The 3-Dimensional nucleon structure
(mainly in momentum space)

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the nucleon is still a very mysterious object ..... and the most abundant piece of matter in the Universe
simple physical ideas...

TMDs = Transverse Momentum Dependent Parton Distribution Functions (TMD-PDF) or Transverse Momentum Dependent Fragmentation Functions (TMD-FF)

TMD-PDFs give the number density of partons, with their intrinsic motion and spin, inside a fast moving proton, with its spin.

\[ S \cdot (p \times k_\perp) \quad s_q \cdot (p \times k_\perp) \quad S \cdot s_q \quad \ldots \]

“Sivers effect”  “Boer-Mulders effect”
there are 8 independent TMD-PDFs

\[ f^q_1(x, k^2_\perp) \]  unpolarized quarks in unpolarized protons
unintegrated unpolarized distribution

\[ g^q_{1L}(x, k^2_\perp) \]  correlate \( s_L \) of quark with \( S_L \) of proton
unintegrated helicity distribution

\[ h^q_{1T}(x, k^2_\perp) \]  correlate \( s_T \) of quark with \( S_T \) of proton
unintegrated transversity distribution

only these survive in the collinear limit

\[ f^\perp q_{1T}(x, k^2_\perp) \]  correlate \( k_\perp \) of quark with \( S_T \) of proton (Sivers)

\[ h^\perp q_1(x, k^2_\perp) \]  correlate \( k_\perp \) and \( s_T \) of quark (Boer-Mulders)

\[ g^\perp q_{1T}(x, k^2_\perp) \]
\[ h^\perp q_{1L}(x, k^2_\perp) \]
\[ h^\perp q_{1T}(x, k^2_\perp) \]
different double-spin correlations
TMD-FFs give the number density of hadrons, with their momentum, originated in the fragmentation of a fast moving parton, with its spin.

\[ s_q \cdot (p_q \times p_\perp) \]  “Collins effect”

there are 2 independent TMD-FFs for spinless hadrons

\[ D_1^q(z, p_\perp^2) \]  unpolarized hadrons in unpolarized quarks

\[ H_1^{\perp q}(z, p_\perp^2) \]  correlate \( p_\perp \) of hadron with \( s_T \) of quark (Collins)
how to “measure” TMDs?

needs processes which relate physical observables to parton intrinsic motion via QCD factorisation

SIDIS

\[ \ell N \rightarrow \ell h X \]

Drell-Yan processes

\[ p N \rightarrow \ell^+ \ell^- X \]

a similar diagram for

\[ e^+ e^- \rightarrow h_1 h_2 X \]

and, possibly, for

\[ p N \rightarrow h X \]
new probes and concepts to explore the nucleon structure

TMDs - Transverse Momentum Dependent (distribution and fragmentation functions)

(polarized) SIDIS and Drell-Yan

\[ f_{a/p}(x, k_\perp; s_a, S) \]
GPDs - Generalized Partonic Distributions

exclusive processes in leptonic and hadronic interactions

\[ P' - P = \Delta \]

\[ H(x, \xi, \Delta_T) \]

\[ q(x, b_T) = \int \frac{d^2 \Delta_T}{(2\pi)^2} H_q(x, 0, -\Delta_T^2) e^{-i b_T \cdot \Delta_T} \]

spatial partonic distribution in transverse space
GTMDs - Generalised Transverse Momentum Dependent (partonic distributions)

exclusive processes in leptonic and hadronic interactions

\[ k - \frac{\Delta}{2} \quad \quad \quad k + \frac{\Delta}{2} \]

\[ P' - P = \Delta \]

\[ \int d^2k_\perp H(k, \Delta) = H(x, \xi, \Delta_T) \]
TMDs + GPDs and the full story ...

Figure 1. Representation of the projections of the GTMDs into parton distributions and form factors. The arrows correspond to different reductions in the hadron and quark momentum space: the solid (red) arrows give the forward limit in the hadron momentum, the dotted (black) arrows correspond to integrating over the quark transverse-momentum and the dashed (blue) arrows project out the longitudinal momentum of quarks. The different objects resulting from these links are explained in the text.

The plan of the paper is as follows. In section 2, we discuss the formal derivation of the LCWF overlap representation of the quark contribution to GTMDs, specializing the results to two light-cone quark models, namely the chiral quark-soliton model (χQSM) and the light-cone constituent quark model (LCCQM). In section 3, we focus on the TMDs, GPDs, PDFs, FFs and charges. In particular, we derive the general formulas obtained from the projections of GTMDs, and then we discuss and compare the predictions from both the χQSM and the LCCQM. In the last section, we draw our conclusions.

Technical details and explanations about the derivation of the formulas are collected in three appendices.

2 Formalism

2.1 Parton Correlation Functions

The maximum amount of information on the quark distribution inside the nucleon is contained in the fully-unintegrated quark-quark correlator \( \tilde{W} \) for a spin-1/2 hadron [2–5].
phase-space parton distribution, \( W(k, b) \)

(S. Meissner, Metz, Schlegel)

**GTMD**

\[ H(k, \Delta) \]

\[ \Delta = 0 \]

**TMD**

\[ q(x, k_{\perp}) \]

\[ \int d^2 k_{\perp} \]

**Wigner function**

(Belitsky, Ji, Yuan)

\[ W(k, b) \]

\[ \xi = 0 \]

\[ \int d^3 b \]

\[ \int d^2 k_{\perp} db_L \]

\[ H(x, 0, \Delta_T) \]

**FT, \( \Delta \leftrightarrow b \)**

(M. Burkardt)

\[ q(x, b_T) \]

\[ \int d^2 k_{\perp} H(k, \Delta) = H(x, \xi, \Delta_T) \]
Wigner distribution

\[ \rho(x, k_\perp, b_T) \]

Longitudinal momentum

\[ k^+ = xP^+ \]

courtesy of A. Bacchetta
TMDs in SIDIS

TMD factorization holds at large $Q^2$, and $P_T \approx k_\perp \approx \Lambda_{\text{QCD}}$

Two scales: $P_T \ll Q^2$

$\text{d}\sigma^{\ell p \rightarrow \ell h X} = \sum_q f_q(x, k_\perp; Q^2) \otimes \text{d}\hat{\sigma}^{\ell q \rightarrow \ell q}(y, k_\perp; Q^2) \otimes D_q^h(z, p_\perp; Q^2)$

(Collins, Soper, Ji, J.P. Ma, Yuan, Qiu, Vogelsang, Collins, Metz...)
\[
\frac{d\sigma}{d\phi} = F_{UU} + \cos(2\phi) F_{UU}^{\cos(2\phi)} + \frac{1}{Q} \cos \phi F_{UU}^{\cos \phi} + \lambda \frac{1}{Q} \sin \phi F_{LU}^{\sin \phi} \\
+ S_L \left\{ \sin(2\phi) F_{UL}^{\sin(2\phi)} + \frac{1}{Q} \sin \phi F_{UL}^{\sin \phi} + \lambda \left[ F_{LL} + \frac{1}{Q} \cos \phi F_{LL}^{\cos \phi} \right] \right\} \\
+ S_T \left\{ \sin(\phi - \phi_S) F_{UT}^{\sin(\phi - \phi_S)} + \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \right. \\
\left. + \frac{1}{Q} \left[ \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} + \sin \phi_S F_{UT}^{\sin \phi_S} \right] \right. \\
\left. + \lambda \left[ \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} + \frac{1}{Q} \left( \cos \phi_S F_{LT}^{\cos \phi_S} + \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right) \right] \right\}
\]

the \( F_{S_B} \) and \( S_T \) contain the TMDs; plenty of Spin Asymmetries

**HADRON PRODUCTION PLANE**

**LEPTON SCATTERING PLANE**
TMDs in Drell-Yan processes

COMPASS, RHIC, Fermilab, NICA, AFTER...

direct product of TMDs, no fragmentation process
Case of one polarized nucleon only

\[
\frac{d\sigma}{d^4 q \, d\Omega} = \frac{\alpha^2}{\Phi q^2} \left\{ (1 + \cos^2 \theta) \, F_U^{1} + (1 - \cos^2 \theta) \, F_U^{2} + \sin 2\theta \cos \phi \, F_U^{\cos \phi} + \sin^2 \theta \cos 2\phi \, F_U^{\cos 2\phi} + S_L \left( \sin 2\theta \sin \phi \, F_L^{\sin \phi} + \sin^2 \theta \sin 2\phi \, F_L^{\sin 2\phi} \right) + S_T \left[ \left( F_T^{\sin \phi_S} + \cos^2 \theta \, \tilde{F}_T^{\sin \phi_S} \right) \sin \phi_S + \sin 2\theta \left( \sin(\phi + \phi_S) \, F_T^{\sin(\phi + \phi_S)} + \sin(\phi - \phi_S) \, F_T^{\sin(\phi - \phi_S)} \right) \right] \right\}
\]

Collins-Soper frame
Unpolarized cross section already very interesting

\[
\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)
\]

Collins-Soper frame

naive collinear parton model: \( \lambda = 1 \quad \mu = \nu = 0 \)
Collins function from $e^+e^-$ processes

Belle, BaBar, BES-III

\[
\frac{d\sigma^{e^+e^-\rightarrow q\uparrow\bar{q}\uparrow}}{d\cos\theta} = \frac{3\pi\alpha^2}{4s} e_q^2 \cos^2\theta
\]

\[
\frac{d\sigma^{e^+e^-\rightarrow q\uparrow\bar{q}\uparrow}}{d\cos\theta} = \frac{3\pi\alpha^2}{4s} e_q^2
\]

\[
A_{12}(z_1, z_2, \theta, \varphi_1 + \varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^-\rightarrow h_1h_2X}}{dz_1 dz_2 d\cos\theta d(\varphi_1 + \varphi_2)}
\]

\[
= 1 + \frac{1}{4} \frac{\sin^2\theta}{1 + \cos^2\theta} \cos(\varphi_1 + \varphi_2) \times \frac{\sum_q e_q^2 \Delta^N D_{h_1/q\uparrow}(z_1) \Delta^N D_{h_2/\bar{q}\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}
\]

another similar asymmetry can be measured, $A_0$
Experimental results:
clear evidence for Sivers and Collins effects from SIDIS data (HERMES, COMPASS, JLab)
independent evidence for Collins effect from $e^+e^-$ data at Belle, BaBar and BES-III

$$A_{12}(z_1, z_2) \sim \Delta^N D_{h_1/q^\uparrow}(z_1) \otimes \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)$$

**Figure 3.** – Preliminary BaBar measurement of Collins asymmetries (full circle in red). By comparison the superseded Belle off-peak results (open circle in blue), and Belle results on the full data sample (full green circles) are shown. Systematic and statistical errors are added in quadrature.

**Figure 4.** – Collins asymmetry $A_{12}$ (a), and $A_0$ (b), as a function of $(\sin^2 \theta)/(1 + \cos^2 \theta)$, where $\theta = \theta_1$ and $\theta = \theta_2$ have been used in plot (a) and (b), respectively.

The asymmetries are studied in function of symmetric bins ($z_1, z_2$) of the pion fractional energies and in function of $\sin^2 \theta/(1 + \cos^2 \theta)$, and are compared with the Belle analysis. The results are in overall good agreement each other. However, the off-peak data sample is statistically limited, and the update of the measurement with the full BaBar data sample is ongoing.

**REFERENCES**


a similar asymmetry just measured by BES-III
(arXiv 1507:06824)

Collins effect clearly observed both in SIDIS and e+e- processes, by several Collaborations

\( Q^2 = 13 \text{ GeV}^2 \)
TMD extraction from data - first phase
(simple parameterisation, no TMD evolution, limited number of parameters, ...)

unpolarised TMDs - fit of SIDIS multiplicities
(M.A, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (2014) 005)

\[ M_{p}^{\pi^{+}} \]

FIG. 1: The multiplicities \(M_{\pi^{+}}\) obtained from Eqs. (12) and (8), with the parameters of Eq. (15), are compared with HERMES measurements for \(\pi^{+}\) SIDIS production on a proton target [15]. The shaded uncertainty bands correspond to a 5% variation of the total.

\[ Q^{2}=1.80 \text{ GeV}^{2} \]
\[ x_{B}=0.10 \]

\[ Q^{2}=2.90 \text{ GeV}^{2} \]
\[ x_{B}=0.15 \]

\[ Q^{2}=5.20 \text{ GeV}^{2} \]
\[ x_{B}=0.25 \]

\[ Q^{2}=9.20 \text{ GeV}^{2} \]
\[ x_{B}=0.41 \]
strong support for a gaussian distribution

\[
\frac{d^2 n^h(x_B, Q^2, z_h, P_T)}{dz_h dP_T^2} = \frac{1}{2P_T} M^h_n(x_B, Q^2, z_h, P_T) = \frac{\pi \sum_q e_q^2 f_{q/p}(x_B) D_{h/q}(z_h)}{\sum_q e_q^2 f_{q/p}(x_B)} e^{-P_T^2/\langle P_T^2 \rangle} \frac{e^{-p_{\perp}^2/\langle p_{\perp}^2 \rangle}}{\pi \langle P_T^2 \rangle}
\]

\[
\langle P_T^2 \rangle = \langle p_{\perp}^2 \rangle + z_h \langle k_{\perp}^2 \rangle
\]

\[
f_{q/p}(x, k_{\perp}) = f_{q/p}(x) \frac{e^{-k_{\perp}^2/\langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle}
\]

\[
D_{h/q}(z, p_{\perp}) = D_{h/q}(z) \frac{e^{-p_{\perp}^2/\langle p_{\perp}^2 \rangle}}{\pi \langle p_{\perp}^2 \rangle}
\]

\[
\langle k_{\perp}^2 \rangle = 0.57 \quad \langle p_{\perp}^2 \rangle = 0.12
\]

a similar analysis performed by Signori, Bacchetta, Radici, Schnell, JHEP 1311 (2013) 194; it also assumes gaussian behaviour
TMD extraction: transversity and Collins functions - first phase

\[ \Delta_{T}q(x, k_{\perp}) = \frac{1}{2} N_{q}^{T}(x) [f_{q/p}(x) + \Delta q(x)] \frac{e^{-k_{\perp}^{2}/\langle k_{\perp}^{2} \rangle_{T}}}{\pi \langle k_{\perp}^{2} \rangle_{T}} \]

\[ \Delta^{N}D_{h/q^{+}}(z, p_{\perp}) = 2 N_{q}^{C}(z) D_{h/q}(z) h(p_{\perp}) \frac{e^{-p_{\perp}^{2}/\langle p_{\perp}^{2} \rangle}}{\pi \langle p_{\perp}^{2} \rangle} \]

SIDIS and e+e- data, simple parameterization, no TMD evolution, agreement with extraction using di-hadron FF

extraction of $u$ and $d$ Sivers functions - first phase

(in agreement with several other groups)

\[ x \Delta^N f^{(1)}_q(x, Q) \]

\[
\Delta^N f^{(1)}_q(x, Q) = \int d^2k_\perp \frac{k_\perp}{4M_p} \Delta^N \hat{f}_{q/p}^{1}(x, k_\perp; Q) = -f^{1T}(1)q(x, Q)
\]

parameterization of the Sivers function:

\[
\Delta^N \hat{f}_{q/p}^{1}(x, k_\perp; Q) = 2N(x) h(k_\perp) f_q(x, Q) \frac{1}{\pi \langle k^2_\perp \rangle} e^{-k^2_\perp / \langle k^2_\perp \rangle}
\]

$Q^2$ evolution only taken into account in the collinear part (usual PDF)
Sivers effects induces distortions in the parton distribution (quarks polarised along $y$-direction)

courtesy of A. Bacchetta
$b_\perp$ distribution at different values of $x$ (nucleon tomography)

$x_B = 0.25$

$x_B = 0.09$
femtophotography or tomography of the nucleon
TMDs at LHC - linearly polarised gluons in unpolarized protons

\[ p(P_A) + p(P_B) \rightarrow H(K_H) + \text{jet}(K_j) + X \]

\[ K_{\perp} = (K_{H\perp} - K_{j\perp})/2 \]

\[ q_T = K_{H\perp} + K_{j\perp} \]

Boer, Pisano, Phys. Rev. D91 (2015) 7, 074024
The small $q_T$ region cannot be explained by usual collinear PDF factorization: needs TMD-PDFs

other measured evidence of the Sivers and Collins effects

\[ A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \]

\[ p^\uparrow p \rightarrow \pi X \]

Large \( P_T \)

Single Spin Asymmetry

\[ \sqrt{s} = 19.4 \text{ GeV}/c^2, \text{ E704} \]
\[ \sqrt{s} = 62.4 \text{ GeV}/c^2, \text{ PHENIX} \quad 3.2 < \eta < 3.7 \]
\[ \sqrt{s} = 200 \text{ GeV}/c^2, \text{ STAR} \quad \langle \eta \rangle = 3.3 \]
\[ \sqrt{s} = 200 \text{ GeV}/c^2, \text{ STAR} \quad \langle \eta \rangle = 3.7 \]
\[ \sqrt{s} = 500 \text{ GeV}/c^2, \text{ STAR} \quad 2.7 < \eta < 4.0 \]
TMDs and QCD - TMD evolution
study of the QCD evolution of TMDs and TMD factorisation
in rapid development

Collins-Soper-Sterman resummation - NP B250 (1985) 199
Idilbi, Ji, Ma, Yuan - PL B597, 299 (2004); PR D70 (2004) 074021

Aybat, Rogers, PR D83 (2011) 114042
Aybat, Collins, Qiu, Rogers, PR D85 (2012) 034043
Echevarria, Idilbi, Schafer, Scimemi, arXiv:1208.1281
Echevarria, Idilbi, Scimemi, JHEP 1207 (2012) 002
Aybat, Prokudin, Rogers, PRL 108 (2012) 242003
Anselmino, Boglione, Melis, PR D86 (2012) 014028
Aidala, Field, Gamberg, Rogers, PR D89 (2014) 094002
Echevarria, Idilbi, Kang, Vitev, PR D89 (2014) 074013
Bacchetta, Prokudin, NP B875 (2013) 536
Godbole, Misra, Mukherjee, Raswoot, PR D88 (2013) 014029
Boglione, Gonzalez, Melis, Prokudin, JHEP 1502 (2015) 095
Kang, Prokudin, Sun, Yuan, arXiv:1505.05589

+ many more authors...
different TMD evolution schemes and different implementation within the same scheme


dedicated tools:

TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions

Hautmann, Jung, Kramer, Mulders, Nocera, Rogers, Signori
TMD phenomenology - phase 2

how does gluon emission affect the transverse motion?
a few selected results

TMD evolution of up quark Sivers function

Aybat, Collins, Qiu, Rogers, Phys. Rev. D85 (2012) 034043
at large

cerned with the treatment of the Qiu-Sterman formalism
(or FF) matches to a twist-2 collinear factorization treat-
tment (see, e.g., Ref. [51]) that calculations in covariant
aspects of TMD physics.

TMD evolution of up quark Sivers function

Evolved Bochum Gaussian Fits
Up Quark Sivers Function, \( x = 0.1 \)

Evolved Torino Gaussian Fits
Up Quark Sivers Function, \( x = 0.1 \)

\[ -2\pi k_T^3 F_{1T}^{\uparrow} \]

\[ Q = \sqrt{2.4} \text{ GeV} \]
\[ Q = 2 \text{ GeV} \]
\[ Q = 3 \text{ GeV} \]
\[ Q = 4 \text{ GeV} \]

Aybat, Collins, Qiu, Rogers, Phys.Rev. D85 (2012) 034043

TMD evolution of Sivers function studied also by
Extraction of transversity and Collins functions with TMD evolution

(Kang, Prokudin, Sun, Yuan, arXiv:1505.05589)
The results plotted in Fig. 29 correspond to our estimate of Ref. [18] is found by extraction using the so-called dihadron fragment function. The comparison of extracted transversity (solid lines and shaded region) at $Q^2 = 2.4$ GeV$^2$ with Pavia 2015 extraction [18] (dashed lines and shaded region) is very positive and compatible within error bands. The unfavored fragmentation functions are also compared to our and other extraction methods, ours and Ref. [18], is very positive and compatible within error bands. The good agreement of all three methods is a good sign. We conclude that tensor charge is perhaps very stable with changes of DGLAP evolution of collinear distributions, including the different parametrizations for Collins fragmentation functions, the so-called standard parametrization of Collins fragmentation function that couples to collinear distributions (solid lines) at $Q^2 = 249$ GeV$^2$ and calculations at 68% at $Q^2 = 125$ GeV$^2$, of Ref. [18]. One can see from Fig. 28 that the functions at $Q^2 = 249$ GeV$^2$ are excellent. Even though the good quality, the existing data is much better determined by the existing data, as one can see from Fig. 27. The comparison of extracted transversity (solid lines and shaded region) at $Q^2 = 4$ GeV$^2$ and calculations at 68% at $Q^2 = 249$ GeV$^2$ is 99%. The tensor charge was estimated at 95% C.L. using two different parametrizations for Collins fragmentation functions, the so-called flexible scenario, and calculations at 68% at $Q^2 = 249$ GeV$^2$. The facts that the central values and errors of Collins fragmentation functions, the so-called standard parametrization of Collins fragmentation function that couples to collinear distributions, the so-called flexible scenario, and calculations at 68% at $Q^2 = 249$ GeV$^2$, of Ref. [18] are very well determined by existing experimental data.
predictions for BES-III $e^+e^-$ Collins asymmetry $A_0$ in excellent agreement with data, $Q^2 = 13 \text{ GeV}^2$ (some difficulties without TMD evolution)

(Kang, Prokudin, Sun, Yuan, arXiv:1505.05589)
so far ....

Sivers and Collins effects are well established, many transverse spin asymmetries resulting from them. Sivers function and orbital angular momentum? GPDs and orbital angular momentum?

Evidence for gaussian $k_\perp$ and $p_\perp$ dependence of unpolarised TMD-PDFs and TMD-FFs

Gluon TMDs deserve special attention; they might play a role at LHC

Much progress in studies of TMD factorisation and TMD evolution; phenomenological implementation in progress

Combined data from SIDIS, Drell-Yan, e+e-, with theoretical modelling, should lead to a true 3D imaging of the proton waiting for JLab 12, new COMPASS results, future facilities....
some hadron physics in the world
Electron Ion Collider plans in the world.....
Electron Ion Collider: The Next QCD Frontier

Understanding the glue that binds us all

future facilities and experiments:
D-Y @ COMPASS
JLAB 12 GeV
EIC
BESIII
AFTER
NICA-SPD
exploring the 3D structure of the nucleon
courtesy of A. Bacchetta

thank you!