

Dispersion formalism for $\gamma\gamma^{(*)} \rightarrow \pi\pi$

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(very preliminary) Work in collaboration with
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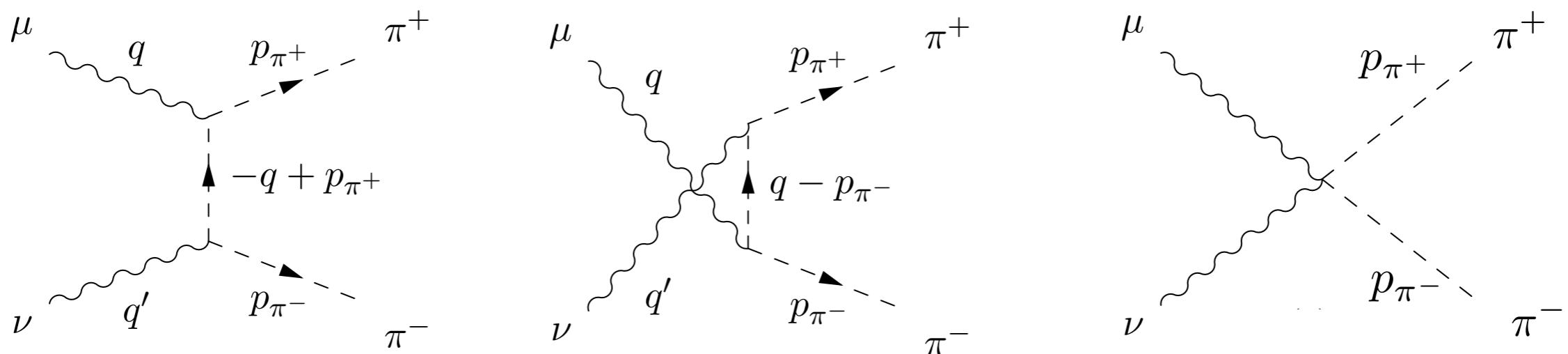


Outline

- Warm up $\gamma\gamma \rightarrow \pi\pi$: main pieces
- $\gamma\gamma^{(*)} \rightarrow \pi\pi$: very preliminary results
- Outlook

First look at $\gamma\gamma^*\rightarrow\pi\pi$ using dispersion relations

- Look first at $\gamma\gamma\rightarrow\pi\pi$ from dispersion relations and identify the most relevant pieces (goal is doubly virtual case)
- Starting point: Low-energy theorem to build up the Born amplitude



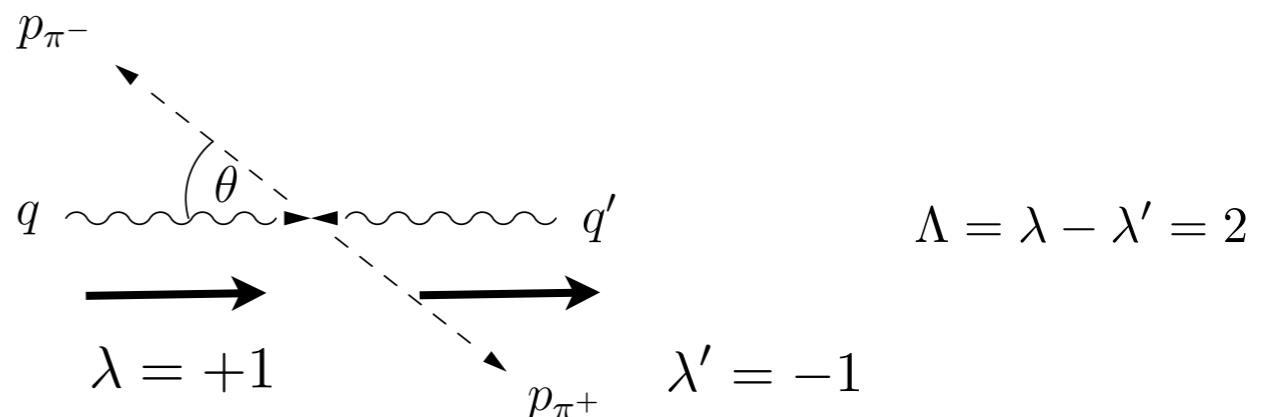
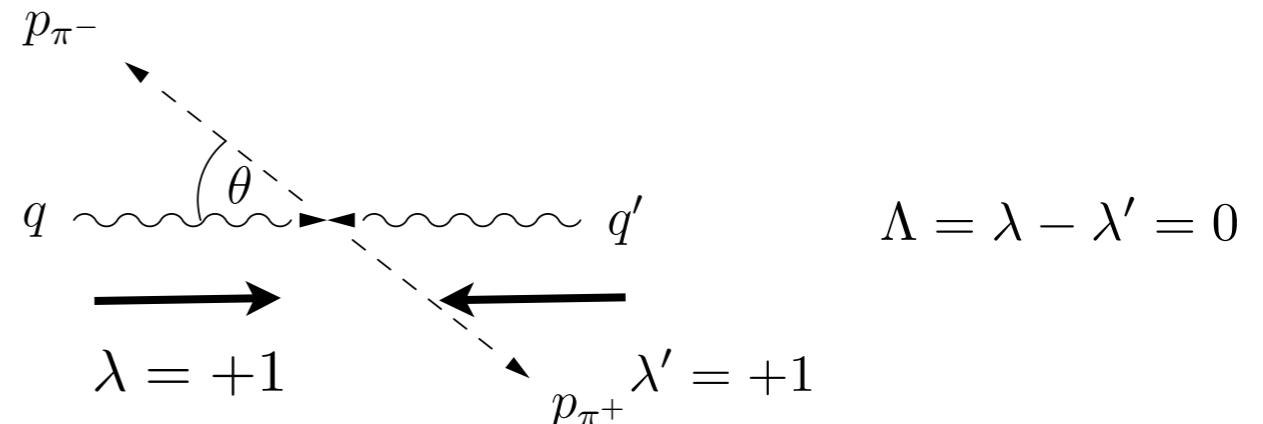
Warm up $\gamma\gamma \rightarrow \pi\pi$: main pieces

a la Morgan and Pennington '87ss

- Look first at $\gamma\gamma \rightarrow \pi\pi$ from dispersion relations and identify the most relevant pieces (goal is doubly virtual case)
- Starting point: Low-energy theorem to build up the Born amplitude

$$\mathcal{M}_{++}^{\text{BORN}} = 2ie^2 \frac{1 - \beta(s)^2}{1 - \beta(s)^2 \cos(\theta)^2}$$

$$\mathcal{M}_{+-}^{\text{BORN}} = 2ie^2 \frac{\beta(s)^2 \sin(\theta)^2}{1 - \beta(s)^2 \cos(\theta)^2}$$



Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces

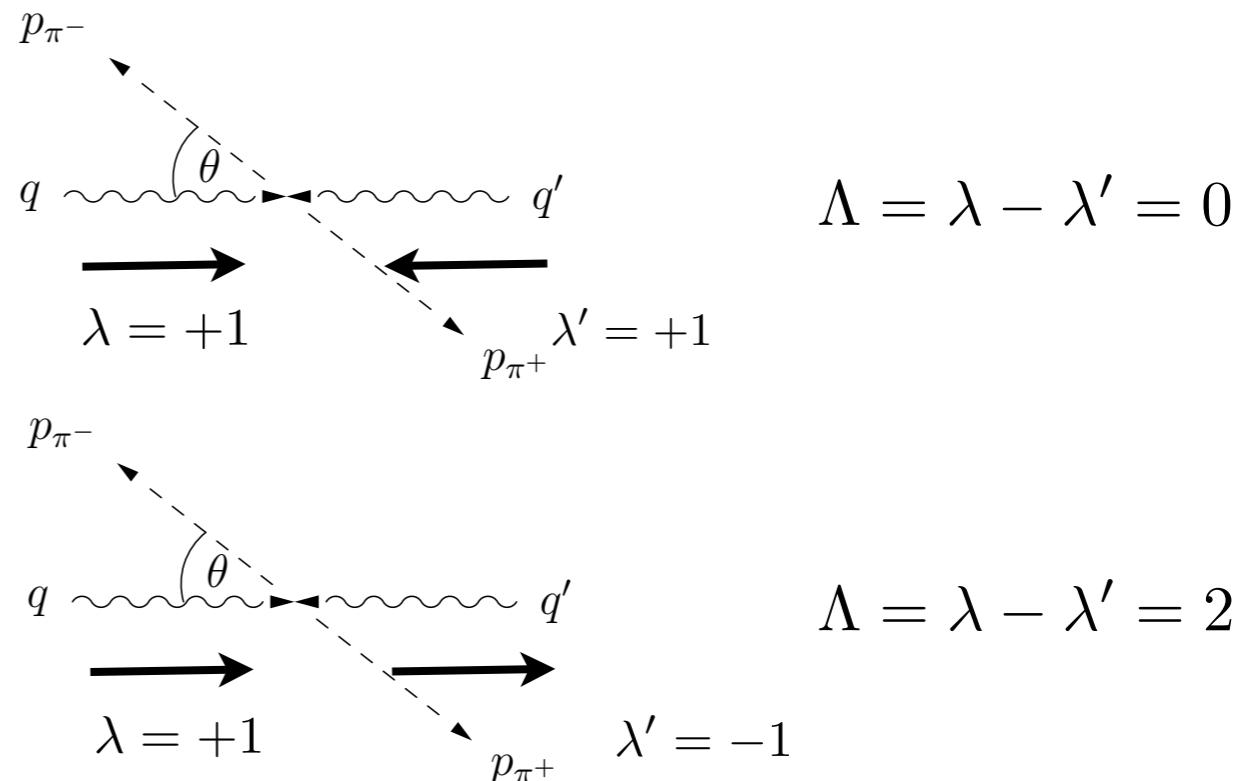
a la Morgan and Pennington '87ss

- Decompose the amplitudes in Partial Waves

$$B_{J\Lambda} = \frac{1}{4ie^2} \int_{-1}^1 d\cos\theta \sqrt{2J+1} \sqrt{\frac{(J-\Lambda)!}{(J+\Lambda)!}} P_J^\Lambda(\cos\theta) \mathcal{M}^\Lambda$$

$$\mathcal{M}_{++}^{\text{BORN}} = 2ie^2 \frac{1 - \beta(s)^2}{1 - \beta(s)^2 \cos(\theta)^2}$$

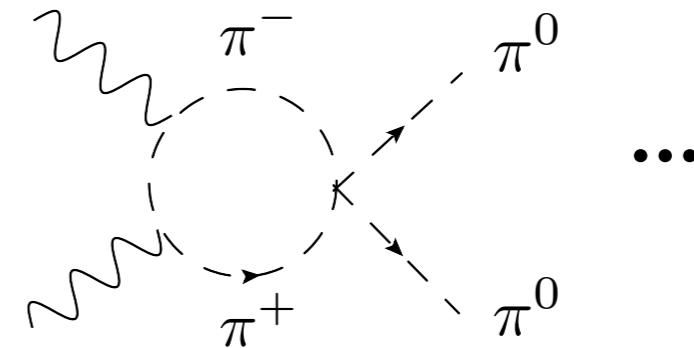
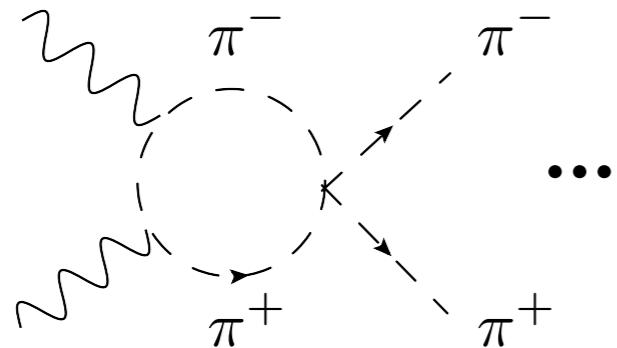
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Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces

a la Morgan and Pennington '87ss

- Include rescattering effects (FSI)



Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces

a la Morgan and Pennington '87ss

- Include rescattering effects using Omnès representation
 - Only $\pi\pi$ (no KK..., no inelasticities)
 - Phase shifts from Peláez *et al.*
 - Left-hand cut with pion only

$$\text{Im } F_{J\Lambda_\gamma}^I(\gamma\gamma \rightarrow \pi\pi) = \rho_{\pi\pi} F_{J\Lambda_\gamma}^{I*}(\gamma\gamma \rightarrow \pi\pi) \mathcal{I}_J^I(\pi\pi \rightarrow \pi\pi)$$


$$\phi_J^{I(\gamma\gamma \rightarrow \pi\pi)}(s) = \delta_{\pi\pi}^{IJ}(s) \quad \text{for } 4m_\pi^2 < s < 4m_K^2$$


$$\Omega_J^I(s) = \exp \left[\frac{s}{\pi} \int_{s_0}^{\infty} ds' \frac{\delta_J^I(s')}{s'(s' - s - i\epsilon)} \right]$$

(the right-hand cut is described by Omnès)

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Unitarized partial wave

$$F_{J\Lambda}^I(s) = \Omega_J^I(s) \left\{ B_{J\Lambda}^I(s) \text{Re} [(\Omega_J^I)^{-1}(s)] - \frac{s(s-s_0)^{\frac{J}{2}}}{\pi} P \int_{s_0}^{\infty} ds' \frac{B_{J\Lambda}^I(s') \text{Im}(\Omega_J^I)^{-1}(s)}{(s'-s_0)^{\frac{J}{2}}(s'-s)} \right\}$$

The diagram illustrates the decomposition of the unitarized partial wave $F_{J\Lambda}^I(s)$. A red arrow points from the term $B_{J\Lambda}^I(s) \text{Re} [(\Omega_J^I)^{-1}(s)]$ to the label "Born (left-hand cut)". A blue arrow points from the term $P \int_{s_0}^{\infty} ds' \frac{B_{J\Lambda}^I(s') \text{Im}(\Omega_J^I)^{-1}(s)}{(s'-s_0)^{\frac{J}{2}}(s'-s)}$ to the label "Omnès (right-hand cut)".

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Unitarized partial wave

$$F_{\Lambda=0}(s, \cos(\theta)) = \sum_{J \geq 0} \sqrt{2J + 1} P_J(\cos(\theta)) F_{J0}(s)$$

Unitarized amplitude

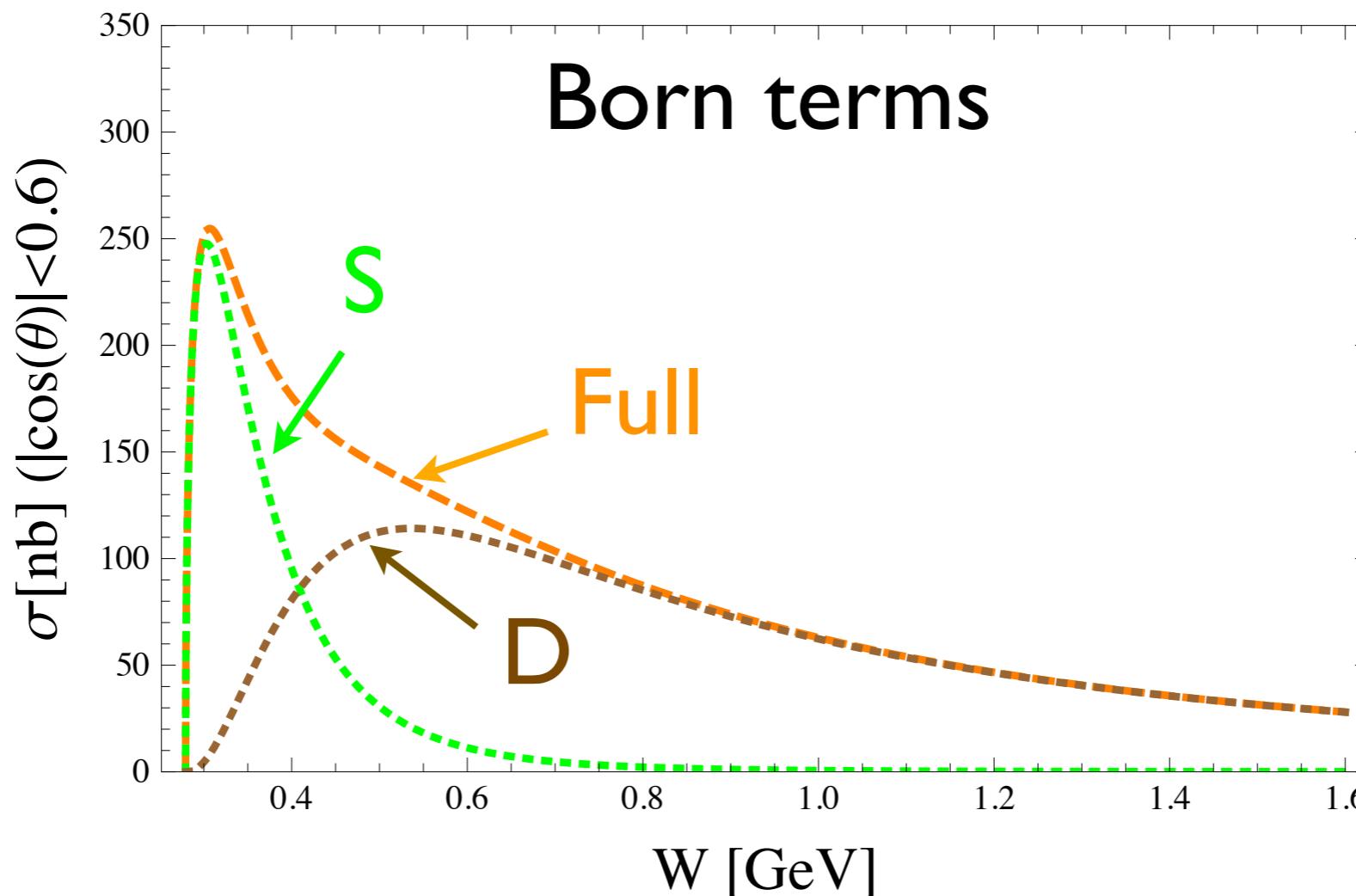
$$F_{\Lambda=2}(s, \cos(\theta)) = \sum_{J \geq 2} \sqrt{2J + 1} \sqrt{\frac{(J - 2)!}{(J + 2)!}} P_J^2(\cos(\theta)) F_{J2}(s)$$

$$\left(\frac{d\sigma}{d \cos(\theta)} \right)_{CM} = \frac{\beta(s)}{32\pi s} \left(|F_{\Lambda=0}|^2 + |F_{\Lambda=2}|^2 \right)$$

Warm up $\gamma\gamma \rightarrow \pi\pi$: main pieces

a la Morgan and Pennington '87ss

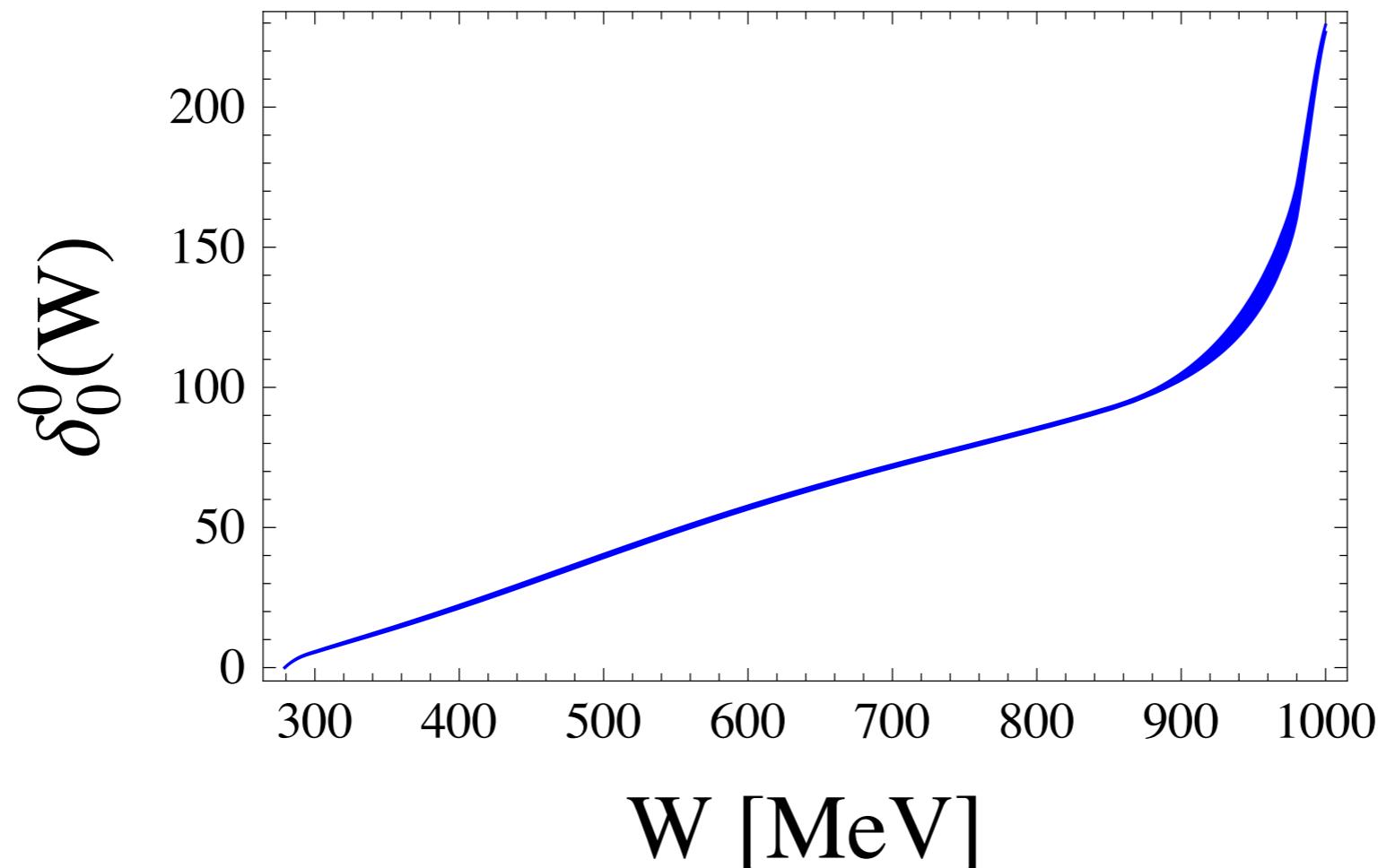
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Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces

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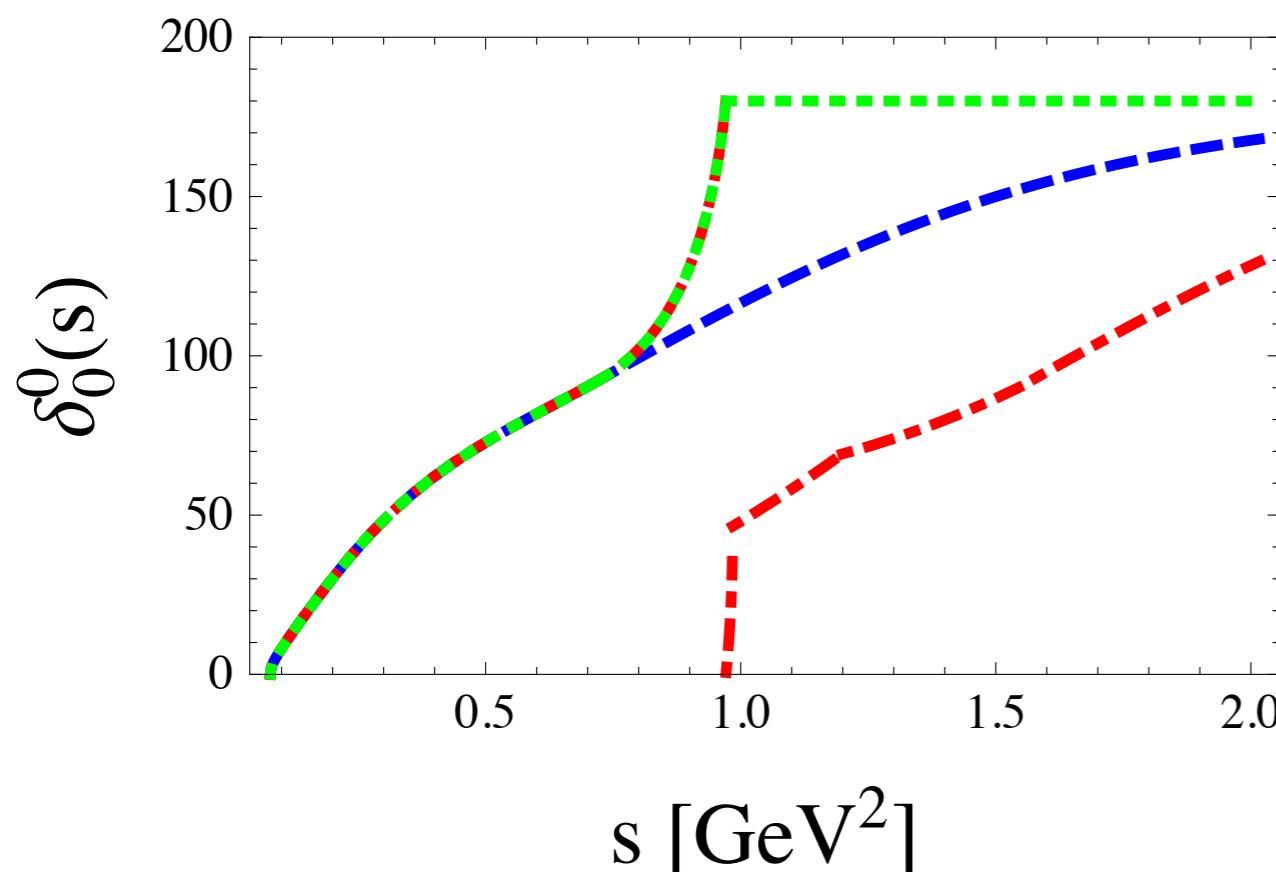
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Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces

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Extrem cases:

$$\phi = \delta_0^0 \text{ for } s < 1 \text{ GeV}^2$$

$$\longrightarrow \phi = 0 \text{ for } s > 1 \text{ GeV}^2$$

$$\longrightarrow \phi = \pi \text{ for } s > 1 \text{ GeV}^2$$

Smooth interpolation

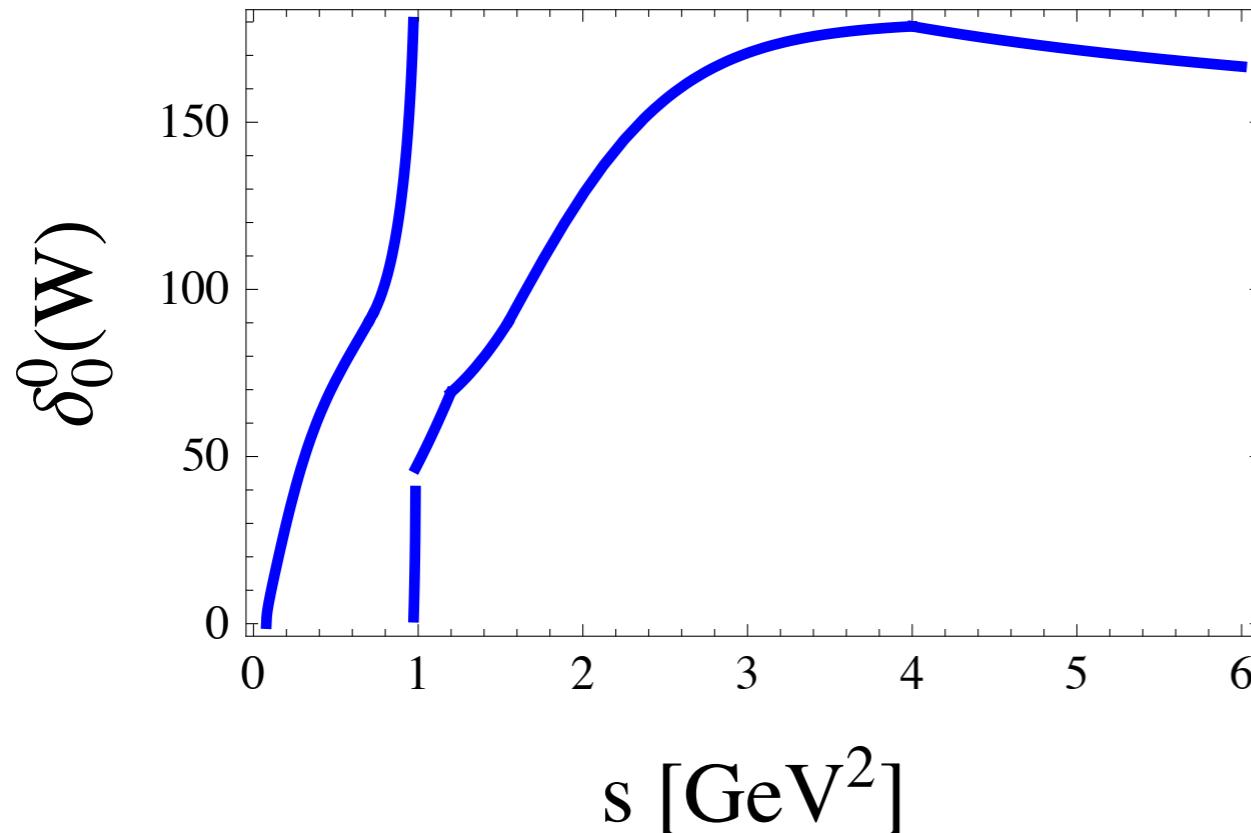
both: blue and red go to π

Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces

a la Morgan and Pennington '87ss

- Include rescattering effects using Omnès representation
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At high energies:



$$\phi = \delta_0^0 \text{ for } s < 1 \text{ GeV}^2$$

$$\phi = \pi \text{ for } s > 1 \text{ GeV}^2$$

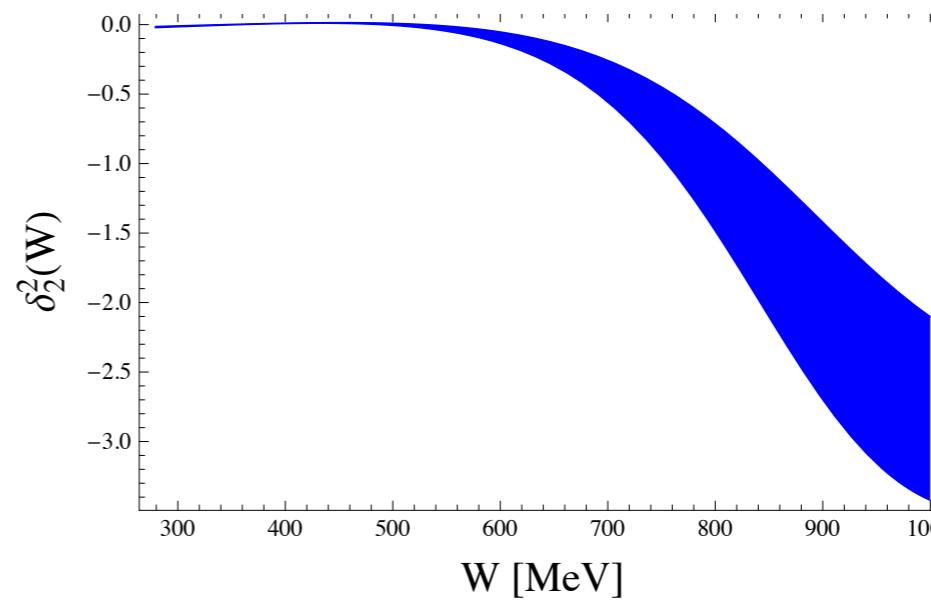
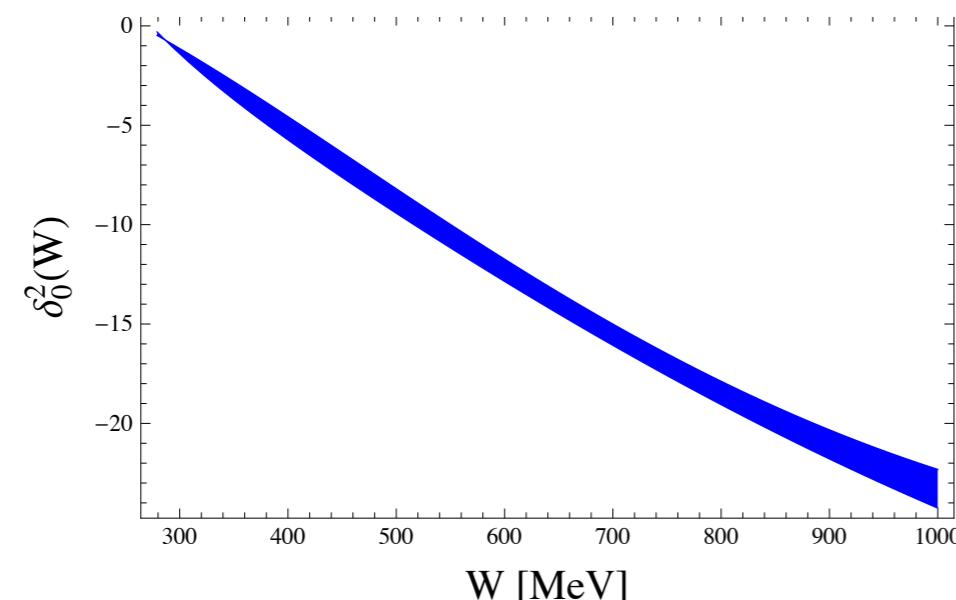
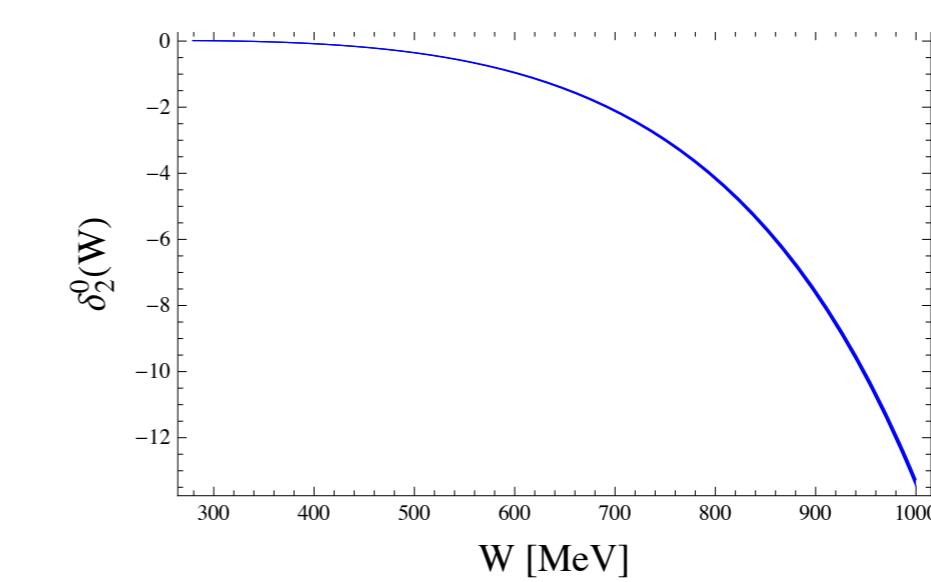
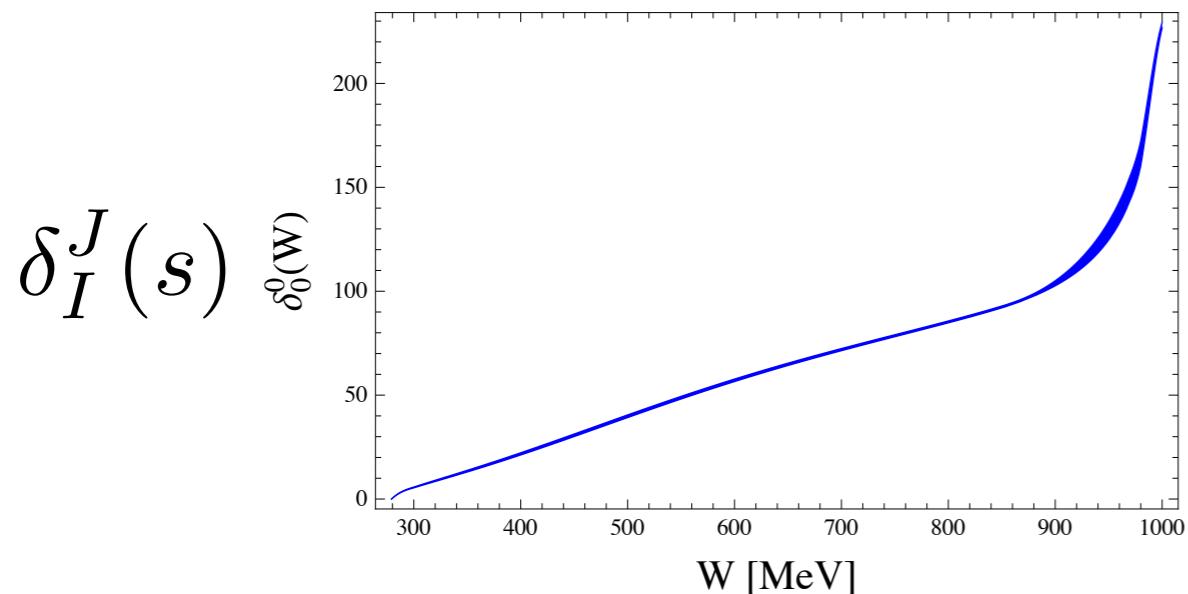
$$\phi = \phi_{\text{Regge}} \sim 130^\circ \text{ for } s > 4 \text{ GeV}^2$$

(using the ρ and ω Regge trajectories)

Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces

a la Morgan and Pennington '87ss

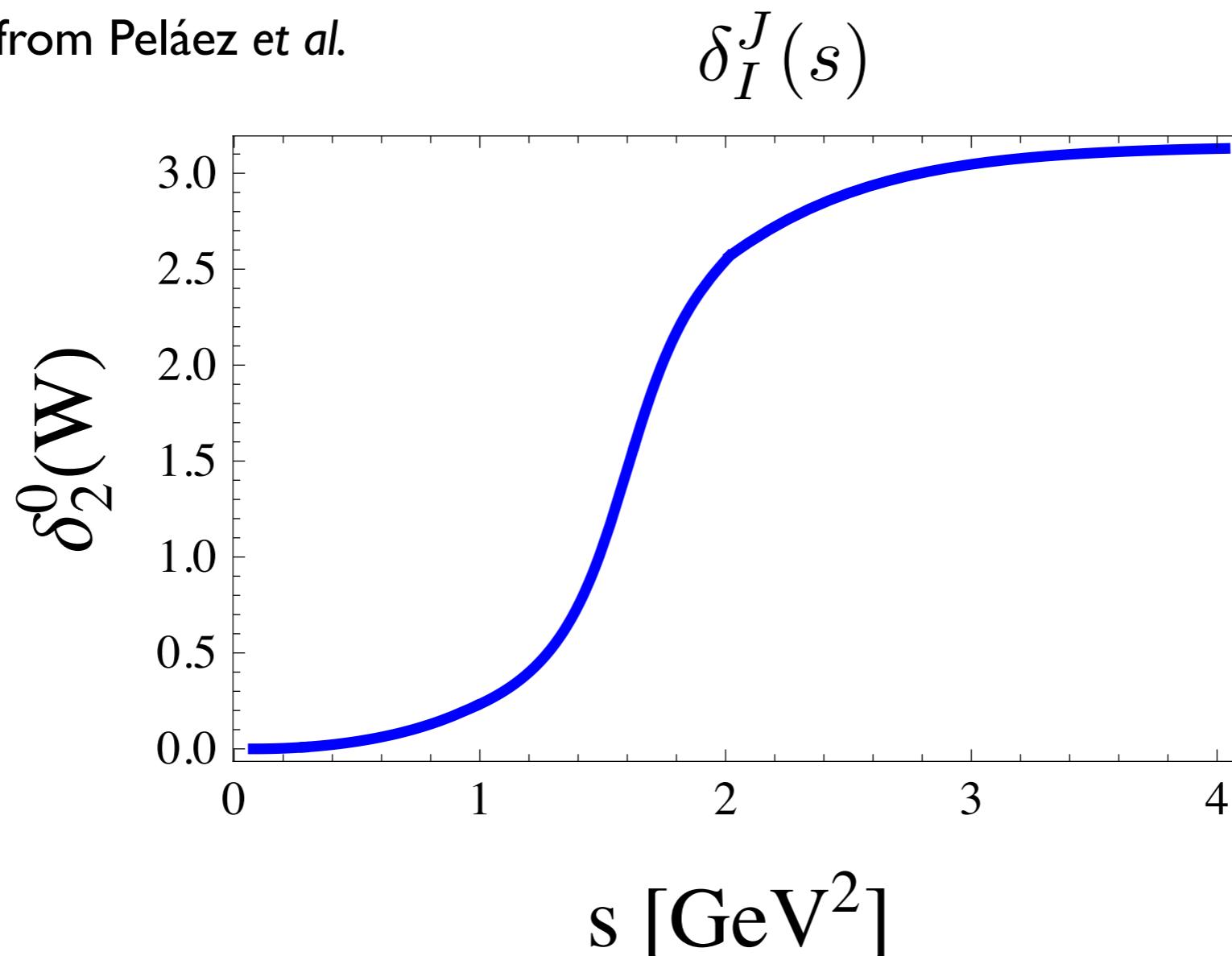
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Warm up $\gamma\gamma \rightarrow \pi\pi$: main pieces

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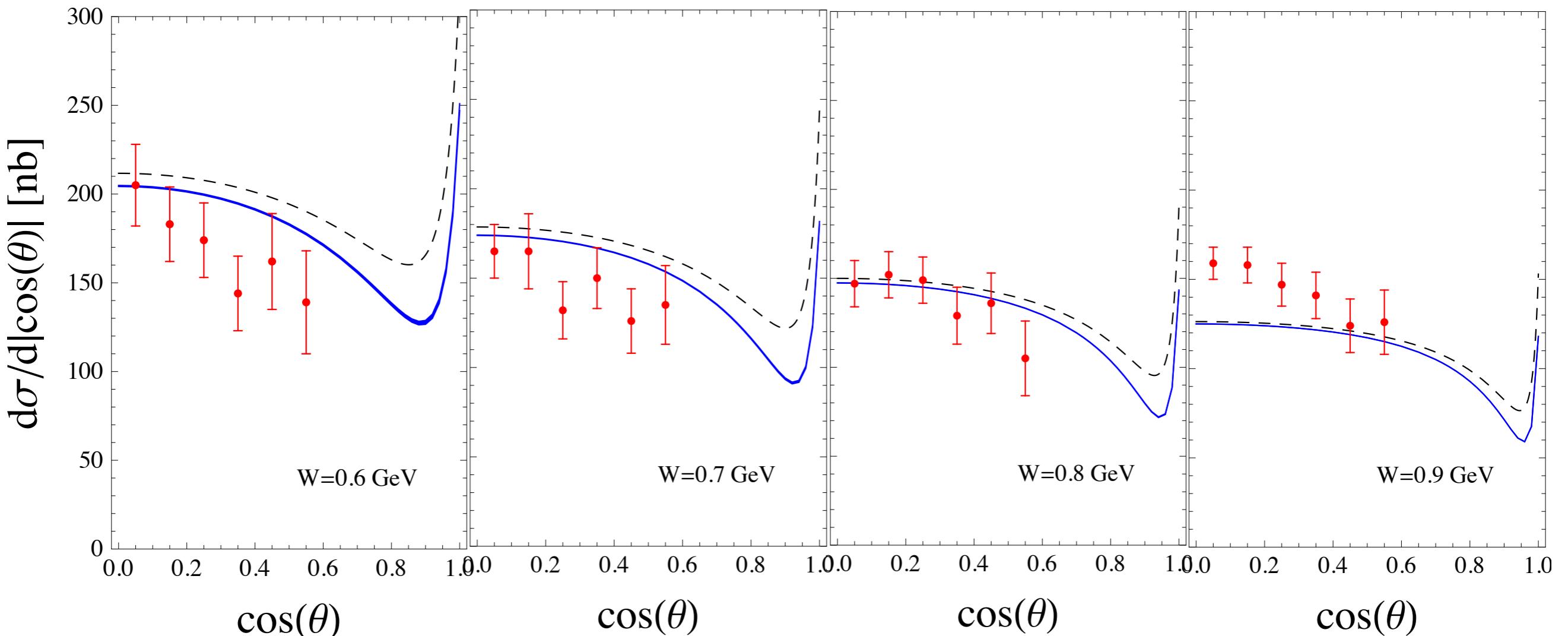
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Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces

preliminary results

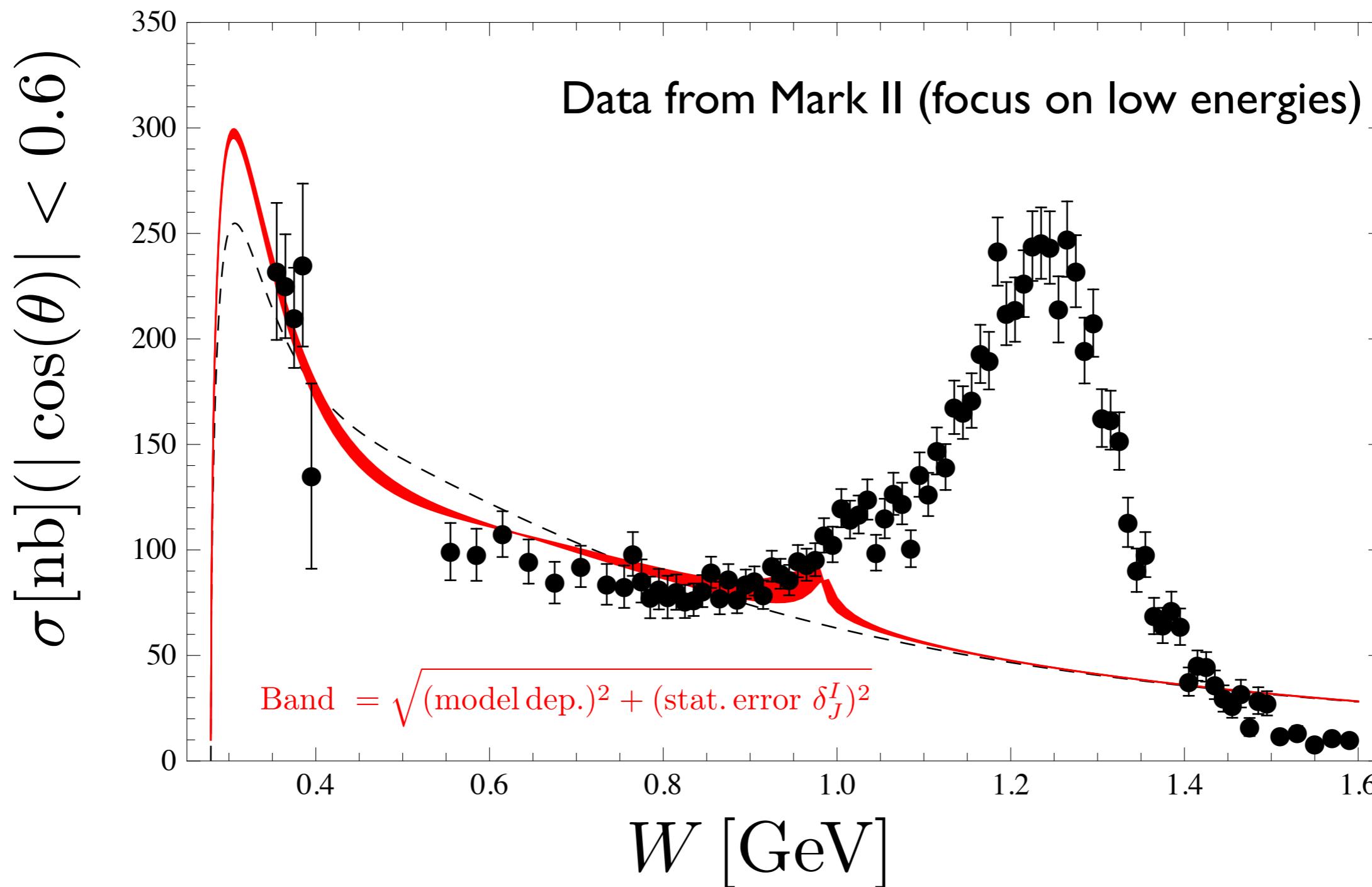
Differential cross section - S wave unitarized



(thickness of blue band is δ_{IJ} statistical error - S Wave only)

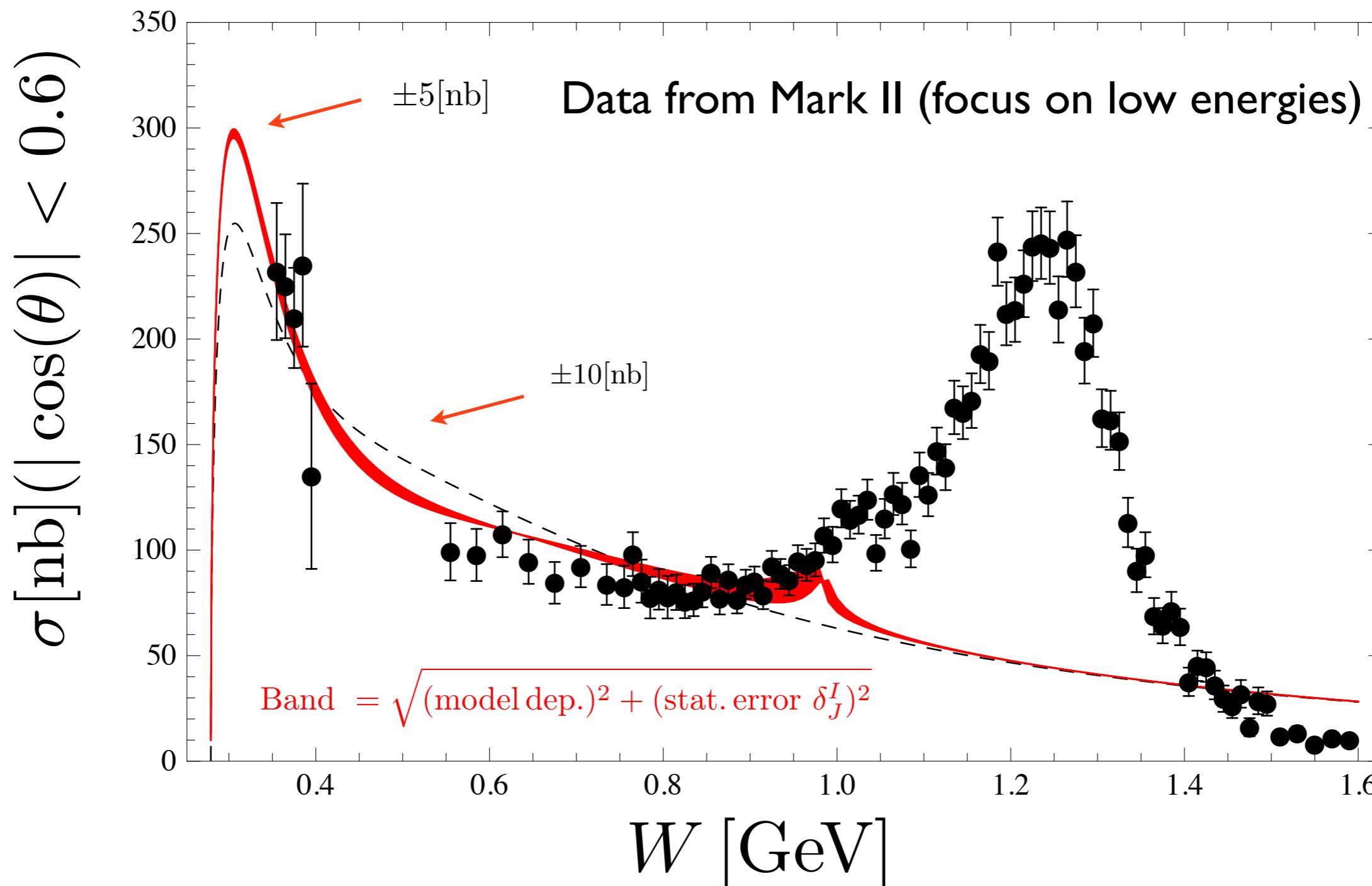
Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces

preliminary results



Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces

preliminary results



Warm up $\gamma\gamma \rightarrow \pi\pi$: main pieces

a la Morgan and Pennington '87ss

- Include $f_2(1270)$ resonance with a Breit-Wigner representation

$$\Gamma(f_2 \rightarrow \pi\pi) = \frac{1}{40\pi} g_{f_2\pi\pi}^2 \frac{\left(\sqrt{m_{f_2}^2/4 - m_\pi^2}\right)^5}{m_{f_2}^4} \quad \leftarrow \quad BR(f_2 \rightarrow \pi\pi) = 0.85$$
$$\Gamma_{f_2} = 185 \text{ MeV}$$
$$\Gamma(f_2 \rightarrow \gamma\gamma) = \frac{\pi\alpha^2}{5} g_{f_2\gamma\gamma}^2 m_{f_2} \quad \leftarrow \quad BR(f_2 \rightarrow \gamma\gamma) = 1.6 \cdot 10^{-7}$$

} [PDG]

$$F_{J=2,\Lambda=2}^{(f_2)}(s) = -\pi\alpha \sqrt{\frac{2}{15}} \frac{g_{f_2\gamma\gamma} g_{f_2\pi\pi}}{m_{f_2}^2} \frac{s^2 \beta^2}{s - m_{f_2}^2 + im_{f_2}\Gamma(s)}$$

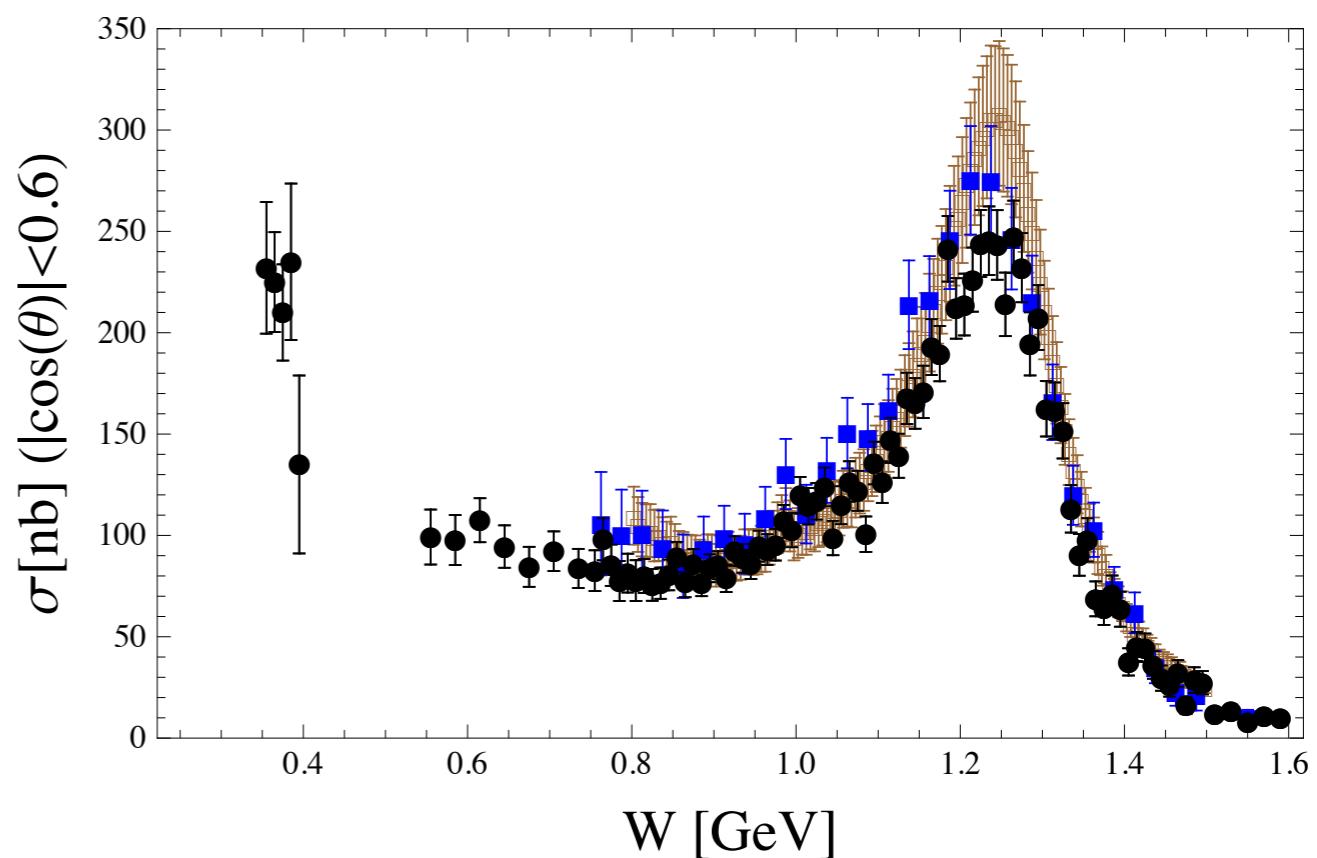
(notice no helicity 0 component)

Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces

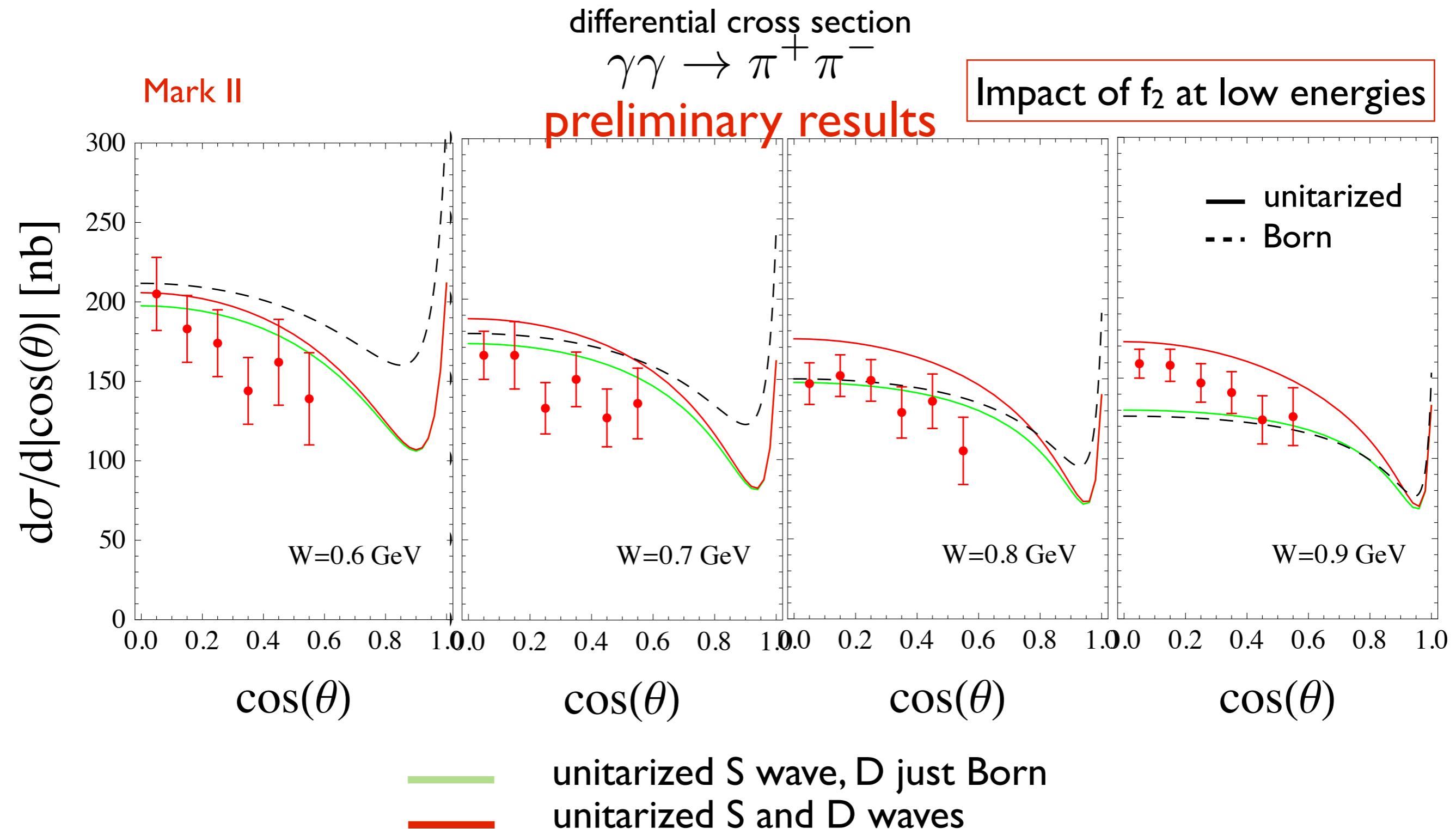
a la Morgan and Pennington '87ss

- Born diagrams + gauge invariance
- Rescattering of FS: Omnès representation
- Only $\pi\pi$ (no KK..., no inelasticities)
- Phase shifts from Peláez *et al.*
- Left-hand cut with pion only
- $f_2(1270)$ as Breit-Wigner resonance (l=0 from helicity 2 photons
-no helicity=0 component-
 D_2 wave)

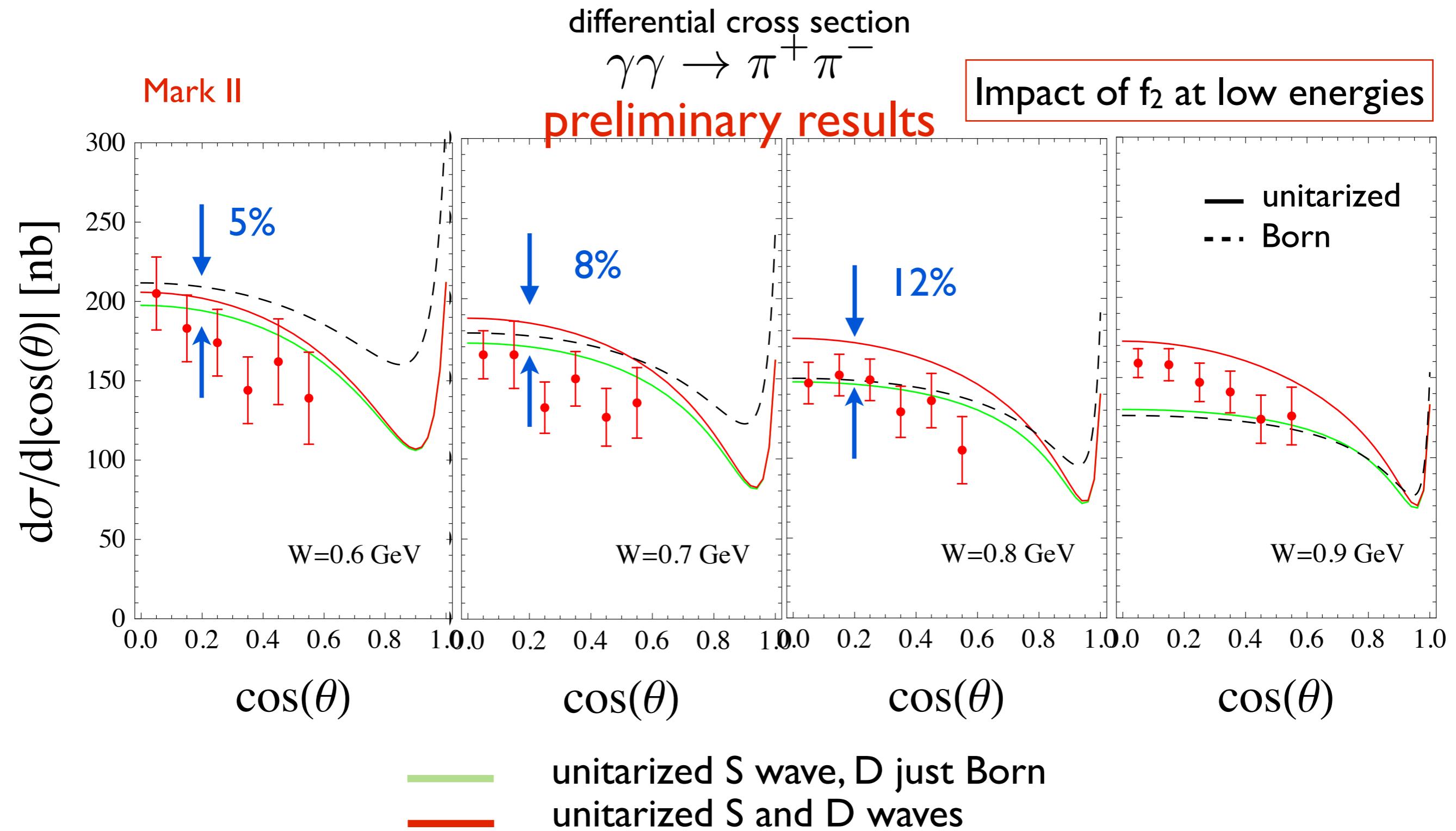
Mark II
CELLO
BELLE



Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces



Warm up $\gamma\gamma \rightarrow \pi^+\pi^-$: main pieces



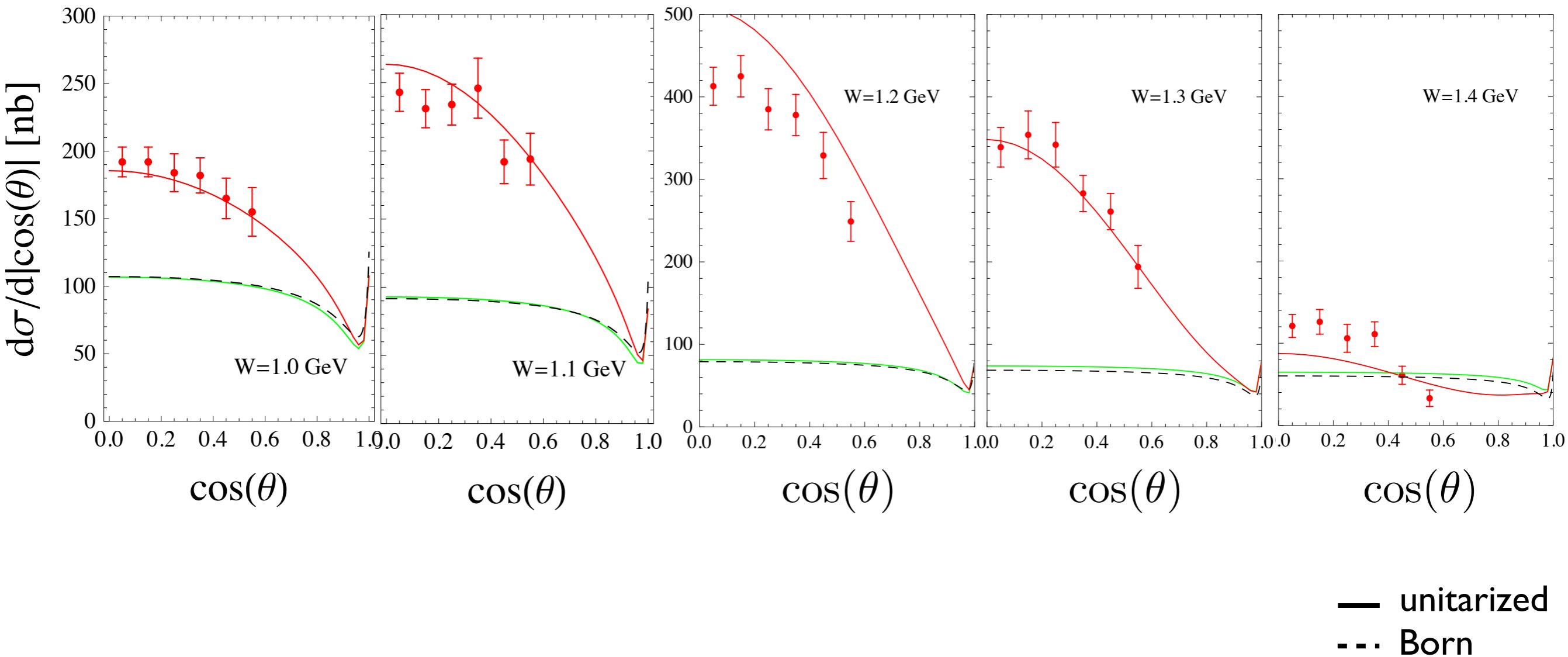
Warm up $\gamma\gamma \rightarrow \pi\pi\pi$: main pieces

differential cross section
 $\gamma\gamma \rightarrow \pi^+ \pi^-$

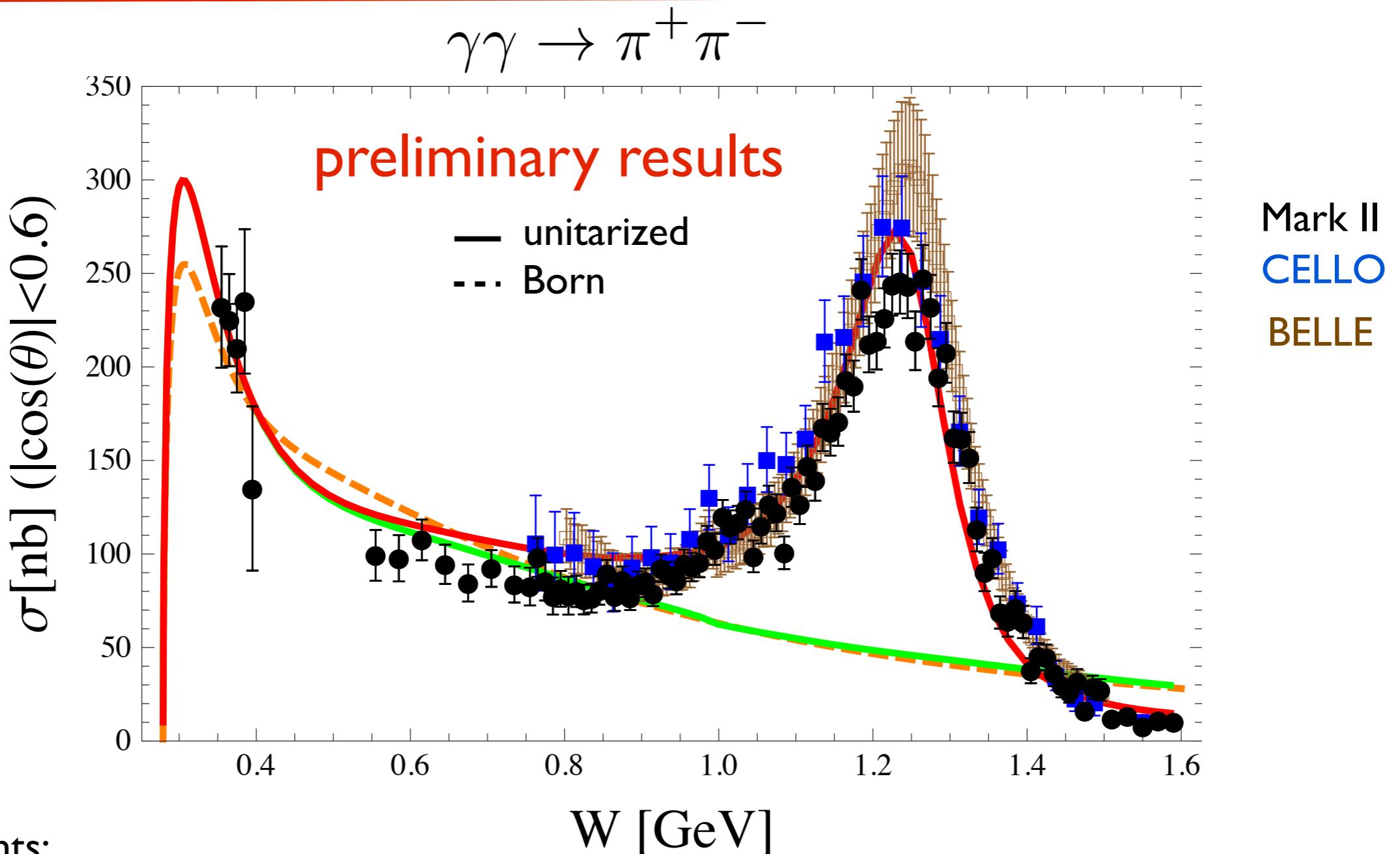
Mark II

preliminary results

Impact of f_2 at high energies



Warm up $\gamma\gamma \rightarrow \pi^+\pi^-$: main pieces



Improvements:

- KK threshold: better description around 1 GeV
(less dependence on the phase shift)
- include helicity 0 f_2
- Include D Wave through phase shifts

[Pennington et al'08,'14]
[Mao et al, '09]
[García-Martin, '10]
[Hoferichter et al '11]

$\gamma\gamma \rightarrow \pi\pi$: conclusions

- Warm up into $\gamma\gamma \rightarrow \pi\pi$ (very preliminary results)
- Identification of main ingredients
- For the first time: both exp error + model dep.
- Interesting region at low energies (role of f_2)

$\gamma\gamma \rightarrow \pi\pi$: conclusions

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- D Wave unitarization through phase shift: correct wave interference)
- Use data at low-energies for subtracting
- left-hand cuts including higher vector states

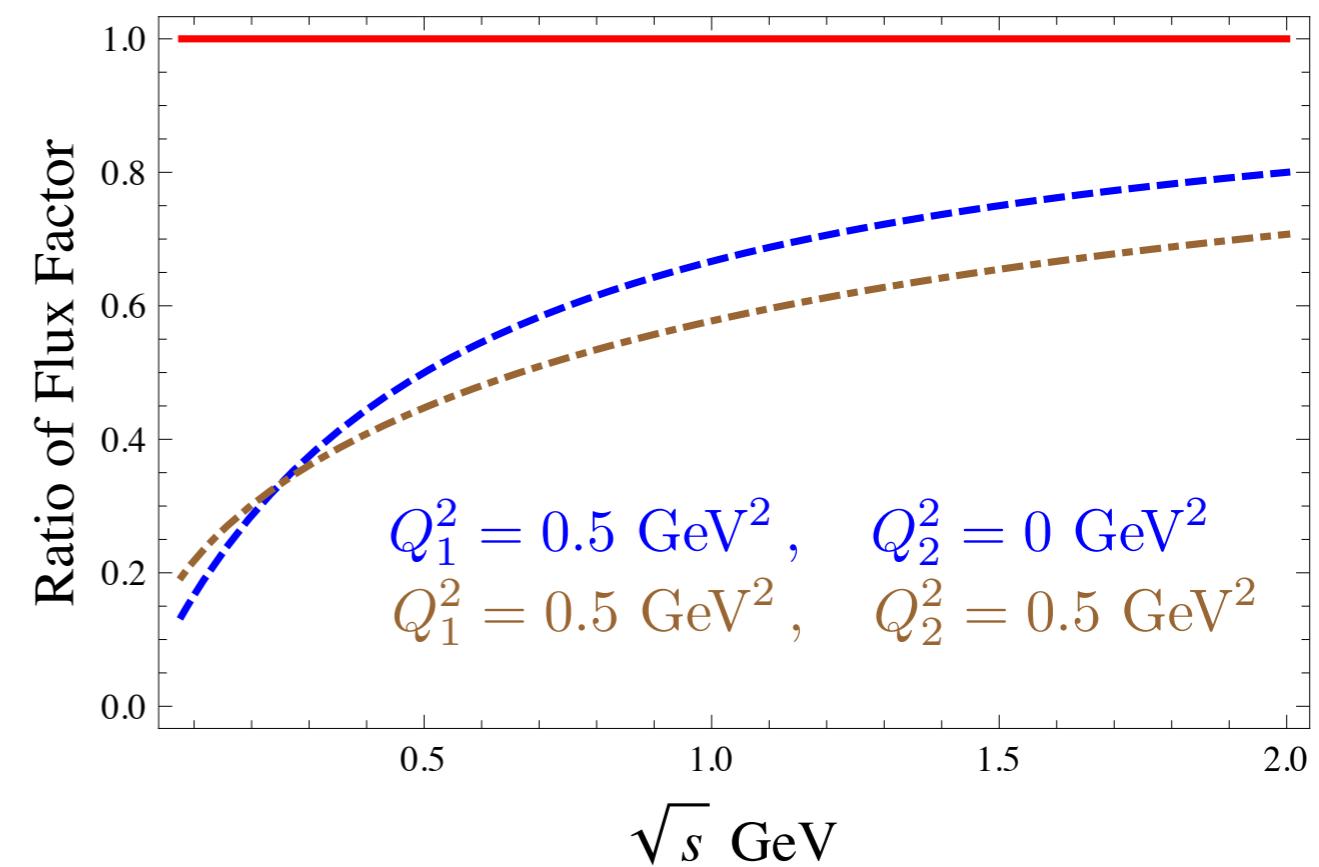
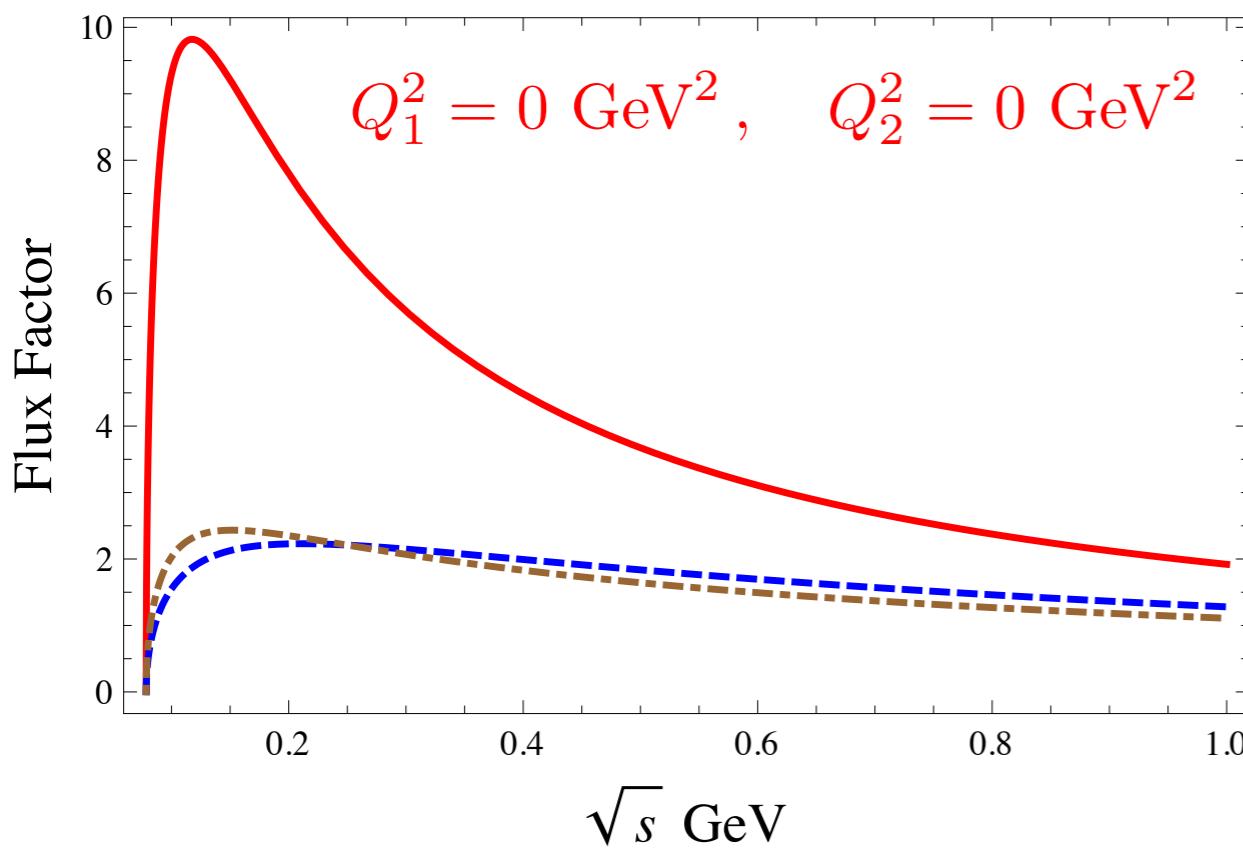
$\gamma^{(*)}\gamma^{(*)} \rightarrow \pi\pi$: flux factor

$$\left(\frac{d\sigma}{d \cos(\theta)} \right)_{CM} = \frac{\beta(s)}{64\pi \sqrt{X(s, Q_1^2, Q_2^2)}} (|F_{\Lambda=0}|^2 + |F_{\Lambda=2}|^2)$$

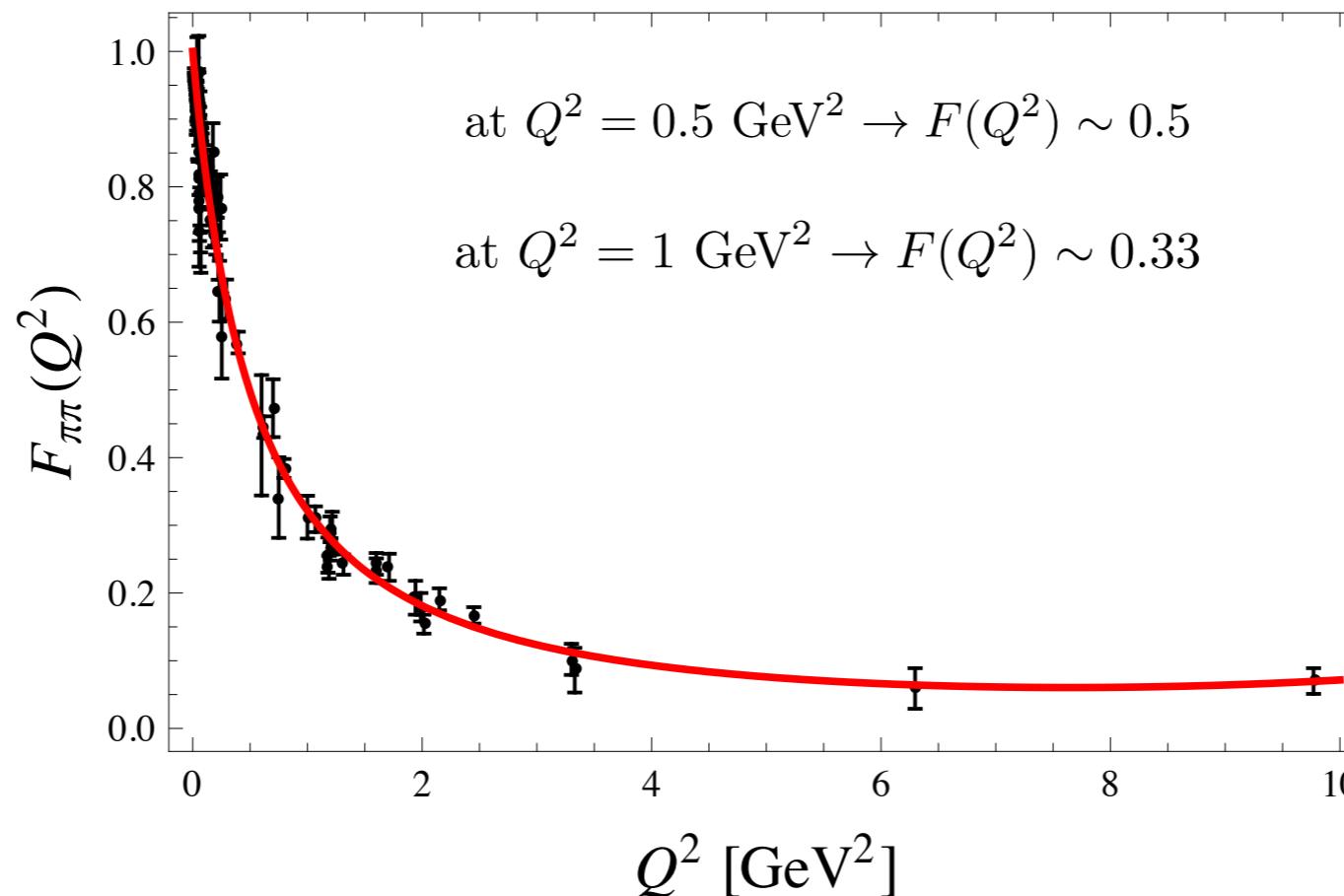
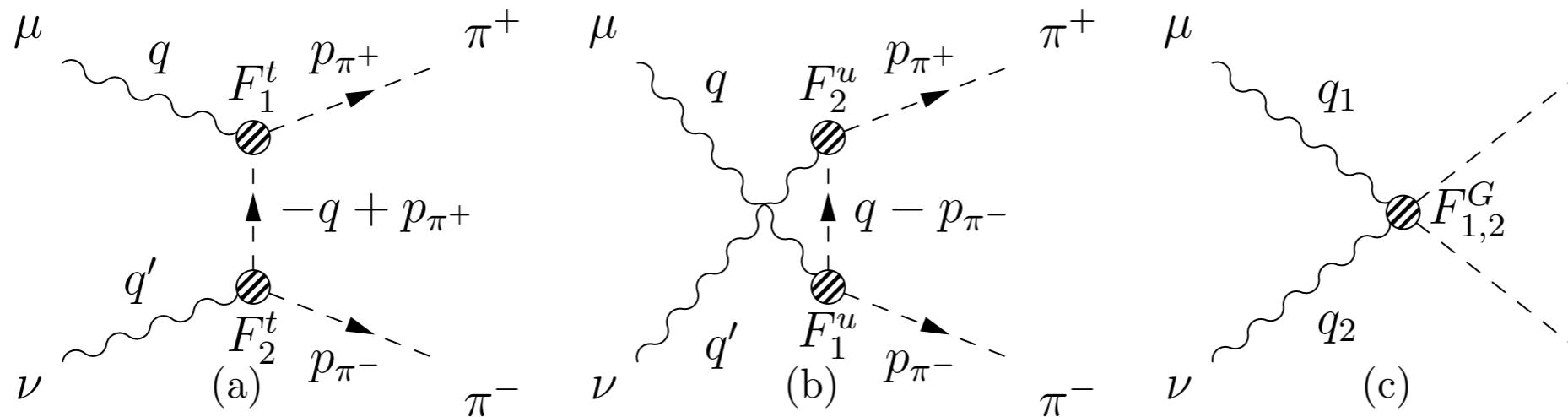
Flux factor

$$\beta = \sqrt{1 - \frac{4m_\pi^2}{s}}$$

$$X(s, Q_1^2, Q_2^2) = \frac{1}{4}(s + Q_1^2 + Q_2^2)^2 - Q_1^2 Q_2^2$$



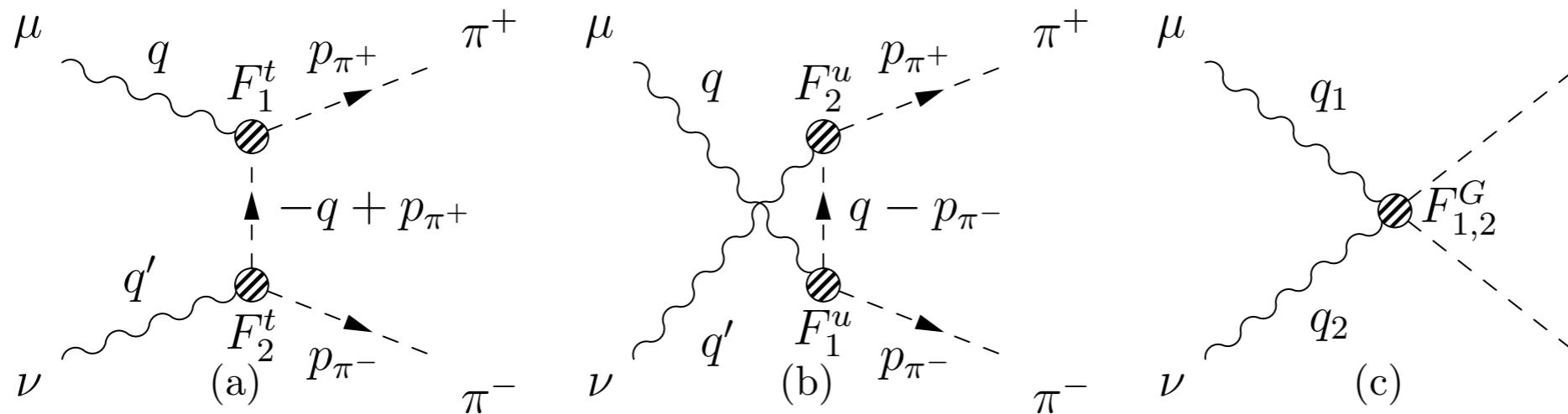
$\gamma^{(*)}\gamma^{(*)} \rightarrow \pi\pi$: form factor



[P.M, S. Peris, JJ Sanz-Cillero, '08]

Brown '73; Bebek '74'76; Dally '77
 Brauel '79; Amendolia (NA7)'86;
 Horn '06'07; Tadevosyan '07

$\gamma^{(*)}\gamma^{(*)} \rightarrow \pi\pi$: amplitudes



$$\mathcal{M}_{++} = F_1 F_2 e^2 \left(2 - s \beta^2 \nu \frac{\sin^2 \theta}{\nu^2 - X \beta^2 \cos^2 \theta} \right)$$

$$\mathcal{M}_{+-} = F_1 F_2 e^2 s \beta^2 \nu \frac{\sin^2 \theta}{\nu^2 - X \beta^2 \cos^2 \theta}$$

$$\beta = \sqrt{1 - \frac{4m_\pi^2}{s}}$$

$$X(s, Q_1^2, Q_2^2) = \frac{1}{4}(s + Q_1^2 + Q_2^2)^2 - Q_1^2 Q_2^2$$

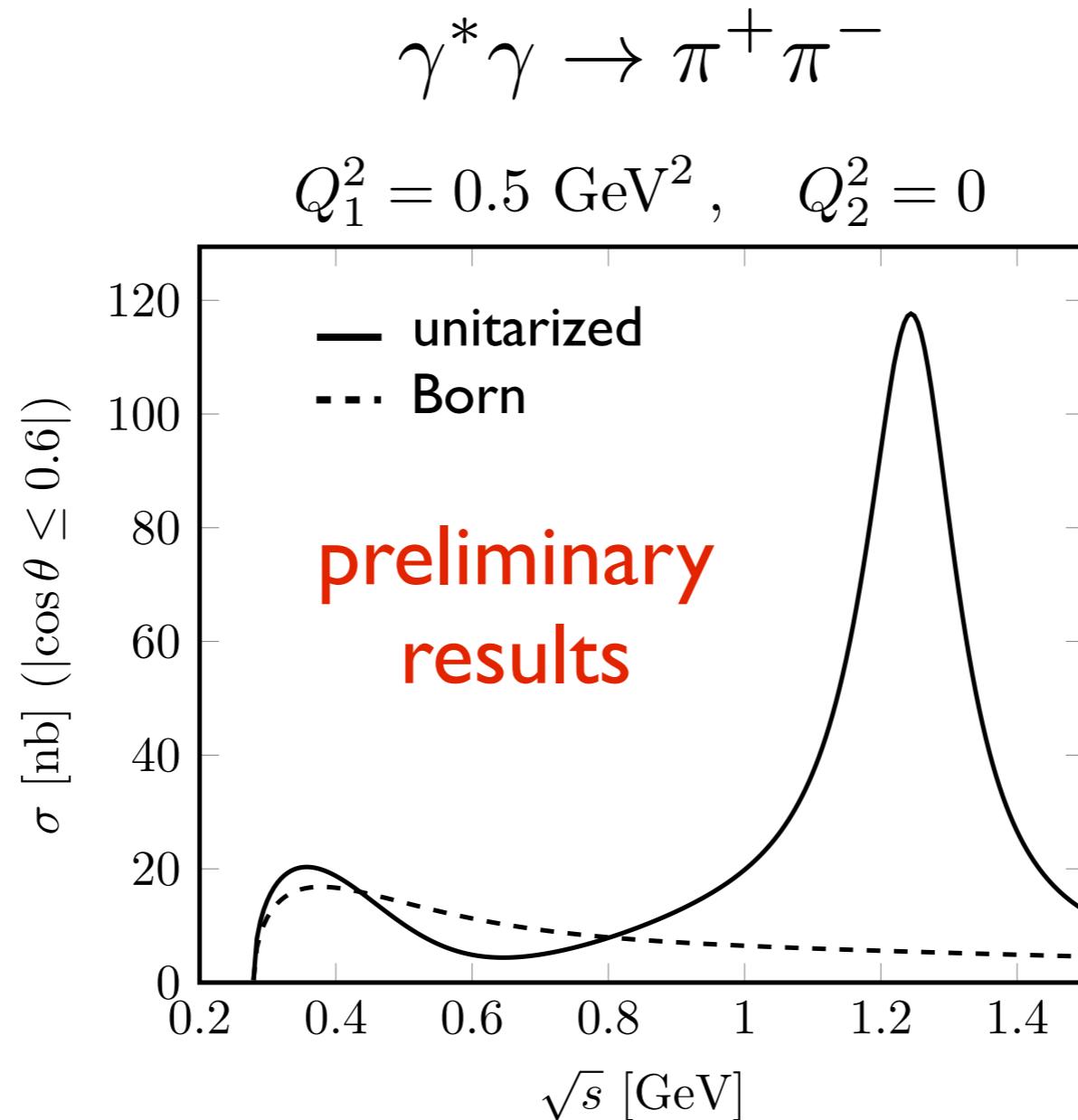
$$\nu(s, Q_1^2, Q_2^2) = \frac{1}{2}(s + Q_1^2 + Q_2^2)$$

F_1, F_2 form factors

$$\mathcal{M}_{00} = 2F_1 F_2 e^2 \sqrt{Q_1^2 Q_2^2} \frac{-\nu + s \beta^2 \cos^2 \theta}{\nu^2 - X \beta^2 \cos^2 \theta}$$

both photons are longitudinal

$\gamma^*\gamma \rightarrow \pi\pi$: main pieces



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$\gamma\gamma^*\rightarrow\pi\pi$: conclusions

- First glance into $\gamma\gamma^*\rightarrow\pi\pi$ (very preliminary results)
- Longitudinal photon contribution
- Interesting region around and up to $Q^2 = 1 \text{ GeV}^2$

$\gamma\gamma^*\rightarrow\pi\pi$: outlook

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Thank you!