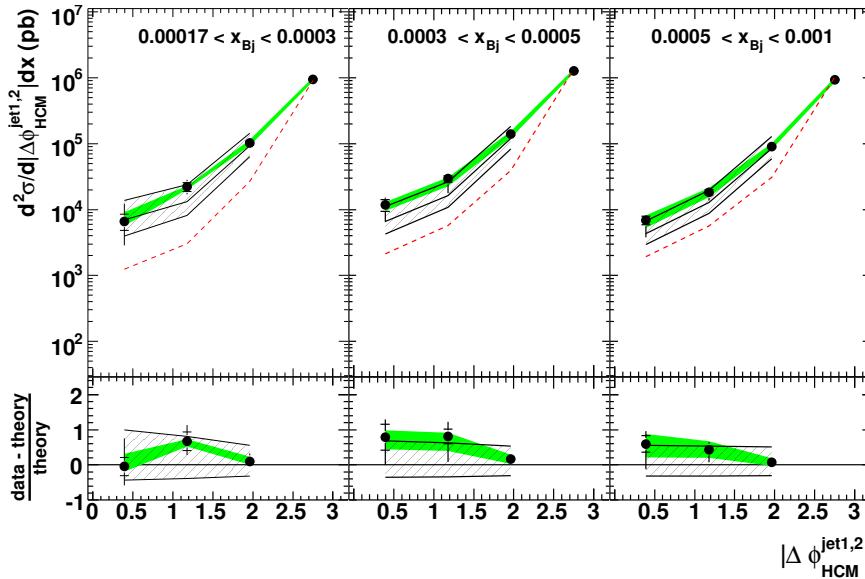
# AZIMUTHAL DISTRIBUTION IN EP 3-JET CROSS SECTION

[ZEUS, 2007]



- grey dashed band: NLO result [NLOJET++]
- NLO results more stable for more inclusive distributions

# DI-JET CORRELATIONS: COMPARISON WITH NLO RESULTS

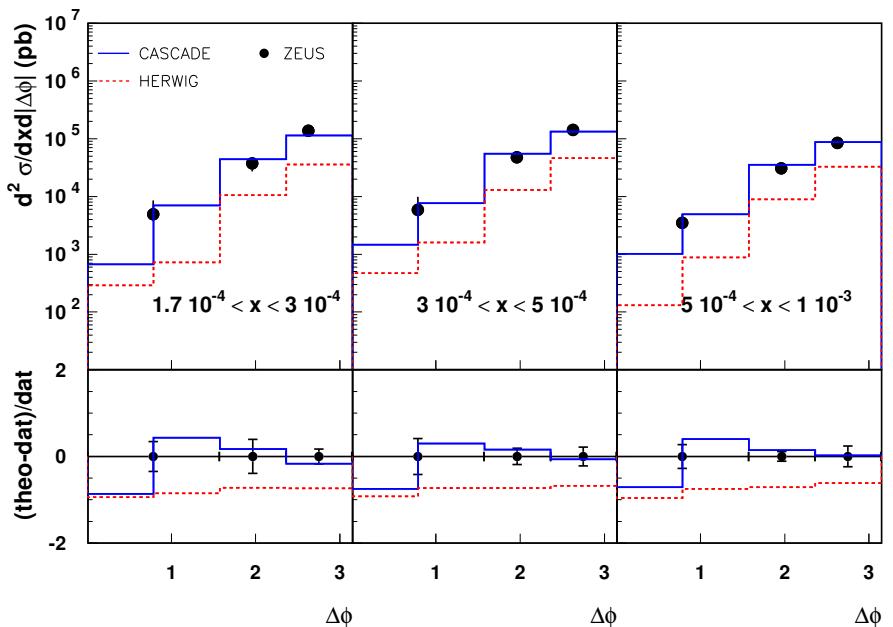
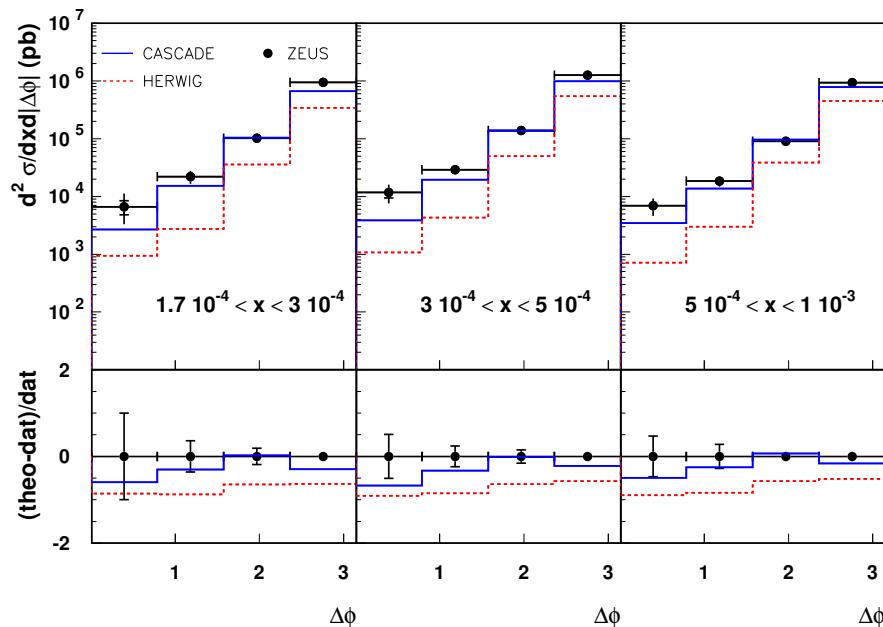


(left) Azimuth dependence and (right) Bjorken-x dependence of di-jet distributions

$$Q^2 > 10 \text{ GeV}^2 \quad , \quad 10^{-4} < x < 10^{-2}$$

- ◊ large variation from order- $\alpha_s^2$  to order- $\alpha_s^3$  prediction as  $\Delta\phi$  and  $x$  decrease
- ⇒ sizeable theory uncertainty at NLO (underestimated by “ $\mu$  error band”)

# ANGULAR JET CORRELATIONS FROM $K_\perp$ -SHOWER (CASCADE) AND COLLINEAR-SHOWER (HERWIG)

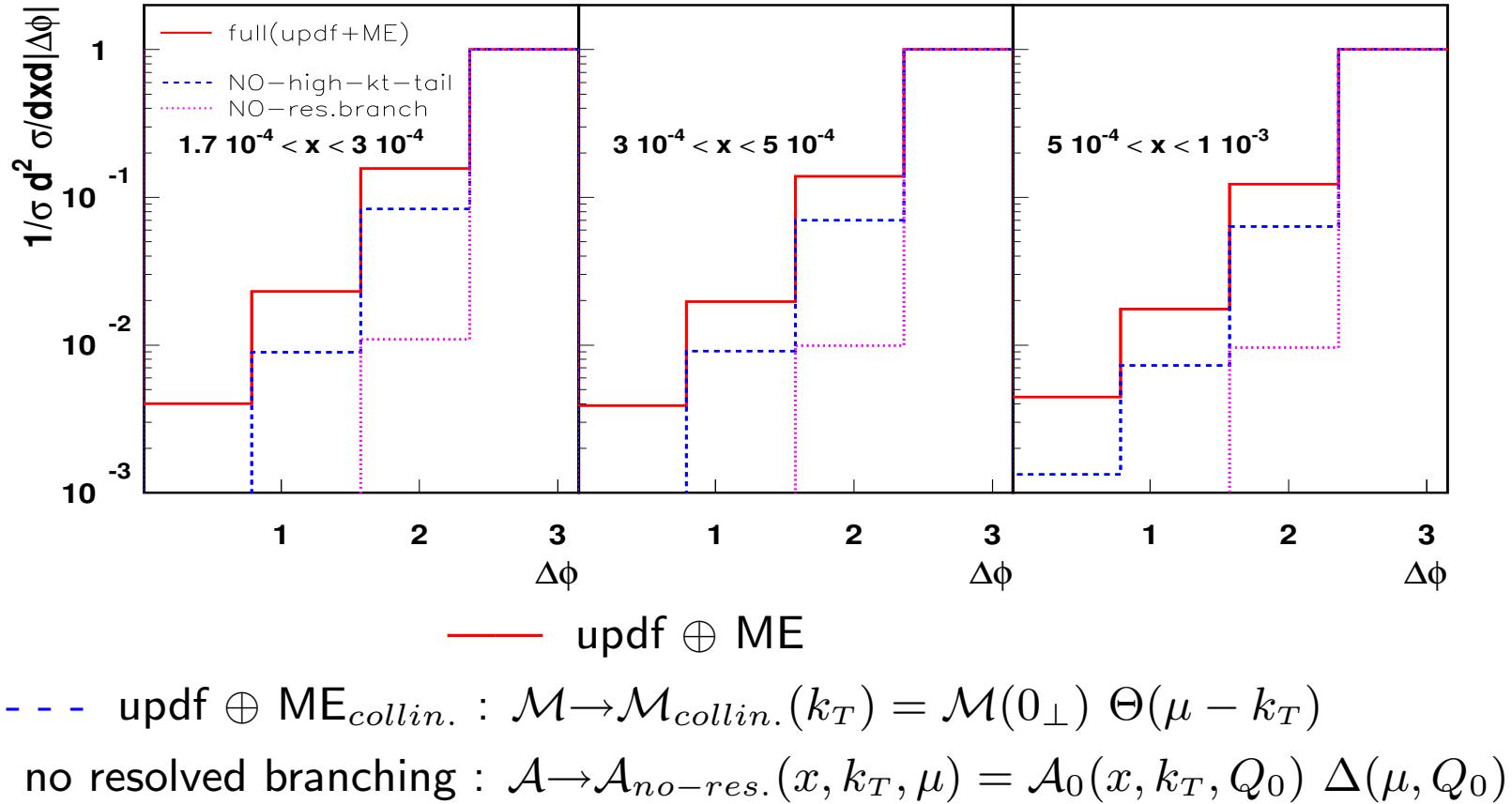


(left) di-jet cross section; (right) three-jet cross section

Jung & H, arXiv:0712.0568 [hep-ph]

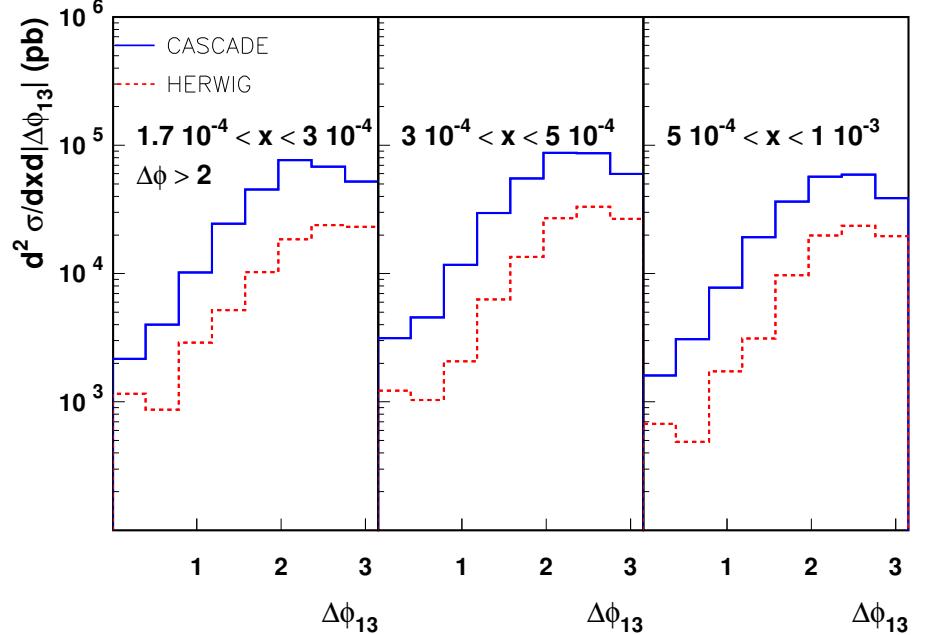
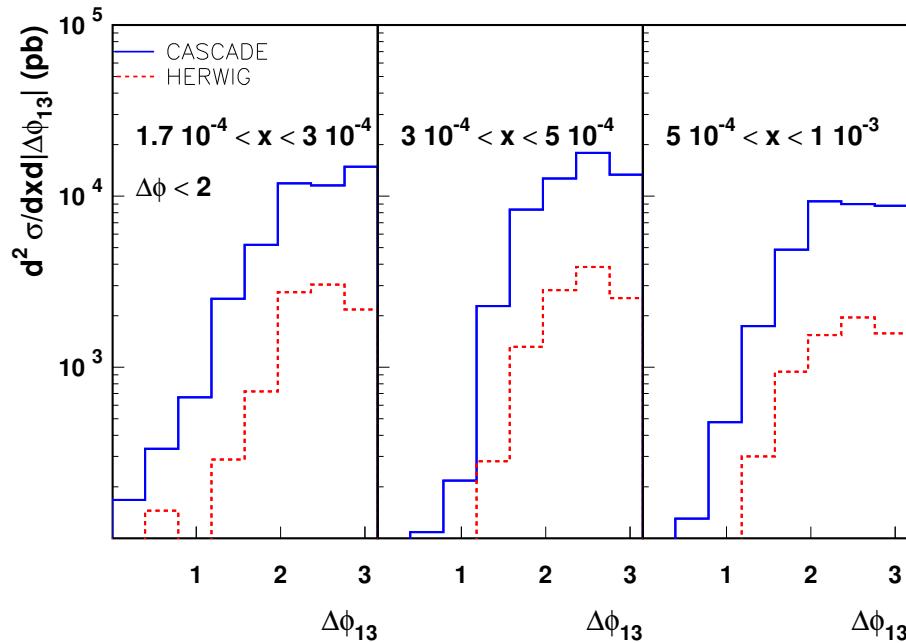
- different shapes from the two MC
  - largest differences at small  $\Delta\phi$
- good description of measurement by  $k_\perp$ -shower

Normalize to the back-to-back cross section:



- ▷ high- $k_\perp$  component in ME essential to describe correlation at small  $\Delta\phi$ 
  - ▷  $k_\perp$ -dependence in u-pdf alone not sufficient
- (cfr., e.g., MC by Höche, Krauss & Teubner, arXiv:0705.4577:  
u-pdf but no ME correction)

## AZIMUTHAL DISTRIBUTION OF THE THIRD JET



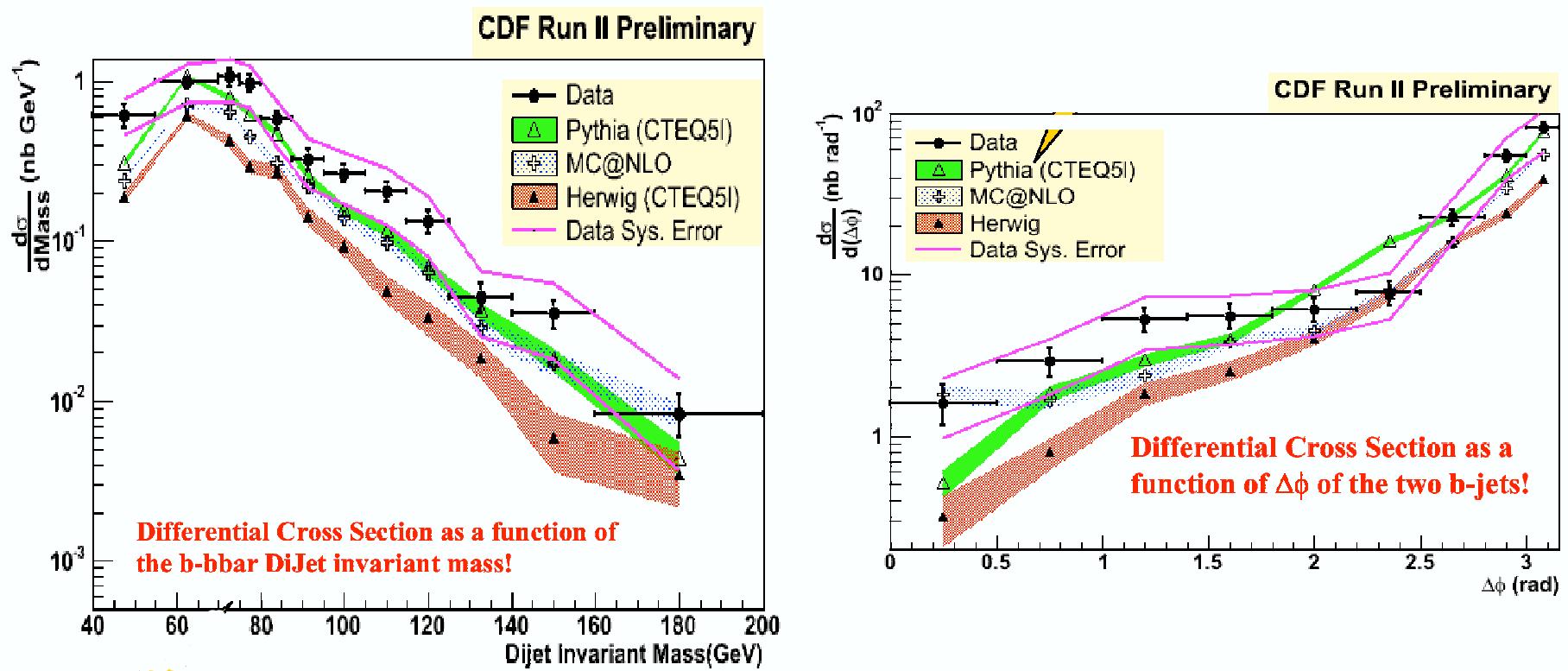
Cross section in the azimuthal angle between the hardest and the third jet  
for small (left) and large (right) azimuthal separations between the leading jets

Jung & H, arXiv:0805.1049 [hep-ph]

- small  $\Delta\phi \Rightarrow$  non-negligible initial  $k_\perp \Rightarrow$  larger corrections to collinear ordering
  - curves become closer at large  $\Delta\phi$

# Tevatron $b$ -jets correlations

[B. Webber, CERN Lectures, 2008 ]



- collinear-shower descriptions not fully satisfactory
- phenomenological studies of  $k_T$ -showers potentially interesting
  - may affect underlying event description

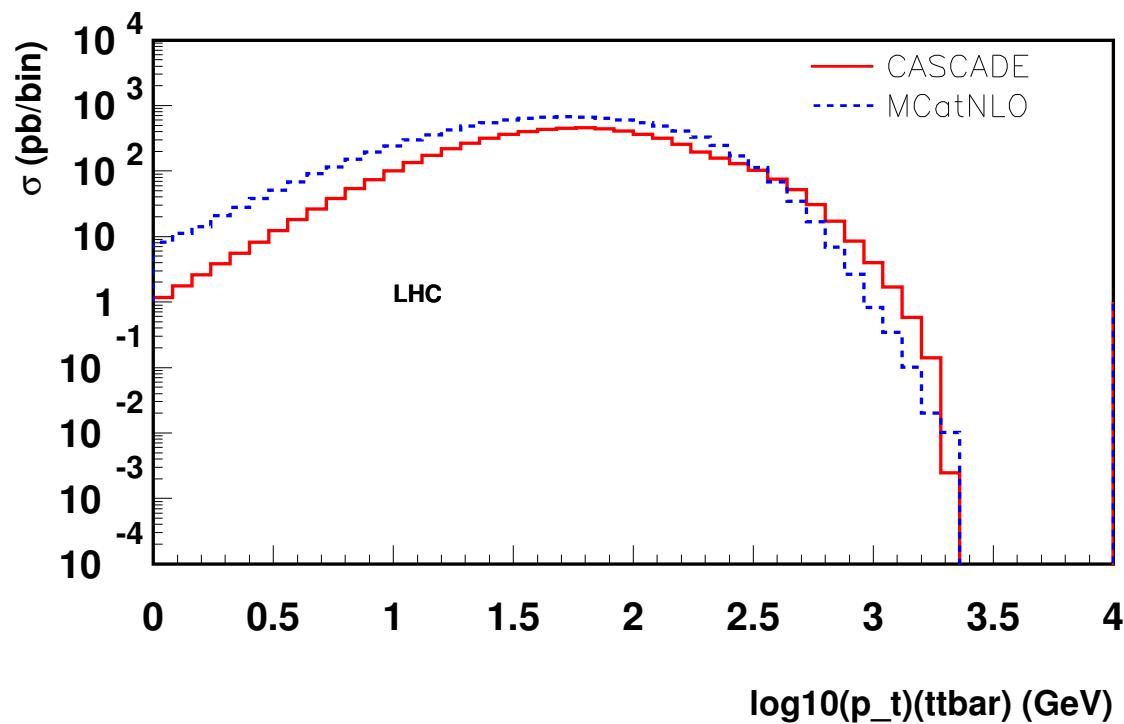
## V. PROSPECTS FOR FURTHER FINAL STATES AND CONCLUSIONS

### ◊ MC and radiative corrections to gluon fusion processes:

- production of  $b, c$  — what size NLO uncertainties at LHC energies?  
[see MC@NLO; Nason et al.]
  - ▷ sizeable corrections from  $g \rightarrow b\bar{b}$  coupling to spacelike jet
  - ▷ coherence effects to  $b\bar{b} + 2 \text{ jets}$  for  $m_b \ll p_T^{(b\bar{b})} \ll p_T^{(\text{jet})}$
- multi-scale effects in  $b\bar{b} + W/Z$  production
- $k_\perp$ -shower vs. MC@NLO for top-antitop pair production  
( $\hookrightarrow$  see  $p_T$  spectrum)
- final states with Higgs
  - possibly 10  $\div$  20 % effects in  $p_T$  spectrum from  $x \ll 1$  terms?  
[Kulesza, Sterman & Vogelsang, 2004]  
see also: Marzani, Ball, Del Duca et al., 2008; H, 2002

$p_T$  distribution of top-antitop pairs  
from  $k_\perp$ -shower and from MC@NLO at LHC energies

[Deak, Jung, Schwennsen, prelim.]



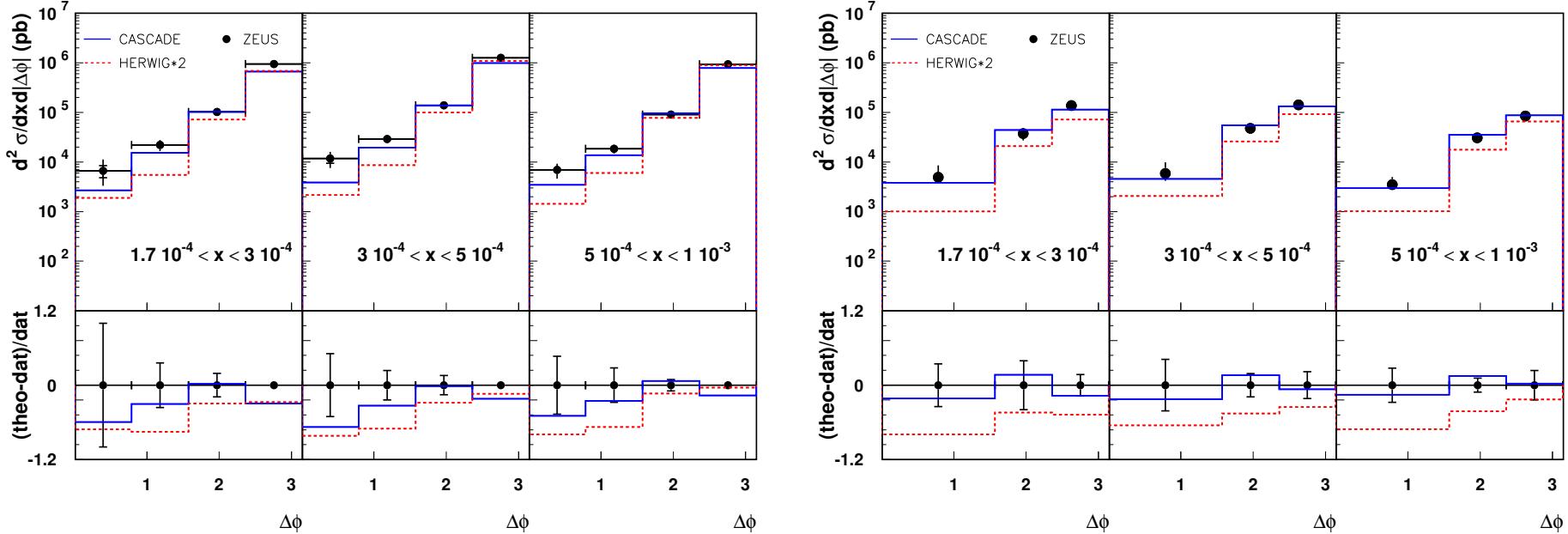
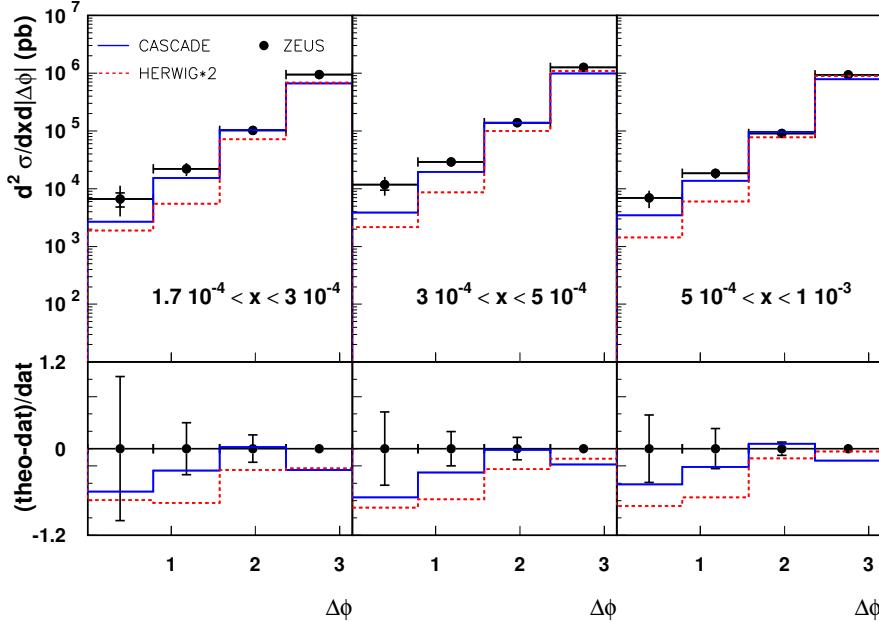
- small- $x$  effects not large in this case
- probe shower in region of finite  $x$  and large virtualities on the order of  $m_{\text{top}}$

## CONCLUSIONS

- Central + forward detectors at the LHC will explore to unprecedented level correlations of high- $p_T$  probes across large rapidity intervals and complex multi-particle final states with many hard scales
- Branching methods based on  $u$ -pdfs and  $k_\perp$ -MEs useful to
  - ▷ simulate high-energy parton showers
  - ▷ investigate possibly new effects from QCD physics
- Systematic theoretical studies of  $u$ -pdf's ongoing
  - ▷ relevant to turn these Monte-Carlo's into general-purpose tools

## EXTRA SLIDES

## HERWIG K-factor of 2 (from two-jet region)



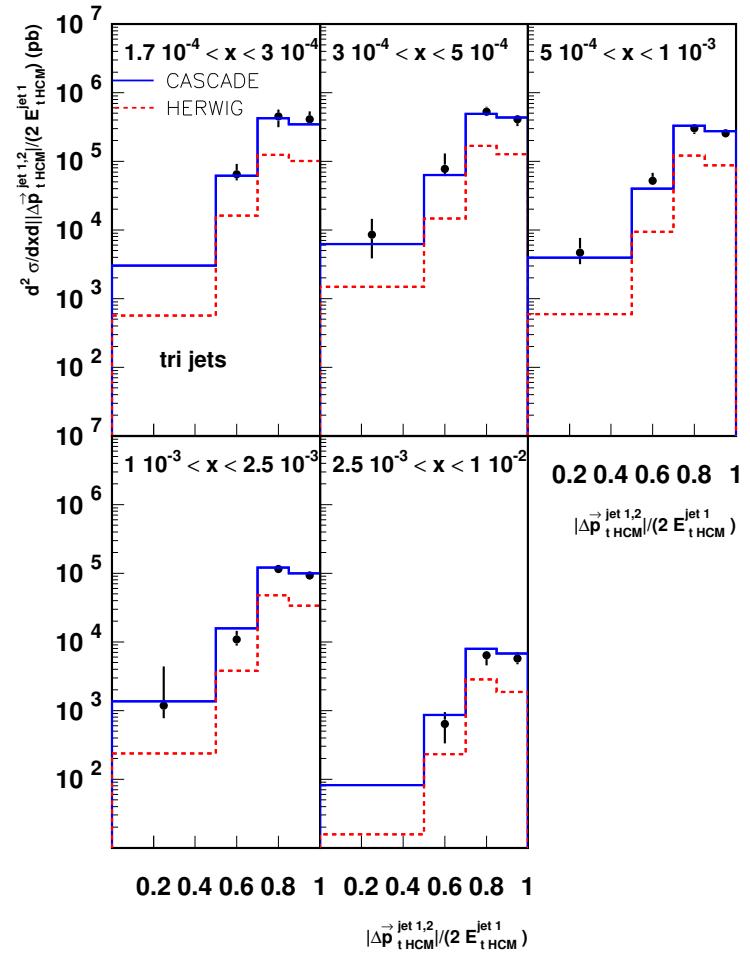
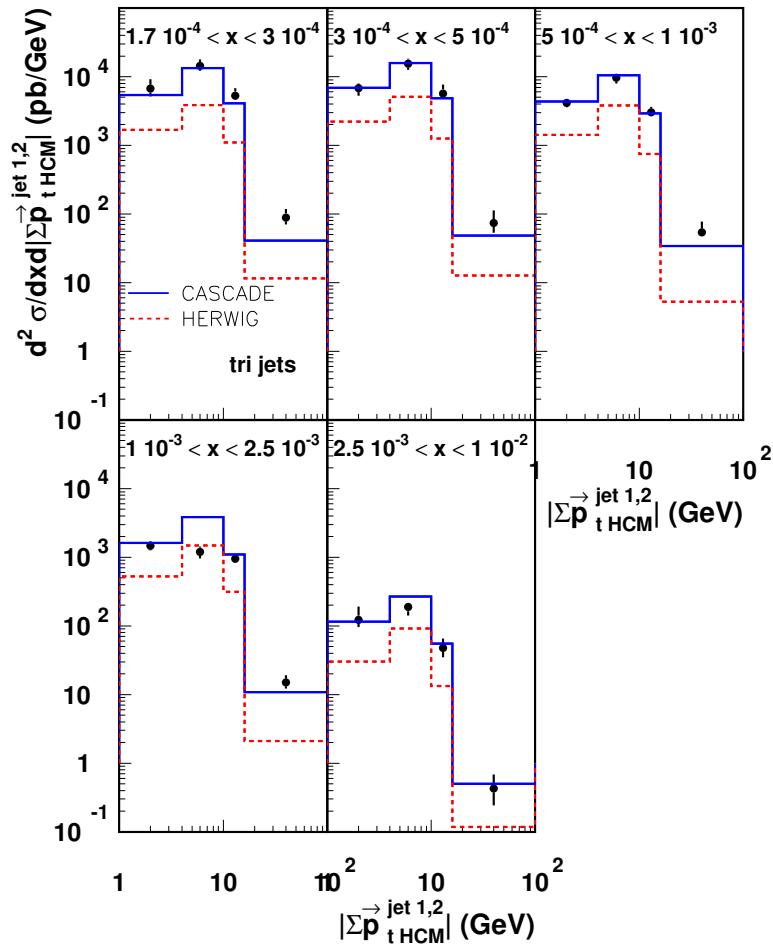
(left) di-jet cross section; (right) three-jet cross section

Jung & H, arXiv:0805.1049 [hep-ph]

- different shapes from the two MC
- small  $\Delta\phi$  not well described by HERWIG
- good description of 3-jet by  $k_\perp$ -shower but not by HERWIG

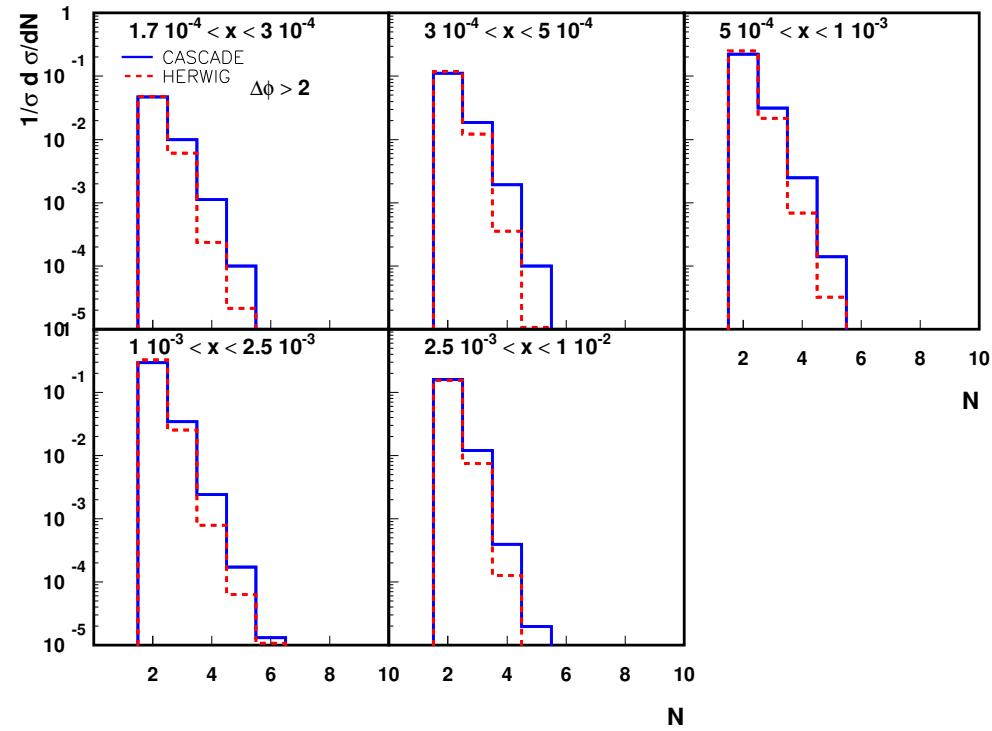
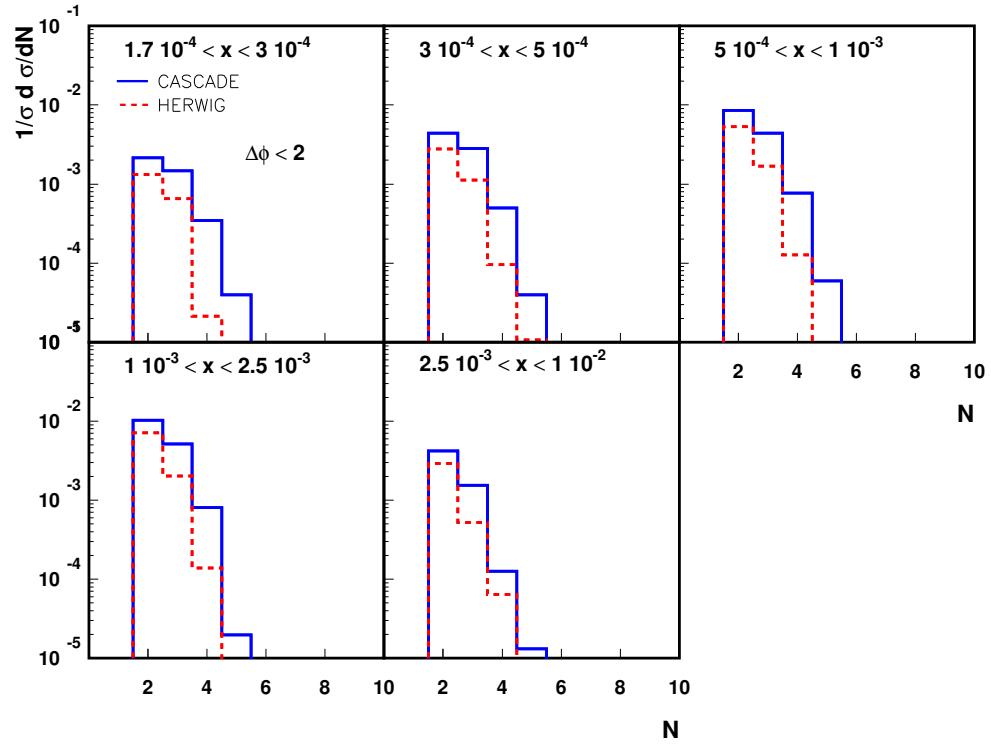
# MOMENTUM CORRELATIONS

[Jung & H, arXiv:0805.1049]



- correlations in the transverse momentum imbalance between the leading jets

## JET MULTIPLICITIES



(left)  $\Delta\phi < 2$ ; (right)  $\Delta\phi > 2$

[Jung & H, arXiv:0805.1049]

- larger contribution from high multiplicity in the MC with u-pdf

$$E_{T,HCM}^{\text{jet}-1} > 7 \text{ GeV} \quad , \quad E_{T,HCM}^{\text{jet}-2,3} > 5 \text{ GeV} \quad , \quad -1 < \eta_{lab} < 2.5$$

- Jet clustering and hadronization:

- ▷ moderate hadronization corrections from jet algorithm used by Zeus and H1  
[arXiv:0705.1931 [hep-ex]; hep-ex/0310019]
- ▷ jet clustering free of non-global logarithms  
[Dasgupta et al., hep-ph/0610242]
- ▷ asymmetric jet cuts to avoid double logs in minimum  $p_T$   
[Banfi and Dasgupta, hep-ph/0312108]
- ▷ nonperturbative corrections in inverse powers of  $Q$  moderate for  $Q^2 > 10 \text{ GeV}^2$

- Radiative effects at higher order:

- ◊ fixed-order beyond NLO is outside present reach for multi-jets in ep and pp
- ◊ enhanced (soft/collinear) higher orders from near back-to-back region  
Y.Delenda et al., arXiv:0706.2172; arXiv:0804.3786; HERWIG
- ◊ largest effects seen at small  $\Delta\phi$  (3 well-separated hard jets)

## Implementations:

Höche, Krauss and Teubner, arXiv:0705.4577 (KMR)  
Golec, Jadach, Placzek, Stephens, Skrzypek, hep-ph/0703317 (CCFM)  
LDCMC Lönnblad & Sjödahl, 2005; Gustafson, Lönnblad & Miu, 2002 (LDC)  
CASCADE Jung, 2004, 2002; Jung and Salam, 2001 (CCFM)  
SMALLX Marchesini & Webber, 1992 (CCFM)

Advantages over standard Monte-Carlo like PYTHIA or HERWIG:

- better treatment of high-energy logarithmic effects
  - likely more suitable for simulating underlying event's  $k_T$

## Current limitations:

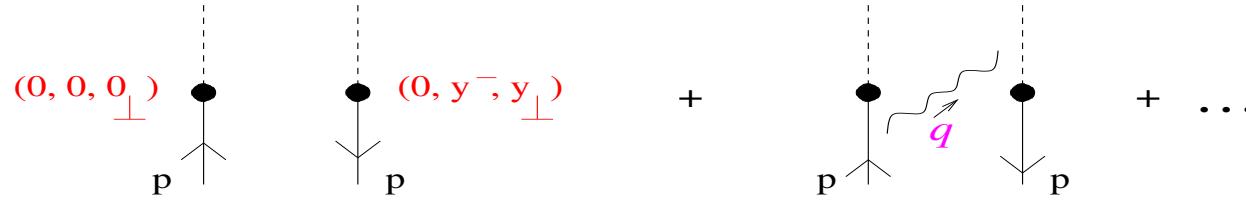
- radiative terms associated to  $x \sim 1$  not automatically included
  - procedure to correct for this not yet systematic
    - e.g.: LO-collin. evolution in Höche et al
  - quark contributions in initial state included partially
    - see also:  $k_\perp$  kernel for sea-quark evolution [Catani & H]
  - limited knowledge of u-pdf's [Jung et al., arXiv:0706.3793;
  - scale of  $\alpha_s$  in branching [J. R. Andersen et al., 2006]

## Summary on 3-jet

- ▷ U-pdfs  $\oplus$   $k_\perp$ -dependent hard MEs describe multi-jet measurements including correlations.
- ▷ coherence effects in angular distributions non-negligible at high energy (small  $x$ ) and small  $\Delta\phi$   
(near large  $\Delta\phi$ , Coulomb/radiative mixed terms also possibly relevant)
- ▷ Furthermore:
  - Results similar to HERWIG if reduced to  $k_\perp$ -ordered phase space
  - Similar to fixed NLO where corrections are not large
- ▷ Non-forward jets  $\Rightarrow$  results less dependent on details of u-pdf evolution models

◇ Suppose a gluon is absorbed or emitted by eikonal line:

$$\mathbf{n} = (0, \mathbf{I}, \mathbf{0}_\perp)$$



$$f_{(1)} = P_R(x, k_\perp) - \delta(1-x) \delta(k_\perp) \int dx' dk'_\perp P_R(x', k'_\perp)$$

where  $P_R = \frac{\alpha_s C_F}{\pi^2} \left[ \frac{1}{1-x} \frac{1}{k_\perp^2 + \rho^2} + \{\text{regular at } x \rightarrow 1\} \right]$   $\rho = \text{IR regulator}$

$\overbrace{\quad\quad\quad}^{\uparrow}$   
endpoint singularity  $(q^+ \rightarrow 0, \forall k_\perp)$

◇ Physical observables:

$$\begin{aligned} \mathcal{O} &= \int dx dk_\perp f_{(1)}(x, k_\perp) \varphi(x, k_\perp) \\ &= \int dx dk_\perp [\varphi(x, k_\perp) - \varphi(1, 0_\perp)] P_R(x, k_\perp) \end{aligned}$$

inclusive case:  $\varphi$  independent of  $k_\perp \Rightarrow 1/(1-x)_+$  from real + virtual

general case: endpoint divergences (incomplete KLN cancellation)

## UPDF's WITH SUBTRACTIVE REGULARIZATION

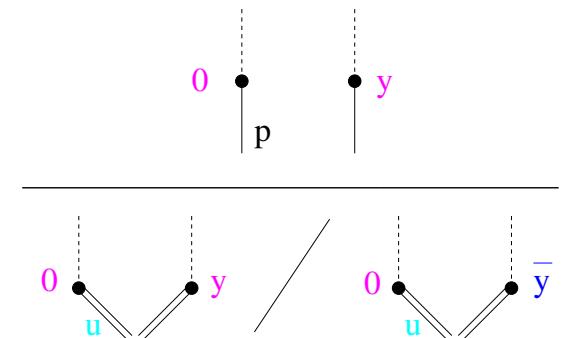
- Endpoint divergences  $x \rightarrow 1$  from incomplete KLN cancellation

Subtractive method: more systematic than cut-off. Widely used in NLO calculations.

Formulation suitable for eikonal-operator matrix elements: Collins & H, 2001.

- gauge link still evaluated at  $n$  lightlike, but multiplied by “subtraction factors”

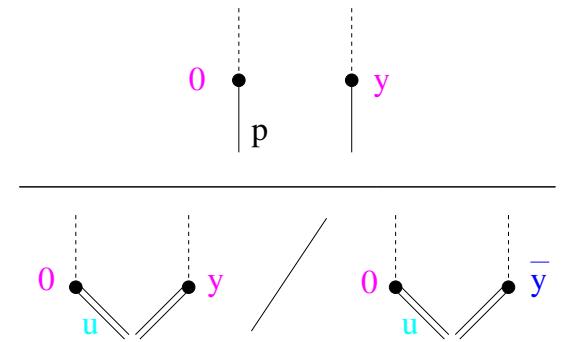
$$\tilde{f}^{(\text{subtr})}(y^-, y_\perp) = \frac{\overbrace{\langle P | \bar{\psi}(y) V_y^\dagger(n) \gamma^+ V_0(n) \psi(0) | P \rangle}^{\text{original matrix element}}}{\underbrace{\langle 0 | V_y(u) V_y^\dagger(n) V_0(n) V_0^\dagger(u) | 0 \rangle / \langle 0 | V_{\bar{y}}(u) V_{\bar{y}}^\dagger(n) V_0(n) V_0^\dagger(u) | 0 \rangle}_{\text{counterterms}}}$$



$$\bar{y} = (0, y^-, 0_\perp); \quad u = \text{auxiliary non-lightlike eikonal } (u^+, u^-, 0_\perp)$$

H, arXiv:0708.1319

◇  $u$  serves to regularize the endpoint; drops out of distribution integrated over  $k_\perp$



One loop expansion:

$$\begin{aligned}
 f_{(1)}^{(\text{subtr})}(x, k_\perp) &= P_R(x, k_\perp) - \delta(1-x) \delta(k_\perp) \int dx' dk'_\perp P_R(x', k'_\perp) \quad (\leftarrow \text{from numerator}) \\
 &- W_R(x, k_\perp, \zeta) + \delta(k_\perp) \int dk'_\perp W_R(x, k'_\perp, \zeta) \quad (\leftarrow \text{from vev's})
 \end{aligned}$$

with  $P_R = \alpha_s C_F / \pi^2 \left\{ 1 / [(1-x)(k_\perp^2 + m^2(1-x)^2)] + \dots \right\}$  = real emission prob.

$W_R = \alpha_s C_F / \pi^2 \left\{ 1 / [(1-x)(k_\perp^2 + 4\zeta(1-x)^2)] + \dots \right\}$  = counterterm

- $\zeta$ -dependence cancels upon integration in  $k_\perp$     [ $\zeta = (p^{+2}/2)u^-/u^+$ ]

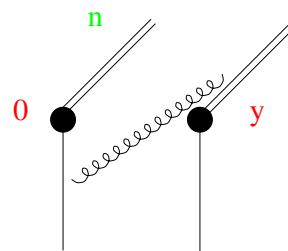
$$\begin{aligned}
 \Rightarrow \mathcal{O} &= \int dx dk_\perp f_{(1)}^{(\text{subtr})}(x, k_\perp) \varphi(x, k_\perp) \\
 &= \int dx dk_\perp \{ P_R [\varphi(x, 0_\perp) - \varphi(1, 0_\perp)] + (P_R - W_R) [\varphi(x, k_\perp) - \varphi(x, 0_\perp)] \}
 \end{aligned}$$

- first term: usual  $1/(1-x)_+$  distribution
- second term: singularity in  $P_R$  cancelled by  $W_R$

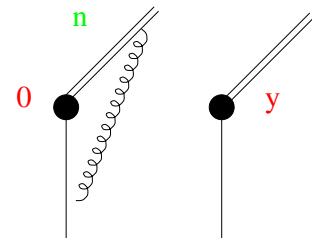
## Order- $\alpha_s$ analysis

[H, hep-ph/0702196]

- ▷ Expand Wilson-line matrix element  
to one loop



(a)



(b)

- ▷ Gauge link at infinity does not contribute  
in covariant gauge

- ▷  $d = 4 - 2\epsilon$  for UV divergences

$$\begin{aligned} \tilde{f}_{(a)+(b)}(y) &= \frac{\alpha_s C_F}{4^{d/2-2} \pi^{d/2-1}} p^+ \int_0^1 dv \frac{v}{1-v} \left[ e^{ip \cdot y v} 2^{d/2-1} \left( \frac{\rho^2}{\mu^2} \right)^{d/4-1} \right. \\ &\times \left. \frac{1}{(-y^2 \mu^2)^{d/4-1}} K_{d/2-2}(\sqrt{-\rho^2 y^2}) - e^{ip \cdot y} \Gamma(2 - \frac{d}{2}) \left( \frac{\mu^2}{\rho^2} \right)^{2-d/2} \right] \end{aligned}$$

$K$  = modified Bessel function;  $\Gamma$  = Euler gamma function

$$\rho^2 = (1-v)^2 m^2 + v \lambda^2$$

- $v \rightarrow 1$ : endpoint singularity
- can relate result to ordinary pdf by expanding in  $y^2$

$\rightarrow$  separate long-distance terms in  $\ln(\mu^2/\rho^2)$   
 and short-distance terms in  $\ln(y^2\mu^2)$

$$\begin{aligned}
 \tilde{f}_{(a)+(b)} &\simeq \frac{\alpha_s C_F}{4^{d/2-2} \pi^{d/2-1}} p^+ \int_0^1 dv \frac{v}{1-v} \left\{ \left[ e^{ip \cdot yv} - e^{ip \cdot y} \right] \Gamma(2 - \frac{d}{2}) (\frac{\mu^2}{\rho^2})^{2-d/2} \right. \\
 &+ e^{ip \cdot yv} 4^{d/2-2} \Gamma(\frac{d}{2} - 2) (-y^2 \mu^2)^{2-d/2} \\
 &+ \sum_{k=1}^{\infty} \frac{\Gamma(2 - d/2) \Gamma(d/2 - 1)}{k! 4^k \Gamma(k + d/2 - 1)} e^{ip \cdot yv} (\frac{\rho^2}{\mu^2})^{d/2+k-2} (-y^2 \mu^2)^k \\
 &+ \left. \sum_{k=1}^{\infty} \frac{4^{d/2-2-k} \Gamma(d/2 - 2) \Gamma(3 - d/2)}{k! \Gamma(k + 3 - d/2)} e^{ip \cdot yv} (\frac{\rho^2}{\mu^2})^k (-y^2 \mu^2)^{2-d/2+k} \right\}
 \end{aligned}$$

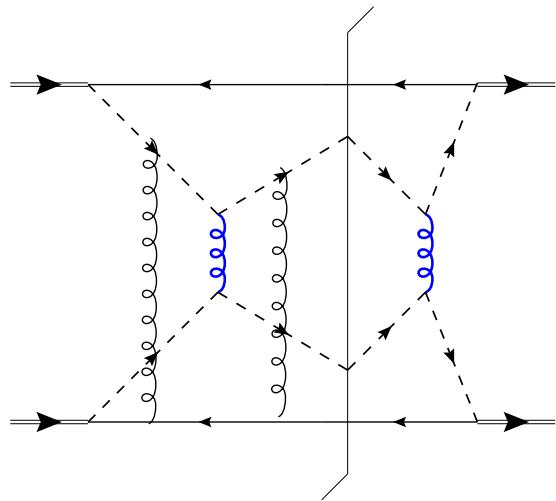
$v \rightarrow 1$  endpoint singularity

- cancels for ordinary pdf (first term in rhs)
- present, even at  $d \neq 4$  and finite  $\rho$ , in subsequent terms

## FURTHER ISSUES AT HIGHER ORDER

- soft gluon exchange with spectator partons

⇒ factorization breaking in higher loops?



Collins, arXiv:0708.4410

Vogelsang and Yuan, arXiv:0708.4398

Bomhof and Mulders, arXiv:0709.1390

◇ likely suppressed for small- $x$ , small- $\Delta\phi$

◇ could affect physical picture near back-to-back region

- Note: Coulomb/radiative mixing terms also appear to break coherence in di-jet cross sections with gap in rapidity

Forshaw, Kyrieleis & Seymour, 2006