http://arxiv.org/pdf/1504.02228.pdf

Towards a space-like determination of the leading hadronic corrections to the muon g-2 ☆

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Abstract

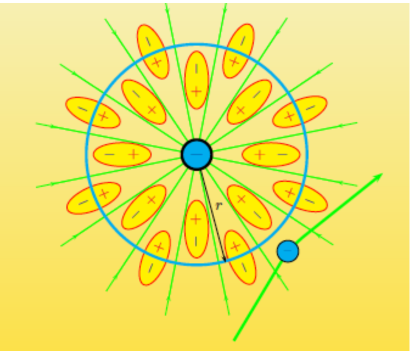
We propose a novel approach to determine the leading hadronic correction to the muon g-2 using measurements of the effective electromagnetic coupling in the space-like region extracted from Bhabha scattering data. Although challenging, we argue that this alternative method may become feasible at flavor factories and possibly competitive with the accuracy of the present determinations obtained with the dispersive approach via time-like data.

17th RadioMonteCarLOW WG – Frascati 21/4/15

Talk dedicated to the memory of Prof. Eduard Kuraev

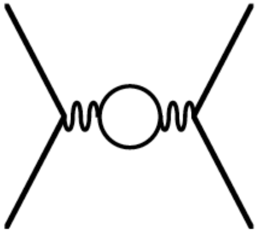
α_{em} running and the Vacuum Polarization

- Due to Vacuum Polarization effects $\alpha_{em}(q^2)$ is a running parameter from its value at vanishing momentum transfer to the effective q^2.



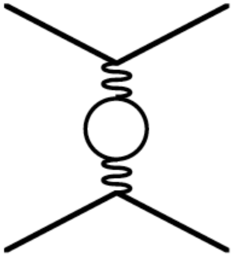
➢ The "Vacuum Polarization" function $\Pi(q^2)$ can be "absorbed" in a redefinition of an effective charge:

$$e^2 \rightarrow e^2(q^2) = \frac{e^2}{1+(\Pi(q^2)-\Pi(0))} \qquad \alpha(q^2) = \frac{\alpha(0)}{1-\Delta\alpha}; \quad \Delta\alpha = -\Re e\left(\Pi(q^2)-\Pi(0)\right)$$

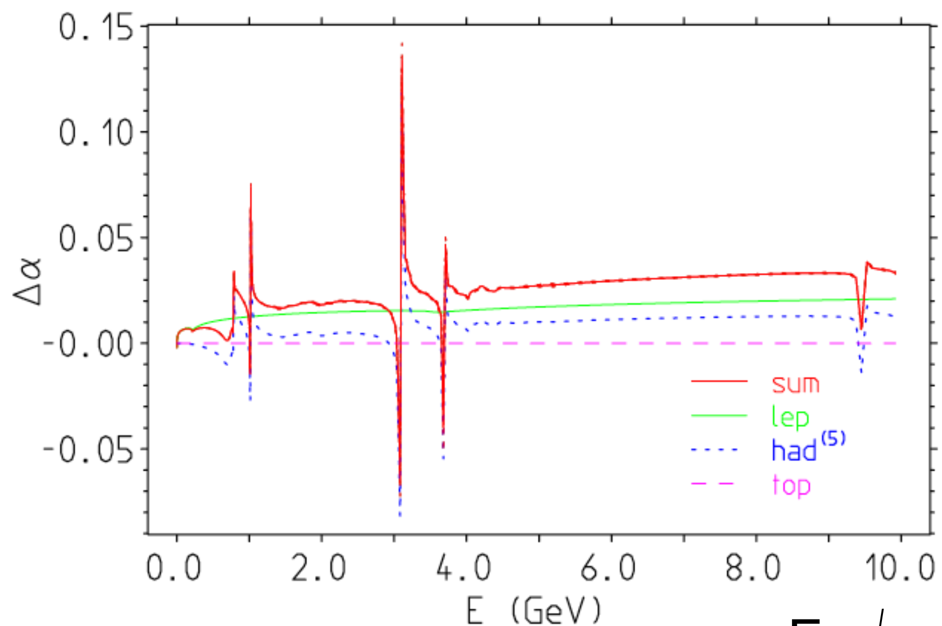


$$\Delta\alpha = \Delta\alpha_l + \Delta\alpha^{(5)}_{had} + \Delta\alpha_{top}$$

➢ $\Delta\alpha$ takes a contribution by non perturbative hadronic effects ($\Delta\alpha^{(5)}_{had}$) which exibits a different behaviour in time-like and spacelike region



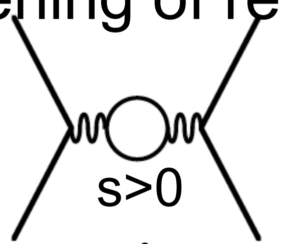
Running of α_{em}



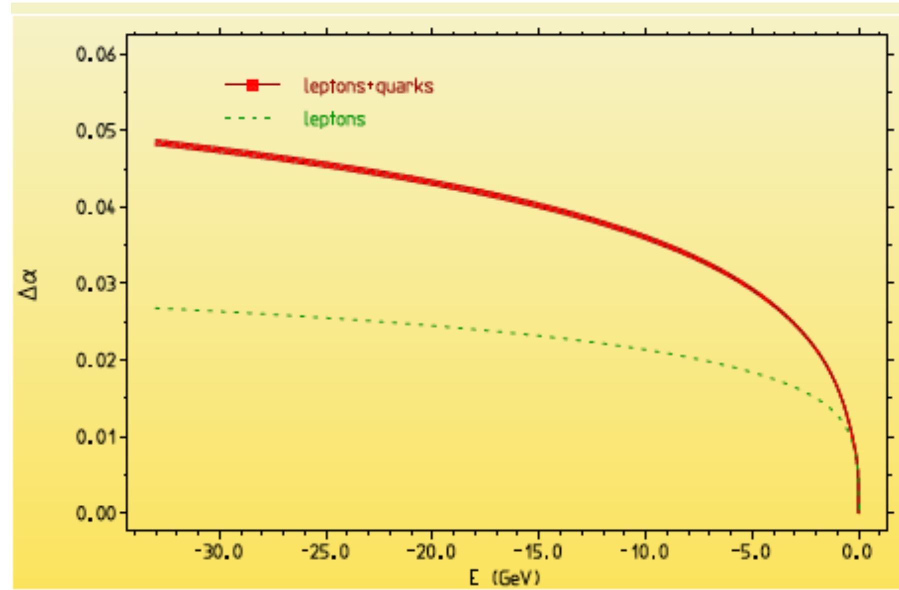
E=√s

Time-like

Behaviour characterized by the opening of resonances



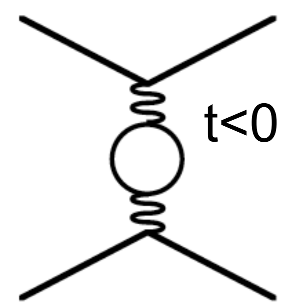
$$\Delta\alpha^{(5)}_{had}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s-M_Z^2-i\varepsilon)}$$



E=-√-t

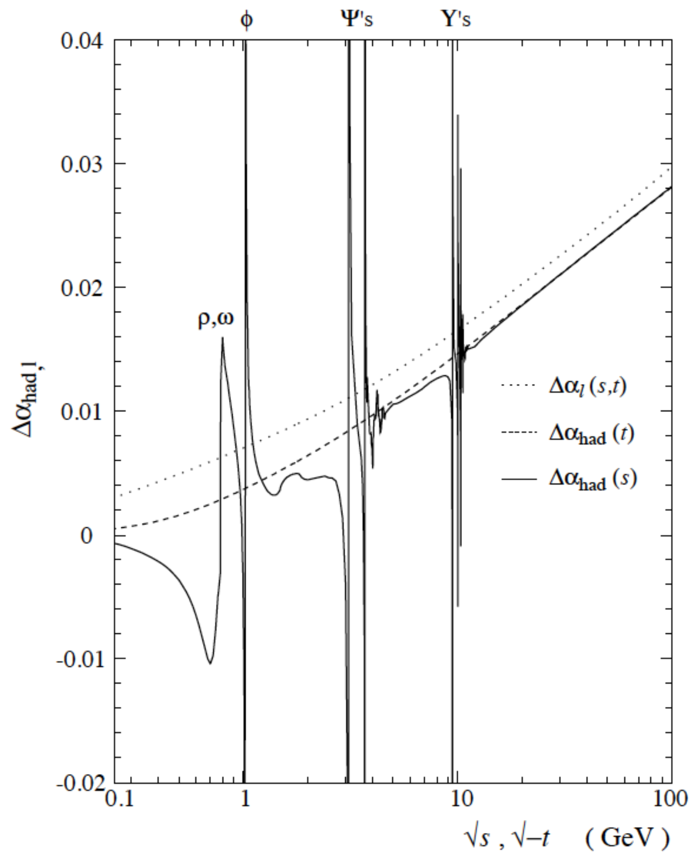
Space-like

Very smooth behaviour



Measurement of α_{em} running

- A direct measurement of $\alpha_{em}(q^2)$ in space/time like region can prove the running of α_{em}
- It can provide a test of "duality" (fare way from resonances)
- It has been done in past by few experiments at e^+e^- colliders by comparing a "well-known" QED process with some reference (obtained from data or MC)

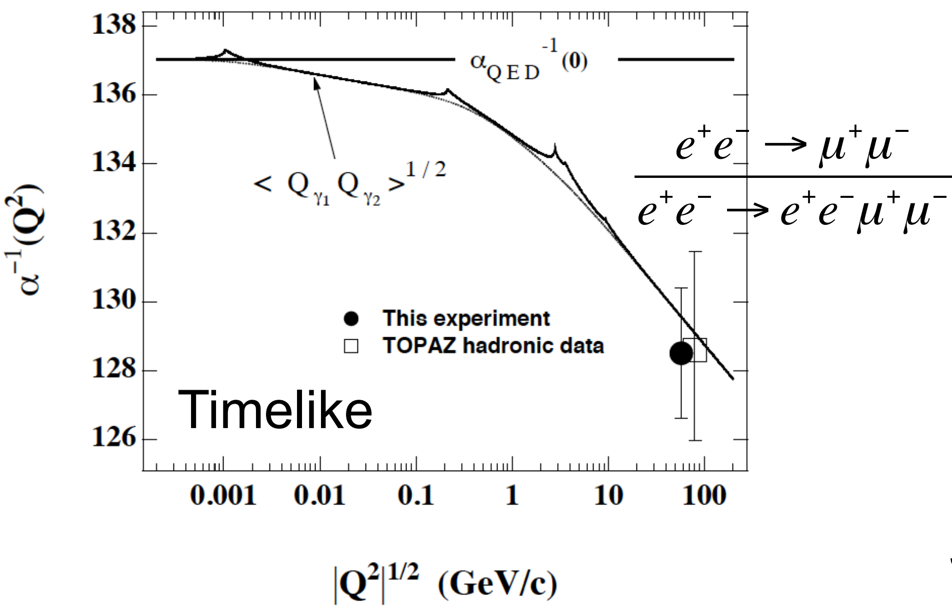


$$\left(\frac{\alpha(q^2)}{\alpha(q_0^2)}\right)^2 \sim \frac{N_{signal}(q^2)}{N_{norm}(q_0^2)}$$

N_{signal} can be Bhabha process, muon pairs, etc…
N_{signal} can be Bhabha process, $\gamma\gamma$ pairs, Theory, etc…

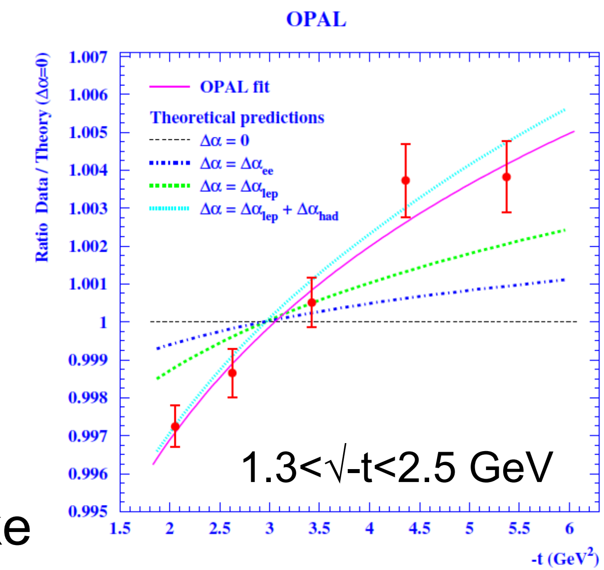
Measurement of α_{em} running

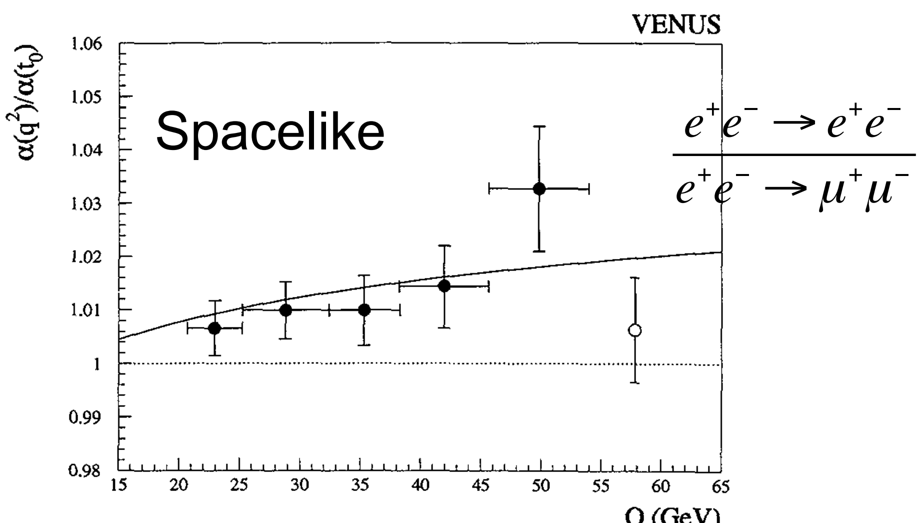
e+e- collider TRISTAN at √s=57.8 GeV,



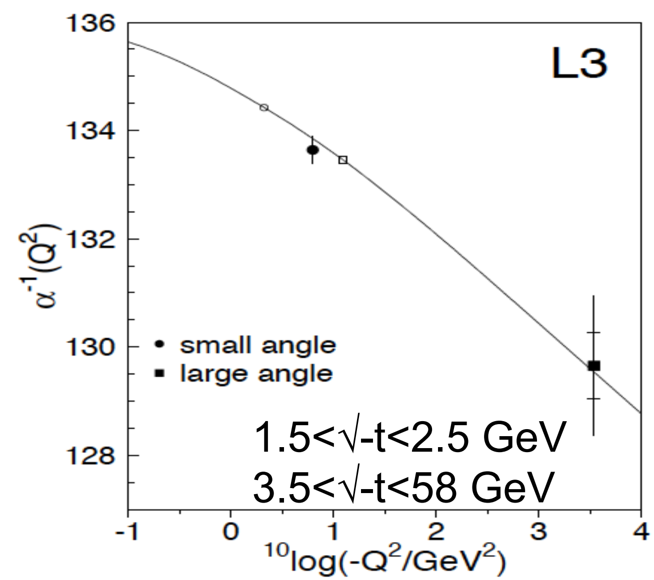
e+e- collider LEP at √s=189 GeV, using **Bhabha** events

Spacelike



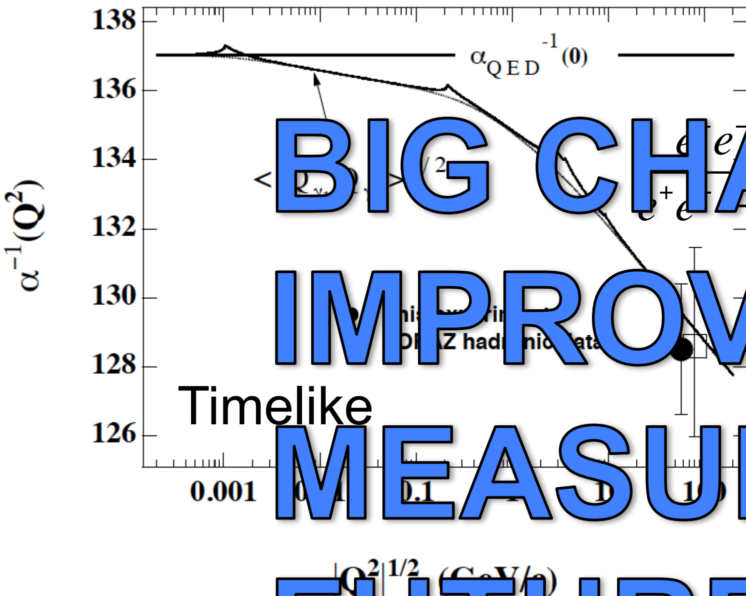


10<√-t<54 GeV

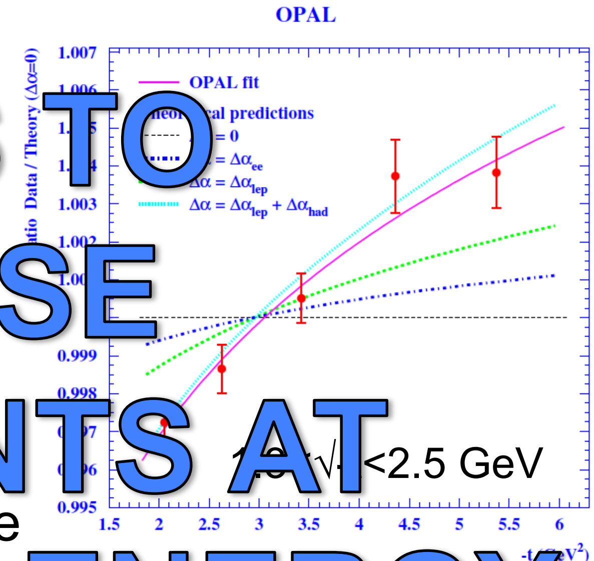


Measurement of α_{em} running

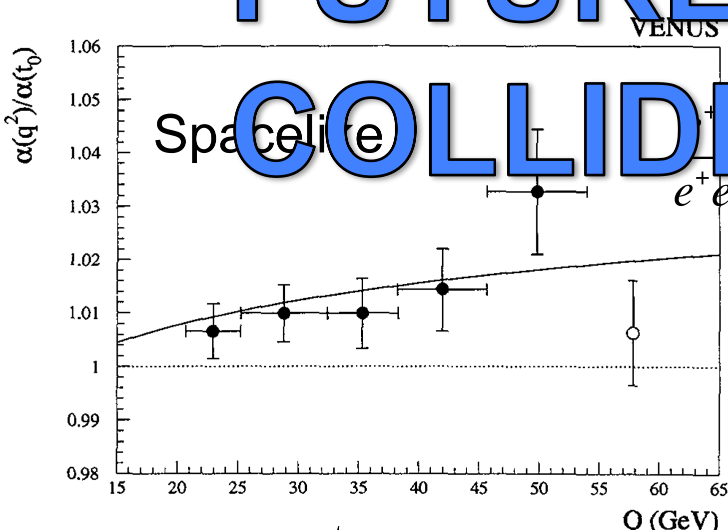
e+e- collider TRISTAN at √s=57.8 GeV,



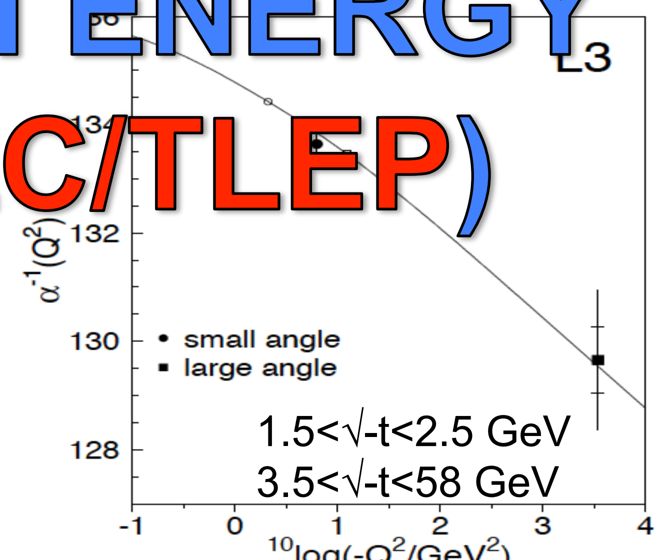
e+e- collider LEP at √s=189 GeV, using **Bhabha** events



BIG CHANCES TO IMPROVE THESE MEASUREMENTS AT FUTURE HIGH ENERGY COLLIDER (ILC/TLEP)



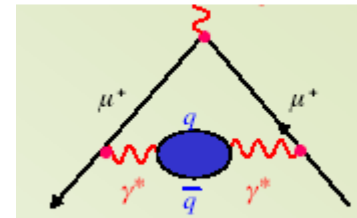
10<√-t<54 GeV



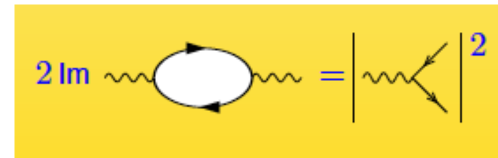
a_μ^{HLO} calculation, traditional way: time-like data

$a_\mu = (g-2)/2$

$$\boxed{a_\mu^{HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} \sigma_{e^+e^-\to hadr}(s) K(s)\, ds}$$



$$a_\mu^{HLO.} = \frac{\alpha}{\pi^2}\int_0^\infty \frac{ds}{s} K(s)\,\mathrm{Im}\,\Pi_{had}(s) \quad \sigma_{e^+e^-\to hadr}(s) = \frac{4\pi}{s}\mathrm{Im}\,\Pi_{had}(s)$$



$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \sim \frac{1}{s}$$

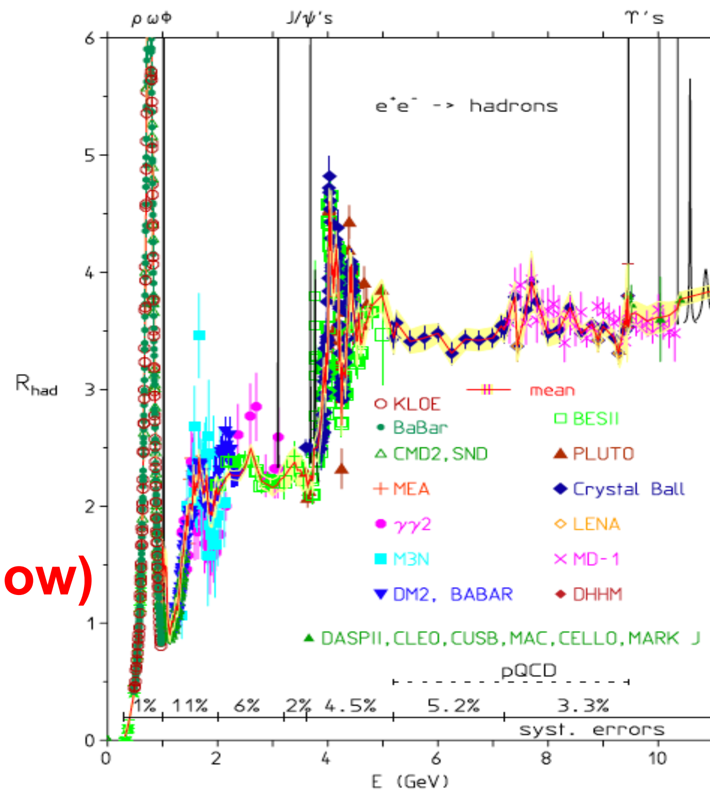
Traditional way: based on precise experimental (time-like) data:

$$a_\mu^{had} = (689.7 \pm 4.4)\cdot 10^{-10}$$

Main contribution in the low energy region

$\delta a_\mu^{exp} \to$ 1.5 10^{-10} = 0.2% on a_μ^{HLO} (from 0.7% now)

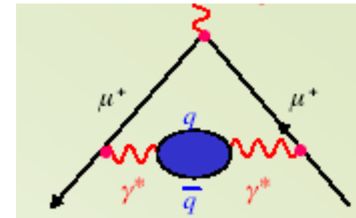
NEW G-2 at FNAL and JPARC



a_μ^{HLO} evaluation in spacelike region: alternative approach

$a_\mu=(g-2)/2$

$$\boxed{a_\mu^{HLO} = -\frac{\alpha}{\pi}\int_0^1 (1-x)\Pi_{had}(-\frac{x^2}{1-x}m_\mu^2)dx}$$



x =Feynman parameter

$$t = \frac{x^2 m_\mu^2}{x-1} \qquad 0 \le -t < +\infty$$

t<0

$$x = \frac{t}{2m_\mu^2}(1-\sqrt{1-\frac{4m_\mu^2}{t}}); \qquad 0 \le x < 1;$$

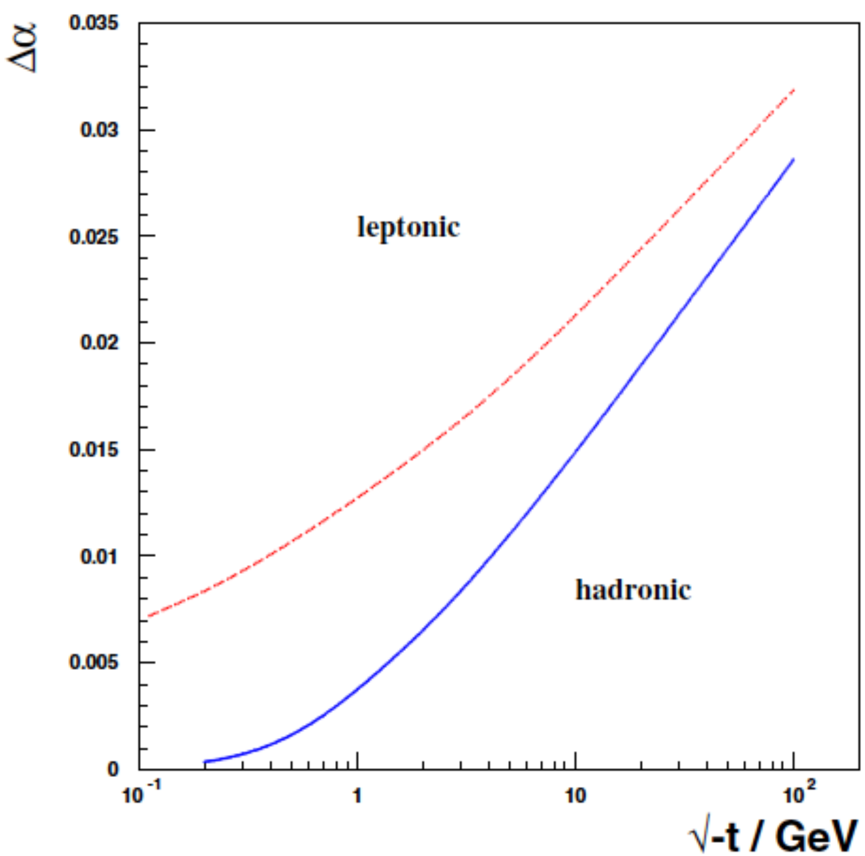
$$t = -s\sin^2(\frac{\vartheta}{2})$$

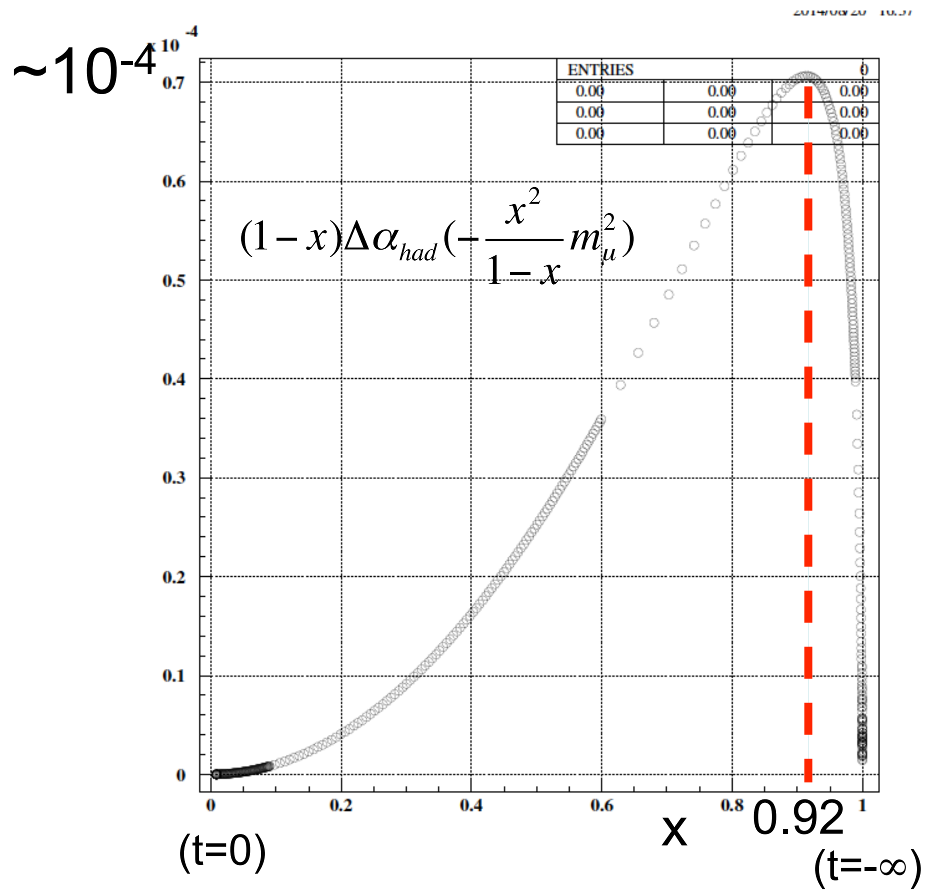
$$\Delta\alpha_{had}(t) = -\Pi_{had}(t) \quad for\ t<0$$

$$\boxed{a_\mu^{HLO} = -\frac{\alpha}{\pi}\int_0^1 (1-x)\Delta\alpha_{had}(-\frac{x^2}{1-x}m_\mu^2)dx}$$

For t<0

Behaviors





$\Delta\alpha$~log(-t)
Dominated at low |t| by leptonic contribution

A. Arbuzov et al., Eur. Phys. J. C 34 (2004) 267

High |t|-values are depressed by 1-x
(a kind of analogy with time-like region)
The integrand is peaked at ~x=0.92
→t=-0.11 GeV^2 (~0.33 GeV) for which
$\Delta\alpha_{had}$(0.92)~ 10^{-3}

Experimental considerations

Using Bhabha at small angle (to emphasize t-channel contribution) to extract $\Delta\alpha$:

$$\left(\frac{\alpha(t)}{\alpha(0)}\right)^2 \sim \frac{d\sigma_{ee\to ee}(t)}{d\sigma^0_{MC}(t)}$$

Where $d\sigma^0_{MC}$ is the MC prediction for Babha process with α(t)=α(0), and there are corrections due to RC…

$$\Delta\alpha_{had}(t) = 1 - \left(\frac{\alpha(t)}{\alpha(0)}\right)^{-1} - \Delta\alpha_{lept}(t)$$

$\Delta\alpha_{lep}(t)$ theoretically well known!

Which experimental accuracy we are aiming at?
$\delta\Delta\alpha_{had}$~1/2 fractional accuracy on $d\sigma(t)/d\sigma^0_{MC}(t)$.

If we assume to measure $\delta\Delta\alpha_{had}$ at 5% at the peak of the integrand ($\Delta\alpha_{had}$ ~10^{-3} at x=0.92) → fractional accuracy on $d\sigma(t)/d\sigma^0_{MC}(t)$ ~ 10^{-4} !

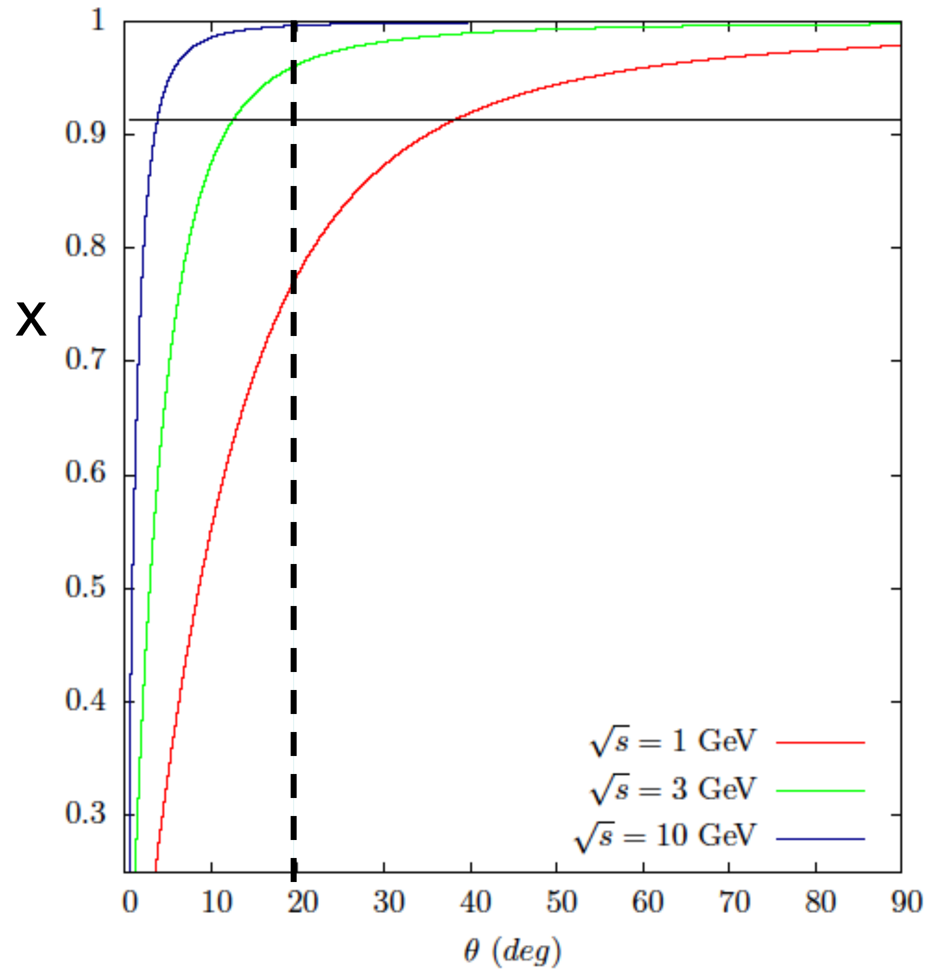
Very challenging measurement (one order of magnitude improvement respect to date) for systematic error

Experimental considerations - II

Most of the region (up to x~0.95) can be covered with a low energy machine (like Dafne/VEPP2000 or tau/charm-B-factories)

Example:
a detector at 20^0 with Ebeam=1 GeV can arrive at x= 0.7. For higher x the angle must be increased (s-channel contribution).

A better situation can be obtained at tau/charm/ B-factories where smaller angles (few degrees) can be required

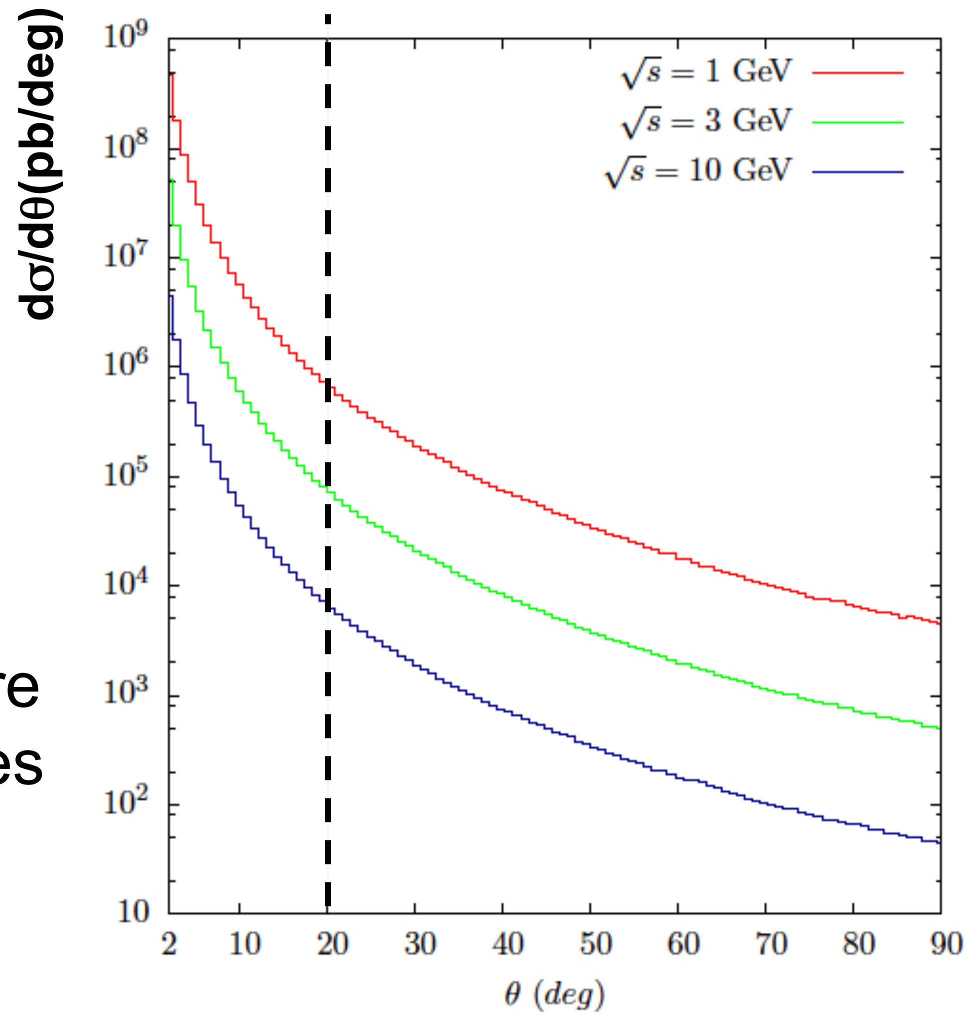


$$t = -s\sin^2(\frac{\vartheta}{2})$$

Statistical consideration

10^{-4} accuracy on Bhabha cross section requires at least 10^8 events which at 20^o mean at least:

- $100 pb^{-1}$ @ 1 GeV
- $1\ fb^{-1}$ @ 3 GeV
- $10\ fb^{-1}$ @ 10 GeV



These luminosities are within reach at flavour factories, where expected integrated luminosities are O(100) of what is required

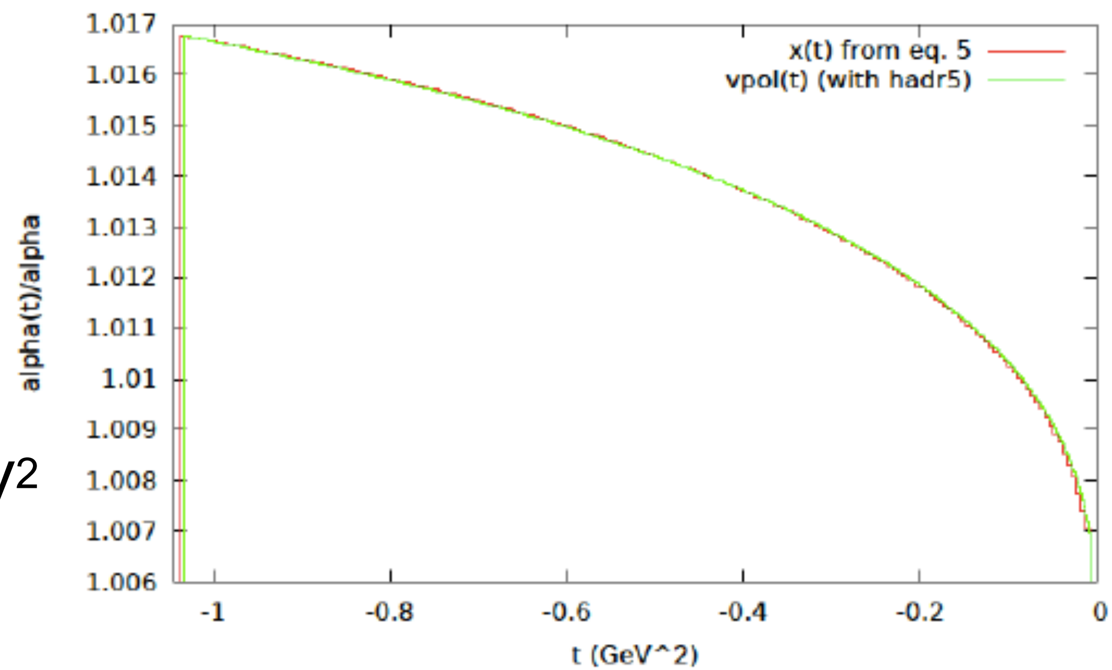
Additional considerations: s-channel

At low energy (<10 GeV) above 10^0 there is still a sizeable contribution from s-channel.
At LO no difficulty to deconvolute the cross section for the s-channel

Test with Babayaga:

s=1 GeV
$10^o<\theta<170^o$

$d\sigma_{born}/dt=1.52$ mb/GeV^2



However this picture changes with Rad. Corr.

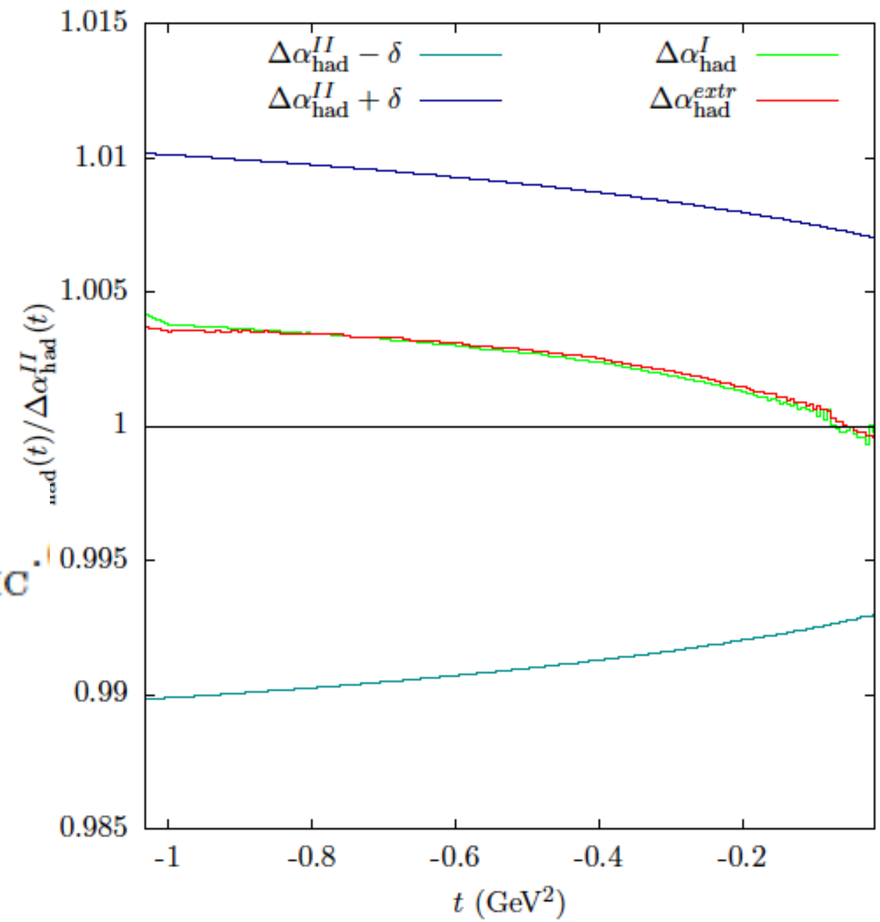
Additional considerations: Rad. Corr.

A Monte Carlo procedure has been developed to check if $\Delta\alpha_{had}(t)$ can be obtained by a minimization procedure with a different $\Delta\alpha_{had}(t)'$ inside

$$\left.\frac{d\sigma}{dt}\right|_{\text{data}} = \left.\frac{d\sigma}{dt}\Big(\alpha(t),\alpha(s)\Big)\right|_{\text{MC}},$$

$\rightarrow$

$$\left.\frac{d\sigma}{dt}\right|_{j,\text{data}} = \left.\frac{d\sigma}{dt}\left(\bar{\alpha}(t)+\frac{i_j}{N}\delta(t),\alpha(s)\right)\right|_{j,\text{MC}}.$$



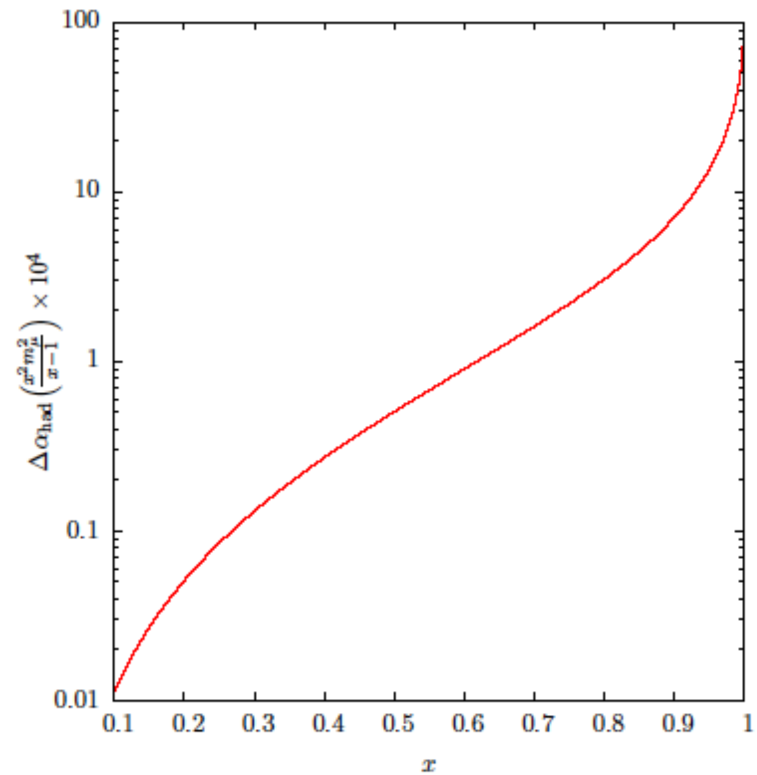
$\Delta\alpha_{had}(t)$ is obtained with$<10^{-4}$ error !

Additional consideration: Normalization

To compare Bhabha absolute cross section from data with MC we need Luminosity of the machine.

Two possibilities:

1) Use Bhabha at very small angle where the uncertainty on $\Delta\alpha_{had}$ can be neglected (for example at E_{beam}=1 GeV and $\theta=5^o$, $\Delta\alpha_{had} \sim 10^{-5}$).
2) Use a process with $\Delta\alpha_{had}=0$, like $e^+e^- \rightarrow \gamma\gamma$. However very difficult to determine it at 10^{-4} accuracy.



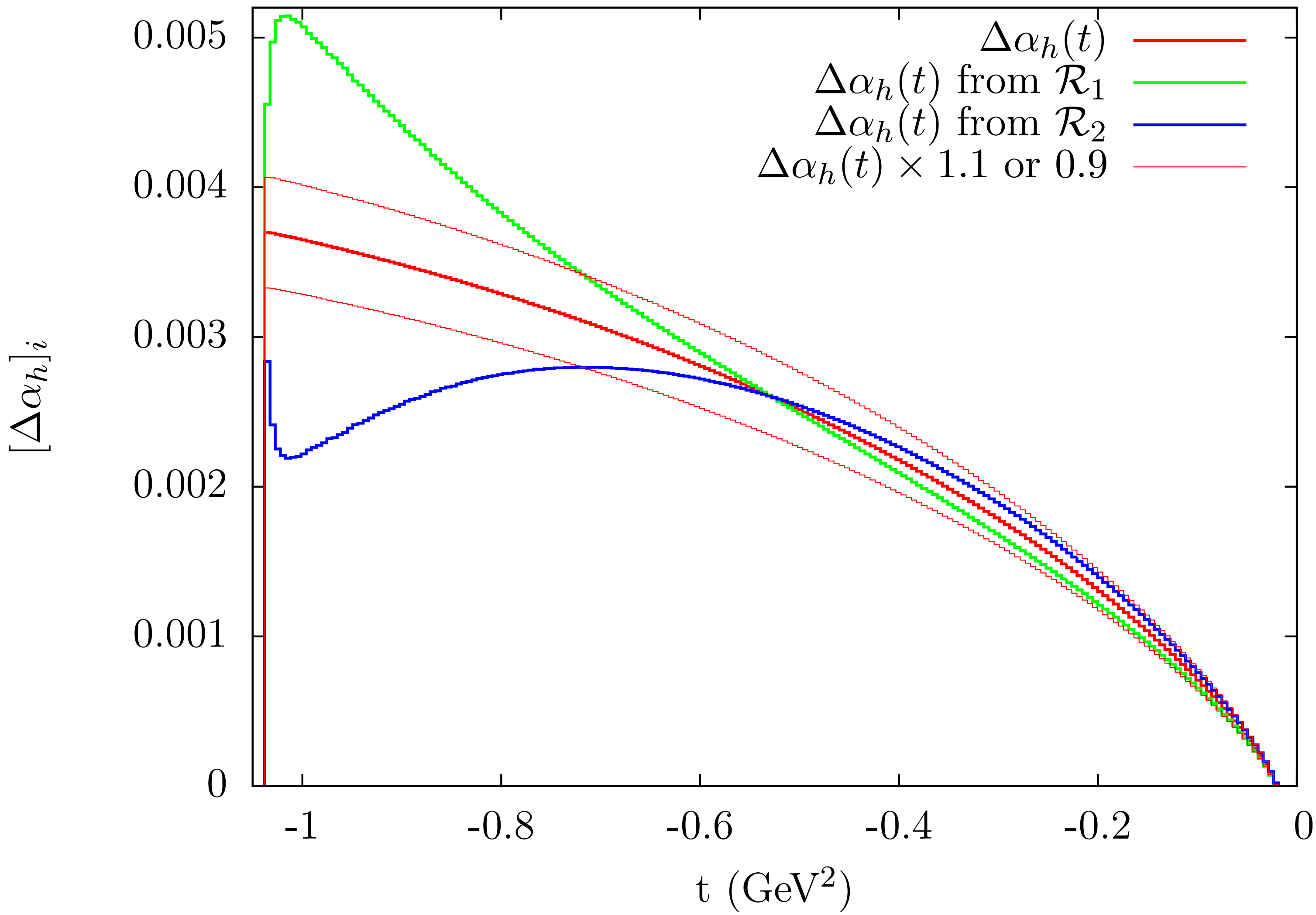
Option 1) looks better to us as some of the common systematics cancel in the measurement !

Conclusions

- Measuring α_{em} running in time-like and space like region appears to be very interesting. (Relatively) high q^2-values can be explored at ILC/TLEP
- An alternative formula for a_μ^{HLO} in spacelike region has been studied in details. It emphasizes low values of t (<1 GeV^2) and can be explored at low energy e+e- machines (VEPP2000/ DAFNE, τ/charm, B-factories)
- It requires to measure the Bhabha cross section at relatively small angles at (better than) 10^{-4} accuracy!
- Reaching such an accuracy demands a dedicated work on theory and detector for the next few years…our WG can give an important contribution on that!!!

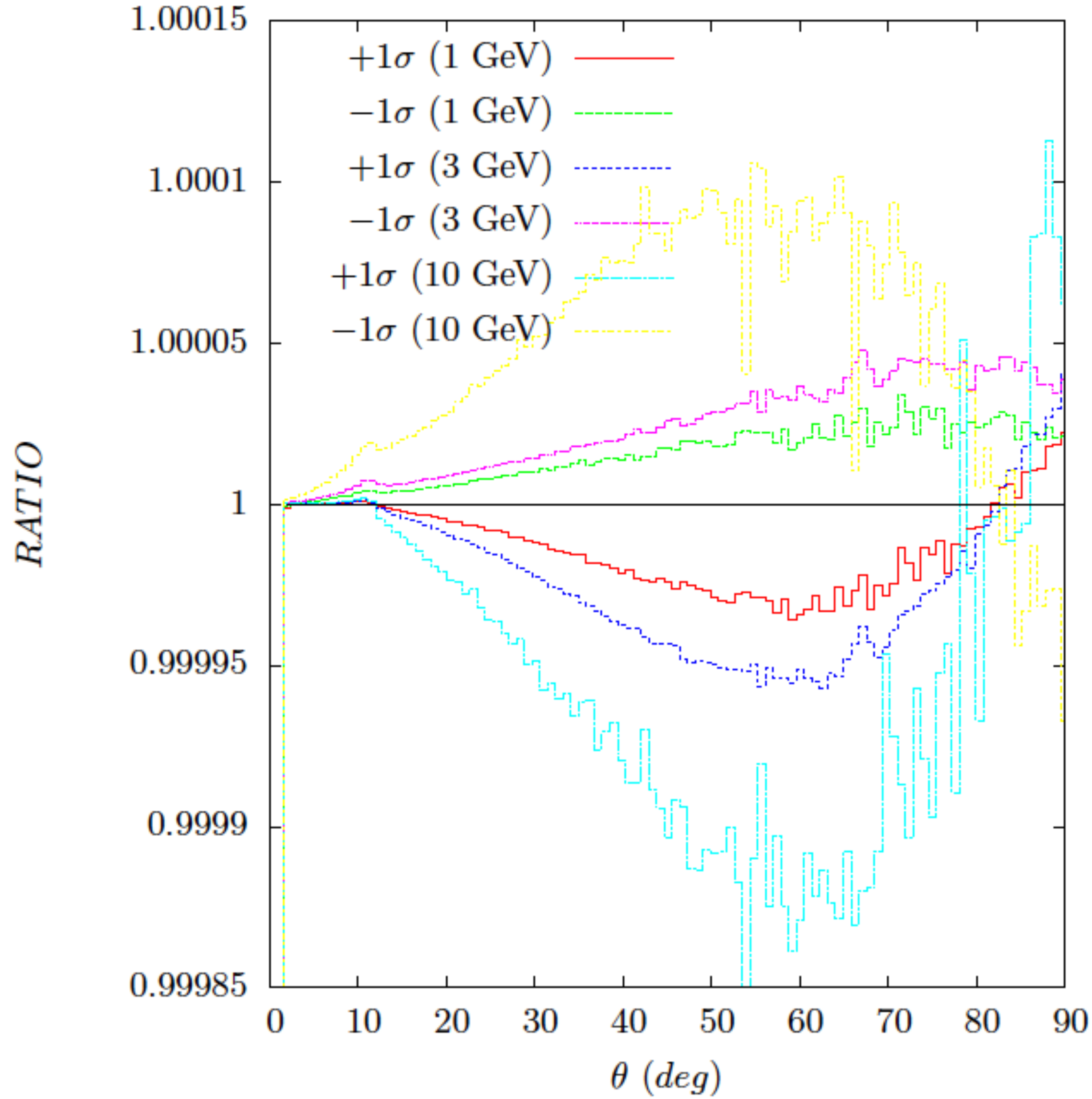
END

test



$\Delta\alpha_h(t)$
$\Delta\alpha_h(t)$ from $\mathcal{R}_1$
$\Delta\alpha_h(t)$ from $\mathcal{R}_2$
$\Delta\alpha_h(t) \times 1.1$ or 0.9
0.005
0.004
0.003
0.002
0.001
0
$[\Delta\alpha_h]_i$
-1
-0.8
-0.6
-0.4
-0.2
0
t (GeV^2)

$\Delta\alpha_{em}^{HAD}(s)$ dependence



Which is the best energy/angle configuration?

$$-t = 9(1-\cos\theta)/2$$

$$x = \frac{t}{2m_{\mu}^{2}}\left(1-\sqrt{1-\frac{4m^{2}}{t}}\right)$$