Meissner-like effect in neutral trapped Fermi gases with arbitrary interaction throughout the BCS-BEC crossover

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Motivations:

⋆ S. Simonucci, P. Pieri, and GCS, published as an article in Nature Physics (September, 2015).

In principle, the Bogoliubov-de Gennes (BdG) eqs

$$\begin{pmatrix} \mathcal{H}(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta(\mathbf{r})^* & -\mathcal{H}(\mathbf{r})^* \end{pmatrix} \begin{pmatrix} u_\nu(\mathbf{r}) \\ v_\nu(\mathbf{r}) \end{pmatrix} = \epsilon_\nu \begin{pmatrix} u_\nu(\mathbf{r}) \\ v_\nu(\mathbf{r}) \end{pmatrix}$$

where $\mathcal{H}(\mathbf{r}) = \frac{(i\nabla + \mathrm{A}(\mathbf{r}))^2}{2m} + V(\mathbf{r}) - \mu$, form the basis for the calculation of physical quantities for superconductors in inhomogeneous situations

$$\Longleftrightarrow$$

the gap parameter $\Delta(\mathbf{r})$ varies with $\mathbf{r}$ and is determined via the self-consistent equation:

$$\Delta(\mathbf{r}) = -v_0 \sum_{\nu} u_{\nu}(\mathbf{r}) v_{\nu}(\mathbf{r})^* \left[1 - 2 f_F(\epsilon_{\nu})\right]$$

where

v_0 = coupling constant of contact interaction

$f_F(\epsilon) = (e^{\epsilon/(k_B T)} + 1)^{-1}$ = Fermi function.

In practice, the use of the BdG equations is limited to a few situations (single vortex, a few vortices, Josephson effect, isolated solitons, $\cdots$) due to limitations in computer time and memory space.

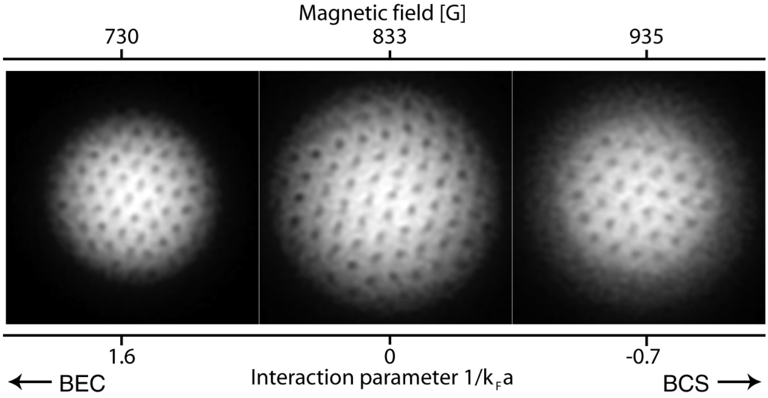
Two kinds of questions:

- Fundamental question: To what extent is the detailed information contained in the BdG eqs really needed?
  ("old" question, raised by Eilenberger in 1968)

  And what happens when the inter-particle coupling gets strong? (BCS-BEC crossover)

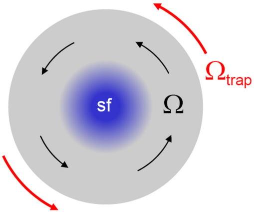
- Practical question: How is it possible to account for experiments with ultra-cold Fermi gases (but with other systems as well), which may reveal the superfluid phase in the presence of complex arrays of vortices ?

The MIT experiment on vortex lattices [Nature **435**, 1047 (2005)]:



Observation of vortex lattices in rotating Fermi gases provides definite evidence for superfluidity !

The Innsbruck experiment [New J. of Phys. **13**, 035003 (2011)]:

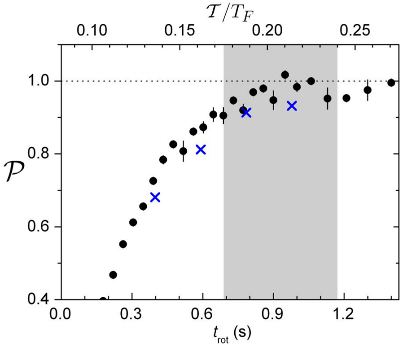


rotating trap

$$\mathcal{P} = \frac{\Theta}{\Theta_{\rm rig}} \times \frac{\Omega}{\Omega_{\rm trap}} \leq 1$$

$\mathcal{P}$ precession parameter

Θ moment of inertia



$\mathcal{P}$ vs temperature at unitarity ($\Omega_{\rm trap}$ is fixed)

BdG eqs $\rightarrow$ differential eqs for $\Delta(\mathbf{r})$:

There are cases when the BdG eqs can be replaced by (simpler to solve) differential equations for $\Delta(\mathbf{r})$:

- Ginzburg-Landau (GL) equation for strongly overlapping Cooper pairs [Gorkov (1959)]:

$$\left[\frac{6\pi^2(k_B T_c)^2}{7\zeta(3)E_F}\left(1-\frac{T}{T_c}\right)+\frac{\nabla^2}{4\,m}\right]\Delta_{\mathrm{GL}}(\mathbf{r})-\frac{3}{4E_F}\,|\Delta_{\mathrm{GL}}(\mathbf{r})|^2\Delta_{\mathrm{GL}}(\mathbf{r})=0$$

which holds close to T_c in the (extreme) BCS limit $(\mu \simeq E_F)$.

BdG eqs $\rightarrow$ differential eqs for $\Delta(\mathbf{r})$:

- Gross-Pitaevskii (GP) equation for dilute composite bosons [Pieri & GCS (2003)]:

$$-\quad \frac{\nabla^2}{4\,m}\,\Phi(\mathbf{r}) + 2\,V(\mathbf{r})\,\Phi(\mathbf{r}) + \frac{8\pi a_F}{2\,m}\,|\Phi(\mathbf{r})|^2\Phi(\mathbf{r}) = \mu_B\,\Phi(\mathbf{r})$$

$$\text{with} \quad \Phi(\mathbf{r}) = \sqrt{\frac{m^2 a_F}{8\pi}}\,\Delta(\mathbf{r})$$

which holds at low T in the BEC limit where

$2\mu \simeq -\varepsilon_0 + \mu_B, \qquad \varepsilon_0 = \frac{1}{m a_F^2} =$ binding energy

$a_F =$ 2-fermion scattering length $\qquad (\mu_B \ll \varepsilon_0)$.

Small parameters for GL and GP eqs:

To derive the GL and GP eqs from the BdG eqs, one exploits the existence of a “small parameter” η:

$$\eta = \frac{|\Delta(\mathbf{r})|}{k_B T_c} \ll 1$$

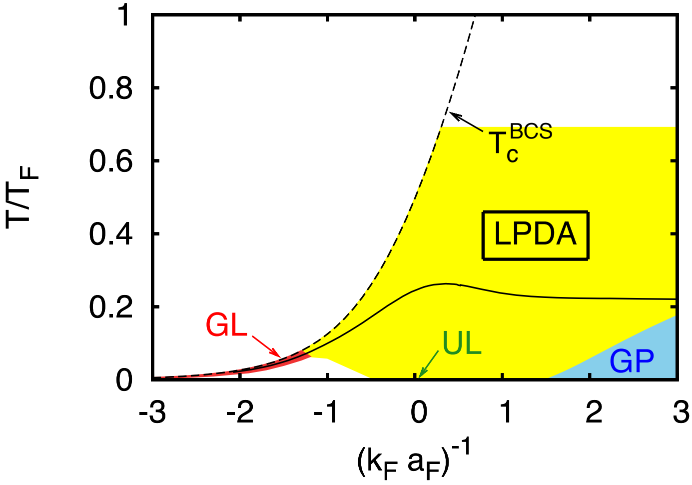
for GL equation

$$\eta = \frac{|\Delta(\mathbf{r})|}{|\mu|} \ll 1$$

for GP equation

Question: Can one do something better than this, and replace the BdG eqs by a differential equation for $\Delta(\mathbf{r})$ over an extended region of the “temperature vs coupling diagram” ?

Temperature vs coupling phase diagram of the BCS-BEC crossover:

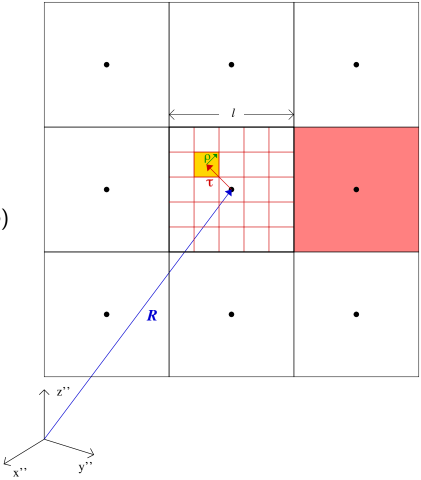


Coarse graining of the BdG equations:

We have considered the following *double coarse-graining procedure* [S. Simonucci and GCS, PRB **89**, 054511 (2014)]:

$$\mathbf{r}'' = \mathbf{R} + \tau + \rho$$

$$\Delta(\mathbf{r}'') = \tilde{\Delta}(\mathbf{R})\, e^{2i\mathbf{Q}(\mathbf{R},\tau)\cdot(\mathbf{R}+\tau+\rho)}$$



The LPDA differential equation for $\Delta(\mathbf{r})$:

After some manipulations and approximations based on the "smoothness" of $\Delta(\mathbf{r})$, from the BdG eqs one arrives at:

$$-\frac{m}{4\pi a_F}\,\Delta(\mathbf{r}) = \mathcal{I}_0(\mathbf{r})\,\Delta(\mathbf{r}) + \mathcal{I}_1(\mathbf{r})\,\frac{\nabla^2}{4m}\Delta(\mathbf{r}) - \mathcal{I}_1(\mathbf{r})\,i\,\frac{\mathbf{A}(\mathbf{r})}{m}\cdot\nabla\Delta(\mathbf{r})$$

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$$\text{with}\quad \mathcal{I}_0(\mathbf{r}) = \int\frac{d\mathbf{k}}{(2\pi)^3}\left\{\frac{\left[1-2f_F(E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r}))\right]}{2\,E(\mathbf{k}|\mathbf{r})} - \frac{m}{\mathbf{k}^2}\right\}$$

$$\mathcal{I}_1(\mathbf{r}) = \frac{1}{2}\int\frac{d\mathbf{k}}{(2\pi)^3}\left\{\frac{\xi(\mathbf{k}|\mathbf{r})}{2\,E(\mathbf{k}|\mathbf{r})^3}\left[1-2f_F(E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r}))\right]\right.$$
$$\left. + \frac{\xi(\mathbf{k}|\mathbf{r})}{E(\mathbf{k}|\mathbf{r})^2}\,\frac{\partial f_F(E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r}))}{\partial E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r})} - \frac{\mathbf{k}\cdot\mathbf{A}(\mathbf{r})}{\mathbf{A}(\mathbf{r})^2}\,\frac{1}{E(\mathbf{k}|\mathbf{r})}\,\frac{\partial f_F(E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r}))}{\partial E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r})}\right\}$$

$$\text{where}\quad \xi(\mathbf{k}|\mathbf{r}) = \frac{\mathbf{k}^2}{2m} - \bar{\mu}(\mathbf{r}) \quad,\quad \bar{\mu}(\mathbf{r}) = \mu - V(\mathbf{r}) - \frac{\mathbf{A}(\mathbf{r})^2}{2m}$$

$$E(\mathbf{k}|\mathbf{r}) = \sqrt{\xi(\mathbf{k}|\mathbf{r})^2 + |\Delta(\mathbf{r})|^2} \quad,\quad E_+^{\mathbf{A}}(\mathbf{k}|\mathbf{r}) = E(\mathbf{k}|\mathbf{r}) - \frac{\mathbf{k}\cdot\mathbf{A}(\mathbf{r})}{m}$$

Recovering GL and GP eqs from LPDA eq:

- GL eq $\quad \Leftarrow \quad (k_F a_F)^{-1} \ll -1 \quad$ and $\quad T \simeq T_c$:

$$\mathcal{I}_0(\mathbf{r}) \cong -\frac{m}{4\pi a_F} + N_0 \frac{(T_c - T)}{T_c} - \frac{7\,\zeta(3)}{8\pi^2} \frac{N_0}{(k_B T_c)^2} |\Delta(\mathbf{r})|^2$$

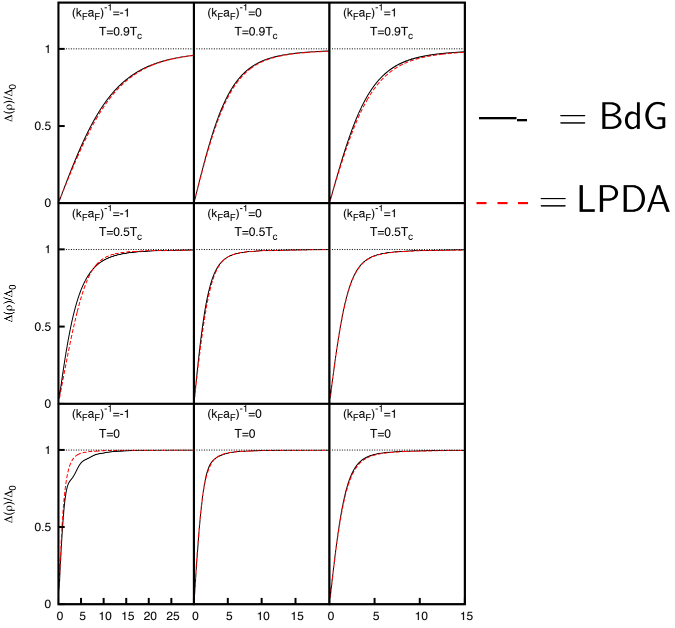
$$\mathcal{I}_1(\mathbf{r}) \cong \frac{k_F^2}{2m} \frac{N_0}{6(k_B T_c)^2} \int_0^{\infty} \frac{dy}{y} \frac{\tanh y}{\cosh^2 y} \qquad \left(N_0 = \frac{m k_F}{2\pi^2} \right)$$

- GP eq $\quad \Leftarrow \quad (k_F a_F)^{-1} \gg +1 \quad$ and $\quad T \ll T_c$:

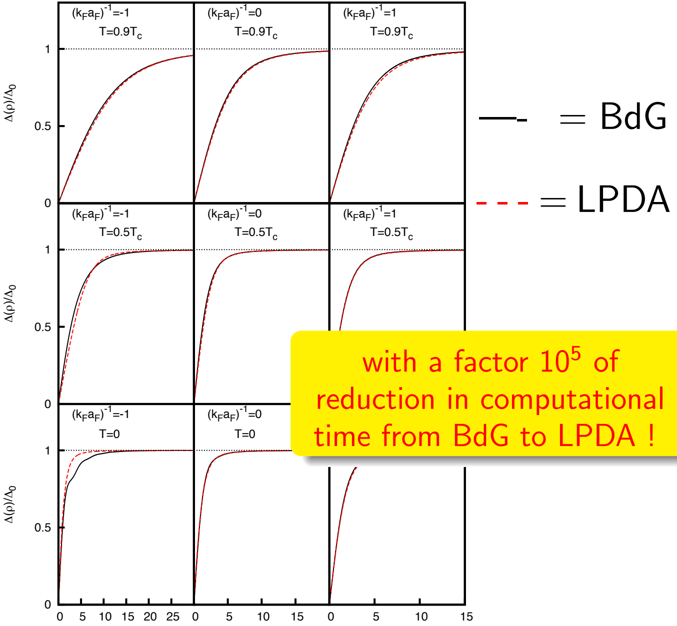
$$\mathcal{I}_0(\mathbf{r}) \cong -\frac{m}{4\pi a_F} + \frac{m^2 a_F}{8\pi} \left[\mu_B - 2\,V(\mathbf{r}) - \frac{m a_F^2}{2\pi} |\Delta(\mathbf{r})|^2 \right]$$

$$\mathcal{I}_1(\mathbf{r}) \cong \frac{m^2 a_F}{8\pi} \qquad \left(\varepsilon_0 = \frac{1}{m a_F^2} = -2\mu + \mu_B \right)$$

Testing the LPDA eq vs the BdG eqs: $\Delta(\mathbf{r})$ for an isolated vortex

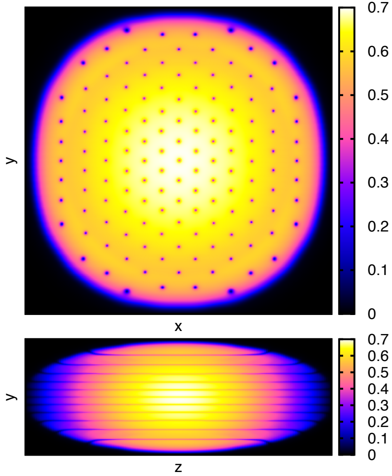


Testing the LPDA eq vs the BdG eqs: $\Delta(\mathbf{r})$ for an isolated vortex



with a factor 10^5 of reduction in computational time from BdG to LPDA !

Comparison with MIT experiment (# 1):



rotating trap with:

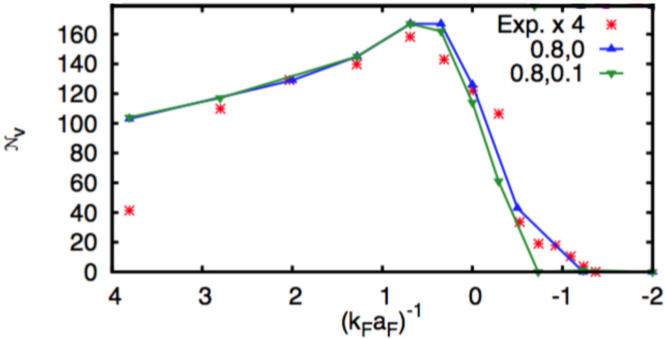
$$\frac{\mathbf{A}(\mathbf{r})}{m} = \boldsymbol{\Omega} \times \mathbf{r}$$

$$\Omega = 0.8\,\Omega_{\mathrm{r}} \quad (T = 0)$$

137 vortices

11 (bending) filaments

Comparison with MIT experiment (# 2):

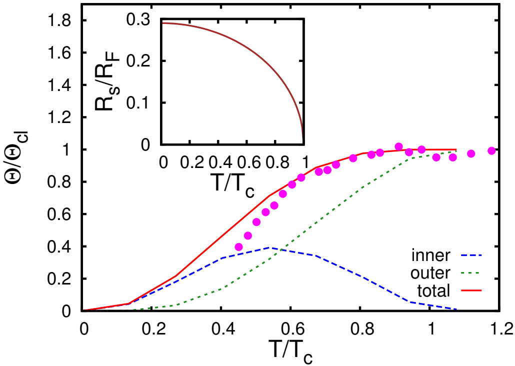


rotating trap with $\Omega = 0.8\,\Omega_{\rm r}$ and $T = (0.0, 0.1)\,T_F$

$*$: experimental values (multiplied by a common factor of 4)

$\leftrightarrow$ Feynman theorem satisfied only in (about) $1/4$ of cloud!

Comparison with Innsbruck exper. (# 1):



temperature dependence of $\Theta/\Theta_{\rm cl}$ for $\Omega \to 0$ at unitarity

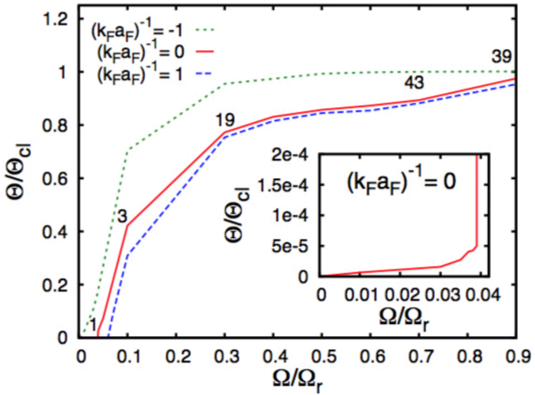
- - - contributed by the "inner" (superfluid) portion of cloud

· · · contributed by the "outer" (normal) portion of cloud

—— sum of the two contributions

• experimental data

Comparison with Innsbruck exper. (# 2):



moment of inertia $\Theta = L/\Omega$ in units of its classical value $\Theta_{\rm cl}$

for various couplings at $T = 0$

not too many vortices are needed to stabilize Θ at $\Theta_{\rm cl}$.

Yrast effect (of order of $1/N \cong 10^{-5}$) before $1^{\rm st}$ vortex enters.

Conclusions & Perspectives:

♣ The LPDA equation for $\Delta(\mathbf{r})$ works well when compared with BdG eqs over a wide portion of coupling-vs-temperature diagram.

♣ It reduces to the GL and GP equations in the appropriate (coupling and temperature) limits.

♣ Finding solutions with large vortex patterns is now possible in terms of the LPDA equation.

♣ Future plans are to consider:

- Imbalanced spin populations
- Correlations beyond mean field
- Time-dependent version

Thank you for your attention !