Single File escape dynamics in microfluidic channels

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Theory ⟷ Experiments

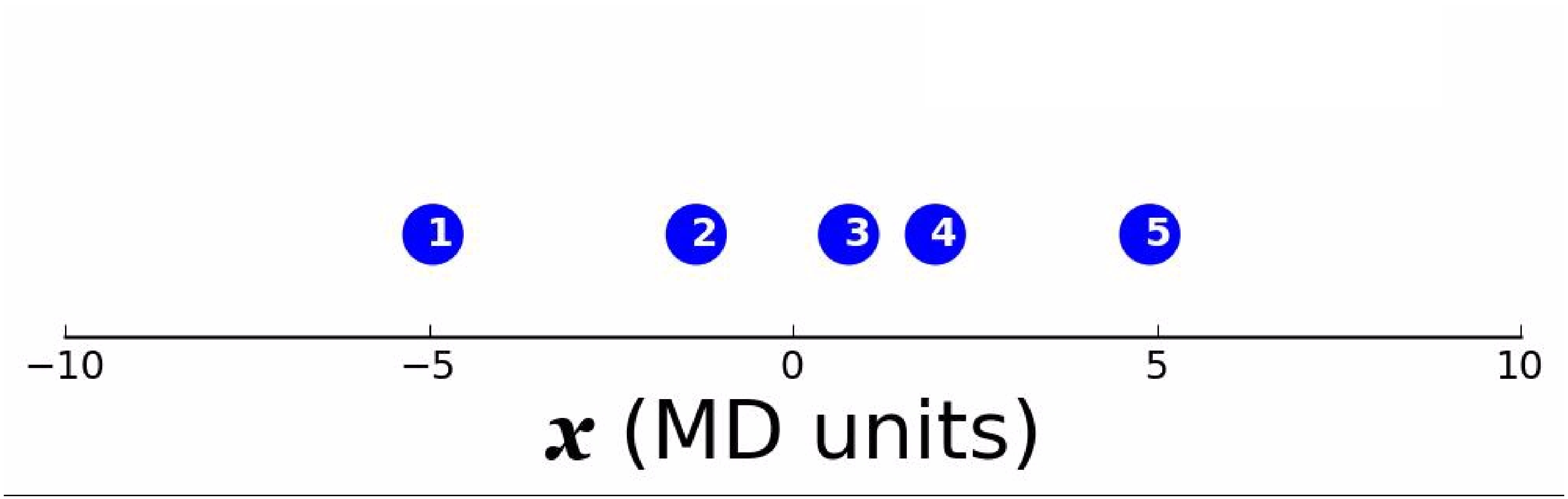
Dipartimento di Fisica
Galileo Galilei

In collaboration with Dr. Stefano Pagliara
Prof. Ulrich Keyser group

UNIVERSITY OF
CAMBRIDGE

Cavendish
Laboratory

Single File Diffusion



SFD ingredients:

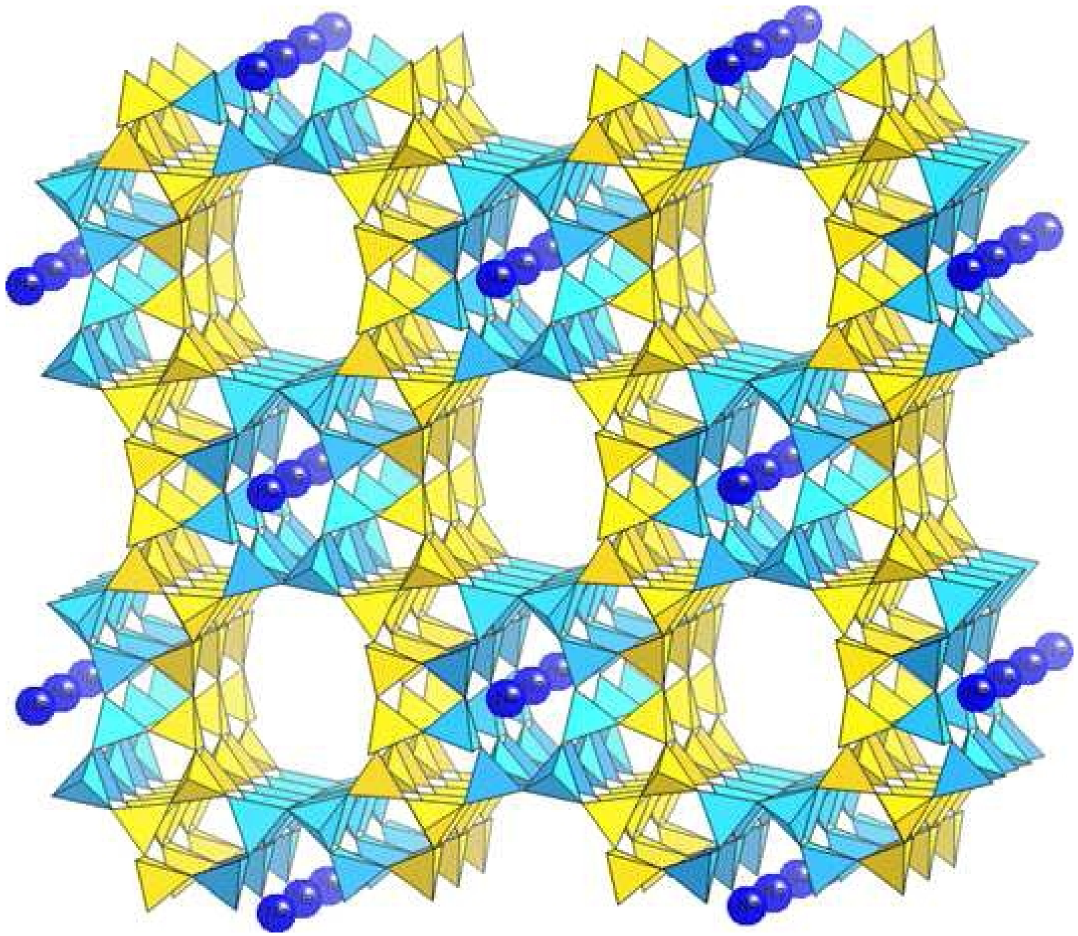
- **Order preservation**
- **Free diffusion between collision events**

SFD - examples

Diffusion in micro/nanoporous materials

Hahn, Karger, Kukla, Phys. Rev. Lett., (1996)

Mukherjee, *et al*., ACS Nano (2010)

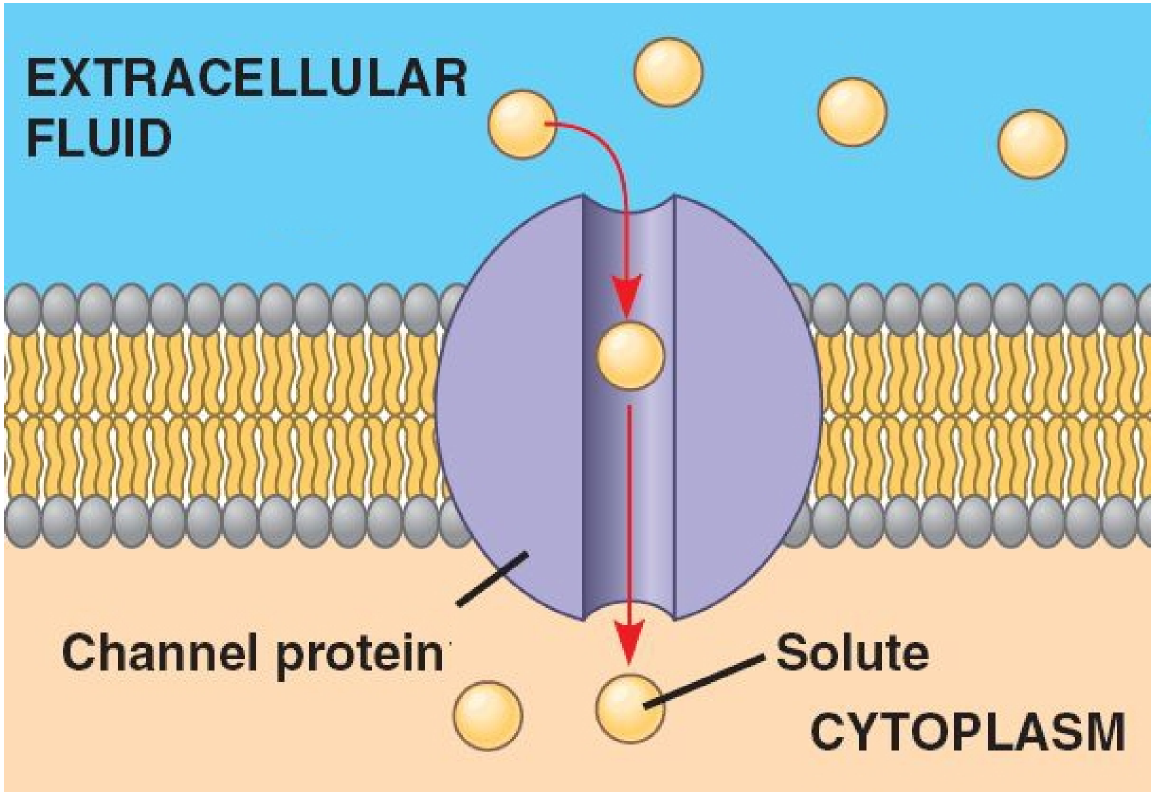


SFD - examples

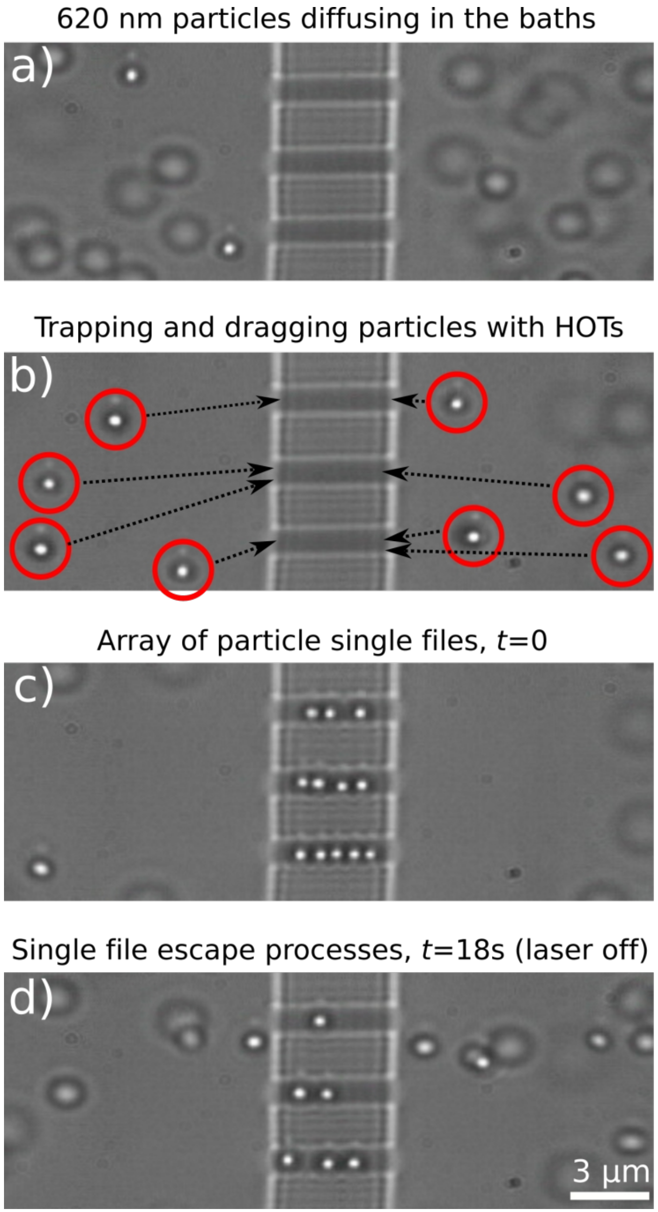
Transport of ions through nanopores

Hodgkin, Keynes,
J. Physiol., (1955)

Jensena, *et al*., PNAS (2010)

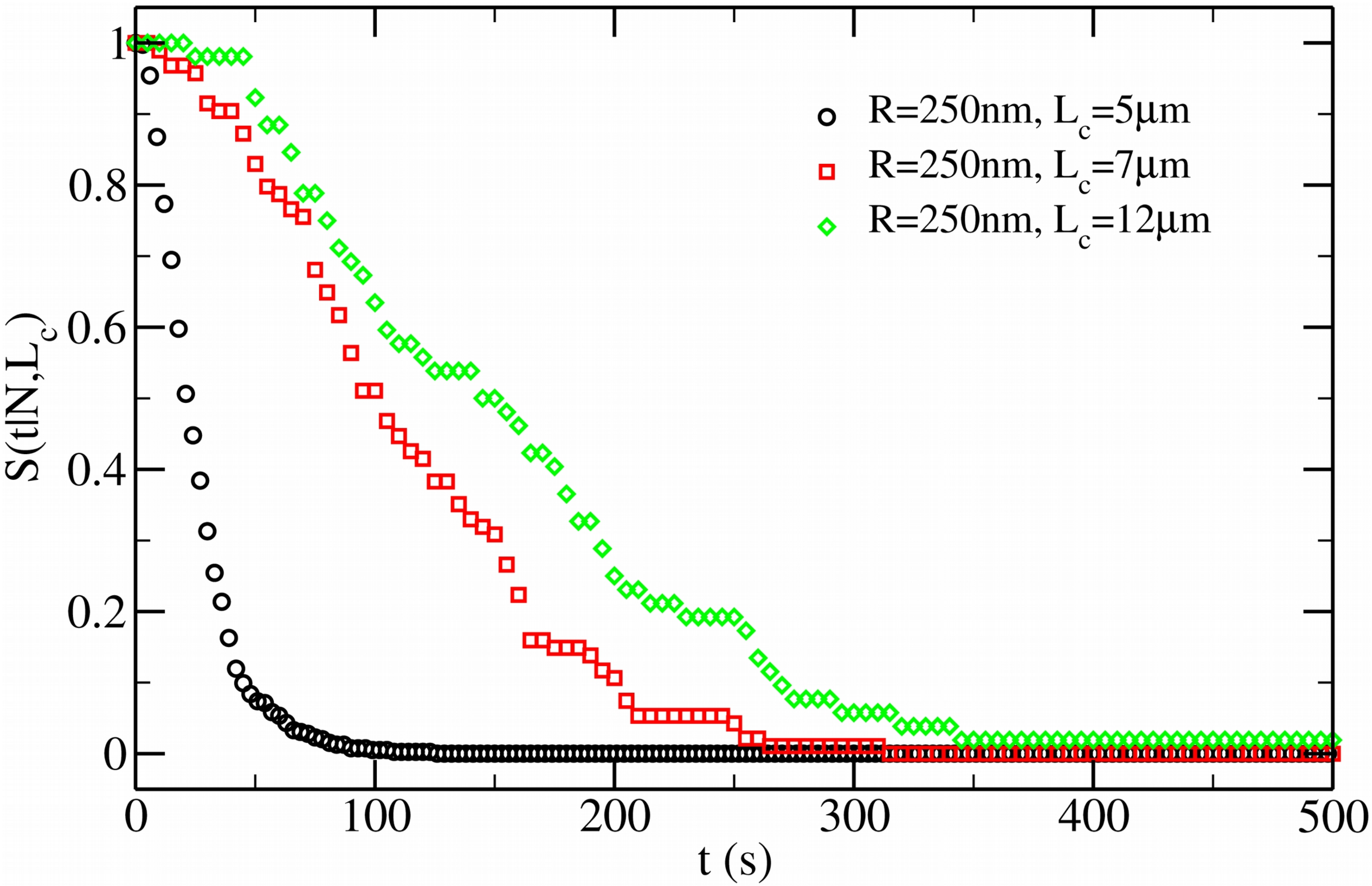


SFD of colloidal particles



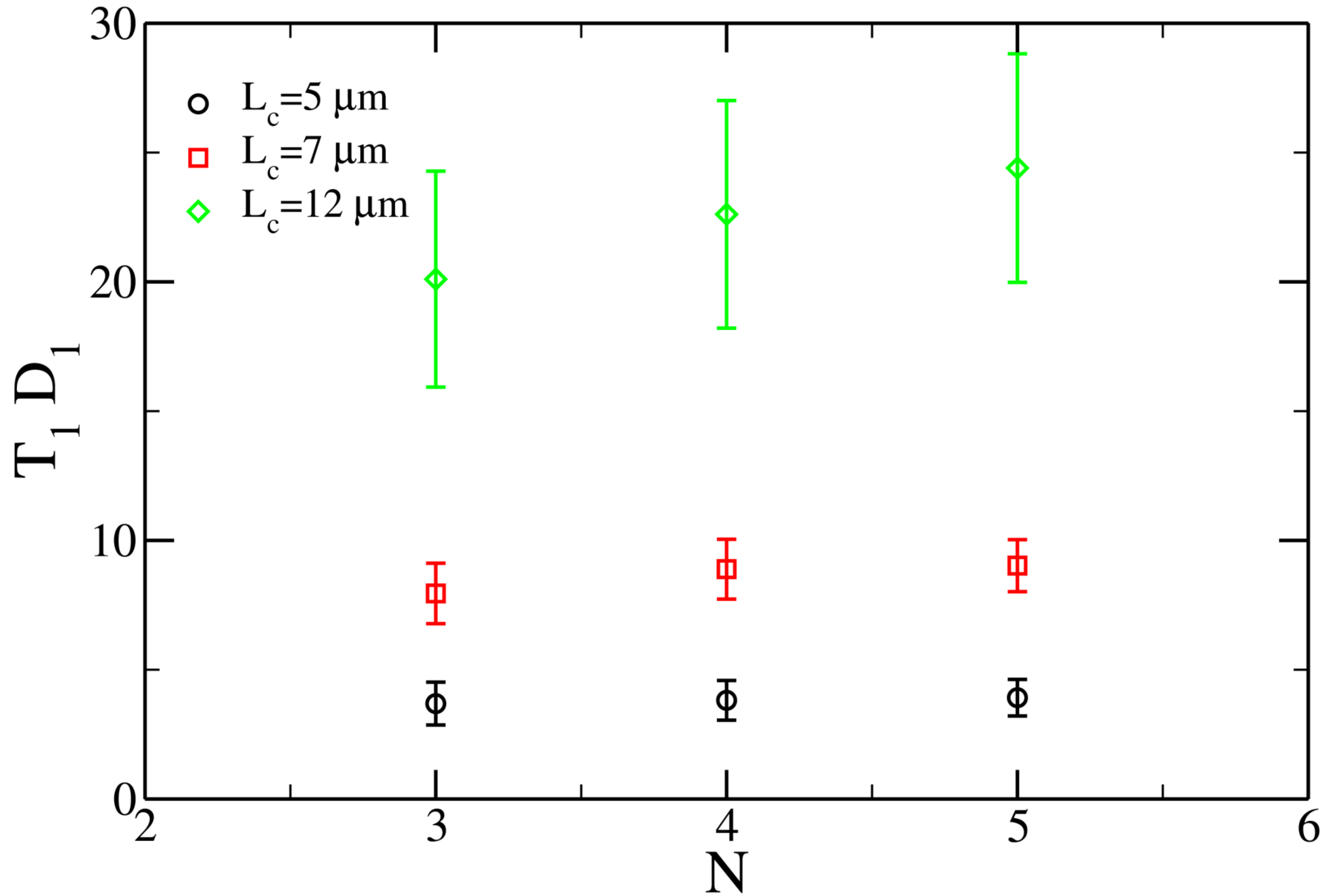
Experimental data

N = 3

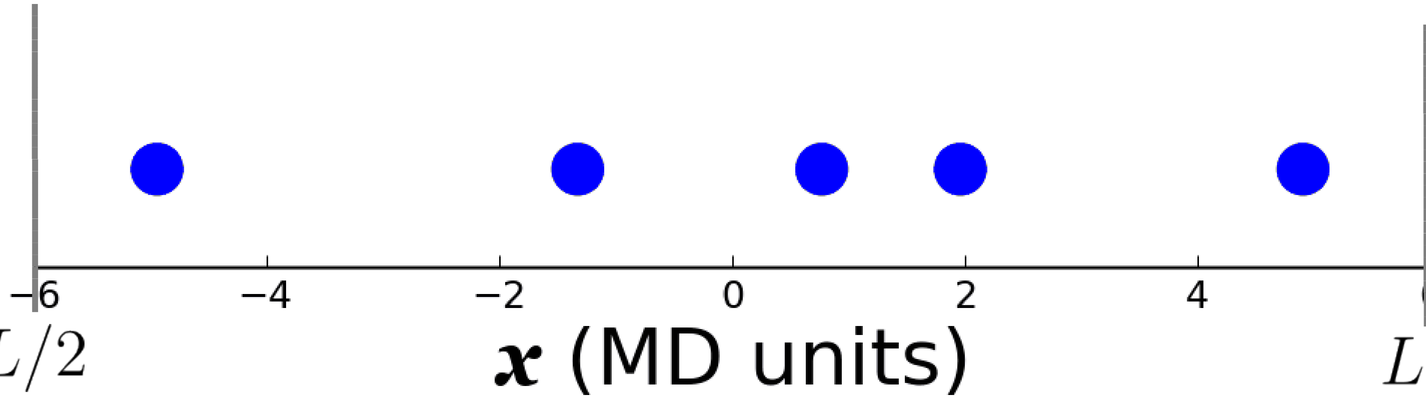


Experimental data

R=250 nm



Emptying process



$-L/2$ absorbing boundary

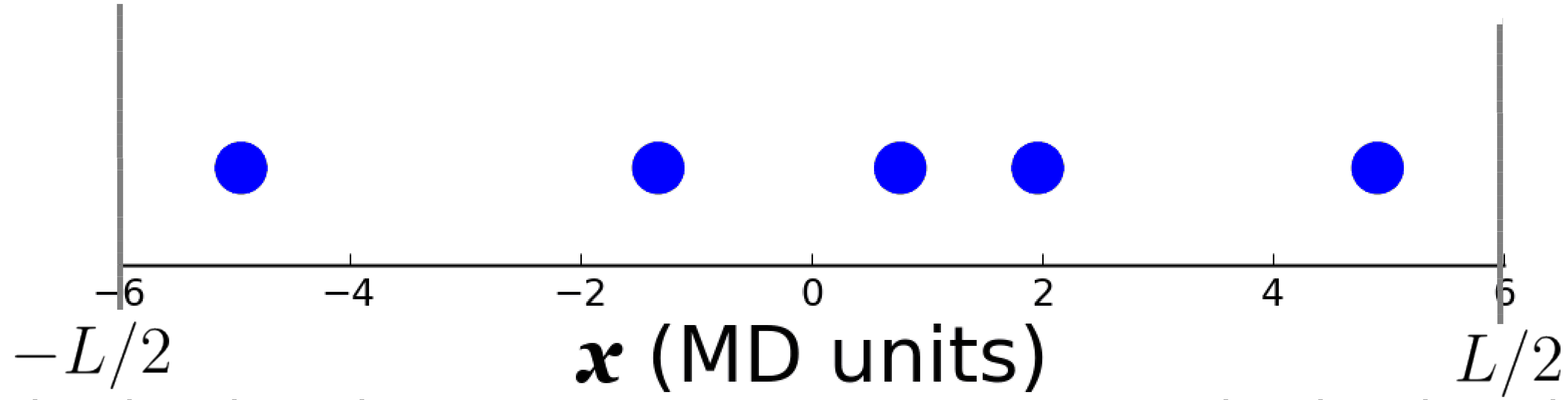
$L/2$ absorbing boundary

We want to characterize the probability of having *at least* one particle inside the channel [$-L_c/2, L_c/2$] $S_1(t|N, L_c, L_0)$

Mean First Passage Time → Characteristic survival time

$$T_1(N, L_c, L_0) = \int_0^{\infty} S(t|N, L_c, L_0)dt$$

Emptying process



absorbing boundary

absorbing boundary

We want to characterize the probability of having *at least* one particle inside the channel [-L_c/2,L_c/2] $S_1(t|N, L_c, L_0)$

For Single File systems we can reconstruct this process exactly, using the *Reflection Principle Method*

Reflection principle method

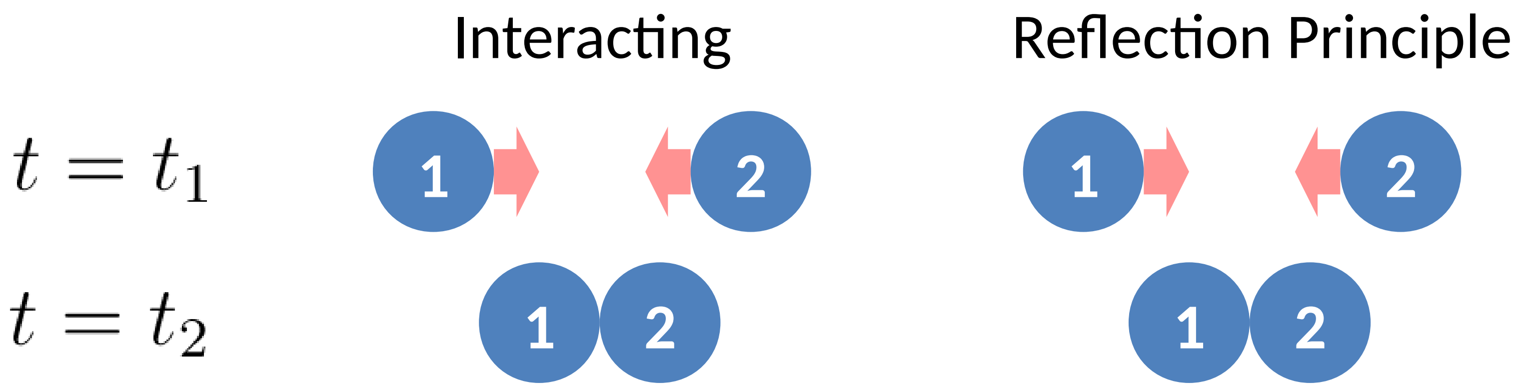
Mapping of the Single File into a non-interactive system

Interacting | Reflection Principle

$t = t_1$

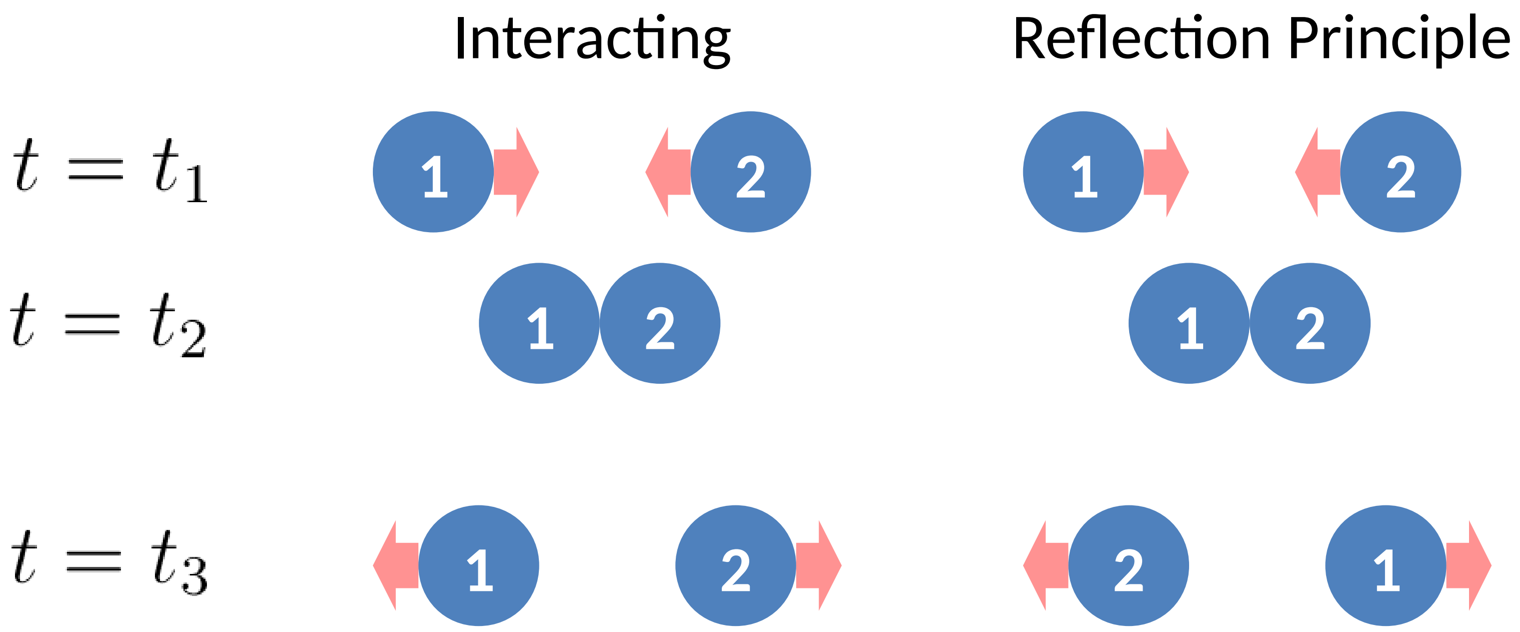
Reflection principle method

Mapping of the Single File into a non-interactive system



Reflection principle method

Mapping of the Single File into a non-interactive system



Reflection principle method

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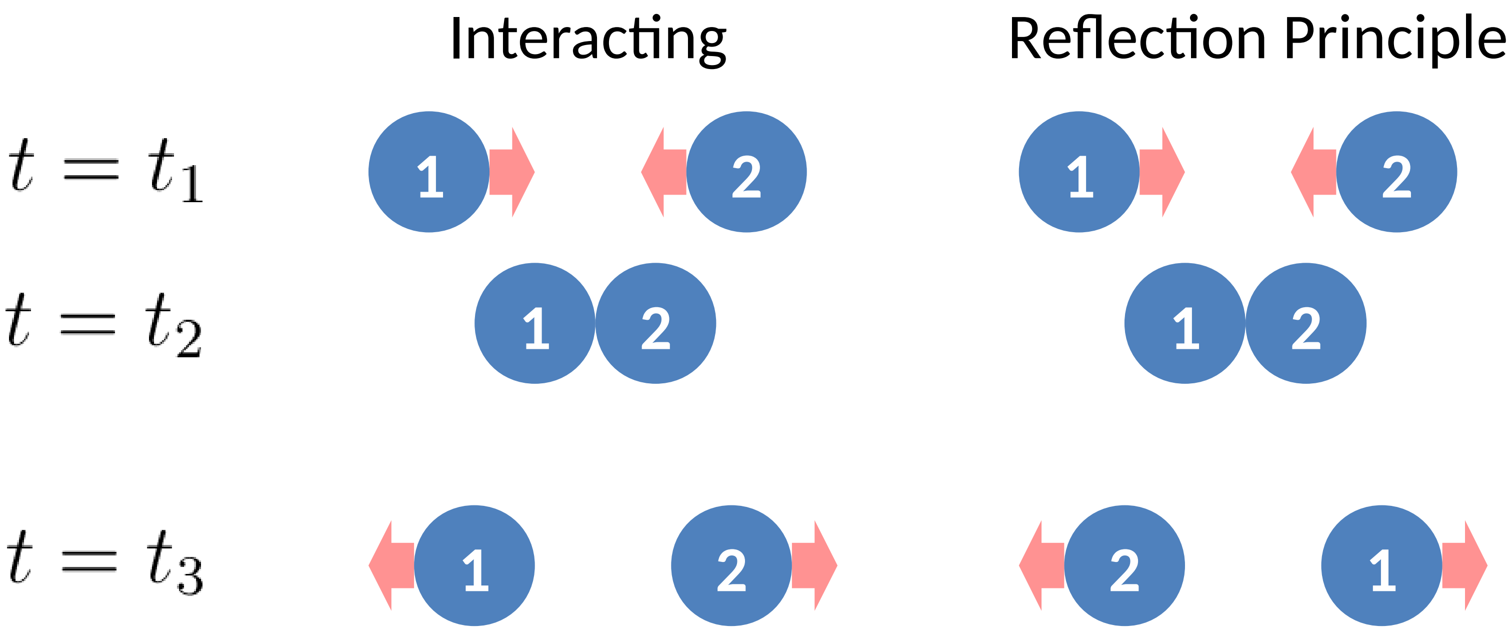
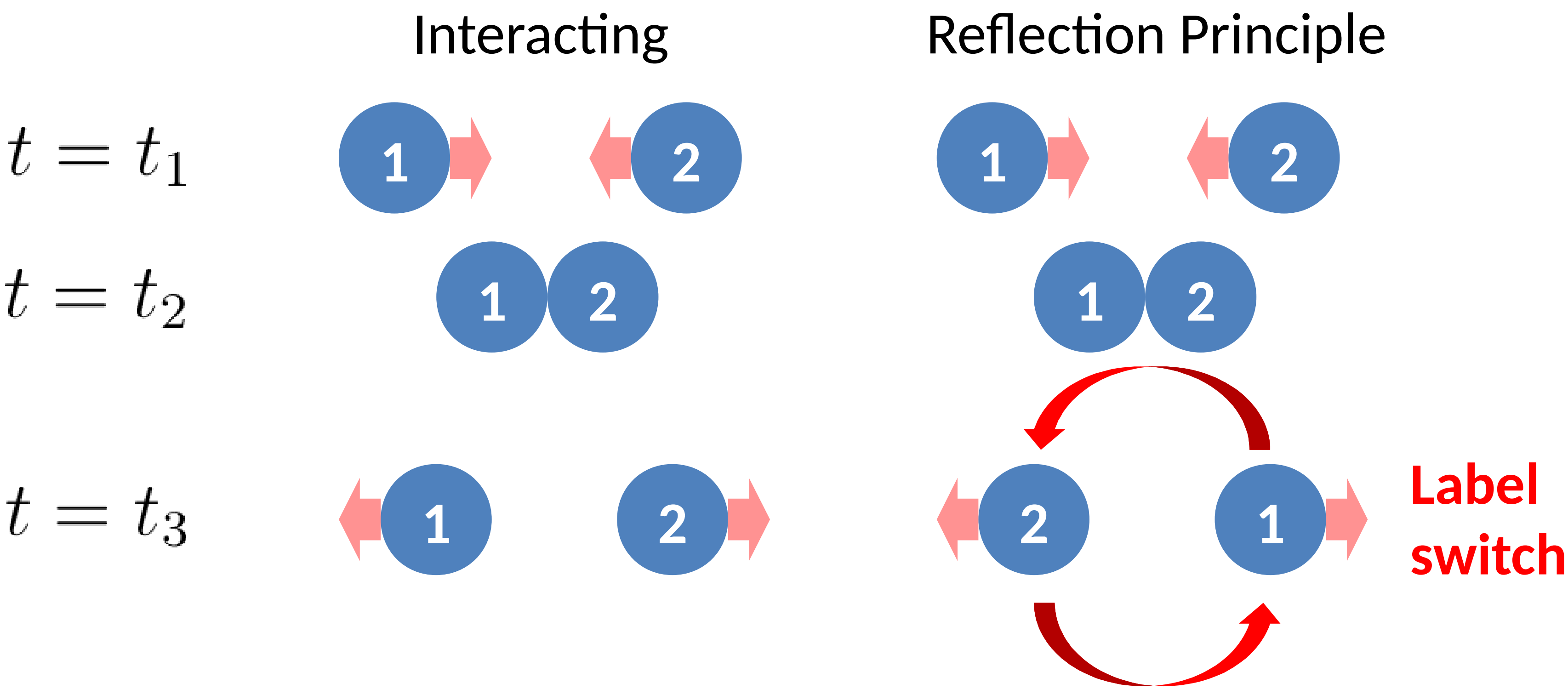


Essential ingredients:

- Identical particles
- Elastic collisions

Reflection principle method

Mapping of the Single File into a non-interactive system



Essential ingredients:

- Identical particles
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SFD - Emptying probability

Using the Reflection Principle method, it is possible to map a Single File system to the non-interactive equivalent

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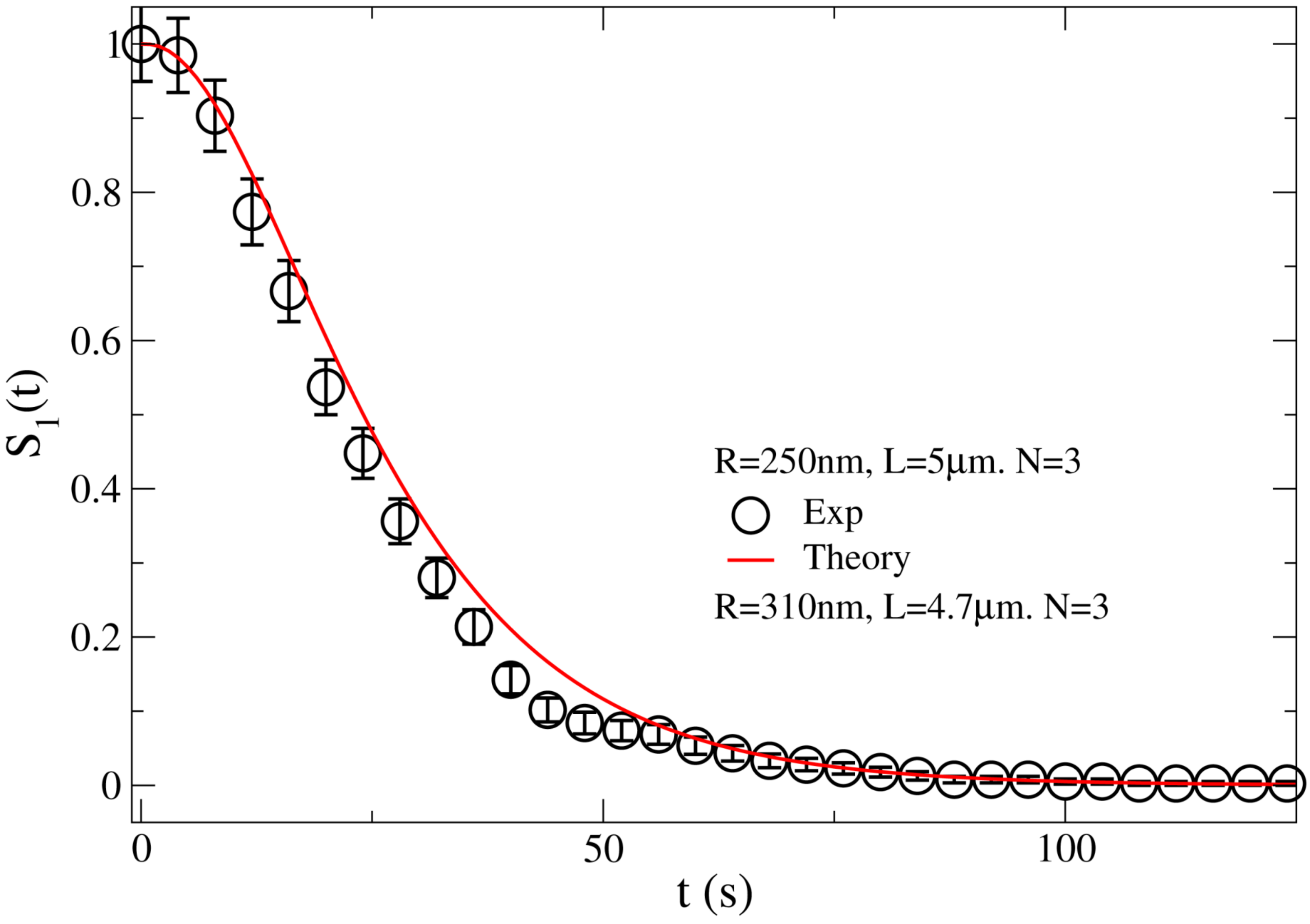
Single File of **point-like particles** (uniform initial conditions):

$$1 - S_1(t|N, L_c, L_0) = [1 - S_1(t|1, L_c, L_0)]^N$$

N particles survival probability

Single particle survival probability

SFD - Emptying probability



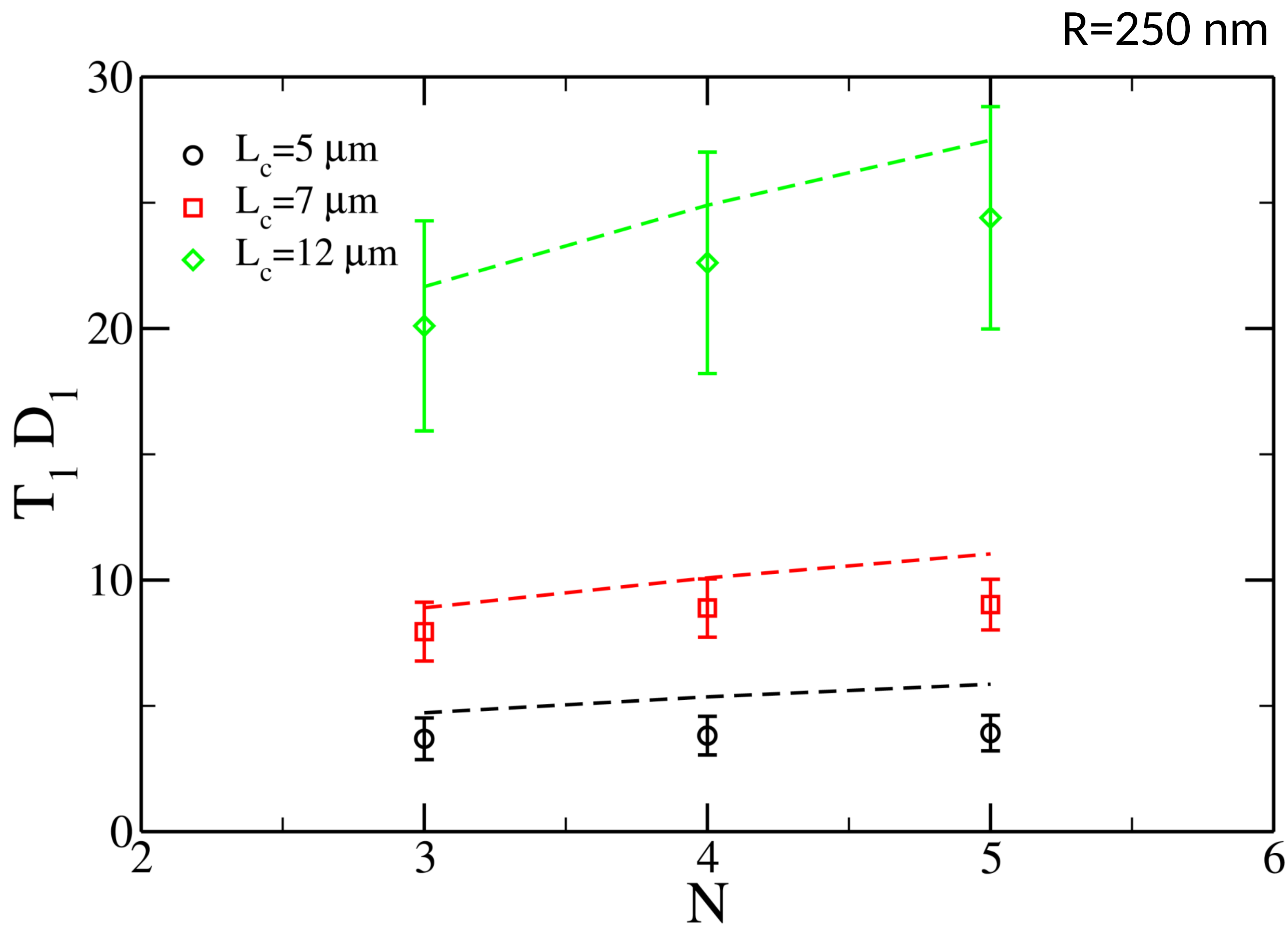
SFD – Mean Emptying Time

It is possible to integrate the last formula to obtain an analytical expression for the Mean Emptying Time, valid for **point-like** particles

$$T_1(N, L_c, L_0) = \frac{L_c^2}{D_1}\, g\left(N, \frac{L_0}{L_c}\right)$$

Valid in presence of small forces $\quad k_B T \gg F_e L_c$

SFD – Mean Emptying Time



SFD – Mean Emptying Time

It is also possible to include excluded volume contributions to the Mean Emptying Time using an effective theory, defining an **effective channel length**

$$L_{eff}(N, L_c, L_0, \Gamma, R) = \frac{\sum_{k=1}^{N} [T_k - T_{k+1}] \left(L_c - 2(k-1)R\right)}{T_1}$$

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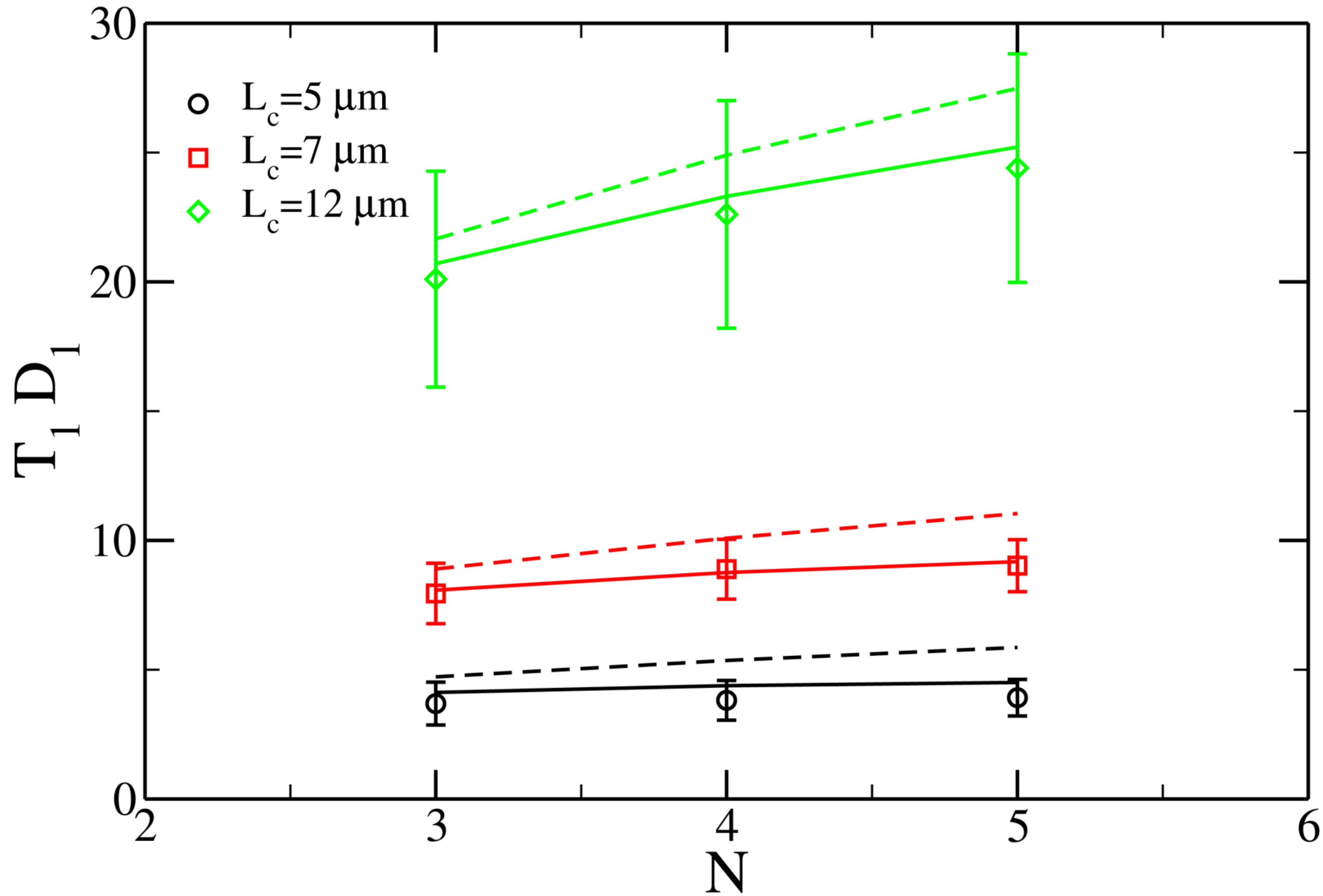
and substituting it into the analytical expression valid for point-like particles

$$T_1(N, L_c, L_0, R) = \frac{L_{eff}(N, L_c, L_0, R)^2}{D_1(R, \Phi)} \, g\left(N, \frac{L_0}{L_c}\right)$$

SFD – Mean Emptying Time

R=250 nm



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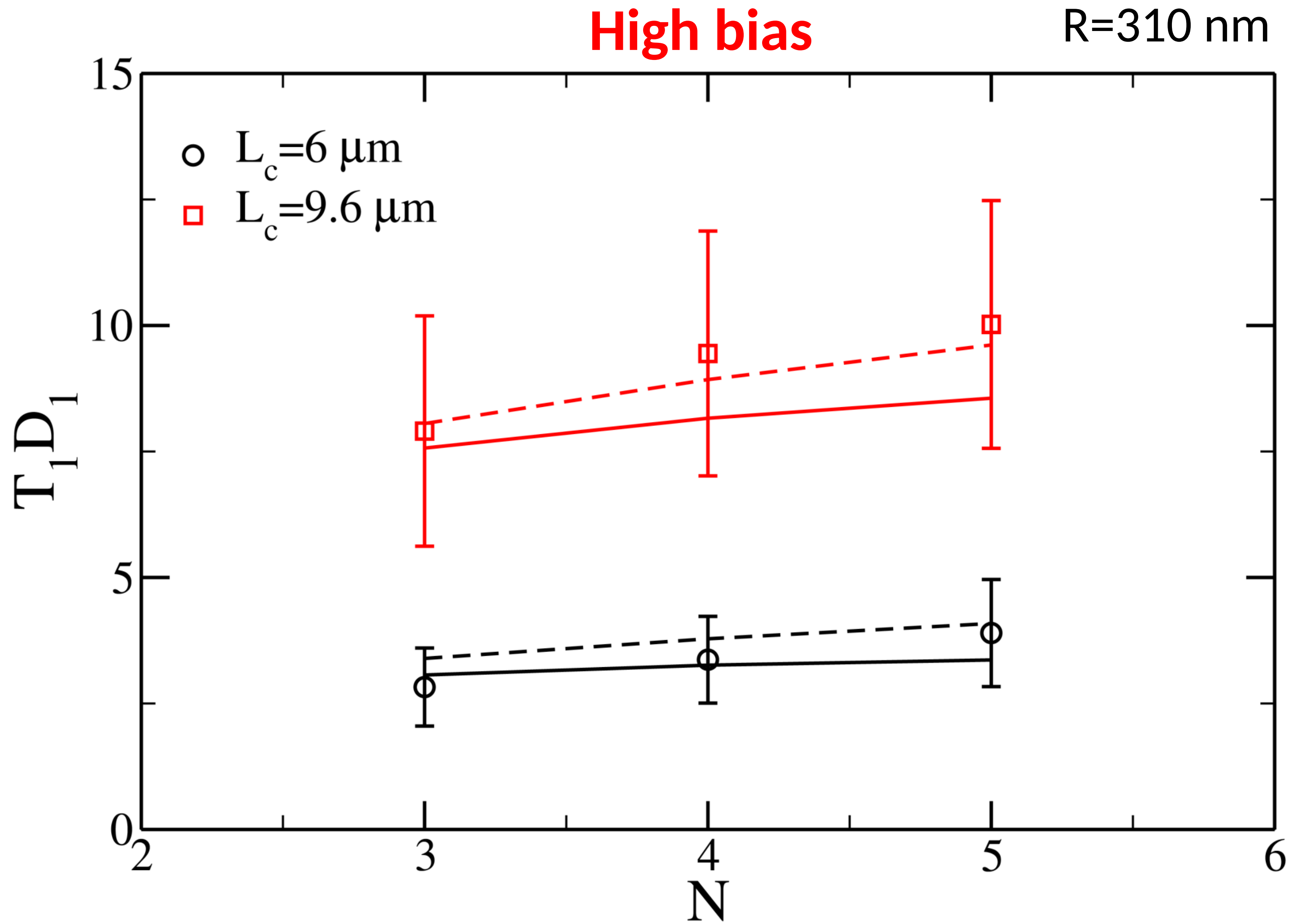

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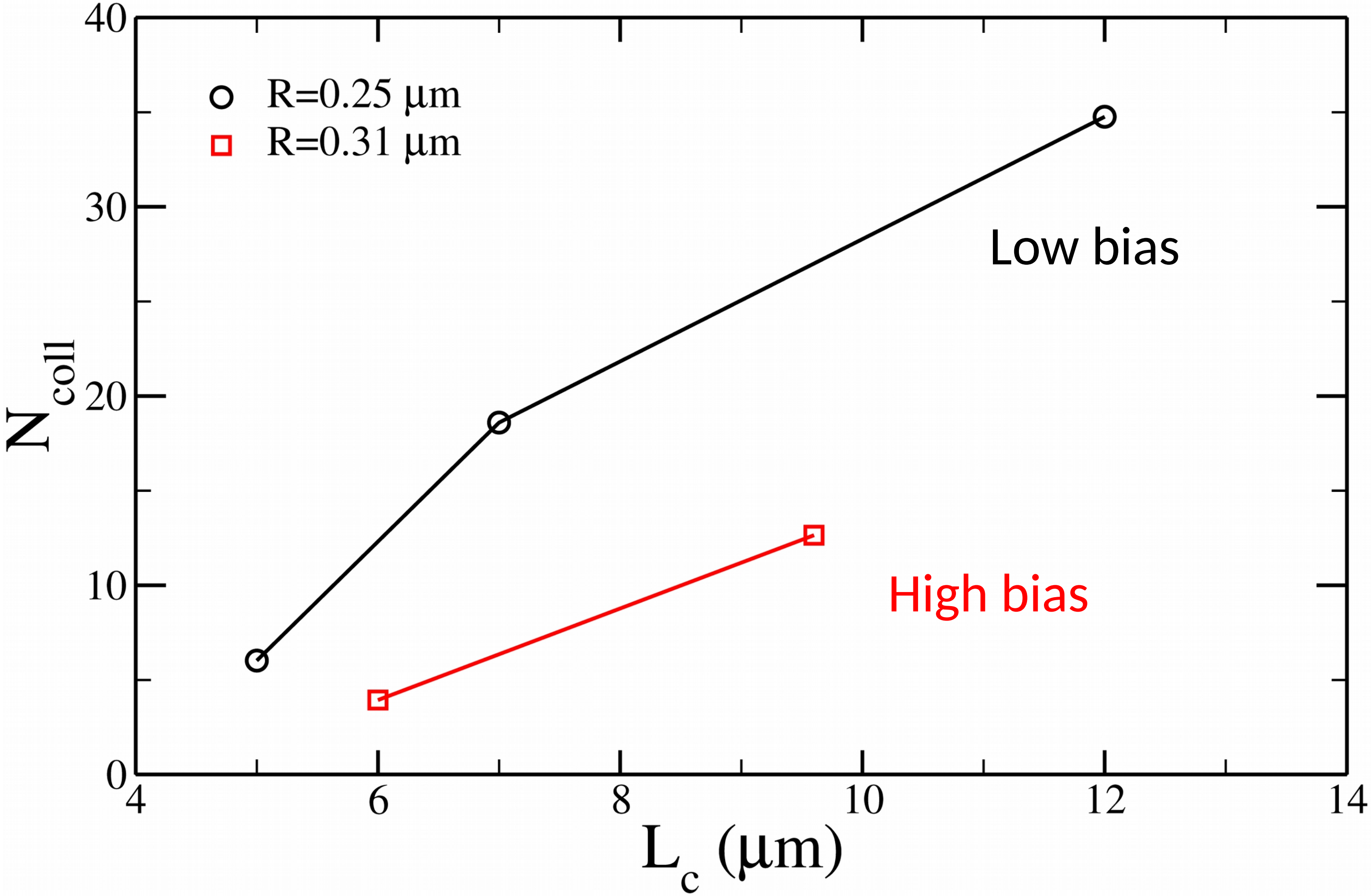


Conclusions

- ✓ We studied the escape properties of Single File systems of colloidal particles in presence of absorbing boundaries
- ✓ We studied the emptying process, finding an analytical solution for the Mean Emptying Time either in the presence and in the absence of an external force
- ✓ We provided an effective theory to account for excluded volume contributions to the Mean Emptying Time
- ✓ These results are in excellent agreement with experimental data of colloidal particles in microfluidic channels

Many thanks to:

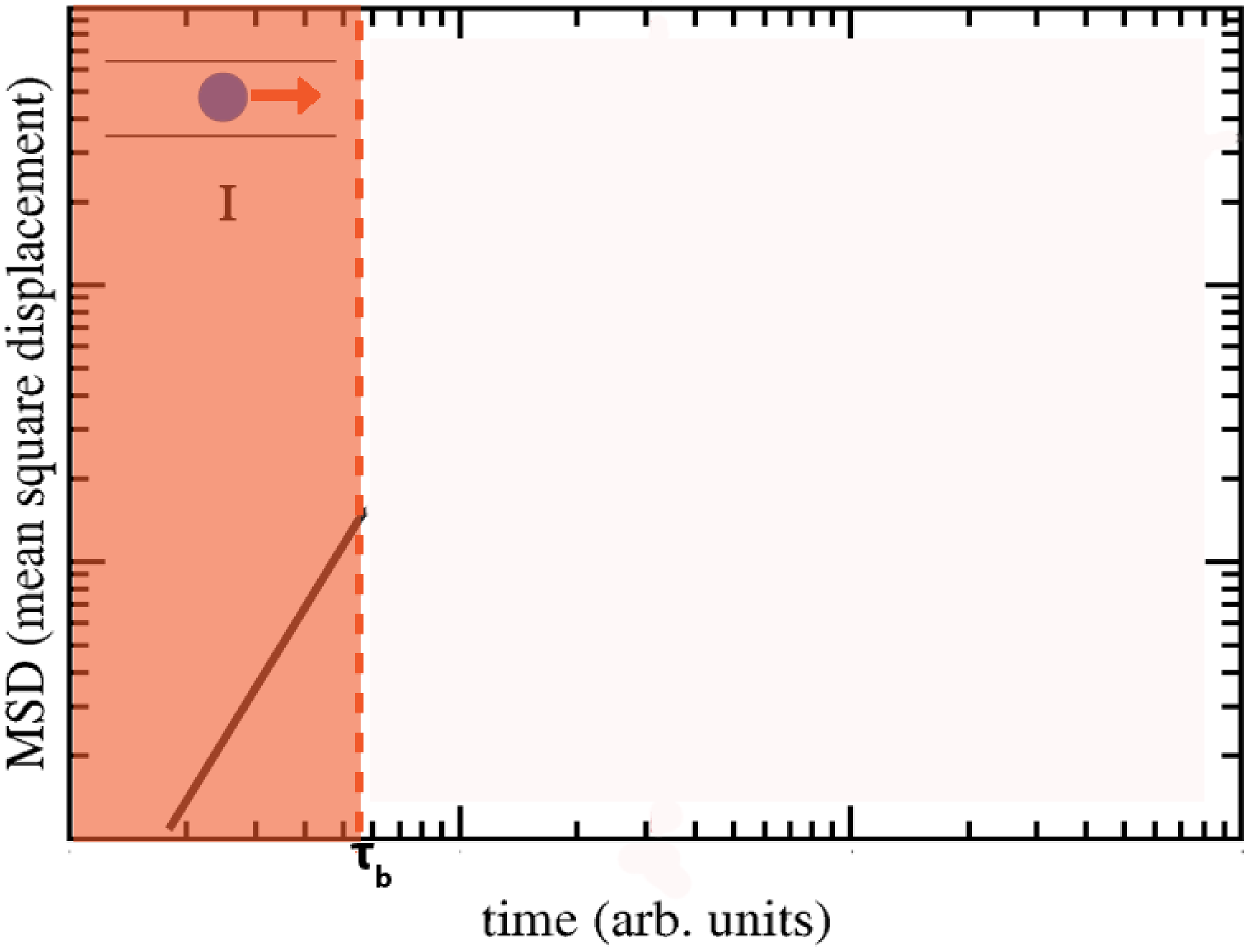
My supervisors

Fulvio Baldovin
Enzo Orlandini
Matteo Pierno

All the people of

Dr. Pagliara and Prof. Keyser @ Cavendish Lab, Cambridge

SFD - MSD sketch

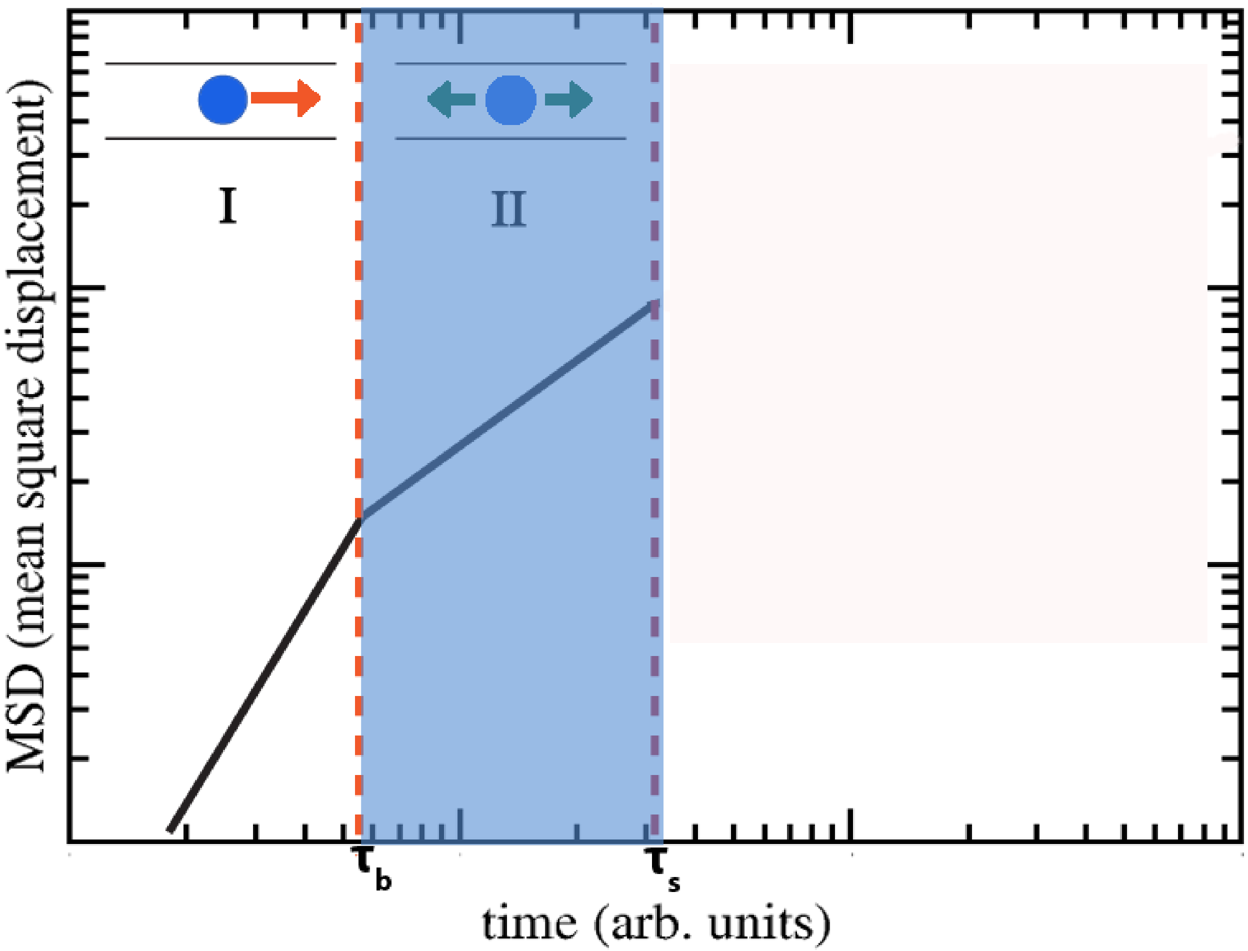


Ballistic

I

Jepsen, D., (1965)
Harris, T. E., (1965)
Levitt, D., (1973)
Kollmann, M., (2003)

SFD - MSD sketch



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Ballistic + Early time diffusion

I II

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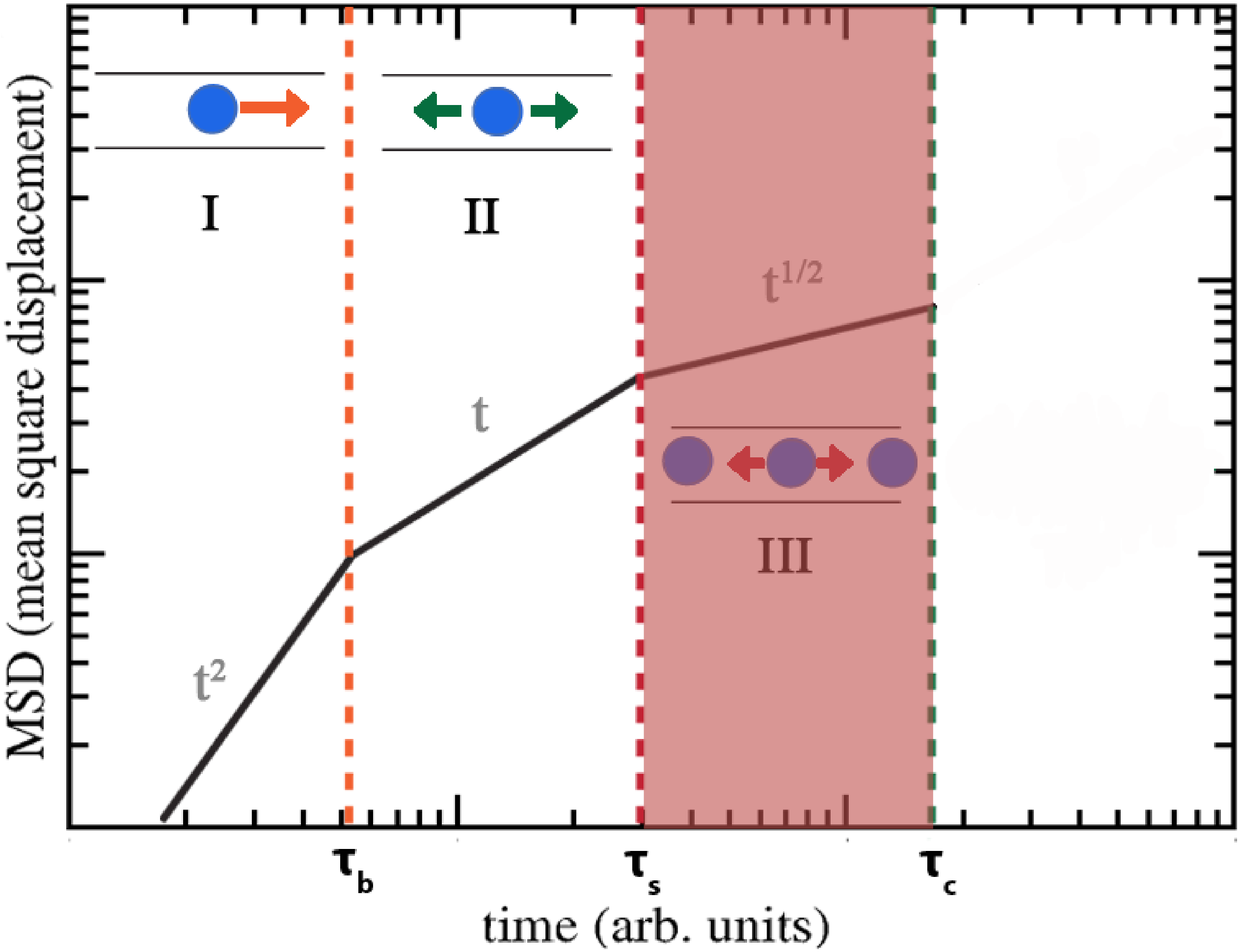

Jepsen, D., (1965)
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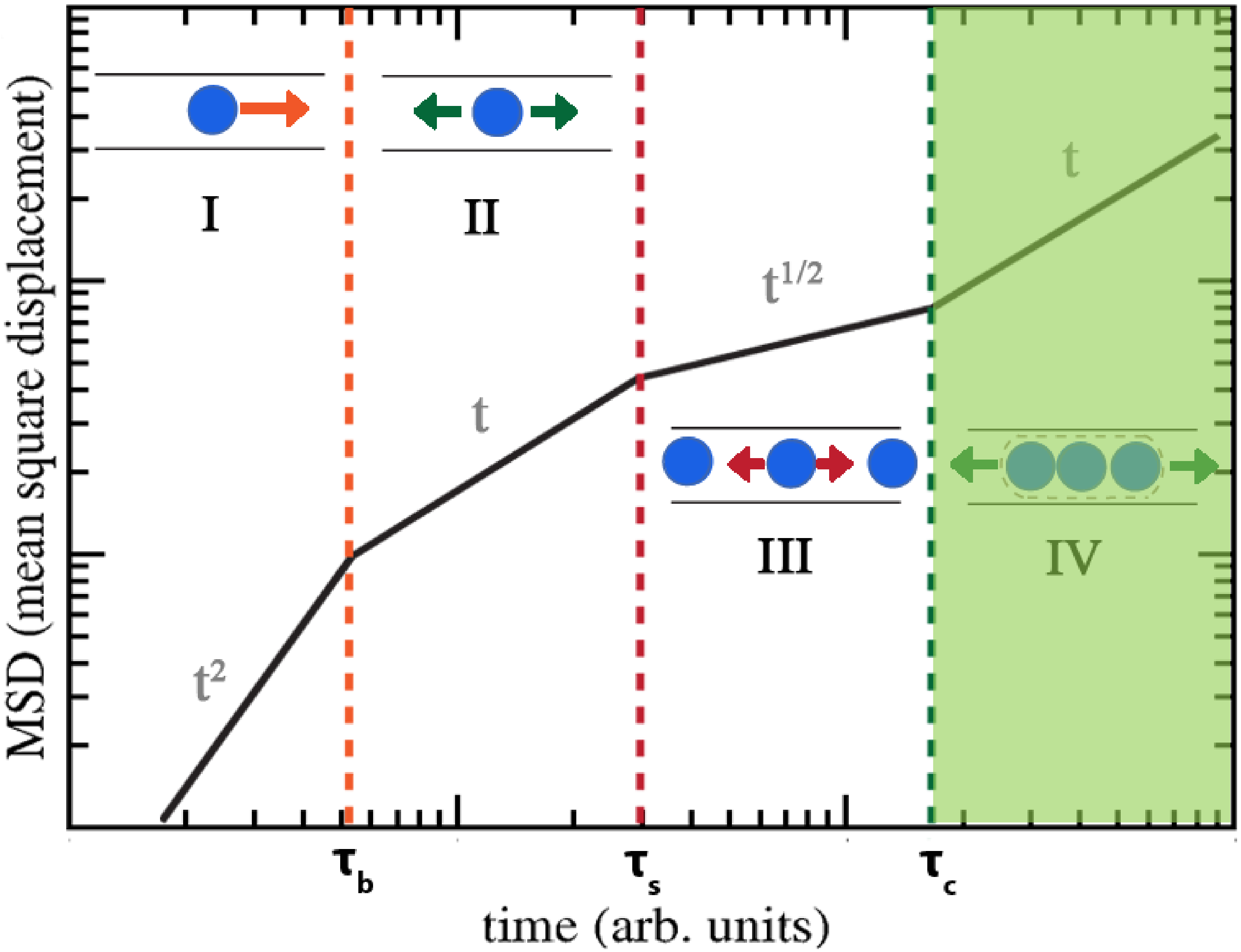
Ballistic + Early time diffusion + **Subdiffusion**

I II III

SFD - MSD sketch



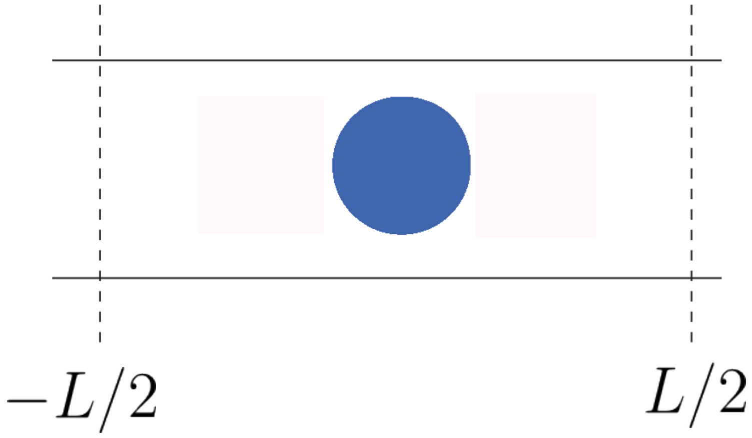
Rodenbeck, C., *et al.*, (1998)
Barkai, E., Silbey, R., (2010)
Lizana, L. *et al.*, (2010)
Delfau, J.B., *et al.*, (2011)

Ballistic + Early time diffusion + Subdiffusion + **Long time diffusion**

I **II** **III** **IV**

First passage statistics

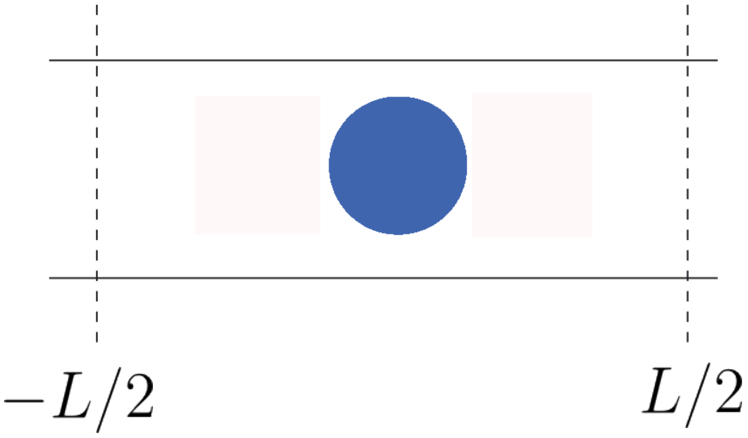
Survival probability



Probability that a particle, started from x_0, is still inside $[-L/2, L/2]$ at time *t*

First passage statistics

Survival probability



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Mean First Passage Time → Characteristic survival time

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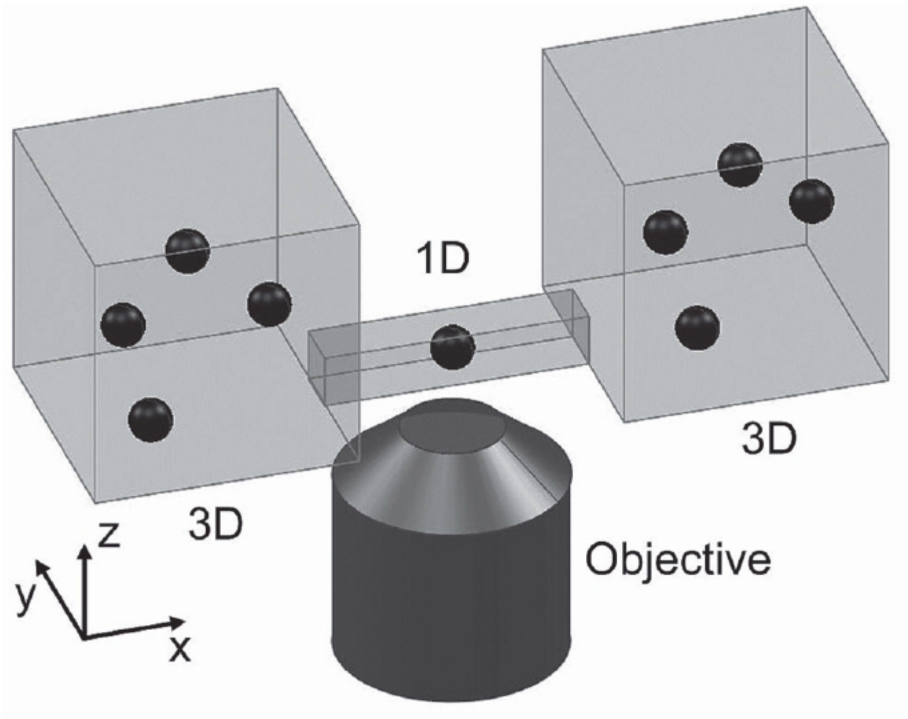
SFD of colloidal particles - experimental setup



from Pagliara, Schwall, Keyser (2012)

Tracking experiment

- Custom-made microscope
- Holographic optical tweezers
- 500 nm polystyrene particles
- Tracking routines
  (Crocker, Grier, 1996)

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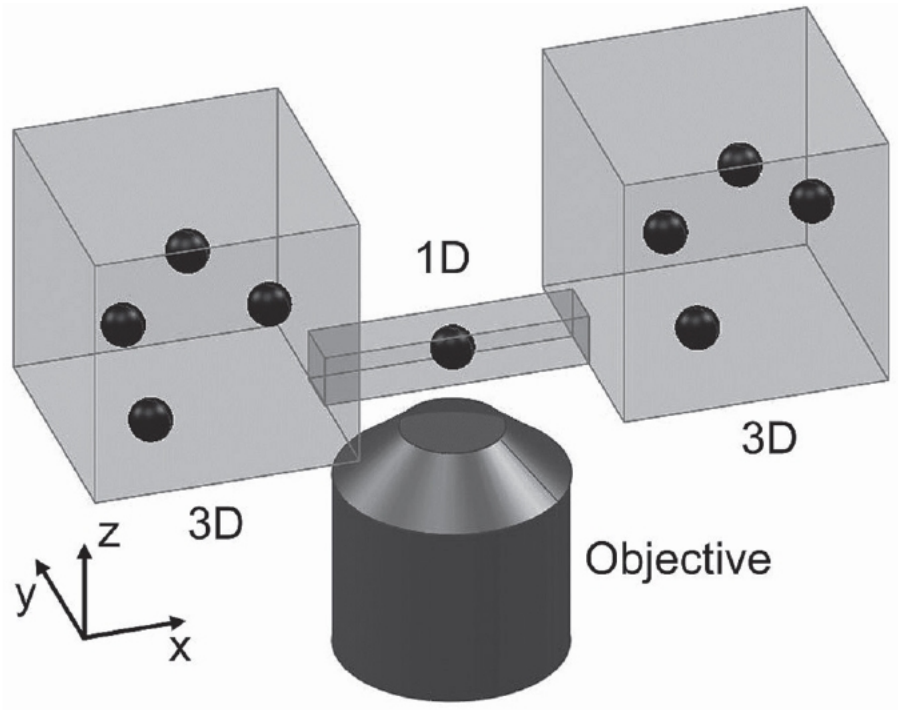

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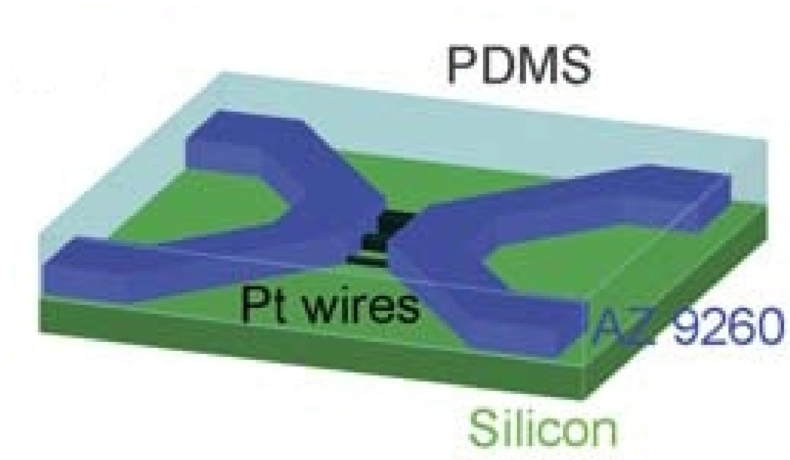
Tracking experiment

- Custom-made microscope
- Holographic optical tweezers
- 500 nm polystyrene particles
- Tracking routines (Crocker, Grier, 1996)

- PDMS chip is obtained by replica molding
- Chamber is made of two reservoires connected by eight sub-micrometric channels



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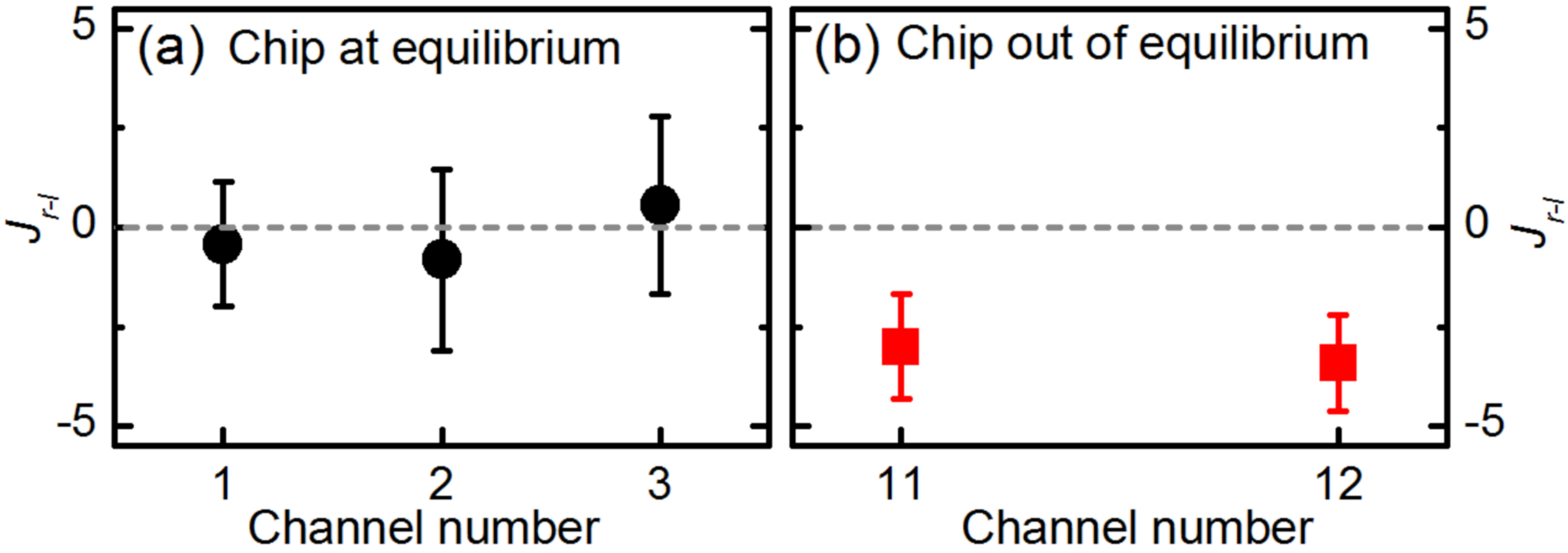
Experimental data



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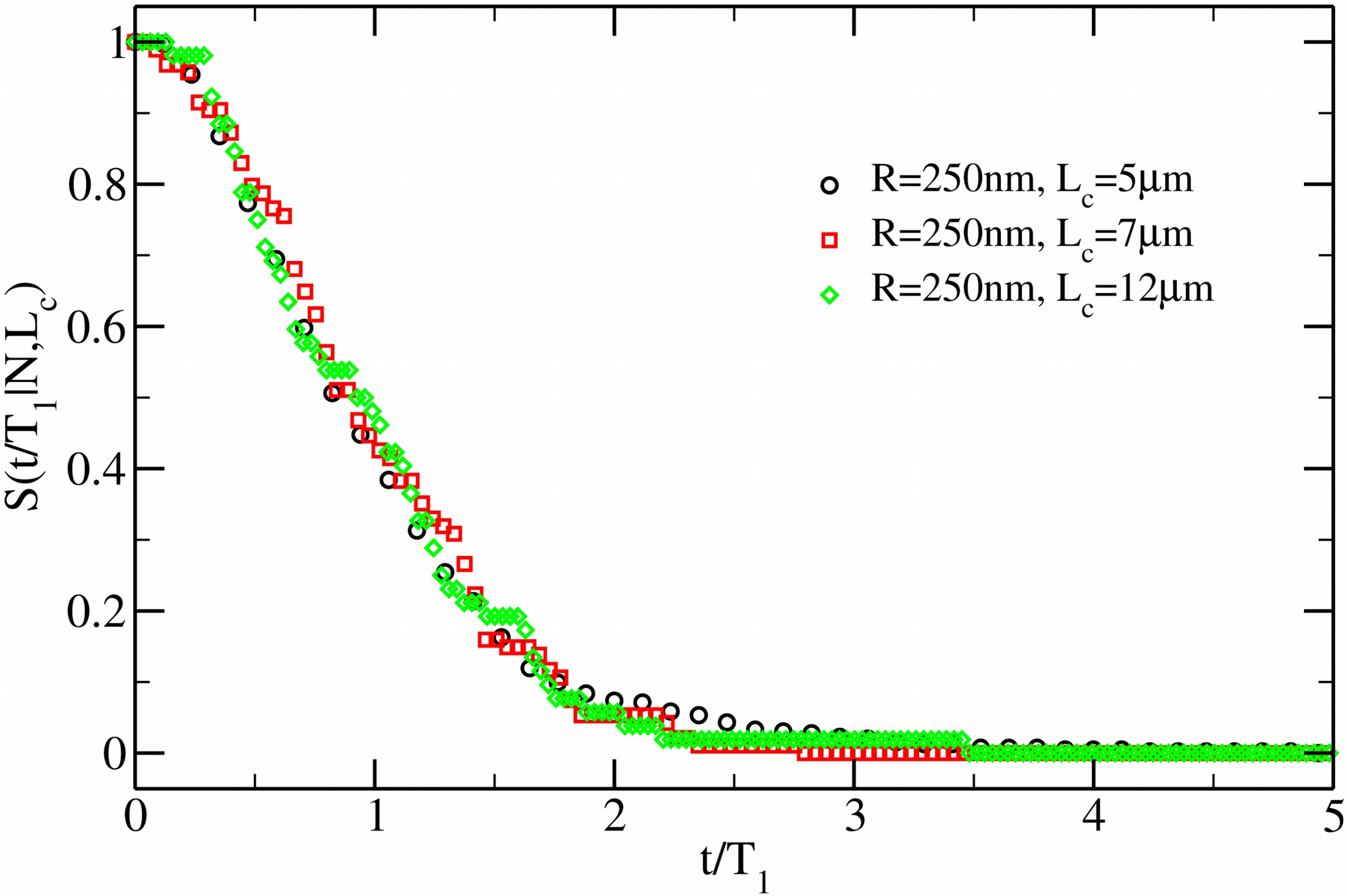

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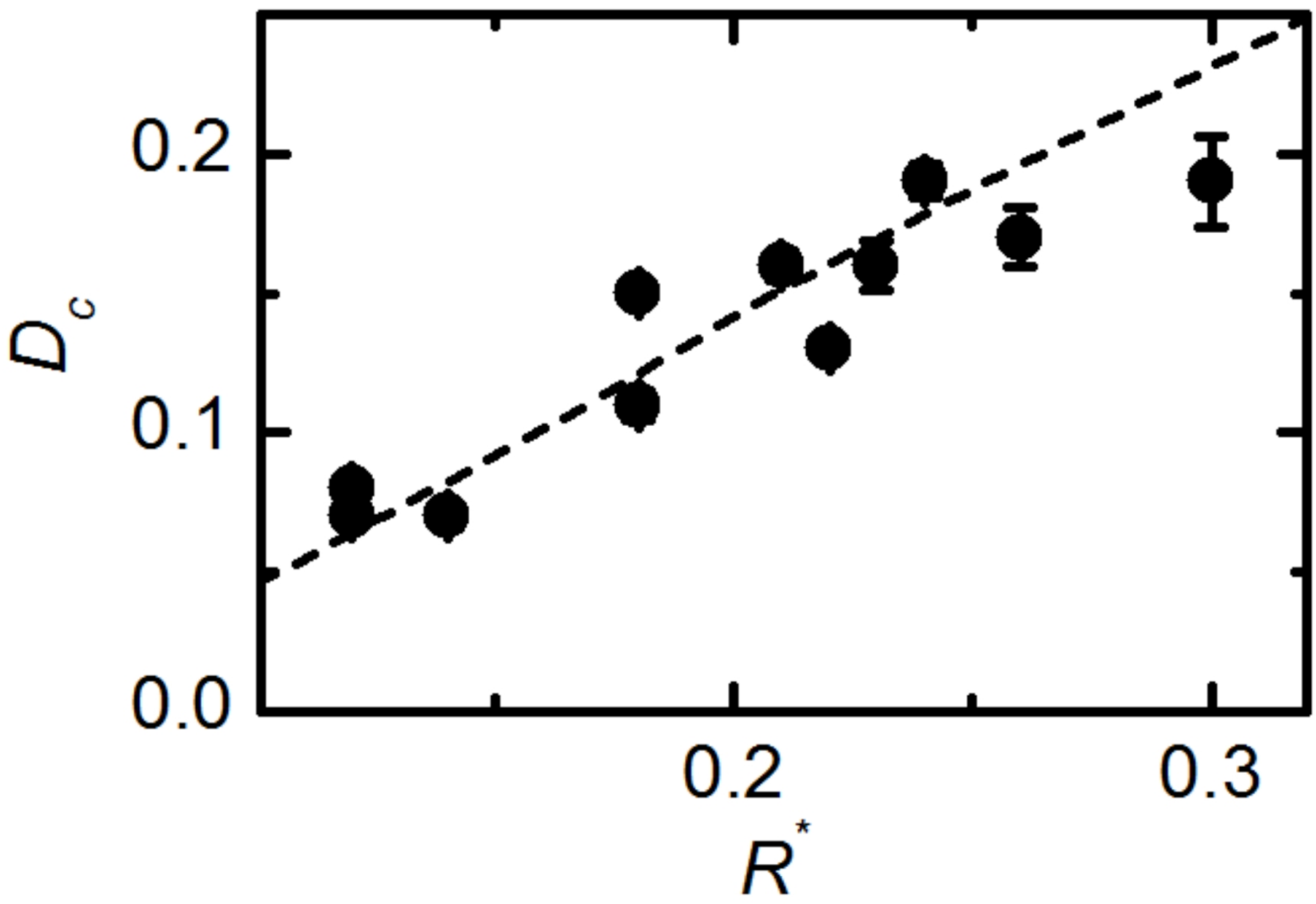

Experimental data