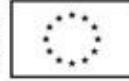




INNOWACYJNA
GOSPODARKA
NARODOWA STRATEGIA SPÓŁNOŚCI

Dotacje na innowacje
Inwestujemy w waszą przyszłość

UNIA EUROPEJSKA
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Fundacja na rzecz Nauki Polskiej

Current status of two and three pion decay modes within TAUOLA

O. Shekhovtsova
INP Cracow / KIPT Kharkov

Frascati, 21.04.2015

Two pion decay modes: $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$

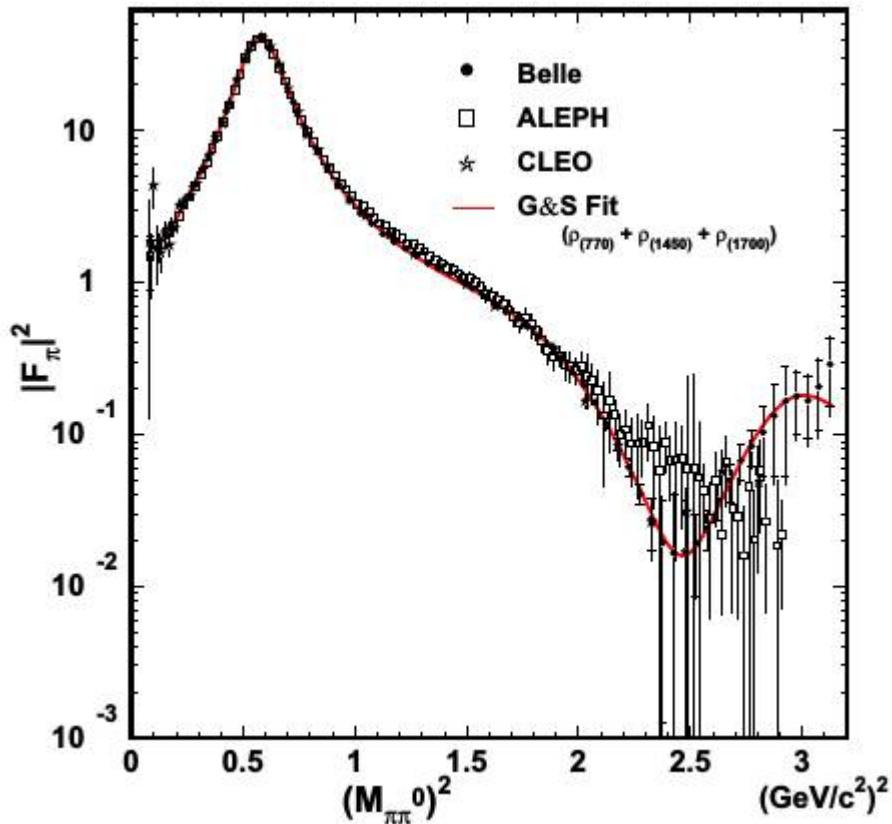
Br 25.52%

TAUOLA two pion FF:

- * Belle
- * RChL Eur.Phys.J.C27 (2003) 587
- * dispersive integral + modified high energy RChL

Two pion decay modes: $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$

Belle data: Phys Rev D 78 (2008) 072006



$$F_\pi(s) = \frac{1}{1 + \beta + \gamma} (\text{BW}_\rho + \beta \cdot \text{BW}_{\rho'} + \gamma \cdot \text{BW}_{\rho''})$$

$$\text{BW}_i^{\text{GS}} = \frac{M_i^2 + d \cdot M_i \Gamma_i(s)}{(M_i^2 - s) + f(s) - i\sqrt{s} \Gamma_i(s)}$$

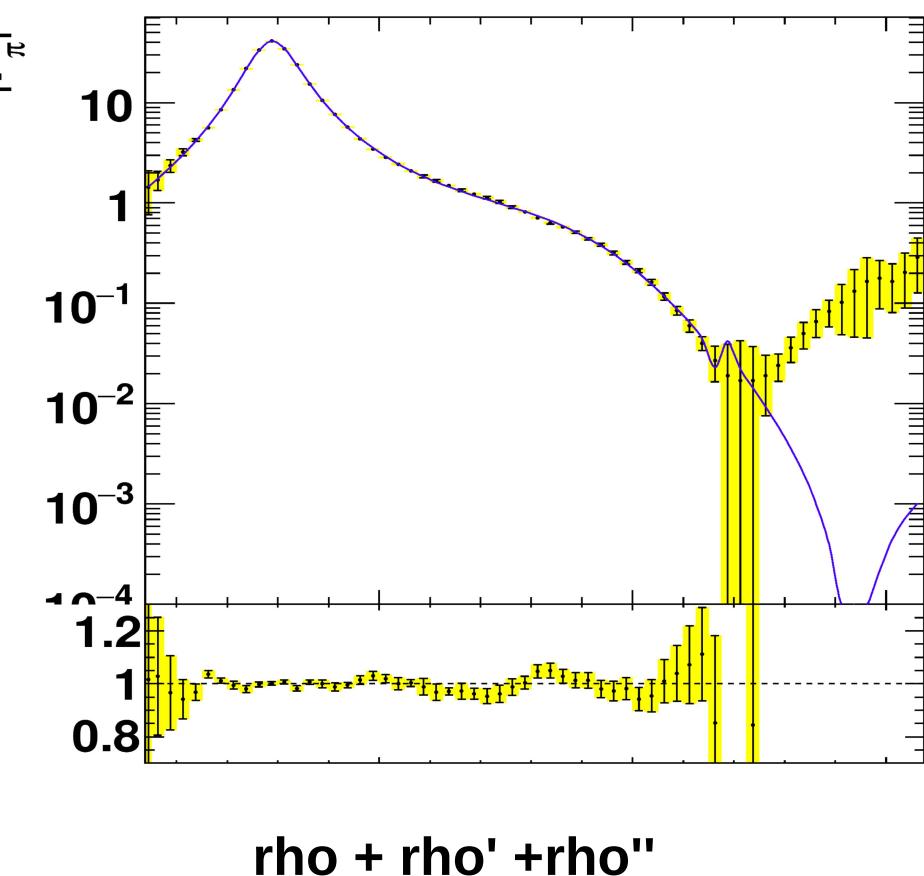
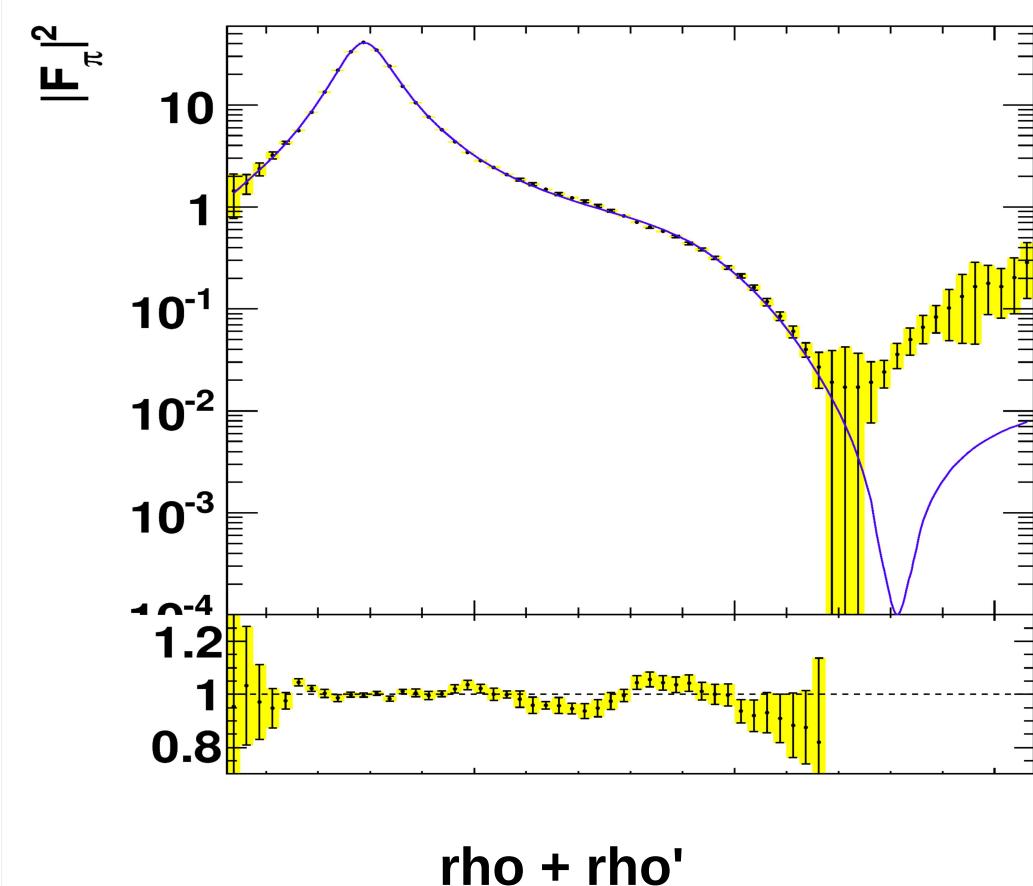
* 2 pion loop

** d : $\text{BW}(0) = 1$

$\sim \text{const} \cdot s^{3/2}$ no ChPT ($\sim s$)

*** complex constant β and γ

$$F_\pi(s) = \frac{1 + \sum_i \frac{F_{V_i} G_{V_i}}{f^2} \frac{q^2}{M_{V_i}^2 - q^2}}{1 + \left(1 + \sum_i \frac{2G_{V_i}^2}{f^2} \frac{q^2}{M_{V_i}^2 - q^2}\right) \frac{2q^2}{f^2} \left[B_{22}^{r,(\pi)} + \frac{1}{2} B_{22}^{r,(K)}\right]}$$



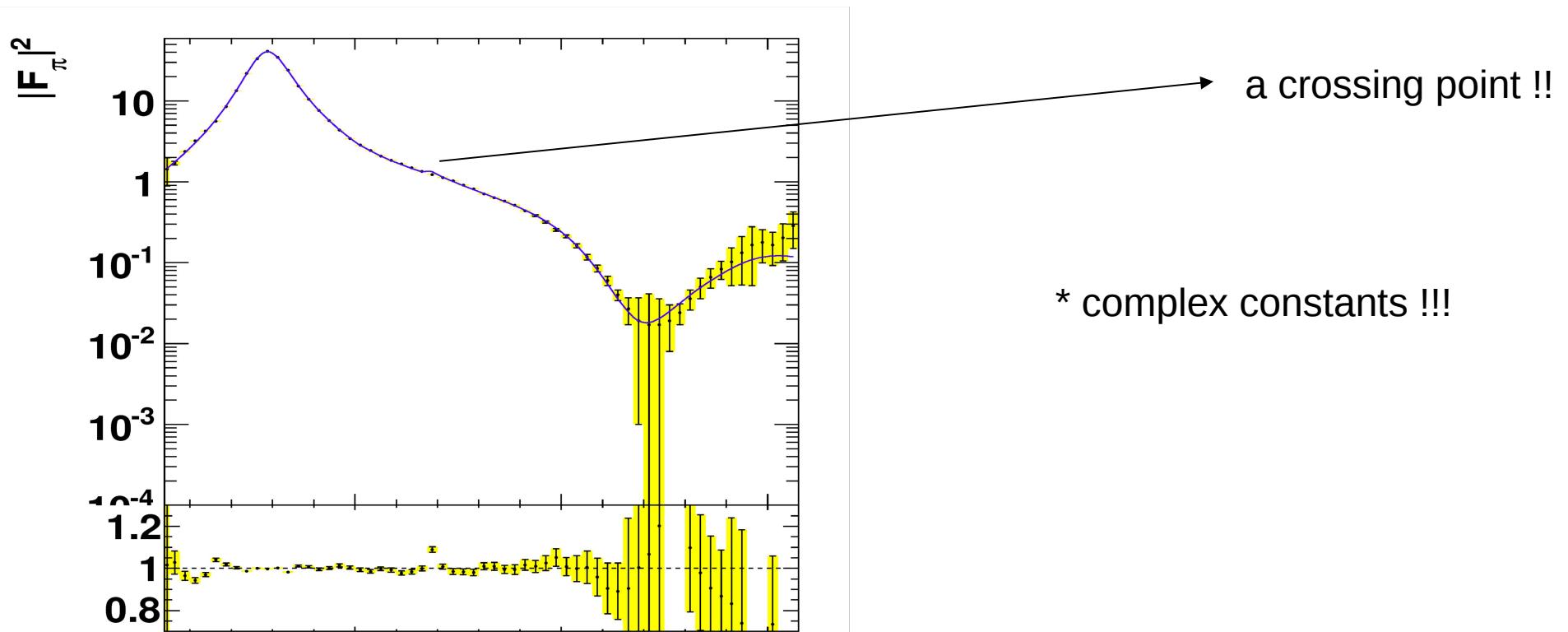
Dispersive integral + modified high energy RChL

D. Gomez-Dumm, P. Roig Eur. Phys. J 73 (2013) 2528

$$F_V^\pi(s) = \exp \left[\alpha_1 s + \frac{\alpha_2}{2} s^2 + \frac{s^3}{\pi} \int_{s_{\text{thr}}}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3 (s' - s - i\epsilon)} \right] \quad \text{low energy}$$

$$F_V^\pi(s) = \frac{M_\rho^2 + (\alpha' e^{i\phi'} + \alpha'' e^{i\phi''}) s}{M_\rho^2 \left[1 + \frac{s}{96\pi^2 F_\pi^2} (A_\pi(s) + \frac{1}{2} A_K(s)) \right] - s} \quad \text{high energy}$$

$$- \frac{\alpha' e^{i\phi'} s}{M_{\rho'}^2 [1 + s C_{\rho'} A_\pi(s)] - s} - \frac{\alpha'' e^{i\phi''} s}{M_{\rho''}^2 [1 + s C_{\rho''} A_\pi(s)] - s}$$



Conclusions/plans for two pions

- * fit with the Belle covariance matrix, to include bin-to-bin correlation
- * kaon loop influence on the Belle parametrization
 - ** ~2% at the rho peak for the RChL parametrization
- * several-pion/kaon loops ($\omega \pi$, $K^* K$)
 - ** *Portoles, J. et al. Nucl.Phys.Proc.Suppl. 131 (2004) 170*

Three pion decay modes

Br 18%

Current status Tauola

* Cleo parametrization

$\tau^- \rightarrow \pi^0 \pi^0 \pi^- \nu_\tau$ D. Asner et al., Phys.Rev. D61 (2000) 012002, hep-ex/9902022

$\tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau$ E. I. Shibata, Nucl.Phys.Proc.Suppl.123 (2003)40 ,
hep-ex/0210039

J.W. Hinson, PhD thesis, Purdue University (2001), PU-99-713

* RChL parametrization V + A contribution (Phys.Rev. D86 (2012) 113008)

B-factories:

- * BaBar a simple model that includes only a1(1260), rho (rho') preliminary data, 2- and 3- pion invariant mass distributions
- * Belle -----> ???

Comparison with Pythia8 physics

Cleo parametrization

$\tau^- \rightarrow \pi^0 \pi^0 \pi^- \nu_\tau$ D. Asner et al., Phys.Rev. D61 (2000) 012002, hep-ex/9902022

Dalitz plots distributions

		Significance	Branching fraction (%)	$ \beta $	phase φ/π
a1(1200) \rightarrow	$\rho\pi$	<i>S</i> -wave	—	68.11	1.00
	$\rho(1450)\pi$	<i>S</i> -wave	1.4σ	0.30 ± 0.64	0.12 ± 0.09
	$\rho\pi$	<i>D</i> -wave	5.0σ	0.36 ± 0.17	0.37 ± 0.09
	$\rho(1450)\pi$	<i>D</i> -wave	3.1σ	0.43 ± 0.28	0.87 ± 0.29
	$f_2(1270)\pi$	<i>P</i> -wave	4.2σ	0.14 ± 0.06	0.71 ± 0.16
	$f_0(600)\pi$	<i>P</i> -wave	8.2σ	16.18 ± 3.85	2.10 ± 0.27
	$f_0(1370)\pi$	<i>P</i> -wave	5.4σ	4.29 ± 2.29	0.77 ± 0.14

$$B_Y^L(s_i) = \frac{m_{0Y}^2}{(m_{0Y}^2 - s_i) - im_{0Y}\Gamma^{Y,L}(s_i)} \quad \Gamma^{Y,L}(s_i) = \Gamma_0^Y \left(\frac{k'_i}{k'_0}\right)^{2L+1} \frac{m_{0Y}}{\sqrt{s_i}}$$

* fitted the beta constants + a1 mass and width

* instead o of the mass (555 MeV) and width (540 MeV) of sigma were fitted, the default version fixes sigma mass = 860 MeV and width = 880 MeV

* mass and width of the resonances were fixed to PDG'98

$\tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau$ is confused; Tauola \leftarrow Cleo parametrization of 2002;
 Pythia8 to PhD thesis parameters from $\pi^0 \pi^0 \pi^-$
No comparison with BaBar data

Resonance Chiral Theory results for three pion decay modes

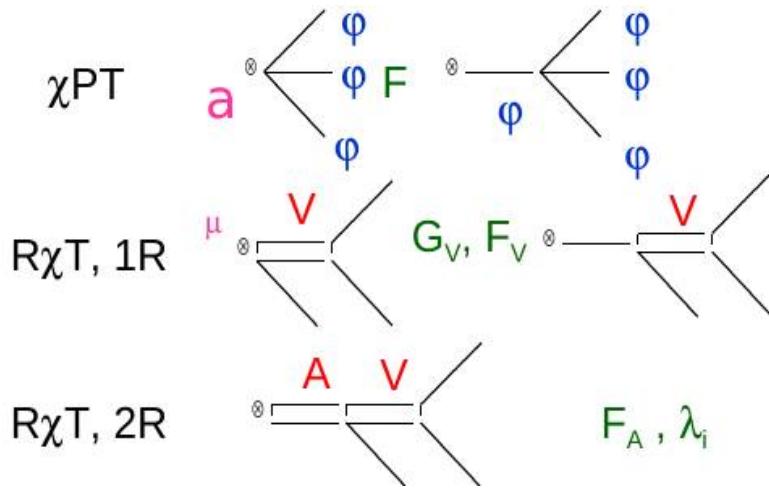
RChL = ChPL + resonances (V, A, S, P) as new active degree of freedom

Phys.Rev. D86 (2012) 113008

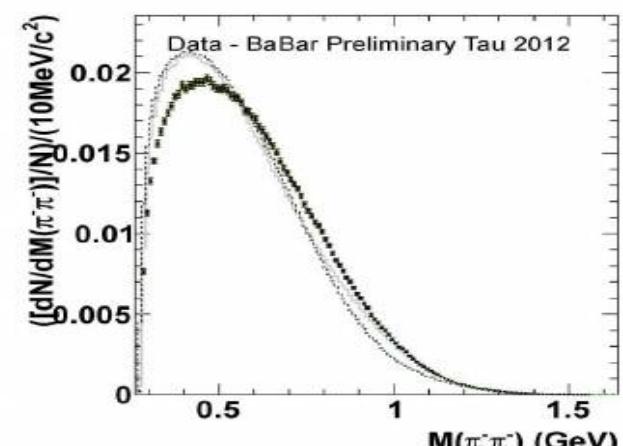
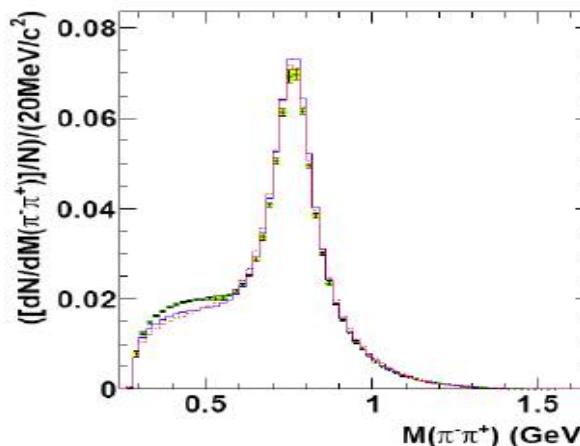
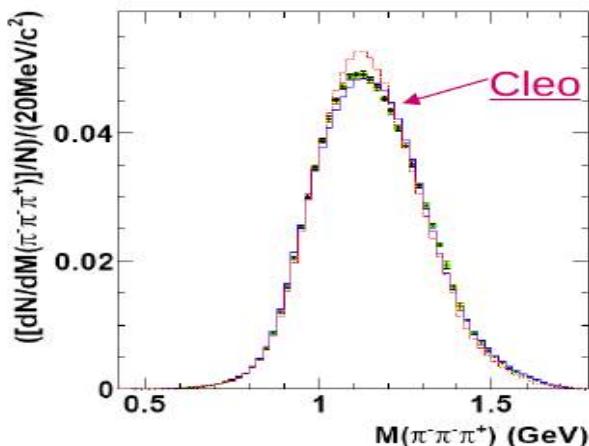
Phys. Rev. D 88, 093012 (2013)

only V, A resonances

$$J^\mu = N \left\{ T_v^\mu [c_1(p_2-p_3)^v F_1 + c_2(p_3-p_1)^v F_2 + c_3(p_1-p_2)^v F_3] + c_4 q^v F_4 - \frac{i}{4\pi^2 F^2} c_5 \epsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma} F_5 \right\}$$



$F_2(Q^2, s_2, s_1) = -F_1(Q^2, s_1, s_2)$
the same for both 3 pion modes
 $F_4 \sim m_\pi^2 / q^2$ is different



Doubts about inclusion of $f_0(500) = \sigma$ in RChT scheme

Cleo inspired contribution + RChT structure of FF

$\pi^- \pi^- \pi^+$

$$F_1^R \rightarrow F_1^R + \frac{\sqrt{2}F_V G_V}{3F^2} [\alpha_\sigma BW_\sigma(s_1)F_\sigma(q^2, s_1) + \beta_\sigma BW_\sigma(s_2)F_\sigma(q^2, s_2)]$$

$$F_1^{RR} \rightarrow F_1^{RR} + \frac{4F_A G_V}{3F^2} \frac{q^2}{q^2 - M_{a1}^2 - iM_{a1}\Gamma_{a1}(q^2)} [\gamma_\sigma BW_\sigma(s_1)F_\sigma(q^2, s_1) + \delta_\sigma BW_\sigma(s_2)F_\sigma(q^2, s_2)]$$

$$BW_\sigma(x) = \frac{m_\sigma^2}{m_\sigma^2 - x - im_\sigma\Gamma_\sigma(x)} \quad \Gamma_\sigma(x) = \Gamma_\sigma \frac{\sigma_\pi(x)}{\sigma_\pi(m_\sigma^2)} \quad F_\sigma(q^2, x) = \exp \left[\frac{-\lambda(q^2, x, m_\pi^2)R_\sigma^2}{8q^2} \right]$$

$\pi^0 \pi^0 \pi^-$

$$F_1^R \rightarrow F_1^R + \frac{\sqrt{2}F_V G_V}{3F^2} \alpha_\sigma^0 BW_\sigma(s_3)F_\sigma(q^2, s_3) \quad \alpha_\sigma = \beta_\sigma, \gamma_\sigma = \delta_\sigma \quad \alpha_\sigma^0 = \alpha_\sigma \cdot \text{Scaling}_{factor}^\gamma$$

$$F_1^{RR} \rightarrow F_1^{RR} + \frac{4F_A G_V}{3F^2} \frac{q^2}{q^2 - M_{a1}^2 - iM_{a1}\Gamma_{a1}(q^2)} \gamma_\sigma^0 BW_\sigma(s_3)F_\sigma(q^2, s_3) \quad \gamma_\sigma^0 = \gamma_\sigma \cdot \text{Scaling}_{factor}^\gamma$$

Our assumptions

1* RChT structure of FF (but not RChT calculation)

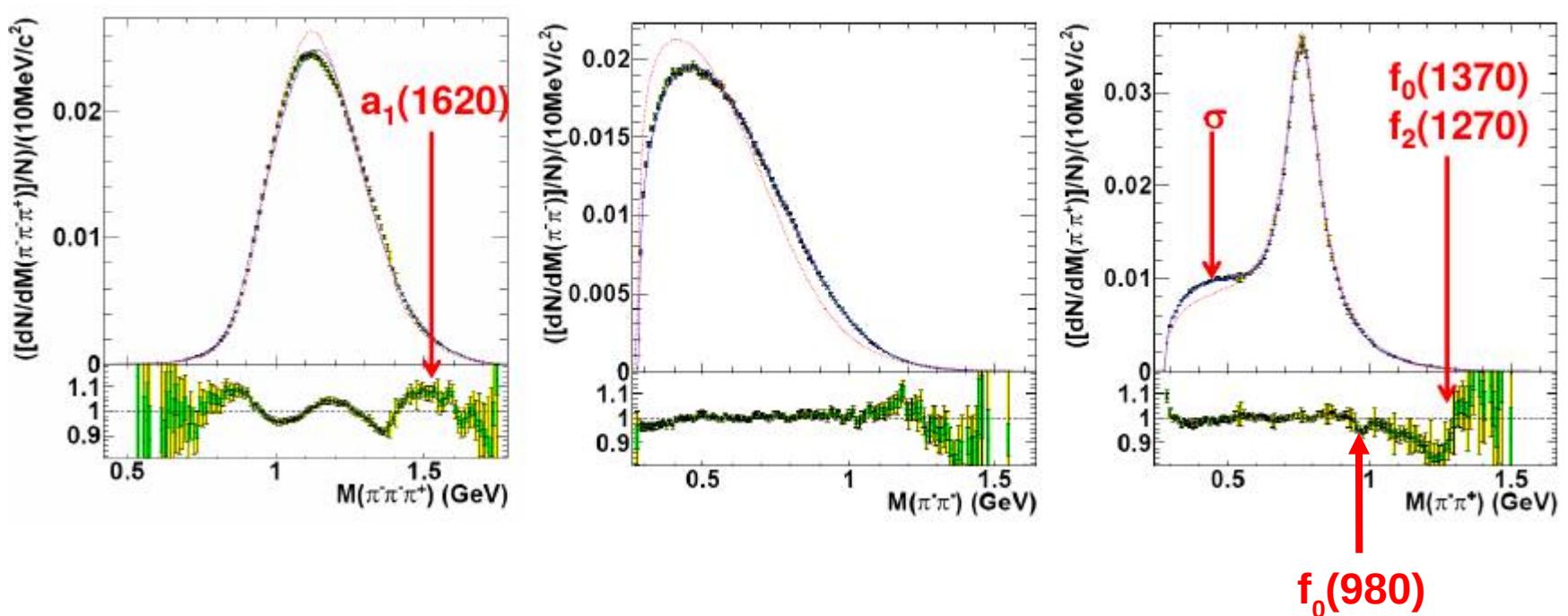
2* simplest BW parametrization: only Im part of loop

3* two sets of parameters, different for $\pi^- \pi^- \pi^+$ and $\pi^0 \pi^0 \pi^-$

4* for $\pi^- \pi^- \pi^+$ we choose not equal parameters

have to be inspected

Numerical results and fit to BaBar data



How include these resonances?

- * $a_1(1260)$ axial-vector, the second one
- * $f_2(1270)$; the lowest tensor resonance
- * $f_0(980)$; belongs to the lowest scalar resonance multiplet

- * $f_0(1370)$; PDG $m = 1200\text{-}1500\text{ MeV}$; $\Gamma = 200 - 500\text{ MeV}$???

The lowest scalar multiplet contribution: $f_0(980)$ and $\sigma(500)$

$$\mathcal{L}^S = c_d \langle S u_\mu u^\mu \rangle + c_m \langle S \chi_+ \rangle$$

$$S(x) = \begin{pmatrix} \frac{a^0}{\sqrt{2}} + \frac{\sigma_0}{\sqrt{3}} + \frac{\sigma_8}{\sqrt{6}} & a^+ & \kappa^+ \\ a^- & -\frac{a^0}{\sqrt{2}} + \frac{\sigma_0}{\sqrt{3}} + \frac{\sigma_8}{\sqrt{6}} & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & \frac{\sigma_0}{\sqrt{3}} - \sqrt{\frac{2}{3}}\sigma_8 \end{pmatrix}$$

ArXiv: 1011.5844; to include f_0

$$\frac{1}{M_S^2 - s} \longrightarrow \frac{\sin^2 \phi_S}{M_{f_0}^2 - s} + \frac{\cos^2 \phi_S}{M_\sigma^2 - s - c_\sigma s^k \bar{B}_0(s, m_\pi^2, m_\pi^2)}$$

PDG 2014: $m = 990 \pm 20$ MeV; $\Gamma = 40\text{-}100$ MeV $\phi_S = -8^\circ$

$B_0(s, m_\pi^2, m_\pi^2)$ - a loop function for $l = 0$, a complex function

A real part of this function enters the nominator

Our assumptions

- 1 * RChT structure of FF (but not RChT calculation)
- 2 * simplest BW parametrization: only Im part of loop
- 3 * two sets of parameters, different for $\pi^- \pi^- \pi^+$ and $\pi^0 \pi^0 \pi^-$
- 4 * for $\pi^- \pi^- \pi^+$ we choose not equal parameters

Study

$$4^* \quad F_1^R \rightarrow F_1^R + \frac{\sqrt{2}F_V G_V}{3F^2} [\alpha_\sigma BW_\sigma(s_1)F_\sigma(q^2, s_1) + \beta_\sigma BW_\sigma(s_2)F_\sigma(q^2, s_2)]$$

chiral prediction limits

$$F(\pi^0 \pi^0 \pi^-) \rightarrow 1 + (16 L_1 + 8L_3)/F^2 s_3 + 8L_2/F^2(s_2 - 2s_1)$$

$$F(\pi^- \pi^- \pi^+) \rightarrow 1 - (16 L_1 + 8L_3)/F^2 (s_2 - 2s_1) + 8L_2/F^2 s_3$$

$$L_1 = \frac{G_V^2}{8M_V^2} - \frac{c_d^2}{6M_S^2} + \frac{\tilde{c}_d^2}{2M_{S_1}^2} \quad L_2 = \frac{G_V^2}{4M_V^2} \quad L_3 = -\frac{3G_V^2}{4M_V^2} + \frac{c_d^2}{2M_S^2}$$

$$F(\pi^0 \pi^0 \pi^-) \rightarrow 1 + 16 L_1 / F^2 (-2s_3 + s_2 - 2s_1)$$

Only V

$$-F(\pi^- \pi^- \pi^+) \rightarrow 1 + 16 L_1 / F^2 (-2s_3 + s_2 - 2s_1)$$

The lowest scalar multiplet contribution: $f_0(980)$ and $\sigma(500)$

$$\mathcal{L}^S = c_d \langle S u_\mu u^\mu \rangle + c_m \langle S \chi_+ \rangle$$

$$S(x) = \begin{pmatrix} \frac{a^0}{\sqrt{2}} + \frac{\sigma_0}{\sqrt{3}} + \frac{\sigma_8}{\sqrt{6}} & a^+ & \kappa^+ \\ a^- & -\frac{a^0}{\sqrt{2}} + \frac{\sigma_0}{\sqrt{3}} + \frac{\sigma_8}{\sqrt{6}} & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & \frac{\sigma_0}{\sqrt{3}} - \sqrt{\frac{2}{3}}\sigma_8 \end{pmatrix}$$

ArXiv: 1011.5844; to include f_0

$$\frac{1}{M_S^2 - s} \longrightarrow \frac{\sin^2 \phi_S}{M_{f_0}^2 - s} + \frac{\cos^2 \phi_S}{M_\sigma^2 - s - c_\sigma s^k \bar{B}_0(s, m_\pi^2, m_\pi^2)}$$

PDG 2014: $m = 990 \pm 20$ MeV; $\Gamma = 40\text{-}100$ MeV $\phi_S = -8^\circ$

$B_0(s, m_\pi^2, m_\pi^2)$ - a loop function for $l = 0$, a complex function
 A real part of this function enters the nominator

Lagrangian with A S meson

Tensor resonance contribution f2(1270)

G. Ecker, C. Zauner arXiv: 0705.0624

$$\mathcal{L} = -\frac{1}{2} \langle T_{\mu\nu} D_T^{\mu\nu, \rho\sigma} T_{\rho\sigma} \rangle + \langle T_{\mu\nu} J_T^{\mu\nu} \rangle$$

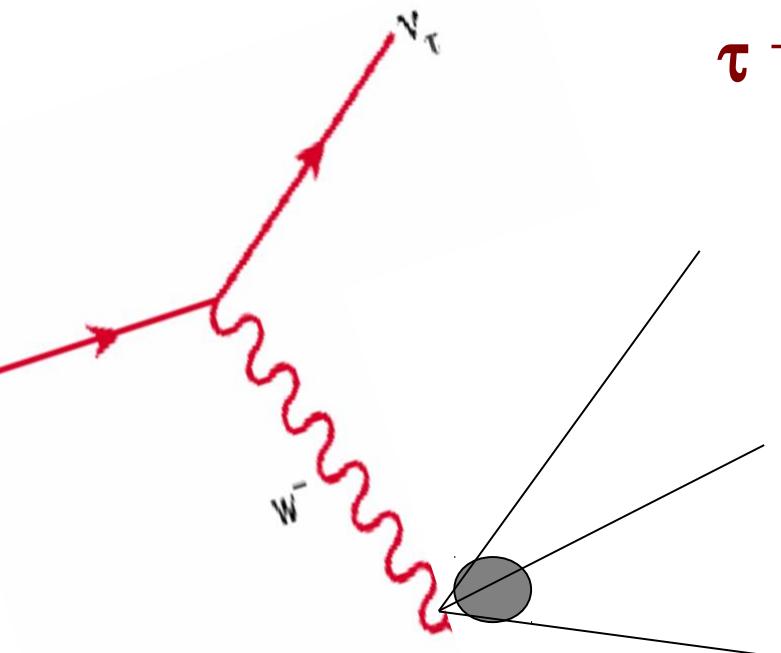
$$T_{\mu\nu} = T_{\mu\nu}^0 \frac{\lambda_0}{\sqrt{2}} + \frac{1}{\sqrt{2}} \sum_{i=1}^8 \lambda_i T_{\mu\nu}^{8,i}$$

$$\frac{1}{\sqrt{2}} \sum_{i=1}^8 \lambda_i T^{8,i} = \begin{pmatrix} \frac{a_2^0}{\sqrt{2}} + \frac{f_2^8}{\sqrt{6}} & a_2^+ & K_2^{*+} \\ a_2^- & -\frac{a_2^0}{\sqrt{2}} + \frac{f_2^8}{\sqrt{6}} & K_2^{*0} \\ K_2^{*-} & \bar{K}_2^{*0} & -\frac{2f_2^8}{\sqrt{6}} \end{pmatrix}, \quad T^0 = f_2^0.$$

No study T A (meson) \longrightarrow Lagrangian with A S meson

Fit Cleo parametrization to BaBar data

BACK UP



$$\tau^- \rightarrow (3\pi)^- \nu_\tau$$

PDG 2014 Br (ex K0)

$$\begin{aligned} BR(\pi^0 \pi^0 \pi^-) &= (9.3 \pm 0.11)\% \\ BR(\pi^- \pi^- \pi^+) &= (9.02 \pm 0.06)\% \end{aligned}$$

Experiment data

Cleo, Aleph $\pi^0 \pi^0 \pi^-$ 1990–2000

Cleo, Aleph, BaBar, Belle $\pi^- \pi^- \pi^+$

*Only BaBar measured the differential spectrum
and preliminary data is available*



= a1 \rightarrow (intermediate resonance state = ρ , f_0 , π') + π

$$BW(s) = m^2 / (m^2 - s - i m \Gamma(s)) ; \text{vertex constant}$$

Resonance Chiral Theory results for three pion decay modes

RChT = ChPT + resonances (V. A. S. P) as new active degree of freedom

$$\mathcal{L}_{R\chi T} = \mathcal{L}_{pGB}^{(2)} + \sum_{R_1} \mathcal{L}_{R_1} + \sum_{R_1, R_2} \mathcal{L}_{R_1 R_2} + \sum_{R_1, R_2, R_3} \mathcal{L}_{R_1 R_2 R_3} + \dots$$

$$\mathcal{L}_{pGB}^{(2)} = \mathcal{L}_2^{\chi PT} = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle$$

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \quad u(\phi) = e^{\left(\frac{i}{\sqrt{2}F}\phi\right)}$$

$$\begin{aligned} \mathcal{L}_R = \sum_i \left\{ \frac{F_{V_i}}{2\sqrt{2}} \langle V_i^{\mu\nu} f_{+\mu\nu} \rangle + \frac{iG_{V_i}}{\sqrt{2}} \langle V_i^{\mu\nu} u_\mu u_\nu \rangle + \frac{F_{A_i}}{2\sqrt{2}} \langle A_i^{\mu\nu} f_{-\mu\nu} \rangle \right. \\ \left. + c_{d_i} \langle S_i u^\mu u_\mu \rangle + c_{m_i} \langle S_i \chi_+ \rangle + i d_{m_i} \langle P_i \chi_- \rangle \right\}, \end{aligned}$$

Antisymmetric formalism for resonances

$$V_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 \end{pmatrix}_{\mu\nu},$$

Our assumptions

- 1 * RChT structure of FF (but not RChT calculation)
- 2 * simplest BW parametrization → only Im part of loop
- 3 * two sets of parameters, different for $\pi^- \pi^- \pi^+$ and $\pi^0 \pi^0 \pi^-$
- 4* for $\pi^- \pi^- \pi^+$ we choose not equal parameters

Preliminary answers

1* calculation within RChT will check $\mathcal{L}^S = c_d \langle S u_\mu u^\mu \rangle + c_m \langle S \chi_+ \rangle$

+ SA(u) lagrangian

2* width = Im part of loop function → + Re part of I=0 loop function

4* this point will be checked by calculation, however, in ChPT there is only one parameter → most probably we will have the same

3* calculation RchT, preliminary one (one resonance) shows equal parameters

3 pion decay of tau in general case

$\text{BR}(\pi^- \pi^- \pi^0)/\text{BR}(\pi^0 \pi^0 \pi^-) = 1$ for [210] structure, V resonance

$\text{BR}(\pi^- \pi^- \pi^0)/\text{BR}(\pi^0 \pi^0 \pi^-) = 4$ for [300] structure, S(T) resonance

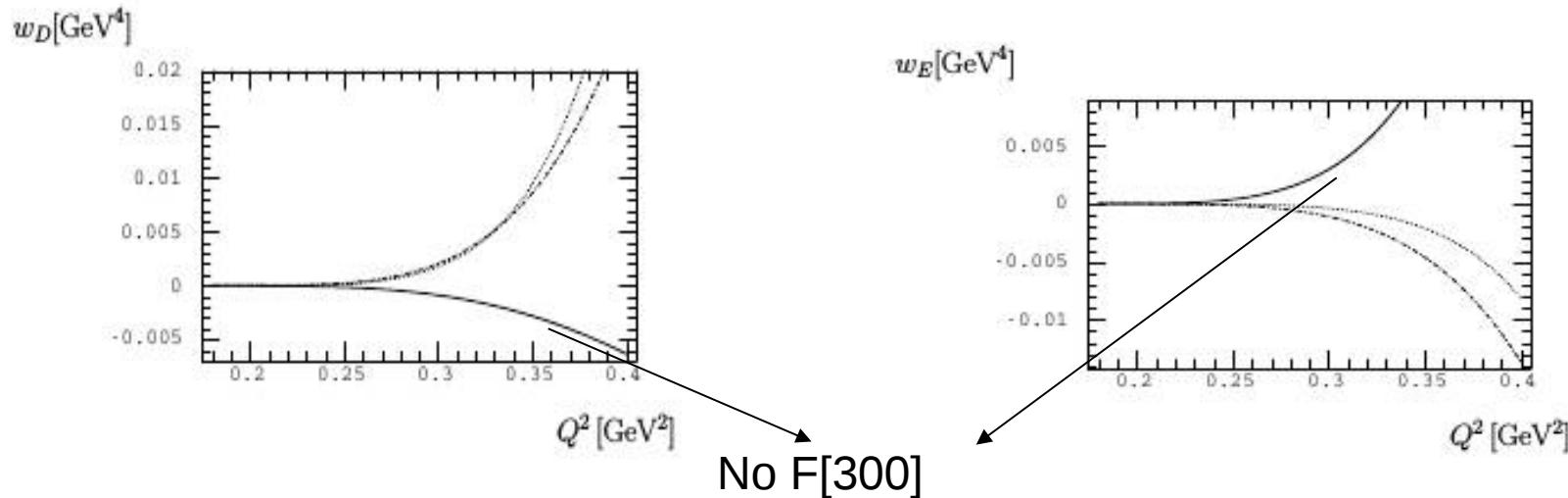
$$F1(\pi^0 \pi^0 \pi^-) = f(F1(\pi^- \pi^- \pi^+))$$

as well as calculation of $F[300]$ and $F[210]$ within ChPT one loop

Implementation for 4 pion case → A. Pais Annals Of Physics 9
(1960) 548

Influence of the $F[300]$ to the integrated structure function

w_D sensitive to $\text{Re}F[300]$, w_E to $\text{Im}F[300]$



Application for Tauola and BaBar data ???