Semi-analytical fluid study of the propagation of an ultrastrong femtosecond laser pulse in a plasma with ultrarelativistic electron jitter

Dušan Jovanović†, Renato Fedele†, Milivoj Belić* and Sergio De Nicola§

†Institute of Physics, University of Belgrade, Belgrade, Serbia
‡Dipartimento di Fisica, Università di Napoli “Federico II” and INFN Sezione di Napoli, Complesso Universitario di Monte S. Angelo, Via Cintia - 80126, Napoli, Italy
*Texas A & M University at Qatar, P.O. Box 23874, Doha, Qatar
§SPIN-CNR, Complesso Universitario di M.S. Angelo, Napoli and INFN Sezione di Napoli, Napoli, Italy
The laser wake field (LWF) excitation is a process based on the ponderomotive effects induced by an electromagnetic wave packet in a plasma. Basically, it performs the process to transform a transverse electric field into a longitudinal one. For typical values of plasma density of the order of $10^{18}$ cm$^{-3}$, the latter is able to accelerate an electron at 1GeV in a few centimeters!
THREE FUNDAMENTALLY DIFFERENT REGIMES

- **Weak Intensity Regime (WIR):** the quiver motion is weakly relativistic \( p_{\perp 0} \ll m_0 c \quad p_{\perp 0} = eE_{\perp 0} / \omega \)

  \( I_{\text{max}} \) ranging from \( 2.5 \times 10^{14} \) to \( 2.5 \times 10^{16} \) W/cm\(^2\)

- **Moderate Intensity Regime (MIR):** the quiver motion is mildly relativistic \( p_{\perp 0} \approx m_0 c \quad p_{\perp 0} = eE_{\perp 0} / \omega \)

  \( I_{\text{max}} \) ranging from \( 1.5 \times 10^{18} \) to \( 3 \times 10^{19} \) W/cm\(^2\)

- **Strong Intensity Regime (SIR):** the quiver motion is ultra-relativistic \( p_{\perp 0} \gg m_0 c \quad p_{\perp 0} = eE_{\perp 0} / \omega \)

  \( I_{\text{max}} \) ranging from \( 10^{20} \) to \( 2.5 \times 10^{22} \) W/cm\(^2\)
To enable the predictions for the multi-petawatt laser pulse behavior, we derive a novel mathematical model that describes both MIR and SIR.

In the classical picture of a slowly varying amplitude of the laser pulse, based on a two-timescale description, this is not possible because the dispersion characteristics of electromagnetic waves in MIR and SIR are too different from each other and can not be described on a common footing.

In the core of a very strong (i.e. SIR) pulse, the electromagnetic wave practically propagates in a vacuum. Such wave is not dispersive, i.e. its group velocity is constant and coincides with its phase velocity.

Conversely, at the edges of such pulse the amplitude is smaller and the wave is dispersive. Under such conditions, the simple envelope description used previously in the MIR, breaks down.
The analysis is based on the Lorentz-Maxwell fluid model in the fully relativistic regime taking the pancake approximation, developed earlier. Our model is derived using a three-timescale description, with an intermediate timescale associated with the nonlinear, intensity-dependent, phase of the electromagnetic pulse.
In MIR, the evolution of the plasma wake and of the laser pulse (depletion, frequency redshifting) was satisfactorily described using a reduced wave equation and a quasistatic plasma response, with a good agreement with full Maxwell-fluid results.

Such fluid calculations provide a valuable insight also into kinetic phenomena, e.g. by establishing the thresholds for the wave breaking that results in the electron trapping.

The analytic studies of the laser-plasma interaction with intensities suitable for LPA have been attempted hitherto only for quasi 1-D, pancake-shaped pulses, using the "quasistatic" approximation and in a cold-fluid description, see the classical papers and references therein.
Then we assume that:

- The transverse variations are much smaller than the longitudinal ones:
  \[ \nabla_\perp \ll \frac{\partial}{\partial z} \]

the solution is slowly varying in the frame that moves with the velocity \( u \mathbf{e}_z \) we have derived our system \textit{wave equation} + \textit{Poisson's equation}.

These equations are valid in an \textit{unmagnetized} plasma, and are written in the following dimensionless quantities (with obvious meaning of the symbols)

\[
\begin{align*}
  t' &= \omega_{pe} t, \\
  \mathbf{r}' &= \frac{\omega_{pe}}{c} (\mathbf{r} - \mathbf{e}_z ut), \\
  n' &= \frac{n}{n_0}, \\
  u' &= \frac{u}{c}, \\
  \mathbf{p}' &= \frac{\mathbf{p}}{m_0 c}, \\
  \mathbf{v}' &= \frac{\mathbf{v}}{c}, \\
  \phi' &= \frac{q \phi}{m_0 c^2}, \\
  \mathbf{A}' &= \frac{q \mathbf{A}}{m_0 c},
\end{align*}
\]
Maxwell's Eqs. parallel and perpendicular to the direction of the e.m. wave propagation (in terms of the vector potential and the scalar potential)

\[
\left[ \frac{\partial^2}{\partial t^2} - 2u \frac{\partial^2}{\partial z \partial t} - (1 - u^2) \frac{\partial^2}{\partial z^2} - \nabla_{\perp}^2 \right] \vec{A}_{\perp} \\
+ \nabla_{\perp} \left( \frac{\partial}{\partial t} - u \frac{\partial}{\partial z} \right) \phi = \vec{v}_{\perp} n, \\
\left( \nabla_{\perp}^2 + \frac{\partial^2}{\partial z^2} \right) \phi = 1 - n.
\]
Electron continuity Eq., longitudinal and perpendicular components of the momentum Eqs.

\[
\left( \frac{\partial}{\partial t} - u \frac{\partial}{\partial z} \right) n + \nabla \cdot (n\vec{v}) = 0,
\]

\[
\left( \frac{\partial}{\partial t} - u \frac{\partial}{\partial z} + \vec{v}_{\perp} \cdot \nabla_{\perp} \right) (p_z + A_z)
\]

\[
- \vec{v}_{\perp} \frac{\partial}{\partial z} (\vec{p}_{\perp} + \vec{A}_{\perp}) + \frac{\partial}{\partial z} (\gamma + \phi) = 0,
\]

\[
\left[ \frac{\partial}{\partial t} + (v_z - u) \frac{\partial}{\partial z} + \vec{v}_{\perp} \cdot \nabla_{\perp} \right] (\vec{p}_{\perp} + \vec{A}_{\perp})
\]

\[
- v_i \nabla_{\perp} (p_i + A_i) + \nabla_{\perp} (\gamma + \phi) = 0,
\]

\[
\gamma = \left( 1 + \frac{\vec{p}^2}{m_0^2 c^2} \right)^{\frac{1}{2}}
\]
The solution of the hydrodynamic equations is sought in a quasistatic regime, i.e. when it is only slowly varying in the moving reference frame, viz.

\[ \frac{\partial}{\partial t} \ll u \frac{\partial}{\partial z} \]

\[ \frac{\partial}{\partial t} = \nabla_{\perp} = 1 - u = 0 \]

In the approximate expressions for the charge and current densities, we use the leading order solution of the electron hydrodynamic equations, which is found as a stationary 1-D solution that is propagating with the speed of light, setting
the leading parts of motion Eqs. are obtained in a simple form

\[
\frac{\partial}{\partial z} \left[ (v_z - 1) n \right] = 0, \\
\frac{\partial}{\partial z} (-p_z + \gamma + \phi) = 0, \\
\frac{\partial}{\partial z} (\vec{p}_\perp + \vec{A}_\perp) = 0,
\]

while from \( \nabla \cdot \vec{A} = 0 \), within the same accuracy, we have \( \partial A_z / \partial z = 0 \).

For \( z \to \pm \infty \), we have \( \phi = \dot{A} = \vec{v} = \vec{p} = 0 \),
\( \gamma = n = 1 \), and using \( \gamma = (1 + p_z^2 + p_\perp^2)^{1/2} \).
\[ n = \frac{(\phi - 1)^2 + \vec{A}_\perp^2 + 1}{2(\phi - 1)^2}, \quad \vec{v}_\perp n = \frac{\vec{A}_\perp}{\phi - 1} \]

\[
\begin{bmatrix}
\frac{\partial^2}{\partial t^2} - 2u \frac{\partial^2}{\partial t \partial z} - (1 - u^2) \frac{\partial^2}{\partial z^2} - \nabla^2_\perp + \frac{1}{1 - \phi}
\end{bmatrix} \vec{A}_\perp
\]

\[ = -\left(\frac{\partial}{\partial t} - u \frac{\partial}{\partial z}\right) \nabla_\perp \phi, \]

\[ \frac{\partial^2 \phi}{\partial z^2} = \frac{(\phi - 1)^2 - 1 - \vec{A}_\perp^2}{2(\phi - 1)^2}. \]
For both MIR and SIR, we seek the solution of the wave equation in the moving frame as the sum of a slowly varying component and a modulated electromagnetic wave, including a phase that is varying on an intermediate scale, viz.

\[
\begin{align*}
\vec{A}_\perp &= \vec{A}_\perp^{(0)}(t_2, \vec{r}_2) \\
&+ \{\vec{A}_{\perp 0}(t_2, \vec{r}_2) \ e^{i[\varphi(t_1, \vec{r}_1) - \omega' t + k'(z + ut)]} + c.c.\}.
\end{align*}
\]

\[
\omega' = \frac{\omega}{\omega_{pe}}, \quad k' = \frac{ck}{\omega_{pe}} = \frac{d_e}{\lambda}
\]

\[
\omega = \sqrt{c^2 k^2 + \omega_{pe}^2}
\]

\[
\alpha_{Re}(\phi) = \frac{\phi}{1 - \phi} + \kappa^2(\phi),
\]

\[
\alpha_{Im}(\phi) = -\nabla_1^2 \varphi + \frac{\partial^2 \varphi}{\partial t_1^2} = -\nabla_\rho^2 \varphi + \mathcal{O}(\epsilon^4).
\]
Regime in which the self-generated magnetic field can be neglected:

\[
\left[ \alpha_{Re}(\phi) + i \alpha_{Im}(\phi) \right] A_{\perp 0} - 2 i \varepsilon^2 \frac{\partial A_{\perp 0}}{\partial t_2} - 2 i \varepsilon (\nabla_\rho \varphi \cdot \nabla_2) A_{\perp 0} - \varepsilon^2 \nabla^2_2 A_{\perp 0} = 0
\]

\[
\frac{\partial^2 \phi}{\partial z_2^2} = \frac{(\phi - 1)^2 - 1 - |A_{\perp 0}|^2}{2(\phi - 1)^2},
\]

\[
(\nabla_\rho \varphi)^2 = \kappa^2(\phi),
\]
Fig. 1 - Evolution of the envelope of the pancake laser pulse with an amplitude that is expected to be used in a future accelerator scheme (SIR). The initial condition was:

\[
A_{\perp 0}(x_2, z_2, 0) = 0.6 a_L(z_2/L_z) \exp\left(-\frac{x_2^2}{2L_x^2}\right) \exp(i \delta k z_2)
\]

\[L_z = 1.6 \text{ and } L_x = 7.5. \quad \delta k = 0.5\]

which gave the maximum stability. The initial electrostatic potential and initial nonlinear phase were adopted to be zero.

In the physical (non-scaled) variables, these initial pulse length and width are 1.8 mm and 300 mm, respectively. Likewise, the dimensionless time \( t_{2\text{max}} = 1.1 \) corresponds, in physical units, to \( 9.69 \times 10^{-12} \text{ s} \), during which time the pulse travels 3 mm. (color online).
Fig. 2 - Evolution of the electrostatic wake potential $\phi(x_2,z_2,t_2)$, produced by the laser pulse displayed in Fig. 1. A very large localized negative potential is created, with $|\phi| \approx 1$, which indicates the almost complete expulsion of electrons in the vicinity of the laser pulse.
Fig. 3 - Evolution of the nonlinear phase $\varphi(x_2, z_2, t_2)$ of the laser pulse displayed in Fig. 1. A noticeable bending of the wave front occurs for $t_2 > 0.5$, simultaneously with the emergence of an electrostatic wake.
CONCLUSIONS

- SIR of pancake-shaped laser pulses through an unmagnetized plasma, by using a (semi)analytic hydrodynamic description studied.

- Novel model nonlinear equations, based on three-timescale description, that appropriately describe all the three intensity regimes derived.

- These equations account for the evolution of the nonlinear phase of the laser wave.

- In the core of a very strong (i.e., SIR) pulse, the electromagnetic wave practically propagates in a vacuum. Such wave is not dispersive, i.e., its group velocity is constant and coincides with its phase velocity.

- Conversely, at the edges of such pulse, the amplitude is smaller and the wave is dispersive. Under such conditions, the simple envelope description, suitable for the MIR, breaks down.

- Kinetic effects, e.g., plasma wave-breaking, trapping of resonant particles, and their subsequent acceleration, not included in the present analysis. They are the subject of an under way study.
Thank you for your attention!