Semi-analytical fluid study of the propagation of an ultrastrong femtosecond laser pulse in a plasma with ultrarelativistic electron jitter

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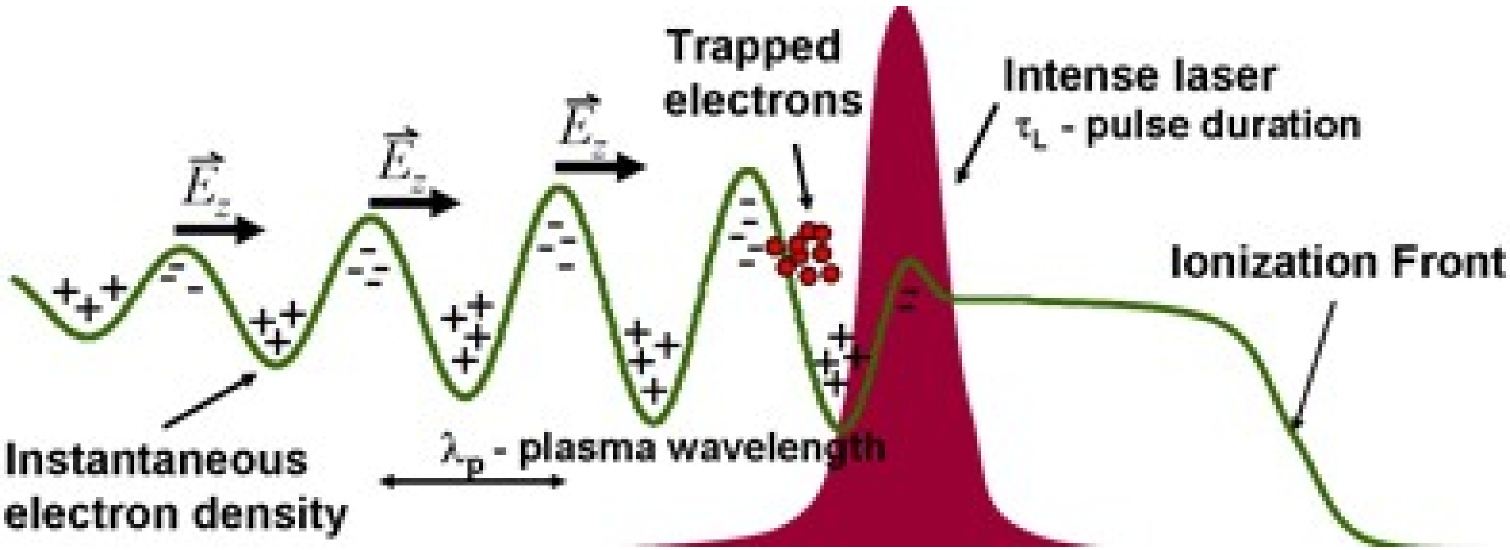
BASIC CONCEPT



The laser wake field (LWF) excitation is a process based on the ponderomotive effects induced by of an electromagnetic wave packet in a plasma. Basically it performs the process to transform a transverse electric field into a longitudinal one. For typical values of plasma density of the order of 10^{18} cm^{-3}, the latter is able to accelerate an electron at 1GeV in a few centimeters!

THREE FUNDAMENTALLY DIFFERENT REGIMES

- Weak Intensity Regime (WIR): the quiver motion is weakly relativistic $p_{\perp_0} \ll m_0 c \qquad p_{\perp_0} = eE_{\perp_0}/\omega$

I_{max} ranging from 2.5×10^{14} to 2.5×10^{16} W/cm^2

- Moderate Intensity Regime (MIR): the quiver motion is mildely relativistic $p_{\perp_0} \lesssim m_0 c \qquad p_{\perp_0} = eE_{\perp_0}/\omega$

I_{max} ranging from 1.5×10^{18} to 3×10^{19} W/cm^2

- Strong Intensity Regime (SIR): the quiver motion is ultra-relativistic $p_{\perp_0} \gg m_0 c \qquad p_{\perp_0} = eE_{\perp_0}/\omega$

I_{max} ranging from 10^{20} to $2.5\ 10^{22}$ W/cm^2

PHYSICAL SCENARIO

- To enable the predictions for the **multi-petawatt laser pulse** behavior, we derive a **novel mathematical model that describes both MIR and SIR**.
- In the **classical picture** of a slowly varying amplitude of the laser pulse, based on a **two-timescale description**, this is not possible because the dispersion characteristics of electromagnetic waves in MIR and SIR are too different from each other and can not be described on a common footing.
- In the **core of a very strong (i.e. SIR) pulse**, the electromagnetic wave practically **propagates in a vacuum**. Such wave is not dispersive, i.e. its group velocity is constant and coincides with its phase velocity.
- Conversely, **at the edges of such pulse the amplitude is smaller and the wave is dispersive.** Under such conditions, the simple envelope description used previously in the MIR, breaks down.

MATEMATICAL MODEL AND PHYSICAL CONSIDERATIONS

- The analysis is based on the **Lorentz-Maxwell fluid model in the fully relativistic regime** taking the pancake approximation, developed earlier.
- Our model is derived using **a three-timescale description**, with an **intermediate timescale associated with the nonlinear, intensity-dependent, phase of the electromagnetic pulse.**

MATEMATICAL MODEL AND PHYSICAL CONSIDERATIONS

- In MIR, the evolution of the plasma wake and of the laser pulse (depletion, frequency redshifting) was satisfactorily described using a reduced wave equation and a quasistatic plasma response, with a good agreement with full Maxwell-fluid results.
- Such fluid calculations provide a valuable insight also into kinetic phenomena, e.g. by **establishing the thresholds for the wave breaking that results in the electron trapping**.
- The analytic studies of the laser-plasma interaction with intensities suitable for LPA have been attempted hitherto only for quasi 1-D, pancake-shaped pulses, using the "***quasistatic***" approximation and in a **cold-fluid description**, see the classical papers and references therein.

MATEMATICAL MODEL AND PHYSICAL CONSIDERATIONS

Then we assume that:

- The transverse variations are much smaller than the longitudinal ones: $$\nabla_\perp \ll \partial/\partial z$$

the solution is slowly varying in the frame that moves with the velocity u **e**z we have derived our system ***wave equation* + *Poisson's equation*.**

These equations are valid in an **unmagnetized** plasma, and are written in the following dimensionless quantities (with obvious meaning of the symbols)

$$t' = \omega_{pe} t, \quad \vec{r}\,' = \frac{\omega_{pe}}{c}(\vec{r} - \vec{e}_z\, ut), \quad n' = \frac{n}{n_0}, \quad u' = \frac{u}{c},$$
$$\vec{p'} = \frac{\vec{p}}{m_0 c}, \quad \vec{v'} = \frac{\vec{v}}{c}, \quad \phi' = \frac{q\phi}{m_0 c^2}, \quad \vec{A'} = \frac{q\vec{A}}{m_0 c},$$

MATEMATICAL MODEL AND PHYSICAL CONSIDERATION

Maxwell's Eqs. parallel and perpendicular to the direction of the e.m. wave propagation (in terms of the vector potential and the scalar potential)

$$\left[\frac{\partial^2}{\partial t^2} - 2u\,\frac{\partial^2}{\partial z\,\partial t} - (1-u^2)\,\frac{\partial^2}{\partial z^2} - \nabla_\perp^2\right]\vec{A}_\perp + \nabla_\perp\left(\frac{\partial}{\partial t} - u\,\frac{\partial}{\partial z}\right)\phi = \vec{v}_\perp n,$$

$$\left(\nabla_\perp^2 + \frac{\partial^2}{\partial z^2}\right)\phi = 1 - n.$$

BEYOND THE SLOWLY-VARYING AMPLITUDE APPROXIMATION

Electron continuity Eq., longitudinal and perpendicular components of the momentum Eqs. ➡

$$\left(\frac{\partial}{\partial t} - u\,\frac{\partial}{\partial z}\right) n + \nabla \cdot (n\vec{v}) = 0,$$

$$\left(\frac{\partial}{\partial t} - u\,\frac{\partial}{\partial z} + \vec{v}_\perp \cdot \nabla_\perp\right)(p_z + A_z) - \vec{v}_\perp\,\frac{\partial}{\partial z}(\vec{p}_\perp + \vec{A}_\perp) + \frac{\partial}{\partial z}(\gamma + \phi) = 0,$$

$$\left[\frac{\partial}{\partial t} + (v_z - u)\frac{\partial}{\partial z} + \vec{v}_\perp \cdot \nabla_\perp\right](\vec{p}_\perp + \vec{A}_\perp) - v_i \nabla_\perp (p_i + A_i) + \nabla_\perp (\gamma + \phi) = 0,$$

$$\gamma = (1 + \vec{p}^{\,2}/m_0^2 c^2)^{\frac{1}{2}}$$

QUASI-STATIC APPROXIMATION

- The solution of the hydrodynamic equations is sought in a quasistatic regime, i.e. when it is only slowly varying in the moving reference frame, viz.

$$\partial/\partial t \ll u\,\partial/\partial z$$

$$\partial/\partial t = \nabla_\perp = 1 - u = 0$$

- In the approximate expressions for the charge and current densities, we use the leading order solution of the electron hydrodynamic equations, which is found as **a stationary 1-D solution that is propagating with the speed of light**, setting

QUASI-STATIC APPROXIMATION

the leading parts of motion Eqs. are obtained in a simple form

$$\frac{\partial}{\partial z}[(v_z - 1)n] = 0,$$

$$\frac{\partial}{\partial z}(-p_z + \gamma + \phi) = 0,$$

$$\frac{\partial}{\partial z}(\vec{p}_\perp + \vec{A}_\perp) = 0,$$

while from $\nabla \cdot \vec{A} = 0$, within the same accuracy, we have $\partial A_z/\partial z = 0$.

for $z \to \pm\infty$, we have $\phi = A = \vec{v} = \vec{p} = 0$
$\gamma = n = 1$, and using $\gamma = (1 + p_z^2 + \vec{p}_\perp{}^2)^{\frac{1}{2}}$

$$n = \frac{(\phi - 1)^2 + \vec{A}_\perp{}^2 + 1}{2(\phi - 1)^2}, \quad \vec{v}_\perp n = \frac{\vec{A}_\perp}{\phi - 1}$$

$$\left[\frac{\partial^2}{\partial t^2} - 2u\,\frac{\partial^2}{\partial t\,\partial z} - (1 - u^2)\frac{\partial^2}{\partial z^2} - \nabla_\perp^2 + \frac{1}{1-\phi}\right]\vec{A}_\perp$$

$$= -\left(\frac{\partial}{\partial t} - u\,\frac{\partial}{\partial z}\right)\nabla_\perp\phi,$$

$$\frac{\partial^2\phi}{\partial z^2} = \frac{(\phi - 1)^2 - 1 - \vec{A}_\perp{}^2}{2(\phi - 1)^2}.$$

For both MIR and SIR, we seek the solution of the wave equation in the moving frame as the sum of a slowly varying component and a modulated electromagnetic wave, including a **phase** that is varying on **an intermediate scale**, viz.

Slowly varying vector potential corresponding to the selfgenerated quasistationary magnetic field.

$$\vec{A}_\perp = \vec{A}_\perp^{(0)}(t_2, \vec{r}_2) + \{\vec{A}_{\perp_0}(t_2, \vec{r}_2)\, e^{i[\varphi(t_1, \vec{r}_1) - \omega' t + k'(z+ut)]} + c.c.\}.$$

$$\omega' = \frac{\omega}{\omega_{pe}}, \quad k' = \frac{ck}{\omega_{pe}} = \frac{d_e}{\lambda}$$

$$\omega = \sqrt{c^2k^2 + \omega_{pe}^2}$$

$$\alpha_{Re}(\phi) = \phi/(1-\phi) + \kappa^2(\phi),$$

$$\alpha_{Im}(\phi) = -\nabla_1^2\varphi + \frac{\partial^2\varphi}{\partial t_1^2} = -\nabla_\rho^2\varphi + \mathcal{O}(\epsilon^4).$$

ZAKHAROV – TYPE SYSTEM OF EQUATIONS TO DESCRIBE PARAMETRIC PROCESSES

Regime in which the self-generated magnetic field can be neglected:

$$[\alpha_{Re}(\phi) + i\,\alpha_{Im}(\phi)]A_{\perp_0} - 2\,i\,\epsilon^2\,\frac{\partial A_{\perp_0}}{\partial t_2}$$

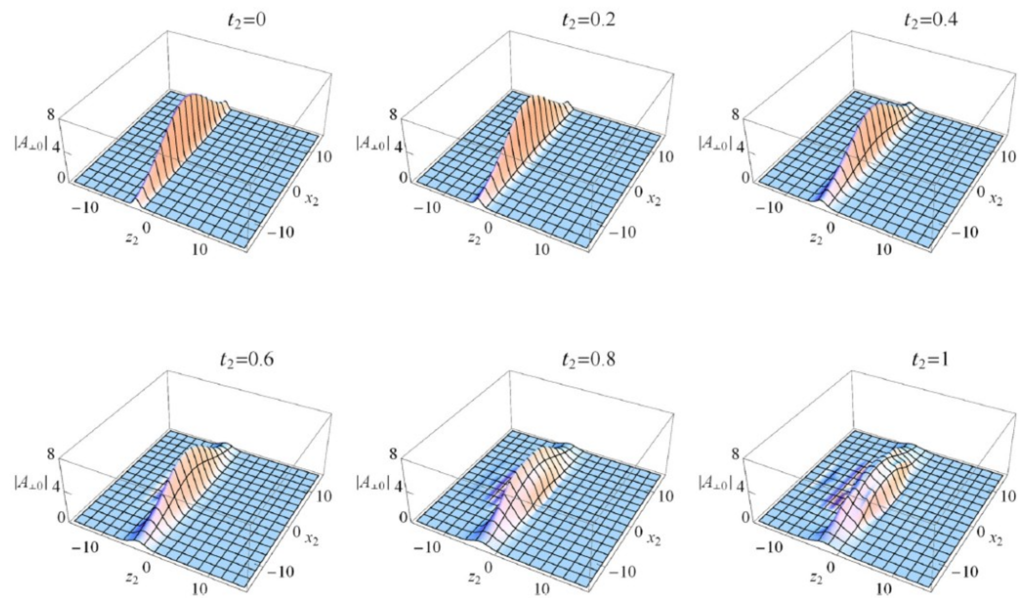
$$-2\,i\,\epsilon(\nabla_\rho\varphi\cdot\nabla_2)A_{\perp_0} - \epsilon^2\,\nabla_2^2 A_{\perp_0} = 0$$

$$\frac{\partial^2\phi}{\partial z_2^2} = \frac{(\phi-1)^2 - 1 - |A_{\perp_0}|^2}{2(\phi-1)^2},$$

$$(\nabla_\rho\varphi)^2 = \kappa^2(\phi),$$

SOME NUMERICAL RESULTS

Fig. 1 – Evolution of the envelope of the pancake laser pulse with an amplitude that is expected to be used in a future accelerator scheme (SIR). The initial condition was:



$$A_{\perp_0}(x_2, z_2, 0) = 0.6\, a_L(z_2/L_z)\ \exp(-x_2^2/2L_x^2)\ \exp(i\,\delta k\, z_2)$$

$L_z = 1.6$ and $L_x = 7.5$. $\delta k = 0.5$

which gave the maximum stability. The initial electrostatic potential and initial nonlinear phase were adopted to be zero.

In the physical (non-scaled) variables, these initial pulse length and width are 1.8 mm and 300 mm, respectively. Likewise, the dimensionless time t_{2max} = 1.1 corresponds, in physical units, to 9.69 10^{-12} s, during which time the pulse travels 3 mm. (color online).

SOME NUMERICAL RESULTS

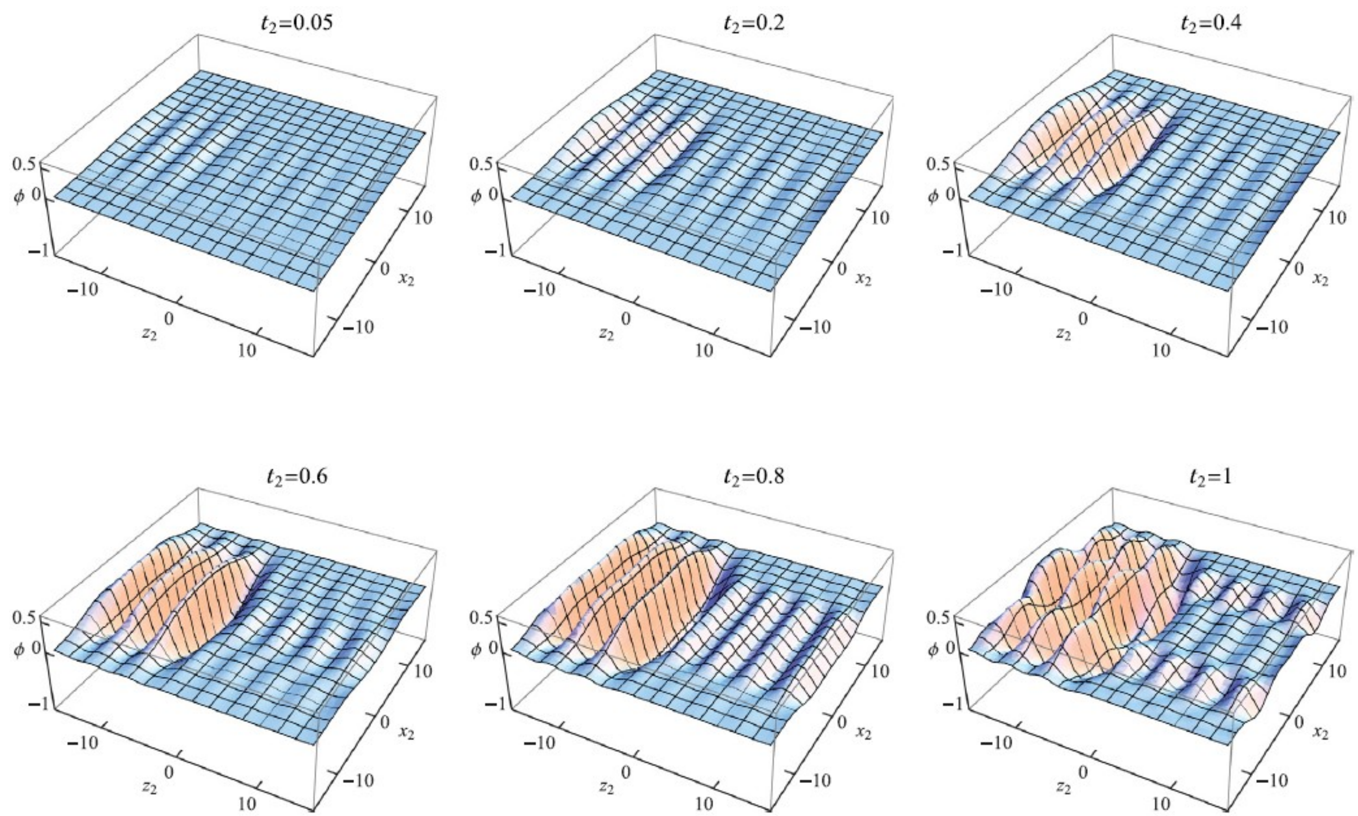


Fig. 2 - Evolution of the electrostatic wake potential $\phi(x_2,z_2,t_2)$, produced by the laser pulse dusplayed in Fig. 1. A very large localized negative potential is created, with $|\phi|\approx 1$, which indicates the almost complete expulsion of electrons in the vicinity of the laser pulse.

SOME NUMERICAL RESULTS

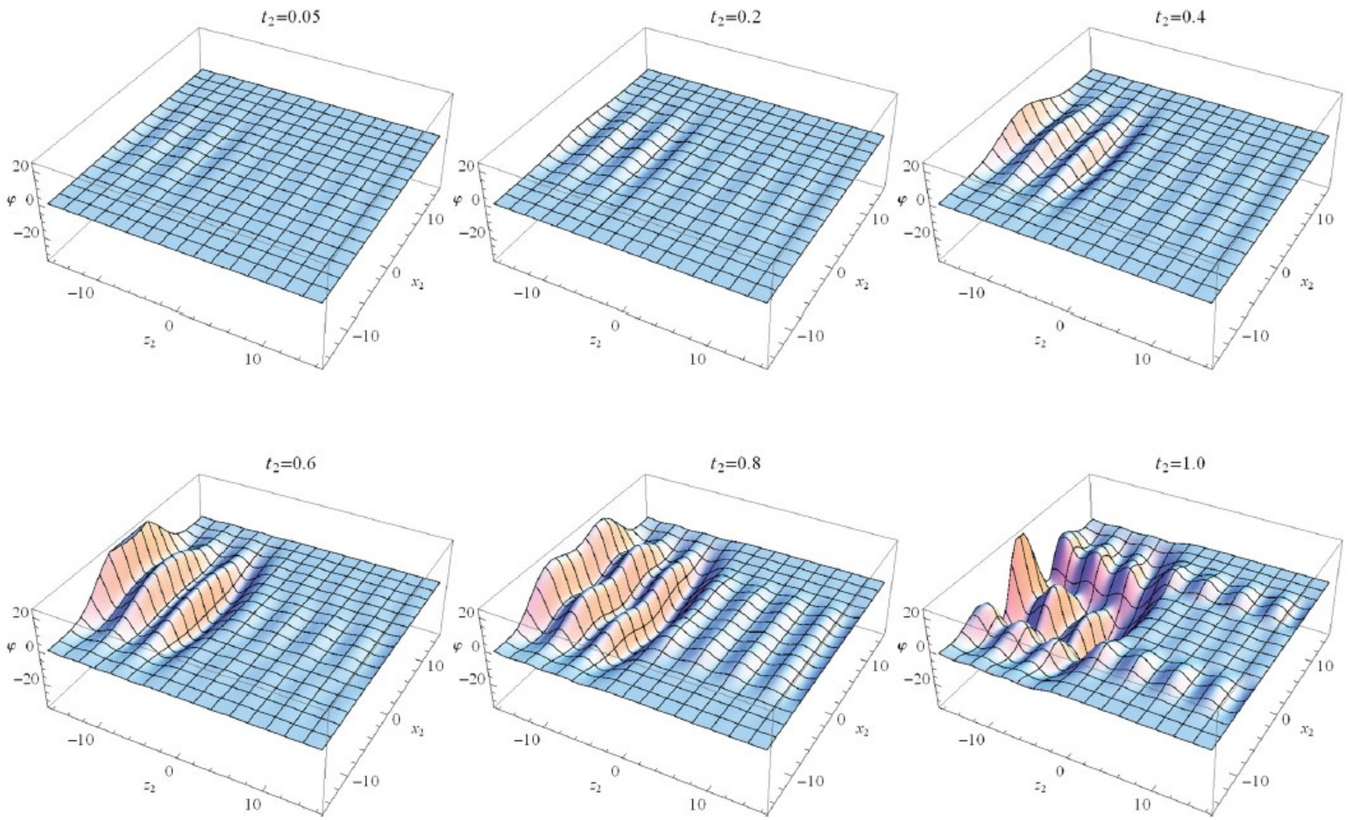


Fig. 3 - Evolution of the nonlinear phase $\varphi(x_2, z_2, t_2)$ of the laser pulse displayed in Fig. 1. A noticeable bending of the wave front occurs for $t_2 > 0.5$, simultaneously with the emergence of an electrostatic wake.

CONCLUSIONS

- SIR of pancake-shaped laser pulses through an unmagnetized plasma, by using a (semi)analytic hydrodynamic description studied.
- Novel model nonlinear equations, based on three-timescale description, that appropriately describe all the three intensity regimes derived.
- These equations account for the evolution of the nonlinear phase of the laser wave.
- In the core of a very strong (i.e., SIR) pulse, the electromagnetic wave practically propagates in a vacuum. Such wave is not dispersive, i.e., its group velocity is constant and coincides with its phase velocity.
- Conversely, at the edges of such pulse, the amplitude is smaller and the wave is dispersive. Under such conditions, the simple envelope description, suitable for the MIR, breaks down.
- Kinetic effects, e.g., plasma wave-breaking, trapping of resonant particles, and their subsequent acceleration, not included in the present analysis. They are the subject of an under way study.

Thank you for your attention!