

Heavy quark masses from QCD Sum Rules

Pere Masjuan
Johannes Gutenberg-Universität Mainz
(masjuan@kph.uni-mainz.de)

Work in collaboration with
Jens Erler (UNAM) and Hubert Spiesberger (Mainz)
preliminary results



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Outline

- Motivation and Introduction
- Using Sum Rules to extract mq
 - state-of-the-art results
 - our proposal
- Conclusions and outlook: the bottom case

Motivation: why m_Q ?

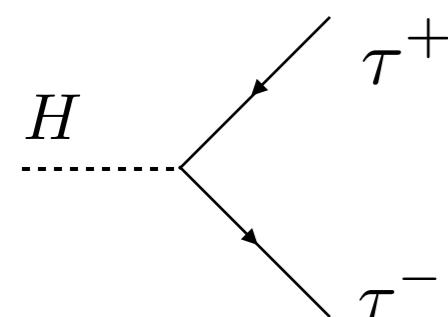
What is a quark mass?

From kinematics:

the position of the production threshold (applies for fundamental particles)

Pole Mass: $M^2 = E^2 - p^2$

But particles are not really isolated (need corrections)



$$\Gamma(H \rightarrow \tau^+ \tau^-) \sim \frac{G_F M_\tau^2}{4\pi\sqrt{2}} M_H$$

QED correction $\left(1 - \frac{\alpha}{\pi} \left(\frac{3}{2} \log \frac{M_H^2}{M_\tau^2} - \frac{9}{4}\right)\right)$

$$M_\tau(M_H) = M_\tau \left(1 - \frac{\alpha}{\pi} \left(\frac{3}{4} \log \frac{M_H^2}{M_\tau^2} + 1\right)\right)$$

What is M_τ^2 ?

$$\alpha = \alpha_{em} = 1/132$$

depends on how
to define the mass

Motivation: why \overline{m} ?

Select the \overline{MS} scheme $\longrightarrow m \rightarrow \overline{m}(\mu)$

$$\overline{m}_q(\mu) = M_q \left(1 - \frac{\alpha}{\pi} \left(\frac{4}{3} + \log \frac{\mu^2}{M_q^2} \right) + \dots \right) \quad \begin{matrix} \alpha = \alpha_s \\ \text{known to} \\ \alpha^4 \end{matrix}$$

$$M_t \sim 170 \text{GeV} \longrightarrow \overline{m}_t(\overline{m}_t) \sim 160 \text{GeV}$$

$$M_b \sim 4800 \text{MeV} \longrightarrow \overline{m}_b(\overline{m}_b) \sim 4200 \text{MeV}$$

large log's, resum them using renormalization group evolution

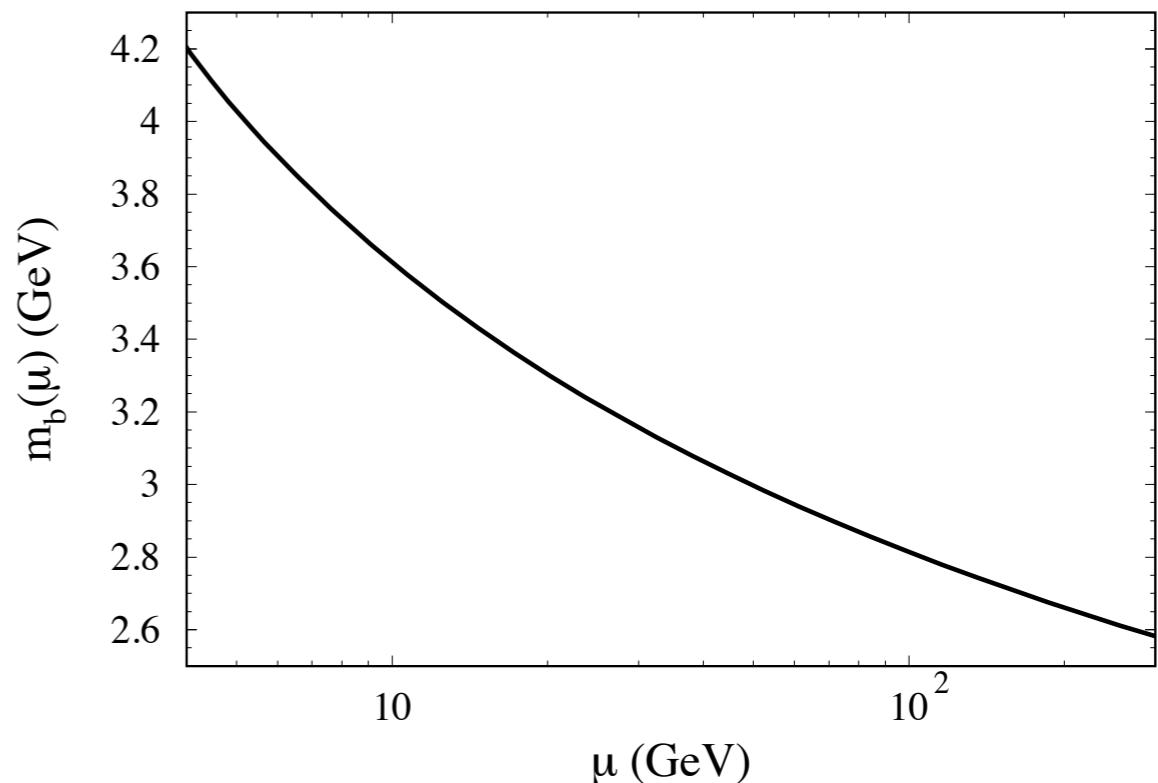
Motivation: why m_Q ?

Renormalization group evolution of quark mass:

$$\mu^2 \frac{d}{d\mu^2} m(\mu) = m(\mu) \gamma(\alpha)$$

$$\gamma(\alpha) = - \sum_{k \geq 0} \gamma_k \left(\frac{\alpha}{\pi} \right)^{k+1}$$

known up to γ_4
[Baikov et al '14]



$$\overline{m}(\mu) = \overline{m}(\mu_0) \left(\frac{\alpha(\mu)}{\alpha(\mu_0)} \right)^{\gamma_0/\beta_0} \left[1 + \left(\frac{\gamma_1}{\beta_0} - \frac{\beta_1 \gamma_0}{\beta_0^2} \right) \left(\frac{\alpha(\mu)}{\pi} - \frac{\alpha(\mu_0)}{\pi} \right) + \dots \right]$$

Motivation: why m_Q ?

Example, Higgs decay

[Kuhn et al '05]

$$M_H = 126 \text{GeV}$$

$$\Gamma(H \rightarrow bb) \sim 3 \frac{G_F M_H}{4\pi\sqrt{2}} \overline{m_b} (M_H)^2 \left(1 + 5.67 \left(\frac{\alpha}{\pi} \right) + 29.1 \left(\frac{\alpha}{\pi} \right)^2 + 41.8 \left(\frac{\alpha}{\pi} \right)^3 - 825.7 \left(\frac{\alpha}{\pi} \right)^4 \right)$$

$$(1 + \dots) \sim 1.25$$

$$\overline{m_b} (M_H)^2 \sim 0.34 M_b^2$$

$$\alpha(M_H) = 0.115$$

larger correction from running of the quark mass

Motivation: why precise mq?

$$\text{Higgs decay} \sim \overline{m_b}(M_H)^2$$

$$\Gamma(B \rightarrow X_u l \nu) \sim G_F^2 m_b^5 |V_{ub}|^2$$

$$\Gamma(B \rightarrow X_c l \nu) \sim G_F^2 m_b^5 f(m_c^2/m_b^2) |V_{cb}|^2$$

Yukawa unification

[Baer et al '00]

$$\frac{\delta m_b}{m_b} \sim \frac{\delta m_t}{m_t} \quad \text{if } \delta m_t \sim 1\text{GeV} \Rightarrow \delta m_b \sim 25\text{MeV}$$

Motivation: why precise mq?

Techniques

Υ -spectroscopy

$$m(\Upsilon(1S)) = 2M_b - \mathcal{C}\alpha^2 M_b + \dots$$

[Ayala et al '14]

lattice: HPQCD '14

$$\overline{m}_c(3\text{GeV}) = 986(6)\text{MeV}$$

$$\overline{m}_b(10\text{GeV}) = 3617(25)\text{MeV}$$

QCD Sum Rules

$$\int \frac{ds}{s^{n+1}} R_q(s) \sim \left(\frac{1}{m_q} \right)^{2n}$$

Motivation: why precise mq?

$\overline{m}_c(\overline{m}_c)$	method	reference
1275.8 ± 5.8	lattice ($N_f = 4$), PS current	HPQCD, 1408.4169
1348 ± 46	lattice (2+1+1), M_D	ETM, 1403.4504
1274 ± 36	lattice ($N_f = 2$), f_D	ALPHA, 1312.7693
1240 ± 50	$c\bar{c}$ X-section DIS	Alekhin et al, 1310.3059
1260 ± 65	$c\bar{c}$ X-section NLO fit	HI and ZEUS, 1211.1182
1262 ± 17	SR $J/\Psi, \Psi(2S - 6S)$	Narison, 1105.5070
1260 ± 36	lattice (2+1), f_D	PACS-CS, 1104.4600
1278 ± 9	SR $J/\Psi, \Psi, R$	Bodenstain et al, 1102.3835
1282 ± 24	1st moment SR $J/\Psi, \Psi, R$	Dehnadi et al, 1102.2264
1280 ± 70	lattice + pQCD in static potential	Laschka et al, 1102.0945
1279 ± 13	1st moment SR $J/\Psi, \Psi, R$	Chetyrkin et al, 1010.6157
1275 ± 25	PDG average	PDG 2014

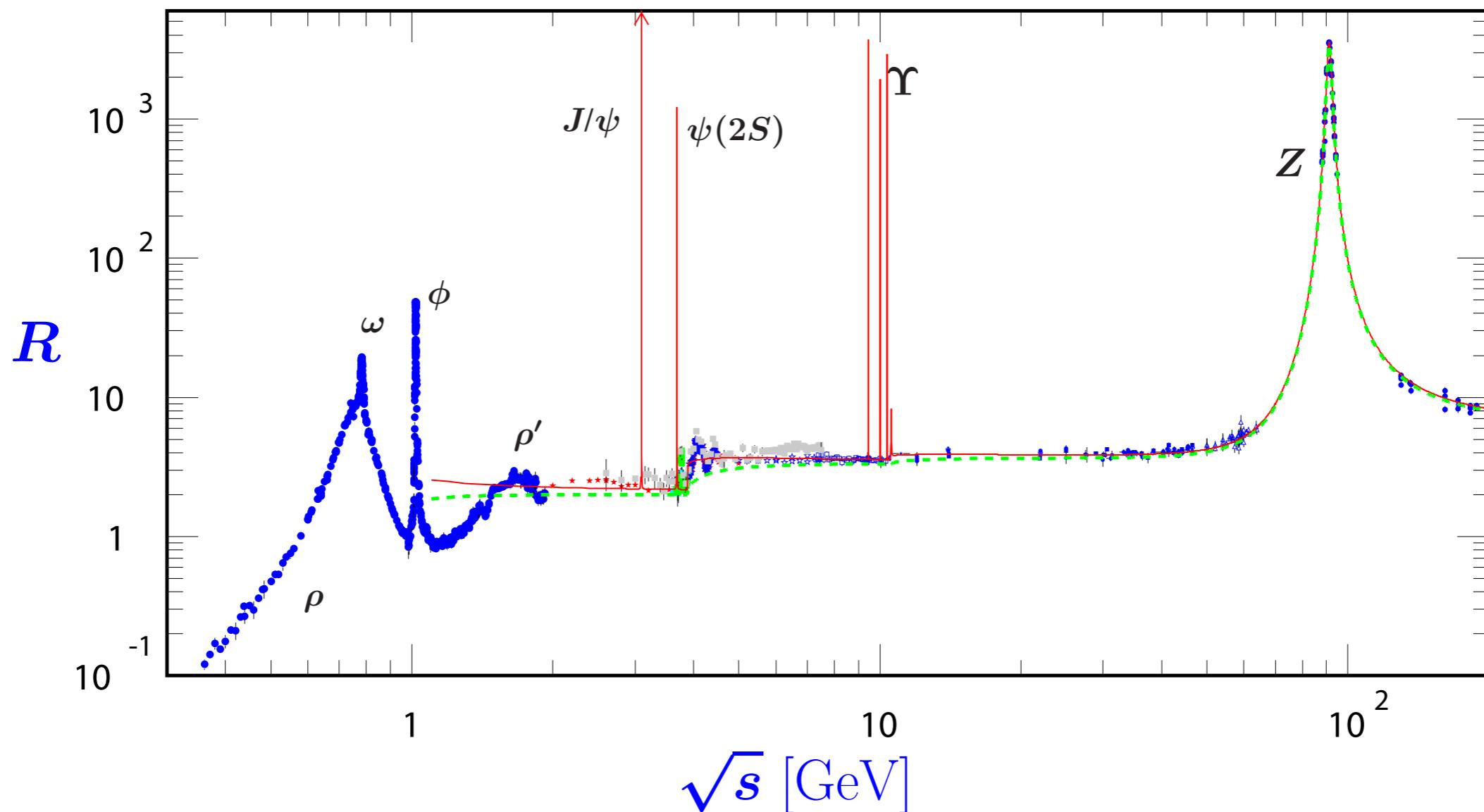
Motivation: why precise m_Q ?

$\overline{m}_b(\overline{m}_b)$	method	reference
4174 ± 24	lattice ($N_f = 4$), PS current	HPQCD, 1408.4169
4201 ± 43	$N^3\text{LO}$ pQCD, M_Y	Ayala et al, 1407.2128
4169 ± 9	SR $\Upsilon(1S - 6S)$	Penin, Zerf, 1401.7035
4247 ± 34	SR, f_B	Lucha et al, 1305.7099
4166 ± 43	lattice + pQCD, M_Y, M_{B_s}	HPQCD, 1302.3739
4235 ± 55	SR $\Upsilon(1S - 6S)$, R	Hoang et al, 1209.0450
4171 ± 9	SR $\Upsilon(1S - 6S)$, R	Bodenstain et al, 1111.5742
4177 ± 11	SR $\Upsilon(1S - 6S)$	Narison, 1105.5070
4180 ± 50	lattice + pQCD in static potential	Laschka et al, 1102.0945
4163 ± 16	2nd moment SR $\Upsilon(1S - 6S)$, R	Chetyrkin et al, 1010.6157
4.180 ± 30	PDG average	PDG 2014

QCD Sum Rules

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

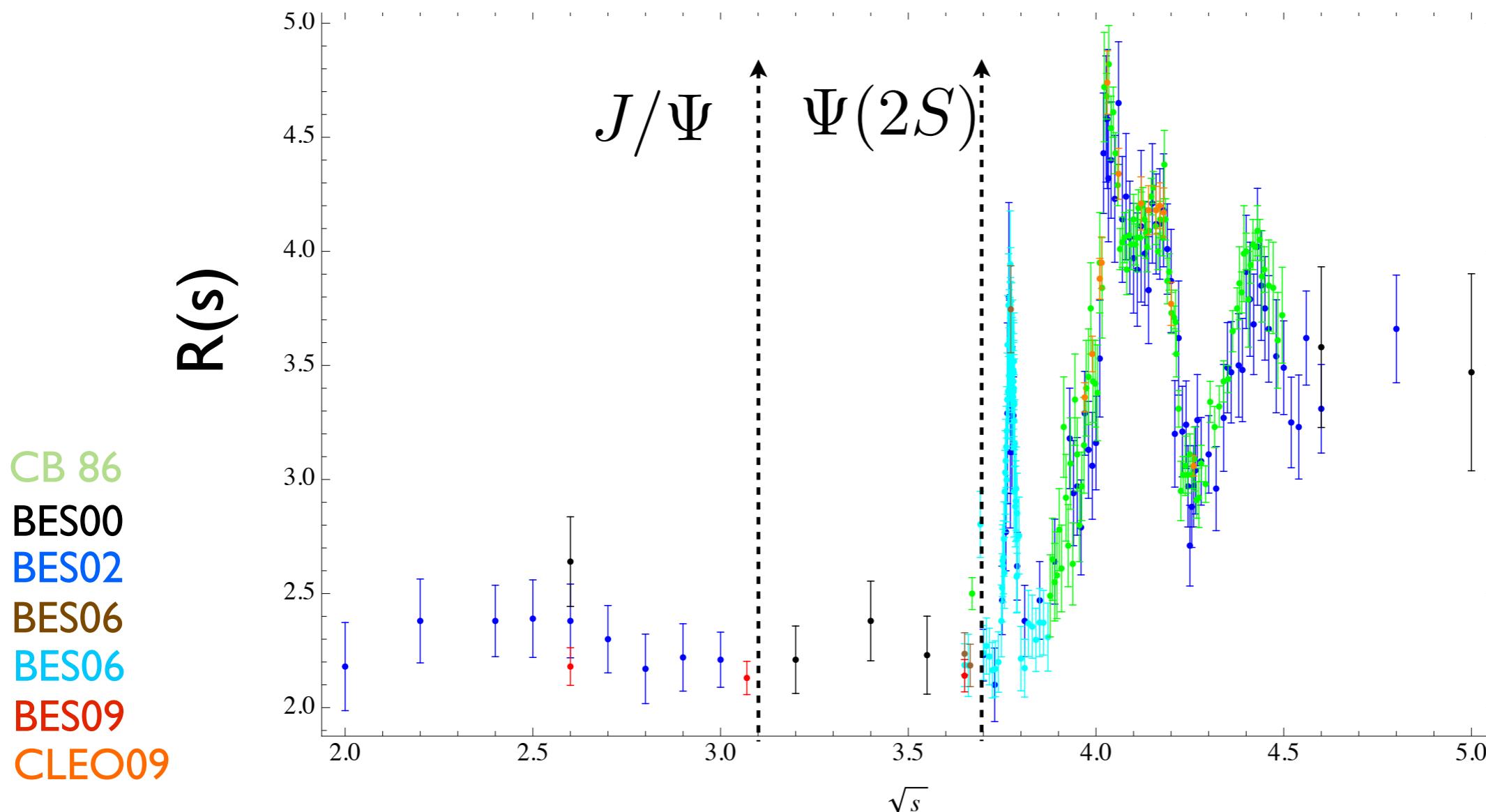
$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = 4\pi\alpha_{\text{em}}(s)^2/3s$$



QCD Sum Rules

$$R(s) = R_{uds}(s) + R_q(s)$$

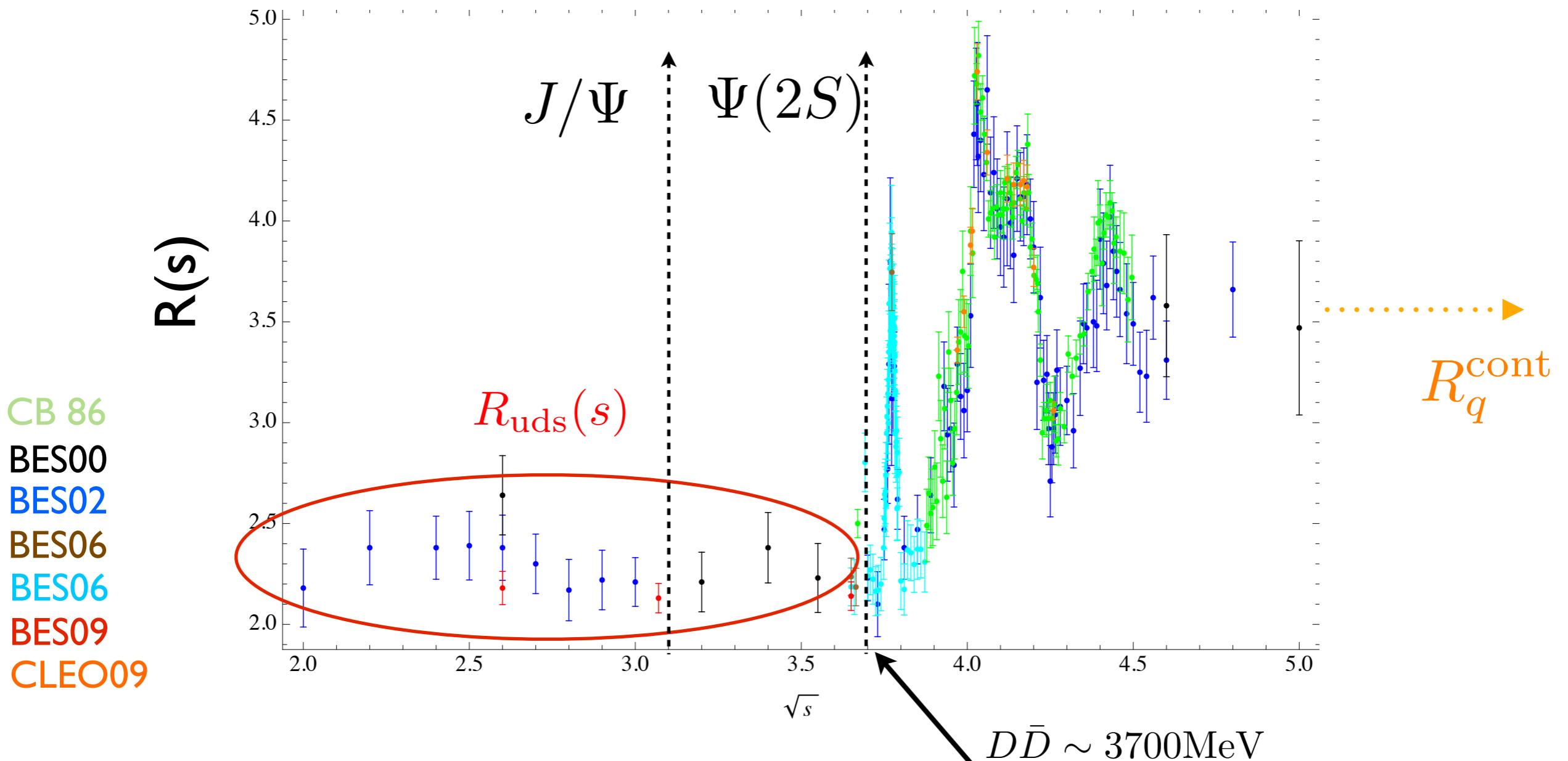
$$R_q(s) = R_q^{\text{Res}}(s) + R_q^{\text{th}}(s) + R_q^{\text{cont}}(s)$$



QCD Sum Rules

$$R(s) = R_{uds}(s) + R_q(s)$$

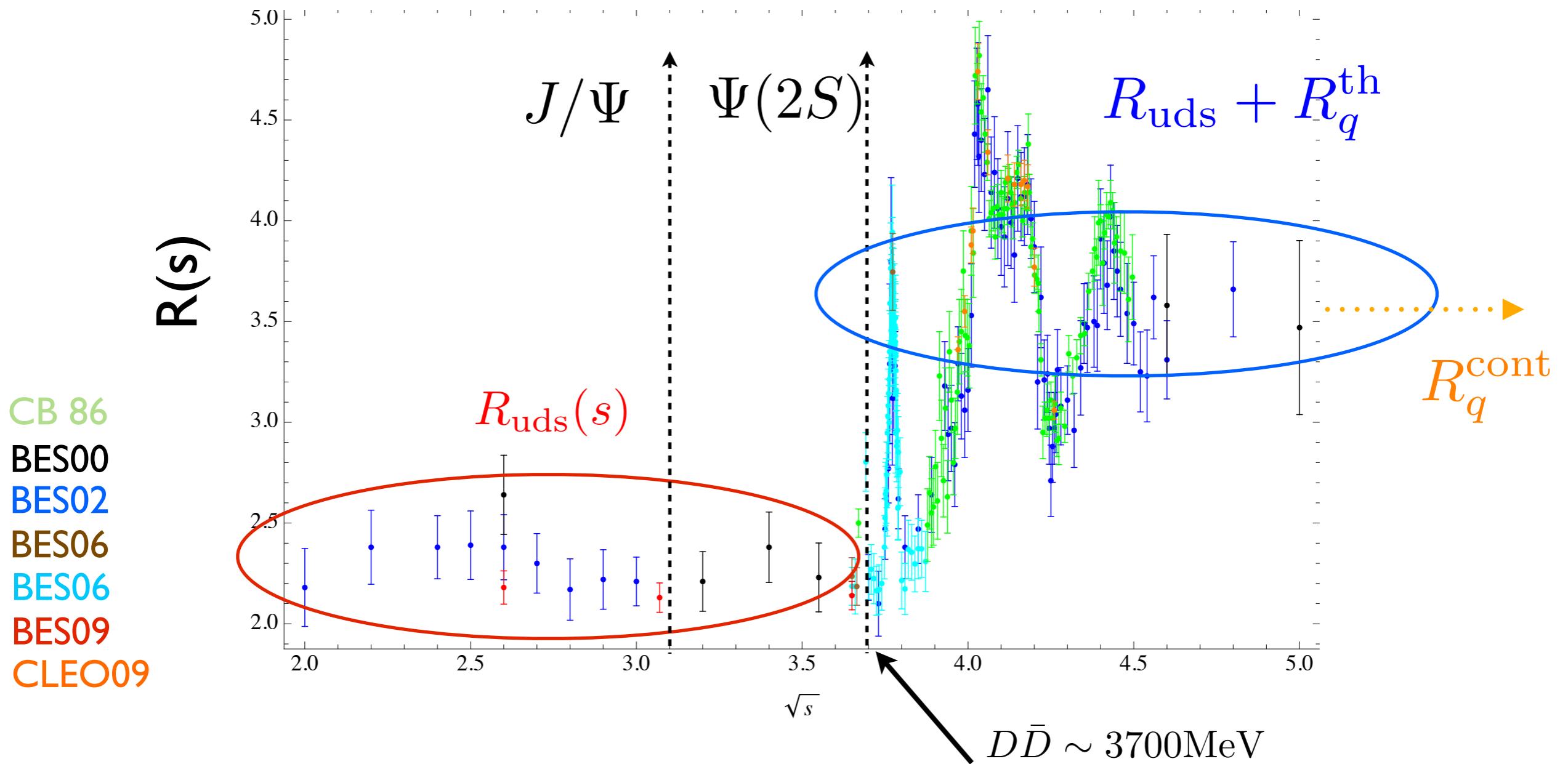
$$R_q(s) = R_q^{\text{Res}}(s) + R_q^{\text{th}}(s) + R_q^{\text{cont}}(s)$$



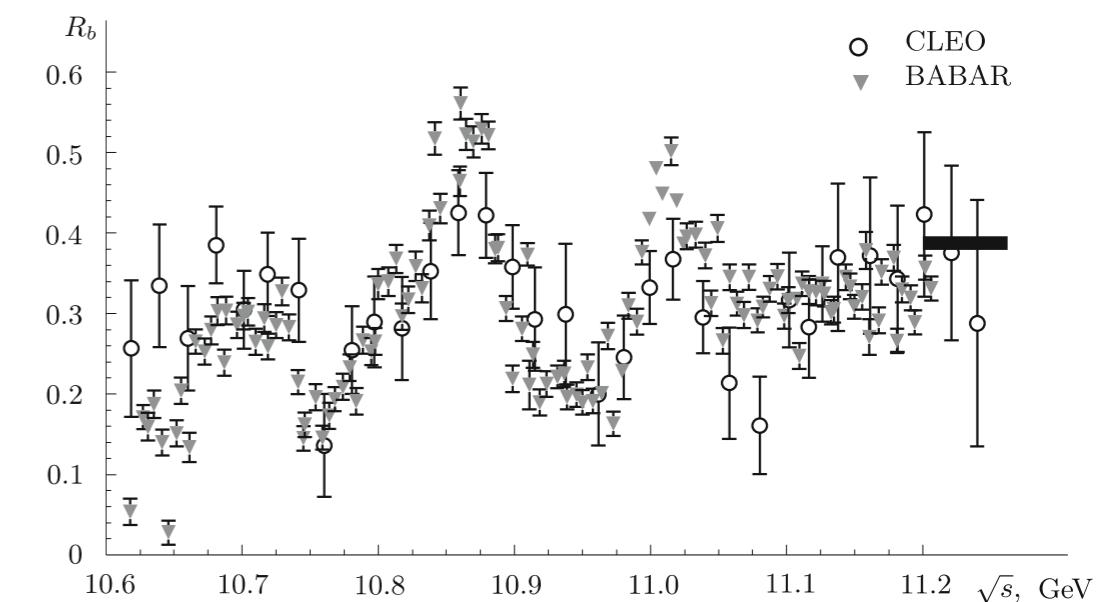
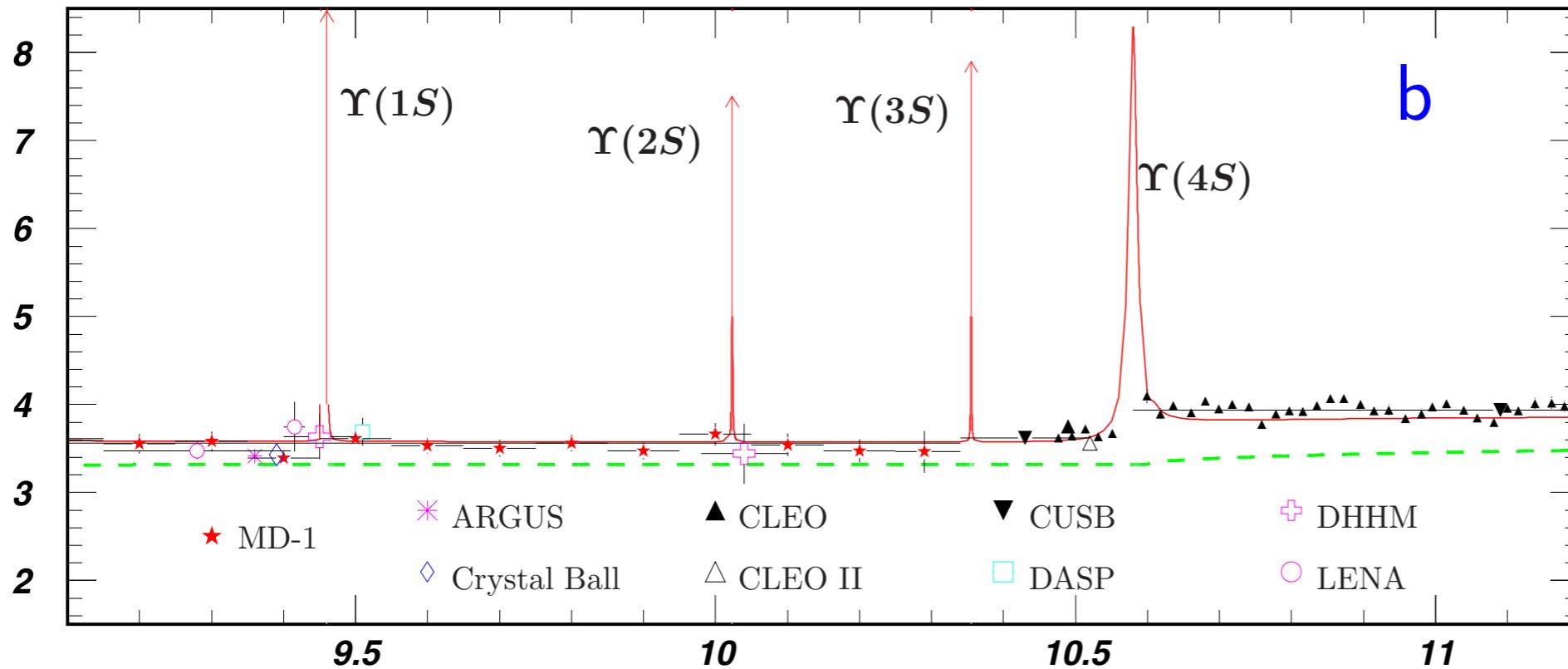
QCD Sum Rules

$$R(s) = R_{uds}(s) + R_q(s)$$

$$R_q(s) = R_q^{\text{Res}}(s) + R_q^{\text{th}}(s) + R_q^{\text{cont}}(s)$$



QCD Sum Rules



QCD Sum Rules

Using the optical theorem:

[SVZ,'79]

$$R(s) = 12\pi \text{Im}[\Pi(s + i\epsilon)]$$

$\Pi_q(s)$ is the correlator of two heavy-quark vector currents can be calculated in pQCD order by order and satisfies a Dispersion Relation:

$$12\pi^2 \frac{\hat{\Pi}_q(0) - \hat{\Pi}_q(-t)}{t} = \int_{4m_q^2}^{\infty} \frac{ds}{s} \frac{R_q(s)}{s+t} \quad \hat{\Pi}_q(s) \text{ in } \overline{MS}$$

For $t \rightarrow 0$

$$\mathcal{M}_n := \left. \frac{12\pi^2}{n!} \frac{d^n}{dt^n} \hat{\Pi}_q(t) \right|_{t=0} = \int_{4m_q^2}^{\infty} \frac{ds}{s^{n+1}} R_q(s)$$

QCD Sum Rules

$\hat{\Pi}_q(s)$ can be Taylor expanded:

$$\Pi_q(t) = Q_q^2 \frac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n \left(\frac{t}{4\hat{m}_q^2} \right)^n$$

QCD Sum Rules

$\hat{\Pi}_q(s)$ can be Taylor expanded:

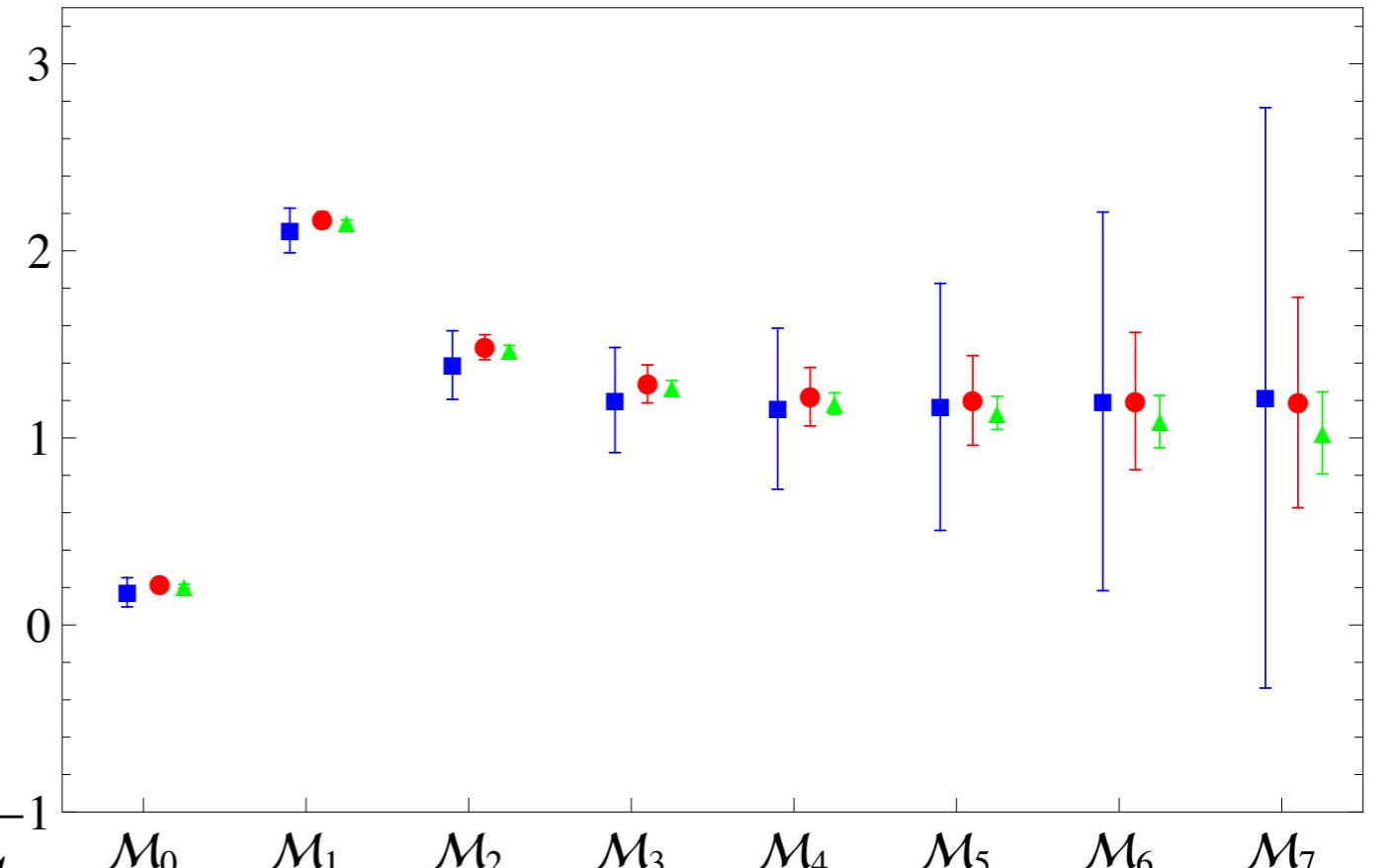
$$\Pi_q(t) = Q_q^2 \frac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n \left(\frac{t}{4\hat{m}_q^2} \right)^n$$

$$\mathcal{M}_n^{\text{pQCD}} = \frac{9}{4} Q_q^2 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n} \bar{C}_n^{-1}$$

$$\hat{\alpha} = \hat{\alpha}(\overline{m}_q)$$

[Maier et al, '08]
 [Chetyrkin, Steinhauser'06]
 [Melnikov, Ritberger'03]

[Kiyo et al '09]
 [Hoang et al '09]
 [Greynat et al '09]



$$\bar{C}_n = \bar{C}_n^{(0)} + \left(\frac{\hat{\alpha}}{\pi} \right) \bar{C}_n^{(1)} + \left(\frac{\hat{\alpha}}{\pi} \right)^2 \bar{C}_n^{(2)} + \left(\frac{\hat{\alpha}}{\pi} \right)^3 \bar{C}_n^{(3)} + \mathcal{O} \left(\frac{\hat{\alpha}}{\pi} \right)^4$$

$$\pm N_C C_F C_A^3 Q_q^2 \left[\frac{\hat{\alpha}_s(\hat{m}_q)}{\pi} \right]^4 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n}$$

QCD Sum Rules

Running of the QCD's coupling constant:

$$\mu^2 \frac{d}{d\mu^2} \frac{\alpha(\mu)}{\pi} = \beta(\alpha) = - \sum_{k \geq 0} \beta_k \left(\frac{\alpha(\mu)}{\pi} \right)^{k+2}$$

known up to k=3
[Ritbergen et al '97]

Integrating:

$$\log \frac{\mu^2}{\Lambda^2} = \int \frac{da}{\beta(a)}$$

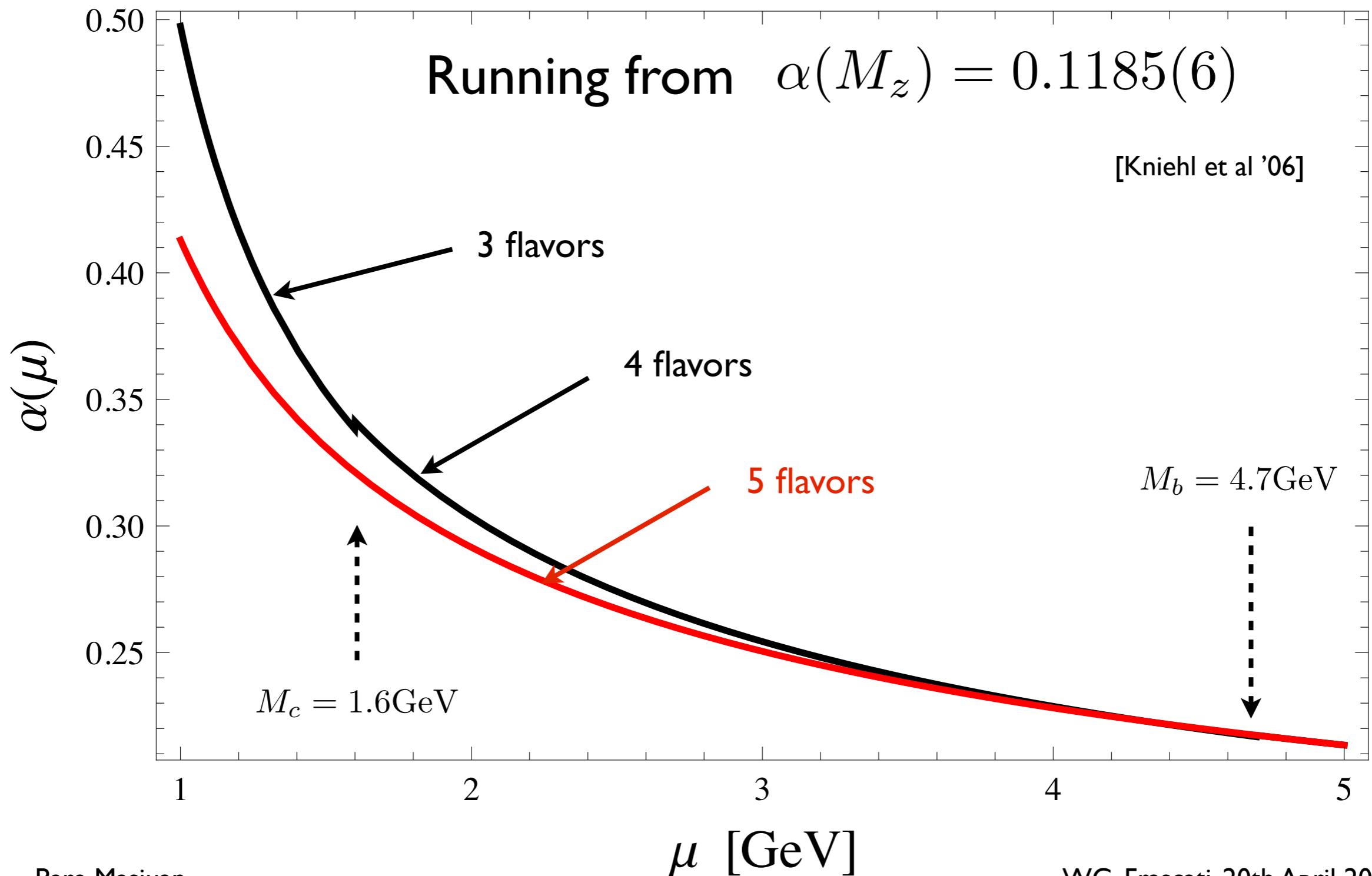
→ integrate numerically
→ integrate and expand
→ expand and integrate

$$a = \frac{\alpha(\mu)}{\pi}$$

Formally of the same order, numerically different

+ threshold effects

Motivation: why precise m_Q ?



Motivation: why precise mq?

From $\alpha(m_\tau) = 0.3142$ to $\alpha(M_z)$ $\Delta\alpha(M_z) = 1.5 \cdot 10^{-4}$

To compare with PDG '14: $\alpha(M_z) = 0.1185(6)$

Uncertainty on threshold effects (due to truncation + mass uncertainty):

$$0.7 \cdot 10^{-4} + 0.2 \cdot 10^{-4} + 1.5 \cdot 10^{-4} \longrightarrow 2 \cdot 10^{-4}$$

(charm thr.) (bottom thr.) (beta truncation) (total)

QCD Sum Rules

Sum Rules:

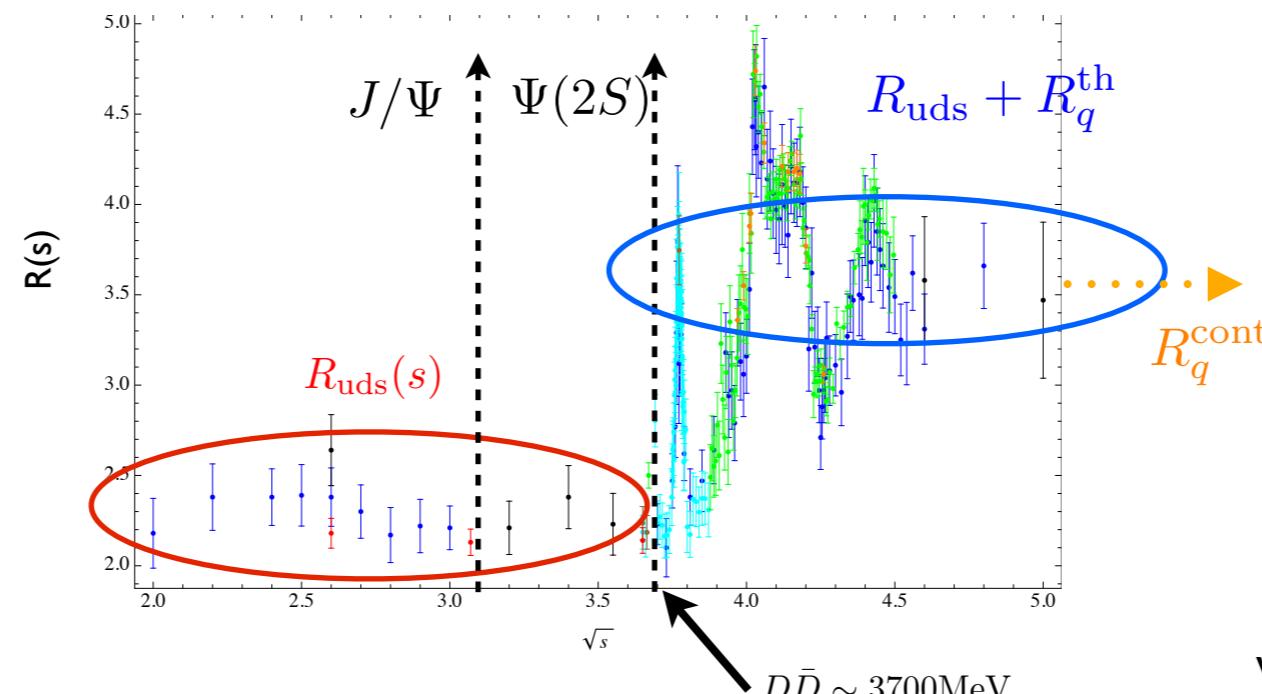
$$\mathcal{M}_n = \int_{4m_q^2}^{\infty} \frac{ds}{s^{n+1}} R_q(s)$$

L.h.s. from theory

$$\mathcal{M}_n^{\text{pQCD}} = \frac{9}{4} Q_q^2 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n} \bar{C}_n$$

R.h.s. from experiment

$$R_q(s) = R_q^{\text{Res}}(s) + R_q^{\text{th}}(s) + R_q^{\text{cont}}(s)$$



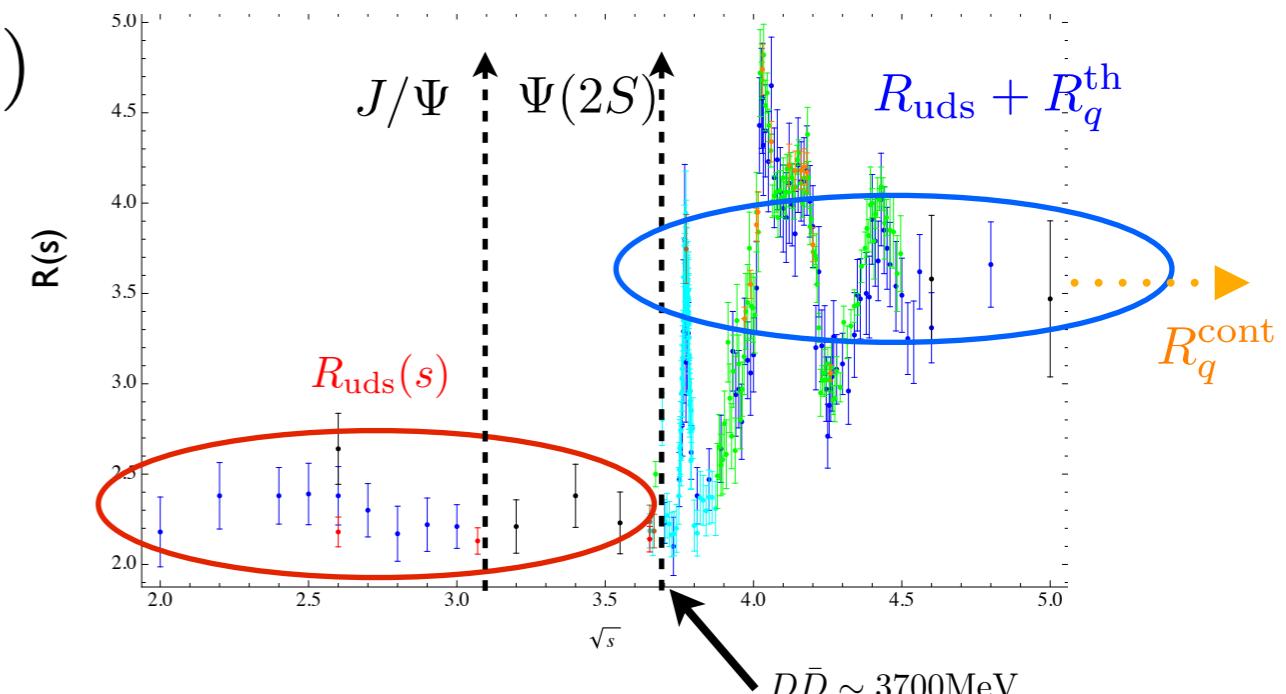
QCD Sum Rules

$$R_q(s) = R_q^{\text{Res}}(s) + R_q^{\text{th}}(s) + R_q^{\text{cont}}(s)$$

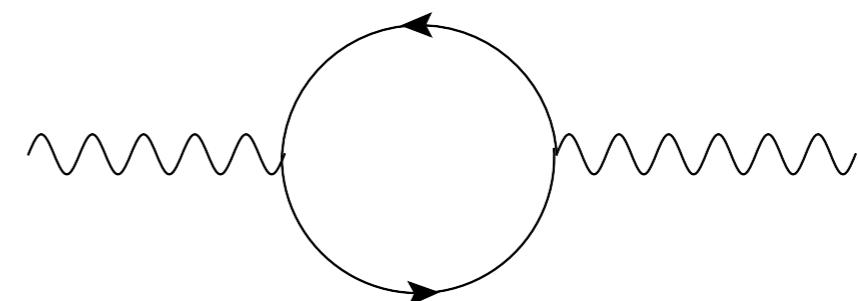
$$R_q^{\text{Res}}(s) = \frac{9\pi M_R \Gamma_R^e}{\alpha_{\text{em}}^2(M_R)} \delta(s - M_R^2)$$

$$R_q^{\text{th}}(s) = R_q(s) - R_{\text{background}}$$

$R_q^{\text{cont}}(s)$ calculated using pQCD
 $(\sqrt{s} \geq 4.8 \text{ GeV})$



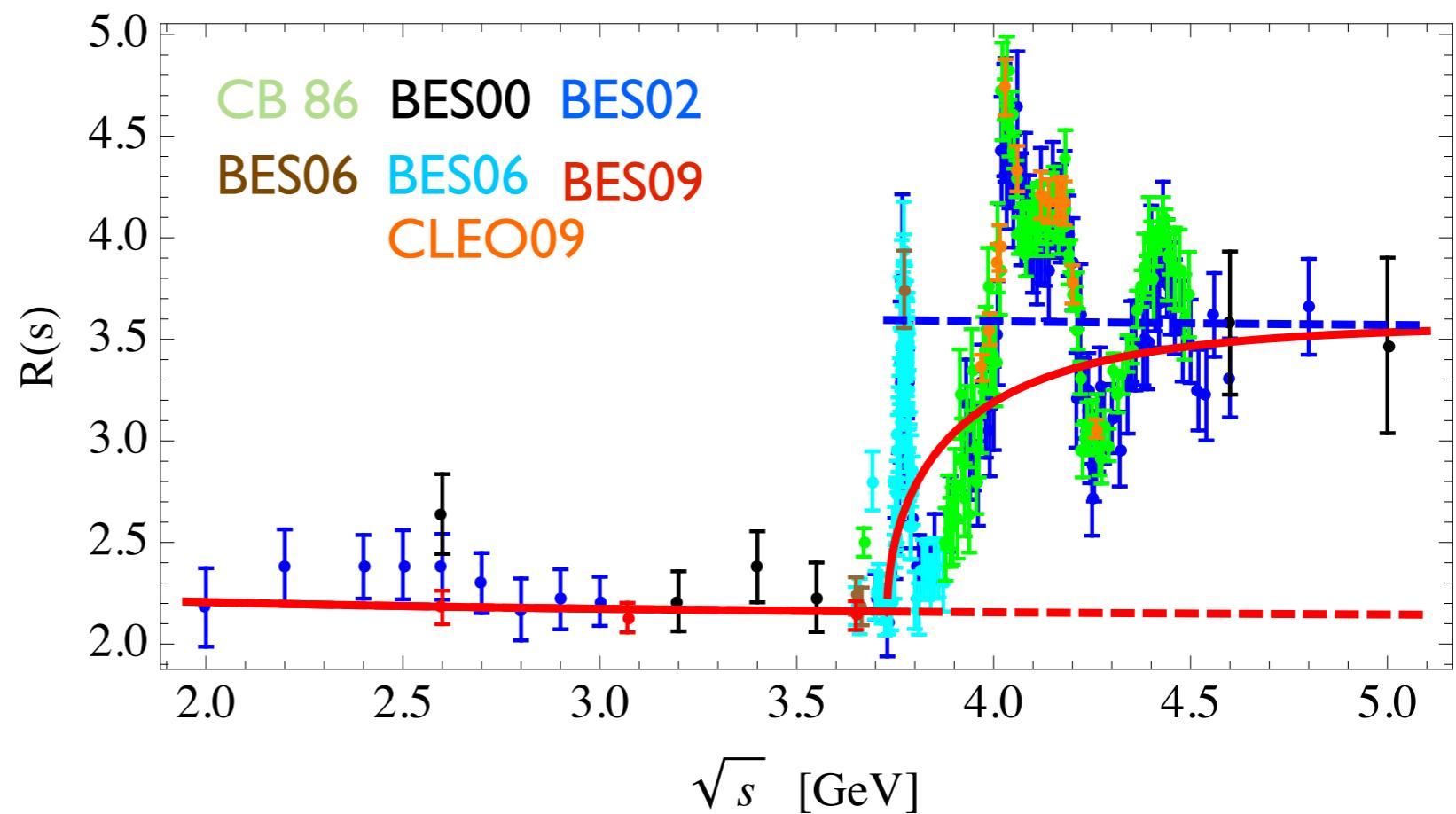
$$(2M_D \leq \sqrt{s} \leq 4.8 \text{ GeV})$$



Background

$$R_{\text{background}} = R_{\text{uds}} + R_{\text{uds(cb)}} + R_{\text{sing}} + R_{\text{QED}}$$

Light flavor
contribution in
charm region

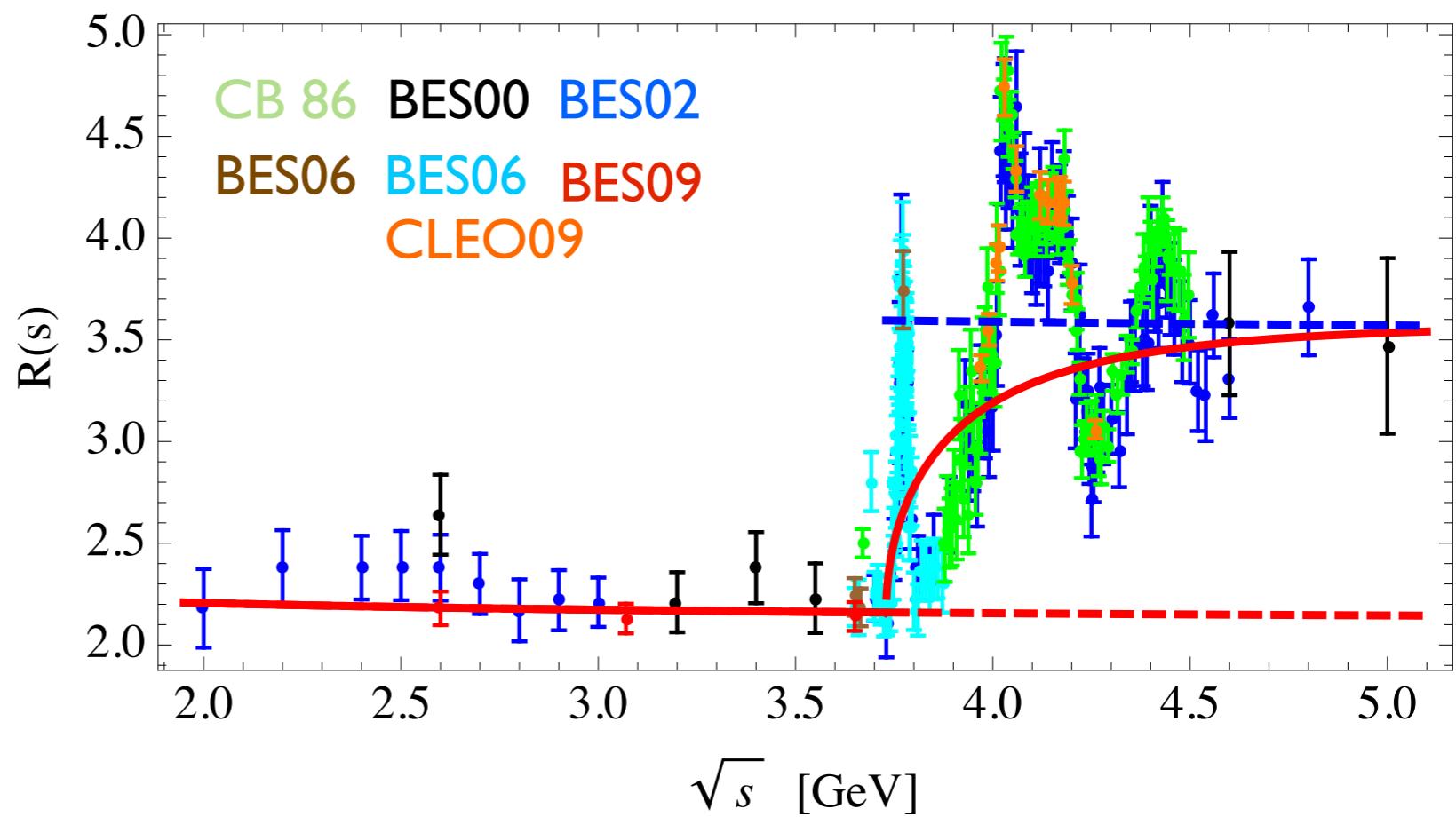
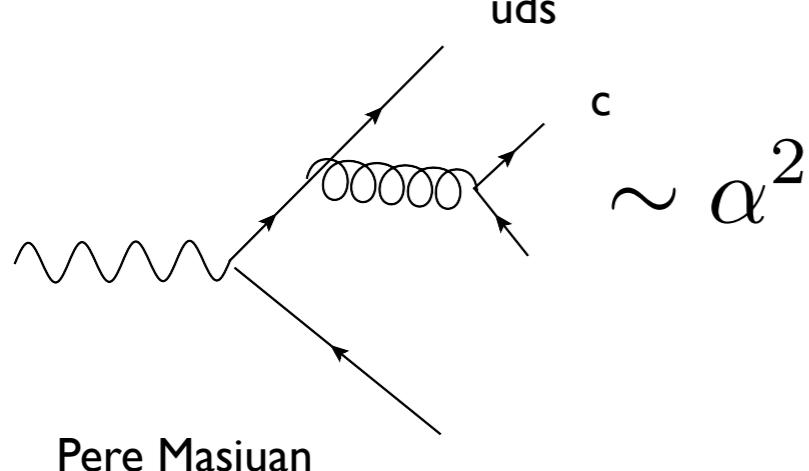


Using pQCD below threshold, calculate R , and extrapolate

Background

$$R_{\text{background}} = R_{\text{uds}} + R_{\text{uds(cb)}} + R_{\text{sing}} + R_{\text{QED}}$$

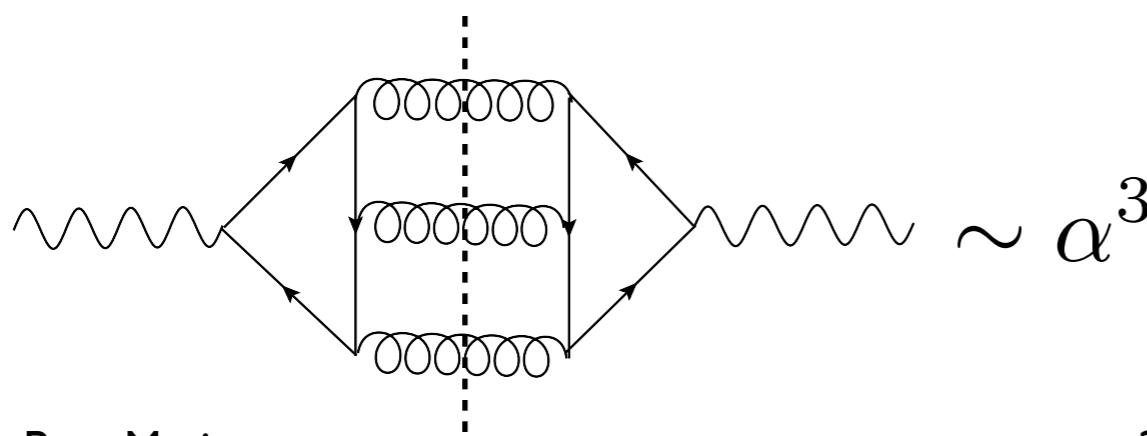
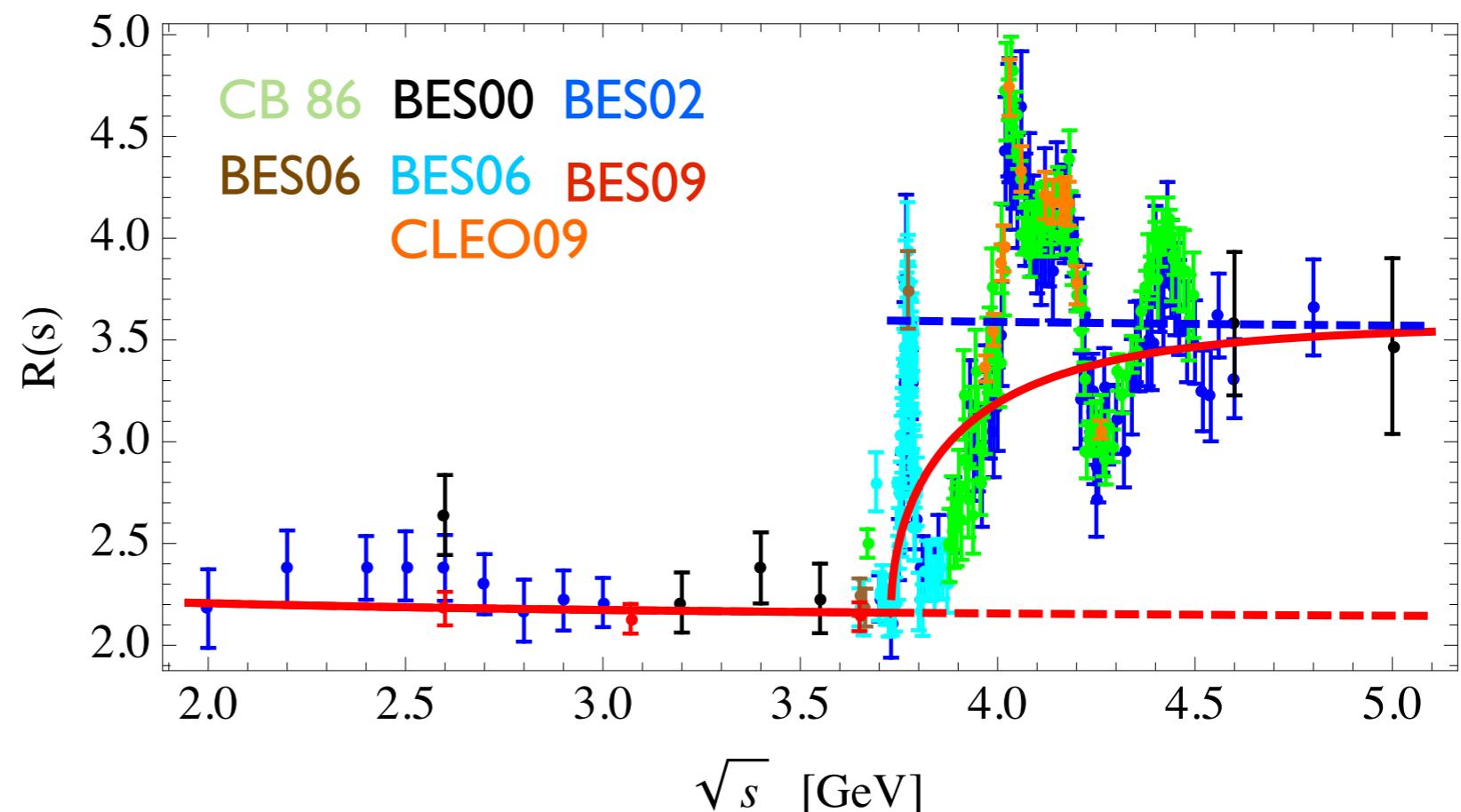
Light flavor
contribution in
charm region
+
secondary
production



Background

$$R_{\text{background}} = R_{\text{uds}} + R_{\text{uds(cb)}} + R_{\text{sing}} + R_{\text{QED}}$$

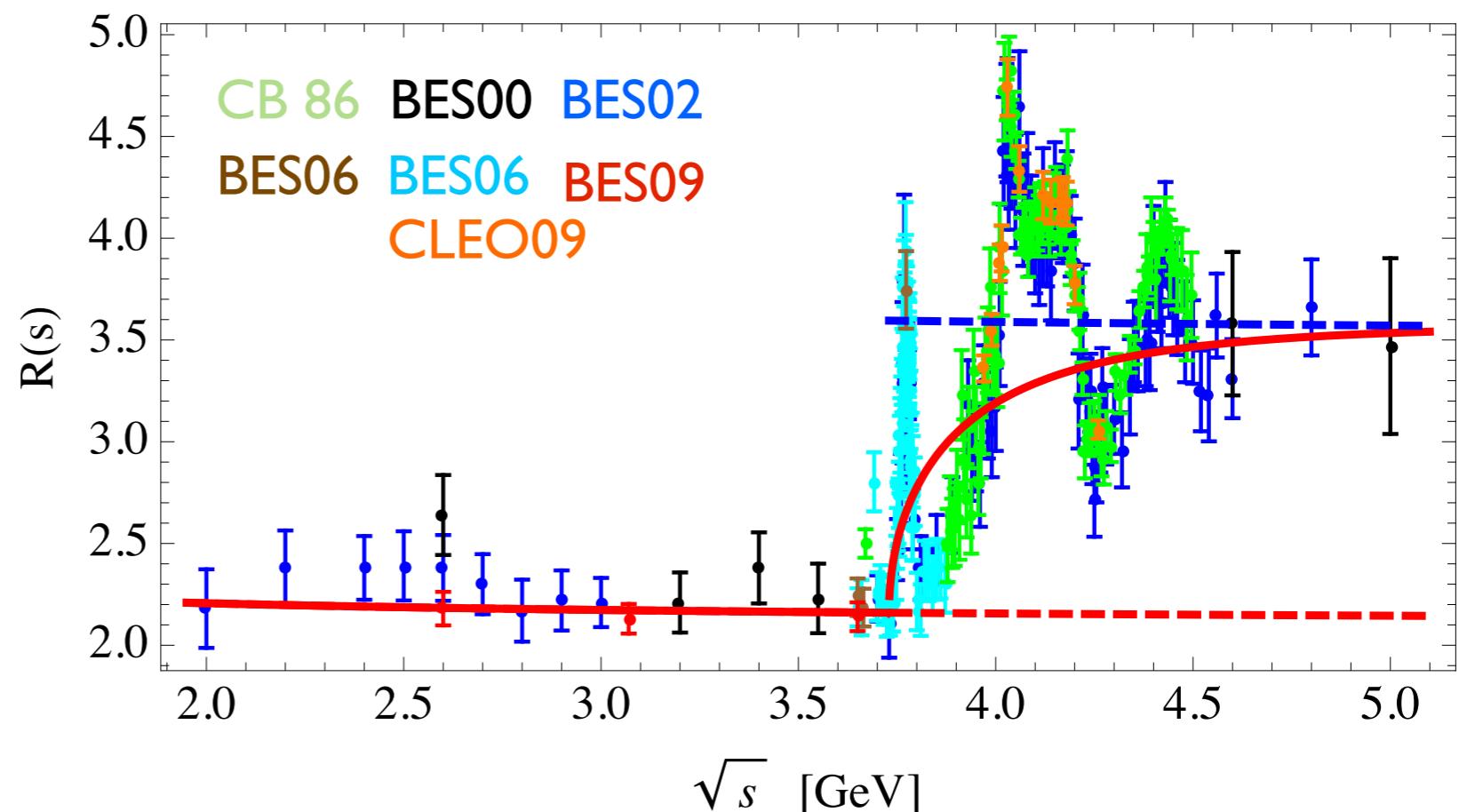
Light flavor
contribution in
charm region
+
secondary
production
+
singlet contribution



Background

$$R_{\text{background}} = R_{\text{uds}} + R_{\text{uds(cb)}} + R_{\text{sing}} + R_{\text{QED}}$$

Light flavor
contribution in
charm region
+
secondary
production
+
singlet contribution
+
2loop QED



Non-perturbative effects

Non-perturbative effects due to gluon condensates to the moments are:

[Chetyrkin et al '12]

$$\mathcal{M}_n^{\text{nonp}}(\mu^2) = \frac{12\pi^2 Q_q^2}{(4\hat{m}_q^2)^{n+2}} \text{Cond } a_n \left(1 + \frac{\alpha_s(\hat{m}_q^2)}{\pi} b_n \right)$$

a_n, b_n are numbers, and $\text{Cond} = \langle \frac{\alpha}{\pi} G^2 \rangle = \frac{3}{\pi^2} (2 \pm 2) \cdot 10^{-3}$ [Dominguez et al '14]

↳ from fits to tau data

$$\left. \frac{\mathcal{M}_n^{\text{nonp}}(\hat{m}_c)}{\mathcal{M}_n^{\text{exp}}} \right|_c \sim 0.5\% - 2\% \longrightarrow \text{up to } 2\% \text{ on } m_c \sim 2\text{MeV}$$

$$\left. \frac{\mathcal{M}_n^{\text{nonp}}(\hat{m}_b)}{\mathcal{M}_n^{\text{exp}}} \right|_b \sim 0\% - 0.05\% \longrightarrow \text{up to } 0.05\% \text{ on } m_b \sim 2\text{MeV}$$

QCD Sum Rules

$$R_q(s) = R_q^{\text{Res}}(s) + R_q^{\text{th}}(s) + R_q^{\text{cont}}(s)$$

$$\frac{9}{4} Q_q^2 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n} \bar{C}_n = \int_{4m_q^2}^{\infty} \frac{ds}{s^{n+1}} R_q(s)$$

[Kuhn et al '07,'09,'12]

<i>n</i>	$m_c(3 \text{ GeV})$	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

QCD Sum Rules

$$R_q(s) = R_q^{\text{Res}}(s) + R_q^{\text{th}}(s) + R_q^{\text{cont}}(s)$$

$$\frac{9}{4} Q_q^2 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n} \bar{C}_n = \int_{4m_q^2}^{\infty} \frac{ds}{s^{n+1}} R_q(s)$$

[Kuhn et al '07,'09,'12]

<i>n</i>	$m_b(10 \text{ GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

QCD Sum Rules

Potential improvements

- Local duality at high energies
- Below threshold: pQCD fitted to data and extrapolated. Higher orders in fit? Energy dependence?
 - the same shift is not applied at high energy tail
- Effects of the truncation of the running?
- Different moments, different results. Correlations?

QCD Sum Rules

Our proposal

- Consider global duality
- Below threshold: the difference between pQCD and data is extra source of error (not used for normalization)
- do not use experimental data on threshold region, only resonances
 - Exp data in threshold only for error estimation
- Include in α an error from the truncation
- Use two different moments to extract the mass

QCD Sum Rules

Our proposal

For a global duality:

$\hat{\Pi}_q(s)$ in \overline{MS}

$$12\pi^2 \frac{\hat{\Pi}_q(0) - \hat{\Pi}_q(-t)}{t} = \int_{4m_q^2}^{\infty} \frac{ds}{s} \frac{R_q(s)}{s+t}$$

$t \rightarrow \infty$ define the \mathcal{M}_0

QCD Sum Rules

Our proposal

For a global duality:

$\hat{\Pi}_q(s)$ in \overline{MS}

$$12\pi^2 \frac{\hat{\Pi}_q(0) - \hat{\Pi}_q(-t)}{t} = \int_{4m_q^2}^{\infty} \frac{ds}{s} \frac{R_q(s)}{s+t}$$

$t \rightarrow \infty$ define the \mathcal{M}_0 (but has a divergent part)

$$\lim_{t \rightarrow \infty} \hat{\Pi}_q(-t) \sim \log(t) \quad \longleftrightarrow \quad \int_{4m_q^2}^{\infty} \frac{ds}{s} R_q(s) \sim \log(\infty)$$

Fortunately, divergence given by the zero-mass limit of $R(s)$

QCD Sum Rules

Our proposal

$$\begin{aligned}\lambda_1^q(s) = & 1 + \frac{\alpha_s(s)}{\pi} \\ & + \left[\frac{\alpha_s(s)}{\pi} \right]^2 \left[\frac{365}{24} - 11\zeta(3) + n_q \left(\frac{2}{3}\zeta(3) - \frac{11}{12} \right) \right] \\ & + \left[\frac{\alpha_s(s)}{\pi} \right]^3 \left[\frac{87029}{288} - \frac{121}{8}\zeta(2) - \frac{1103}{4}\zeta(3) + \frac{275}{6}\zeta(5) \right. \\ & \quad \left. + n_q \left(-\frac{7847}{216} + \frac{11}{6}\zeta(2) + \frac{262}{9}\zeta(3) - \frac{25}{9}\zeta(5) \right) \right. \\ & \quad \left. + n_q^2 \left(\frac{151}{162} - \frac{1}{18}\zeta(2) - \frac{19}{27}\zeta(3) \right) \right]\end{aligned}$$

QCD Sum Rules

Our proposal

Zeroth Sum Rule:

$$\begin{aligned} & \sum_{\text{resonances}} \frac{9\pi\Gamma_R^e}{3Q_q^2 M_R \hat{\alpha}_{em}^2(M_R)} + \int_{4M^2}^{\infty} \frac{ds}{s} \frac{R_q^{\text{cont}}}{3Q_q^2} - \int_{\hat{m}_q^2}^{\infty} \frac{ds}{s} \lambda_1^q(s) \\ &= -\frac{5}{3} + \frac{\hat{\alpha}_s}{\pi} \left[4\zeta(3) - \frac{7}{2} \right] \\ &+ \left(\frac{\hat{\alpha}_s}{\pi} \right)^2 \left[\frac{2429}{48} \zeta(3) - \frac{25}{3} \zeta(5) - \frac{2543}{48} + n_q \left(\frac{677}{216} - \frac{19}{9} \zeta(3) \right) \right] \\ &+ \left(\frac{\hat{\alpha}_s}{\pi} \right)^3 \left[-9.86 + 0.40 n_q - 0.01 n_q^2 \right] \end{aligned}$$

n_q active flavors

QCD Sum Rules

Our proposal

Zeroth Sum Rule:

$$R_q^{\text{cont}}(s) = 3Q_q^2 \lambda_1^q(s) \sqrt{1 - \frac{4\hat{m}_q^2(2M)}{s'}} \left[1 + \lambda_3^q \frac{2\hat{m}_q^2(2M)}{s'} \right]$$
$$s' = s + 4(\hat{m}_q^2(2M) - M^2)$$

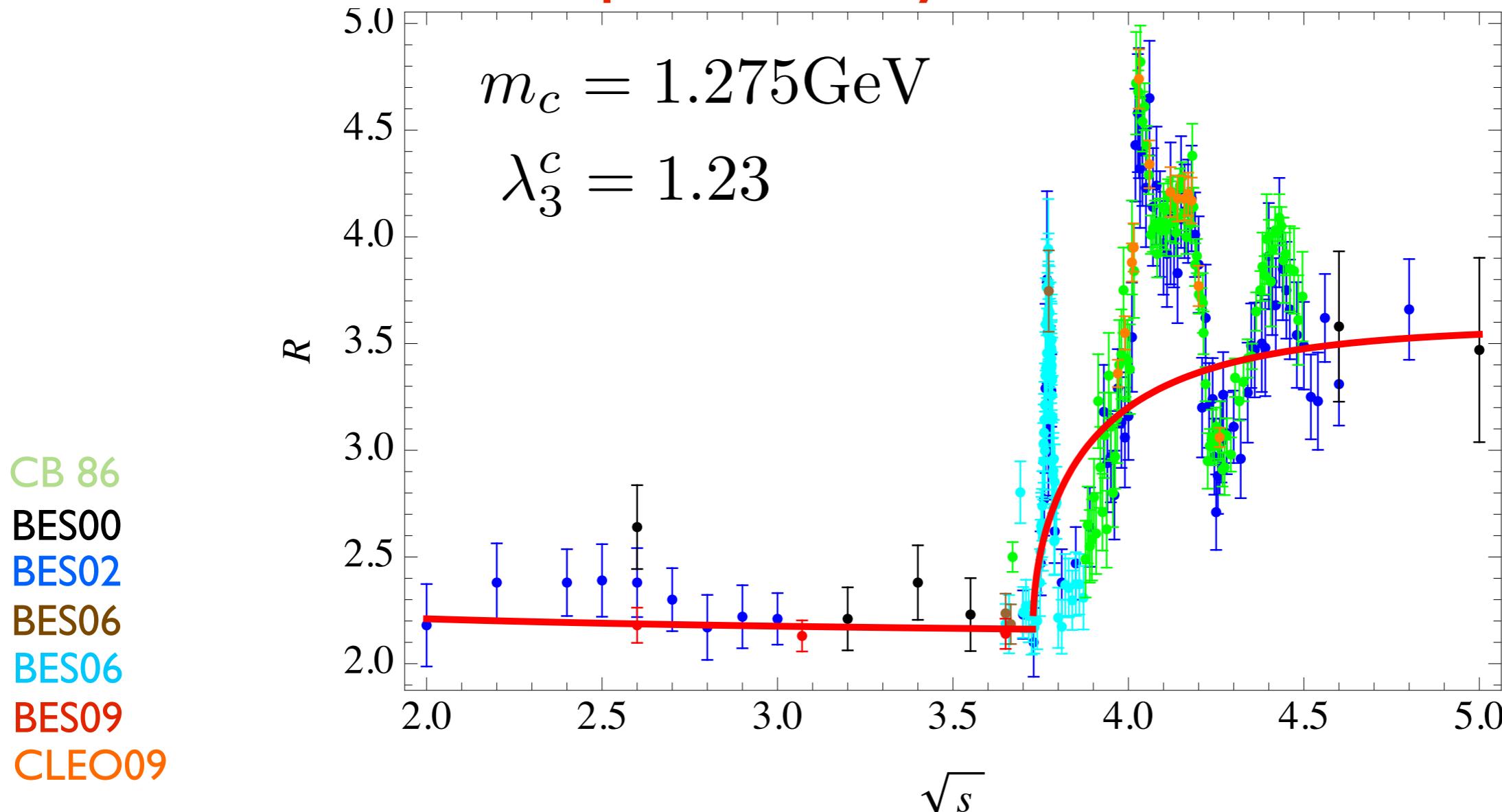
Two parameters to determine: m_q , λ_3^q

We use Zeroth + 2nd moments
(no use of experimental data on R(s) so far)

<i>n</i>	Resonances	Continuum	Total	Theory
Charm preliminary results				
0	1.231 (24)	-3.228(25)	-1.997(35)	Input (11)
1	1.184 (24)	0.962(10)	2.146(26)	2.166(17)
2	1.161 (25)	0.327(5)	1.489(26)	Input (26)
3	1.157 (26)	0.149(3)	1.306(26)	1.290(40)
4	1.167 (27)	0.077(2)	1.244(27)	1.204(61)
5	1.188 (28)	0.042(1)	1.230(28)	1.154(96)
6	1.217 (29)	0.024(1)	1.241(30)	1.104(151)
7	1.253 (31)	0.015(1)	1.267(31)	1.038(240)

QCD Sum Rules

Our proposal
preliminary results



QCD Sum Rules

Our proposal: error budget

$$m_c = 1.275 \text{ GeV}$$

$$\lambda_3^c = 1.23$$

preliminary results



($2M_D \leq \sqrt{s} \leq 4.8 \text{ GeV}$)

n	Data (exp)	$\lambda_3^c = 1.30(12)$ (fit)	$\lambda_3^c = 1.23$ (predict)
0	6.428(196)	6.322(137)	6.240
1	3.516(93)	3.483(78)	3.436
2	1.956(45)	Input	1.928
3	1.106(22)	1.119(27)	1.103
4	0.635(12)	0.651(16)	0.642
5	0.371(6)	0.386(10)	0.380

$$\Delta\lambda_3^c = 0.10 \rightarrow \Delta\overline{m}_c = 1 \text{ MeV}$$

QCD Sum Rules

Our proposal: error budget

We use Zeroth + 2nd moments

preliminary results

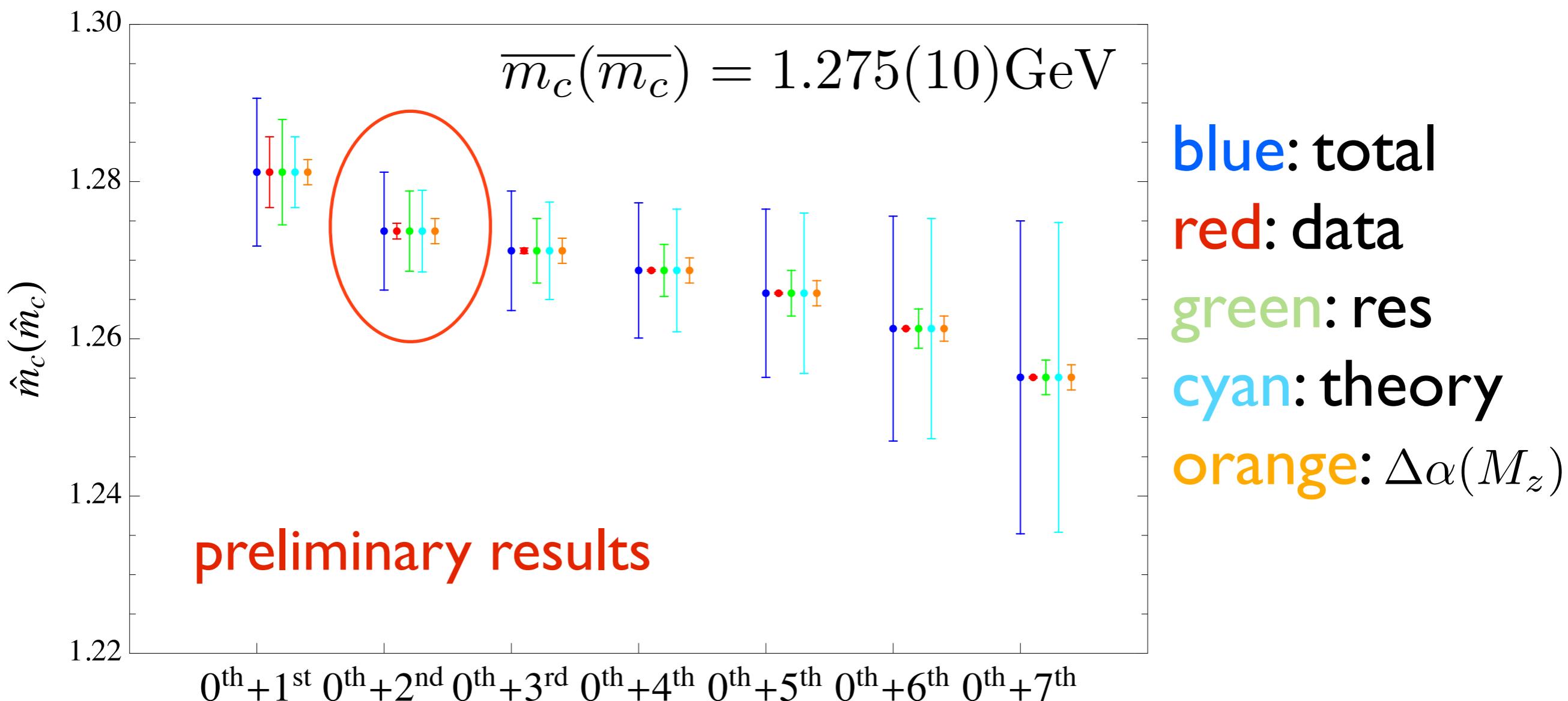
$\Delta\lambda_3$	Δres	Δth	$\Delta\alpha_s(M_z)$	$\Delta Cond$	SideBand	final
1	5	6	2	3	2	1.275(10) GeV

- $\Delta\lambda_3$ from errors on experimental moments on threshold region
- Δres from experimental error on resonance parameters (Γ_R^e)
- Δth is error due truncation of C_n coefficients
- $\Delta(\alpha(M_z))$ is experimental error on $\alpha(M_z)$ and running ($\sim 1 + 1$)
- $\Delta Cond$ is the impact of condensate
- SideBand is the impact of fitting below threshold or using pQCD

QCD Sum Rules

Our proposal: error budget

We use Zeroth + other moments



Outlook: Bottom mass

Our proposal: bottom mass

We use Zeroth + 6th moments

preliminary results

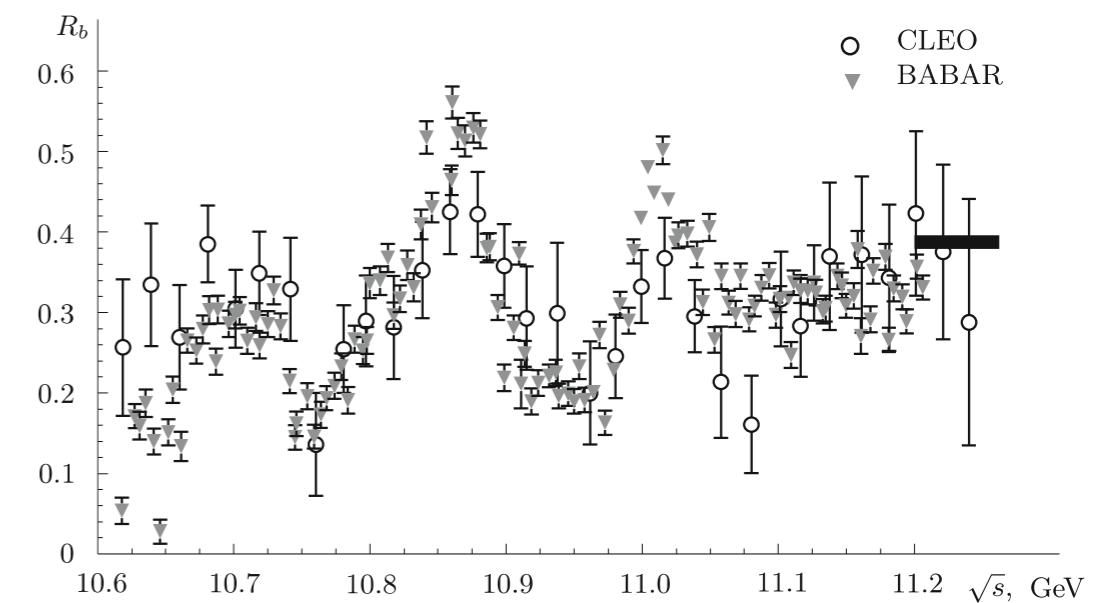
$\Delta\lambda_3$	Δres	Δth	$\Delta\alpha_s(M_z)$	Δ Cond	low fit	final
5	3	5	2	2	5	4195(10) MeV

$$\overline{m}_b(\overline{m}_b) = 4.195(10)\text{GeV}$$

$$\lambda_3^b = 1.81$$

$$\Delta\lambda_3^b = 0.60$$

- By comparing with BABAR data, we find a large discrepancy
- Low fit comes from an artificial 5% error to resolve our puzzle (lack of subthreshold data)



BABAR sys. error $\sim 3\%$

QCD Sum Rules

Our proposal: bottom mass

We use Zeroth + 6th moments

preliminary results

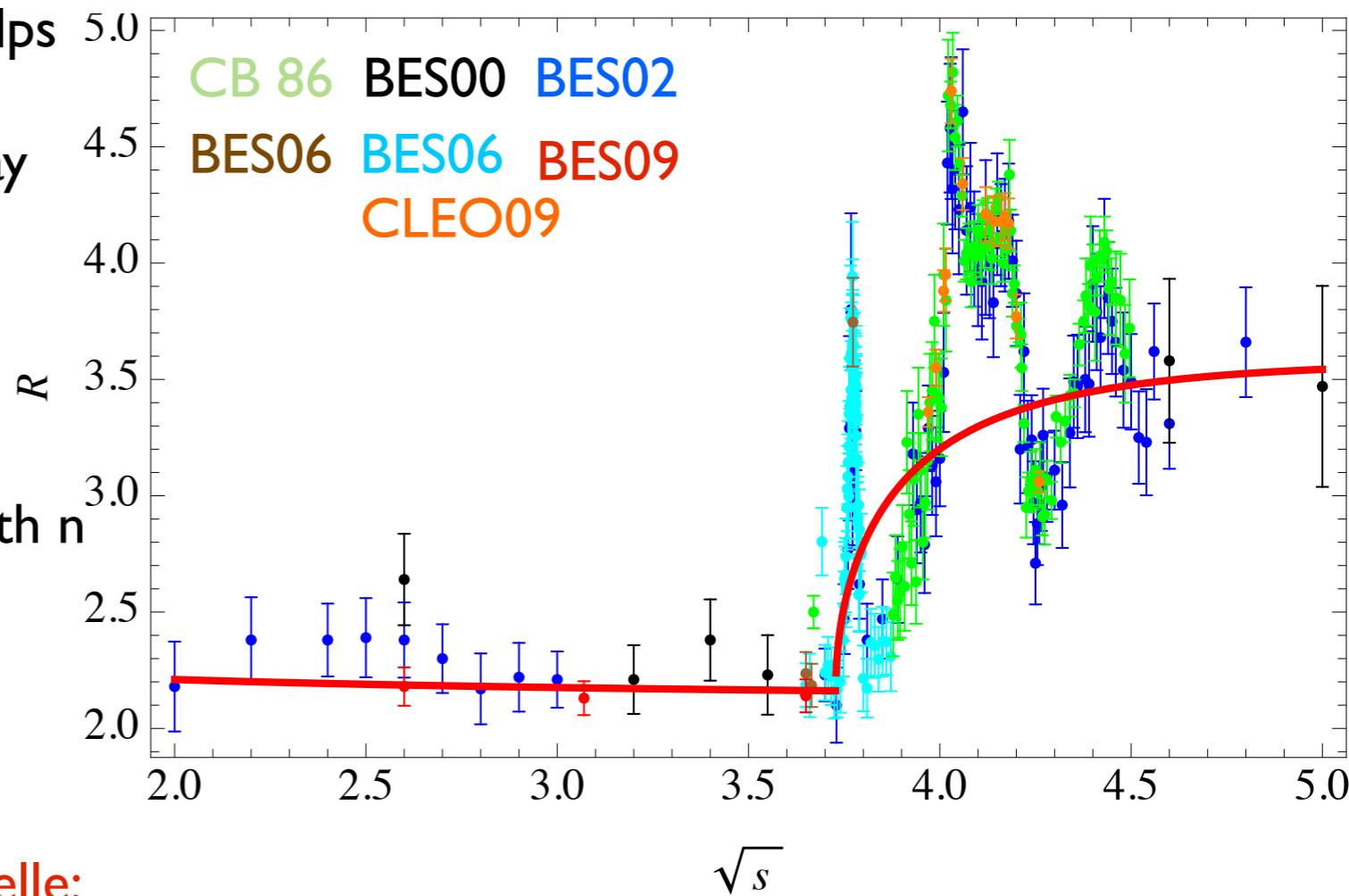
$$\overline{m}_b(\overline{m}_b) = 4.195(10)\text{GeV}$$

$\overline{m}_b(\overline{m}_b)$	method	reference
4174 ± 24	lattice ($N_f = 4$), PS current	HPQCD, 1408.4169
4201 ± 43	$N^3\text{LO}$ pQCD, M_Y	Ayala et al, 1407.2128
4169 ± 9	SR $\Upsilon(1S - 6S)$	Penin, Zerf, 1401.7035
4247 ± 34	SR, f_B	Lucha et al, 1305.7099
4166 ± 43	lattice + pQCD, M_Y , M_{B_s}	HPQCD, 1302.3739
4235 ± 55	SR $\Upsilon(1S - 6S)$, R	Hoang et al, 1209.0450
4171 ± 9	SR $\Upsilon(1S - 6S)$, R	Bodenstain et al, 1111.5742
4177 ± 11	SR $\Upsilon(1S - 6S)$	Narison, 1105.5070
4180 ± 50	lattice + pQCD in static potential	Laschka et al, 1102.0945
4163 ± 16	2nd moment SR $\Upsilon(1S - 6S)$, R	Chetyrkin et al, 1010.6157
4.180 ± 30	PDG average	PDG 2014

QCD Sum Rules

Our proposal: improvements

- Better measurements in subthreshold: helps to normalize the extrapolation
- Better measurements of the electric decay widths of the narrow states
- Smaller error on alpha (from exp.)
- Calculation of the coefficients C_n of the Taylor expansion up to higher orders in both n and alpha
- Study eventual Duality Violations?
- Smaller error on alpha (from theory)



Measurement of R for bottom region at Belle:
both sub- and above threshold

Conclusions and Outlook

- Heavy quark masses are interesting: for being fundamental parameters as well as for their implications on many phenomenological scenarios
- From the different strategies, one of the most precise is the use of SR
- Using SR with new features, we extract them: remark on improvements
 - Results still preliminary
 - Proposal: instead of averaging, take a range of values
- Error sources are understood: seems a clear roadmap for improvements

Thanks!