

# Self-healing percolation

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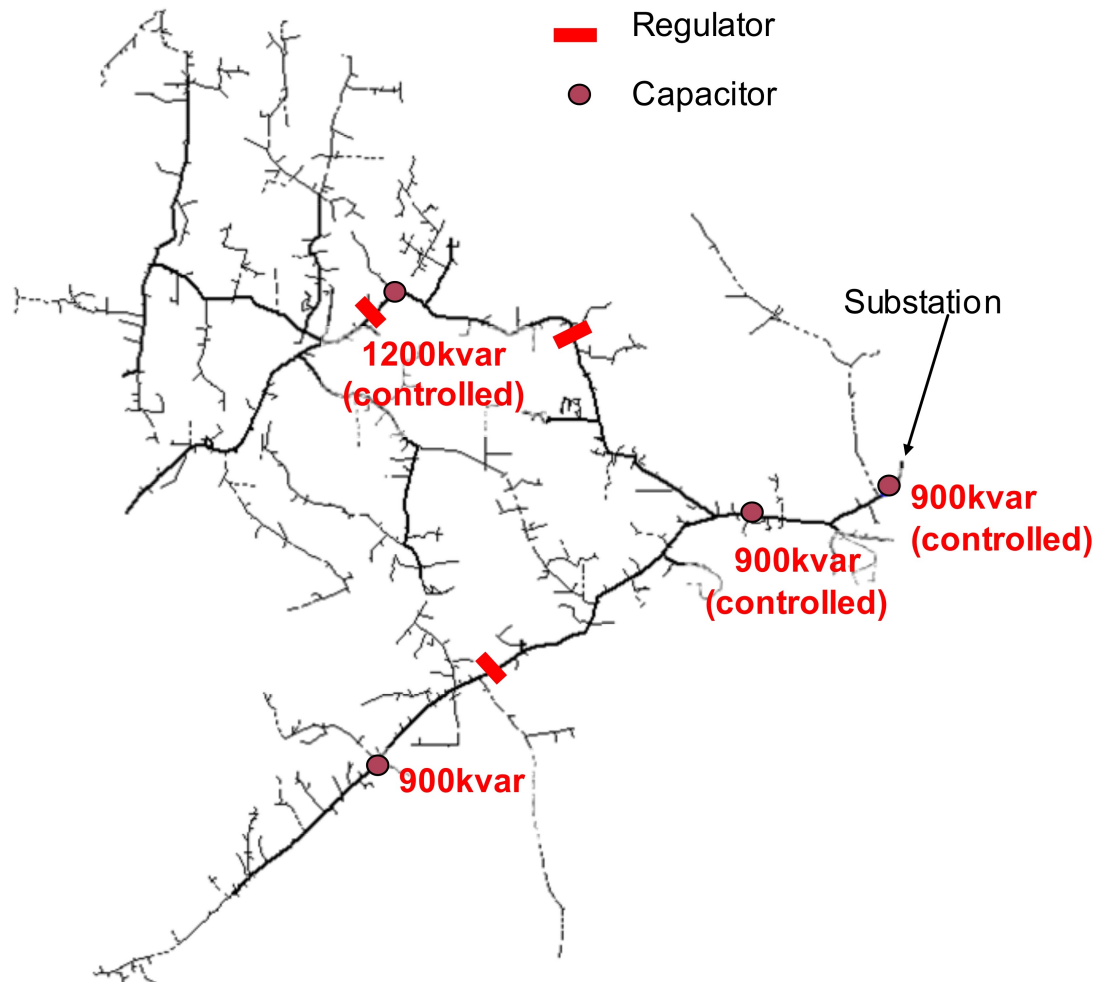
# Summary

- Self Healing Networks
- Cavity Method
- Self Healing Percolation
- Perspectives

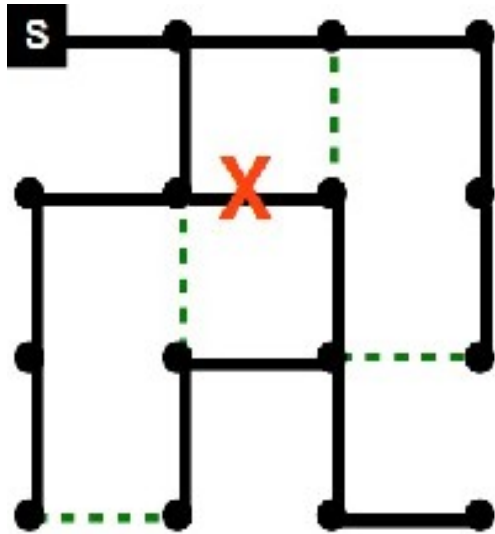


# Self Healing Networks

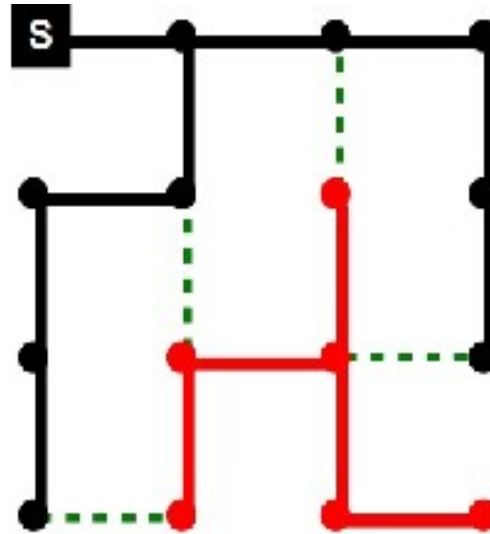
# IEEE test feeder (US, medium voltage)



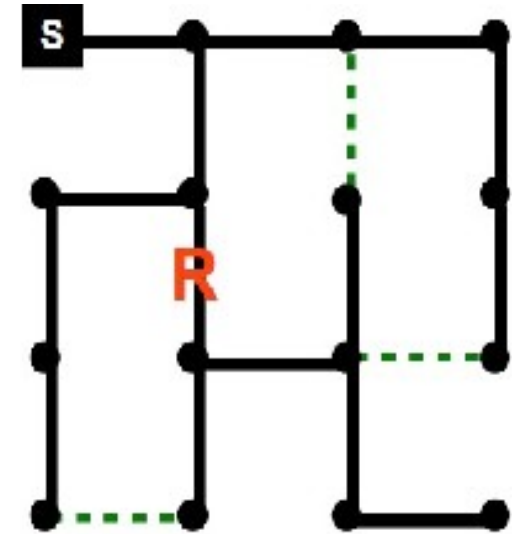
# Self-healing Networks



(a) Initial configuration



(b) Link failure



(c) Recovered Network

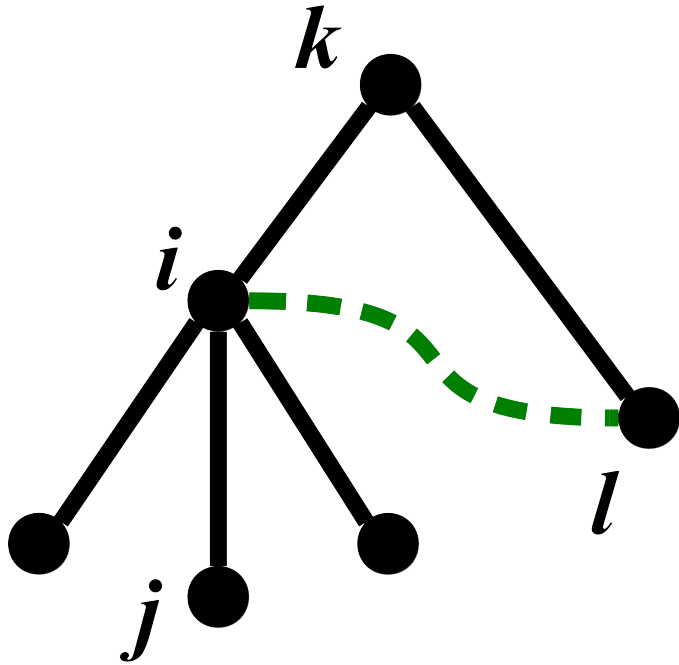
- Inspired by real distribution networks
- Routing Algorithms for Smart Networks

**Self-Healing Networks: Redundancy and Structure**  
**PLoS ONE 9 (2), e87986**



# Cavity Method

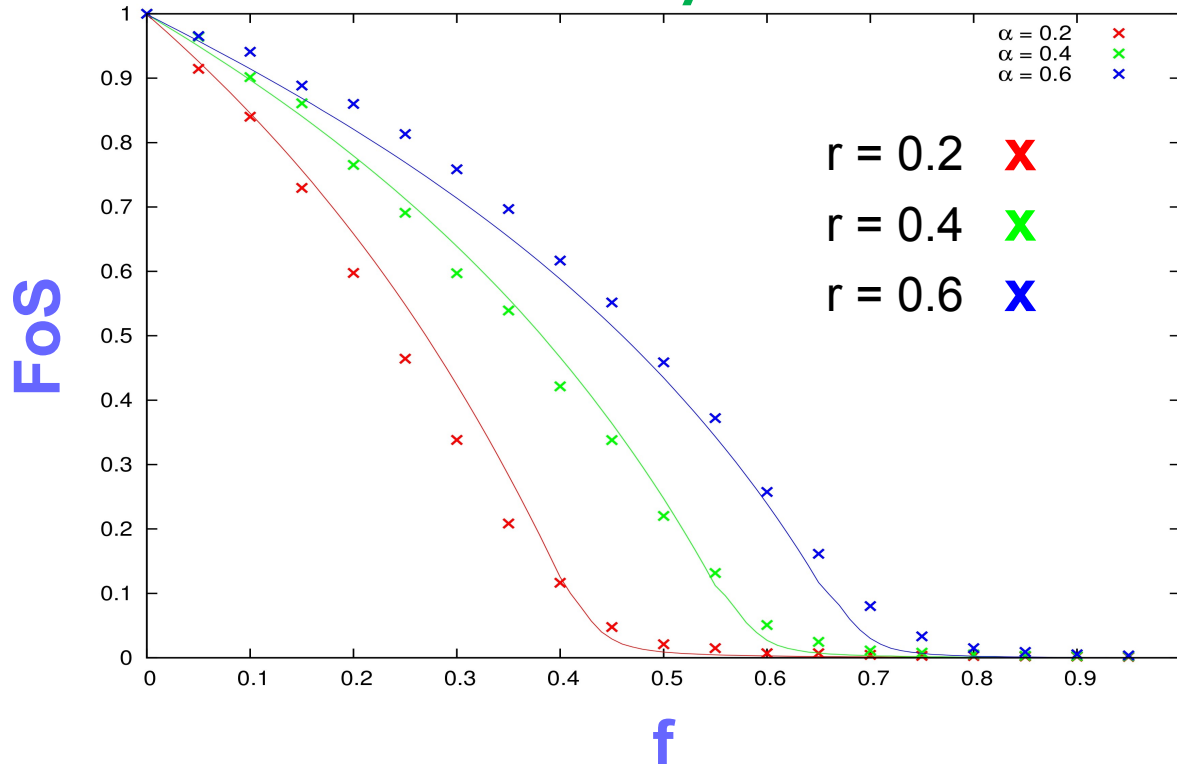
# Message Passing & Cavity Equations



Self consistent equation set for:

- $down_{ij}$  probability of being connected when son  $j$  fails
- $up_{ik}$  probability of being connected when father  $k$  fails
- $redund_{il}$  probability of being connected when neighbour  $l$  fails

# Simulations & theory

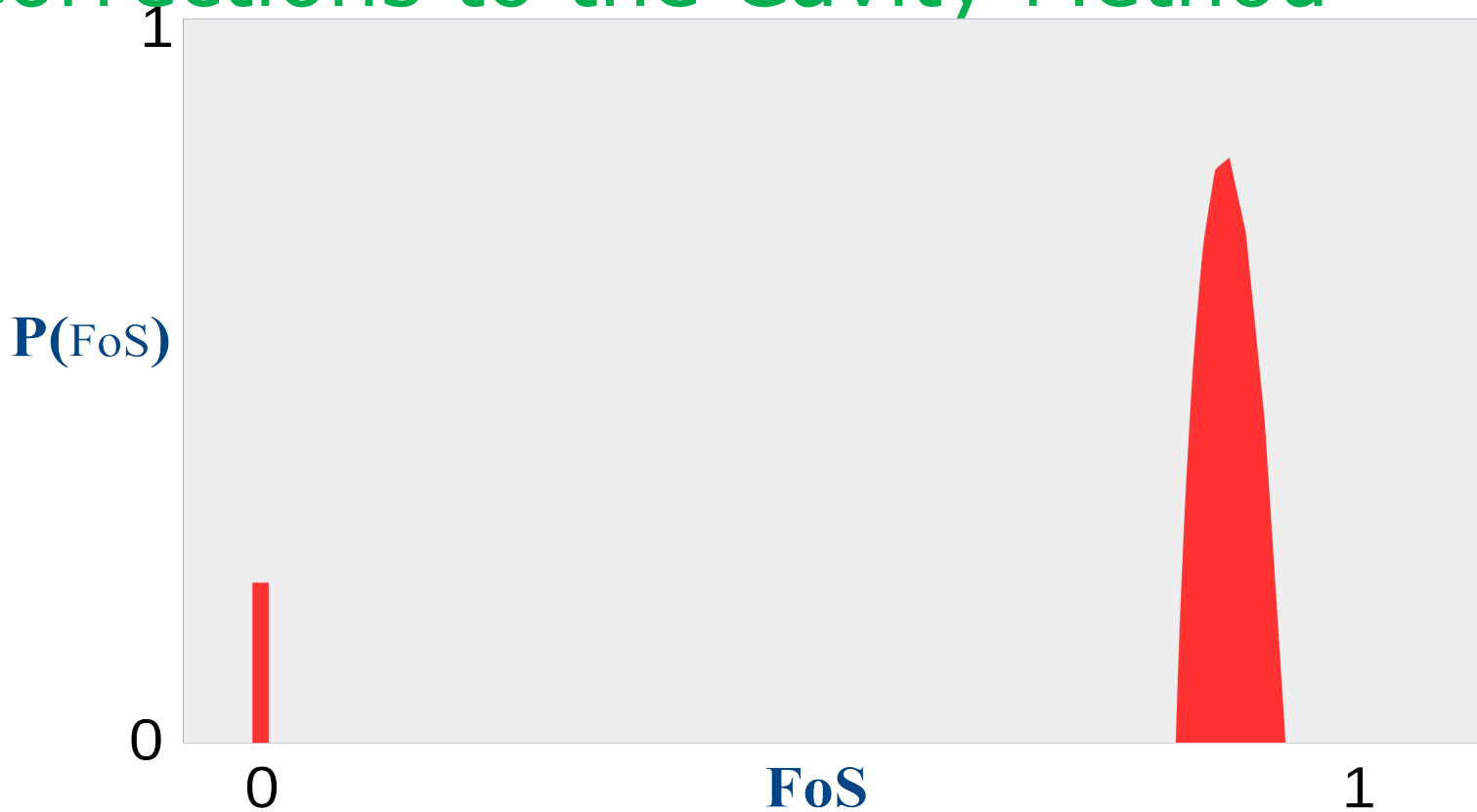


$r$  recoveries –  $f$  failures – **FoS** fraction of served nodes

Self-consistent equations for messages running on the edges of the tree and on redundant edges



# Corrections to the Cavity Method

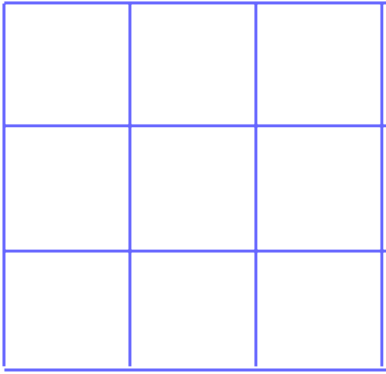


Two kinds of solutions:

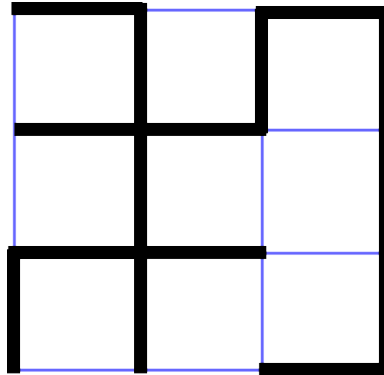
- Blackouts
- Decrease of the **FoS**

# Self-healing Percolation

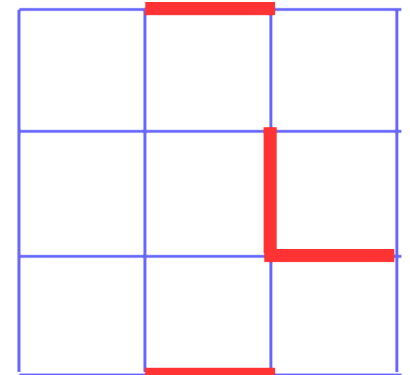
# Self-Healing percolation



World topology



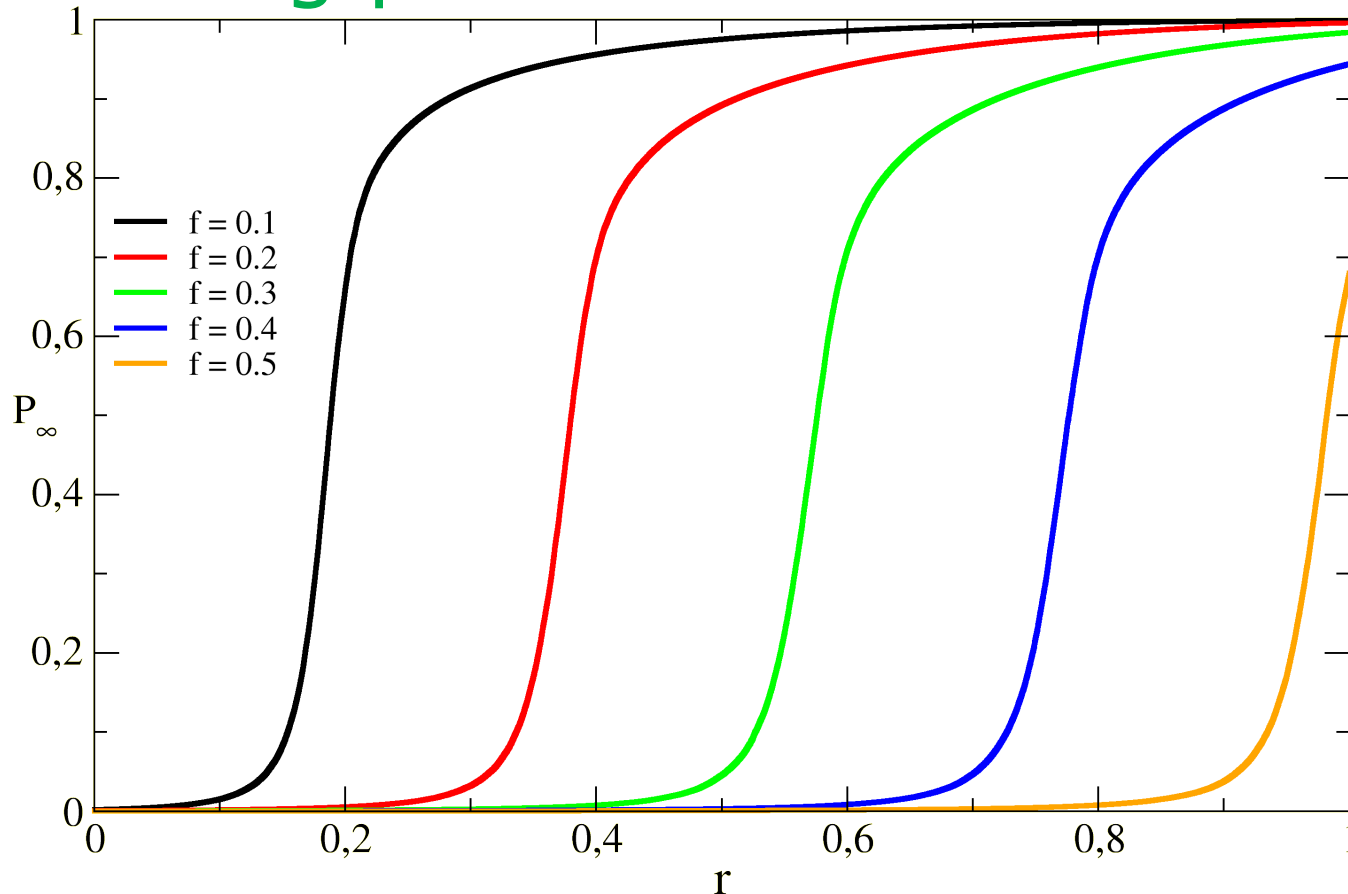
Spanning tree



Redundant links

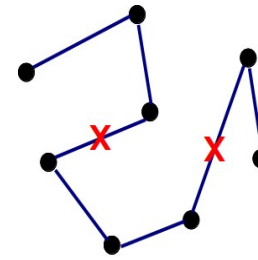
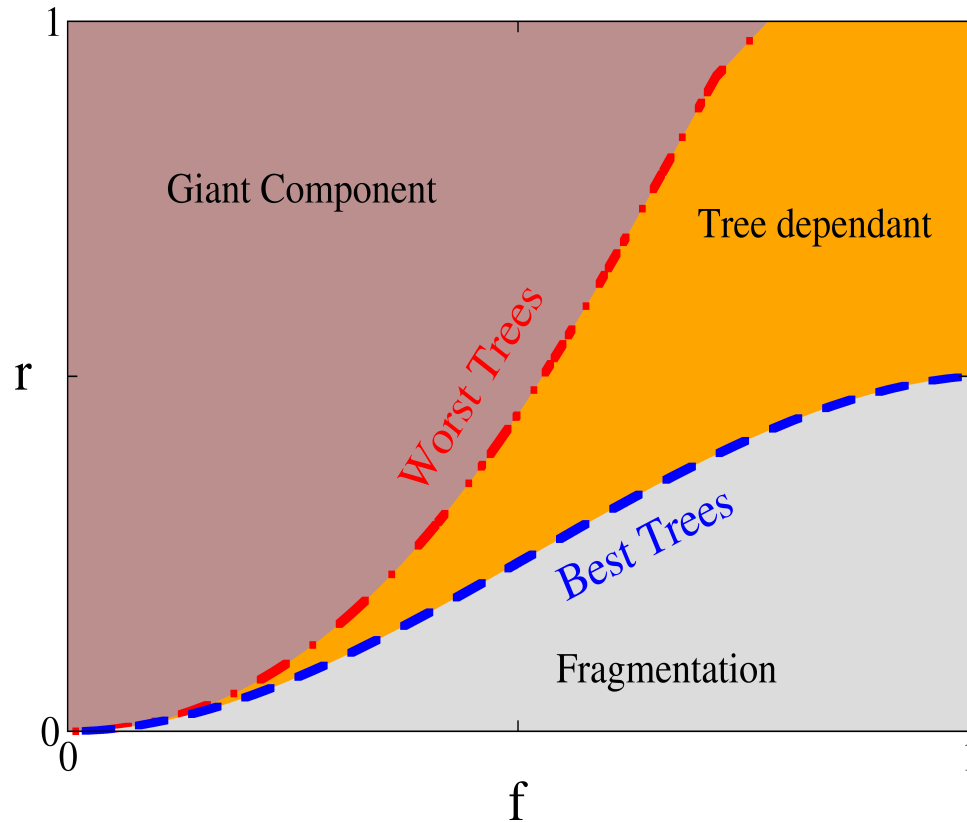
- Failures happen only on the **SPANNING TREE**
- Percolation = Survived links + redundant links

# Self-Healing percolation

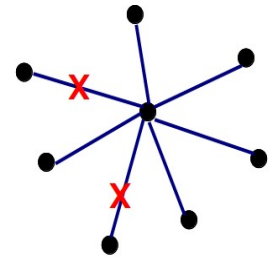
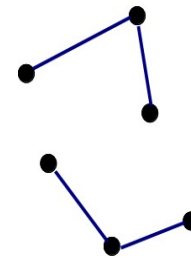


Given the failure of a fraction  $f$  of links, which fraction  $r$  of redundant links is needed to keep most of the system connected (existence of a *giant component*) ?

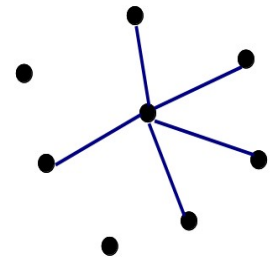
# Topology of the distribution tree



$$s_{\text{MAX}}/N = 3/8$$



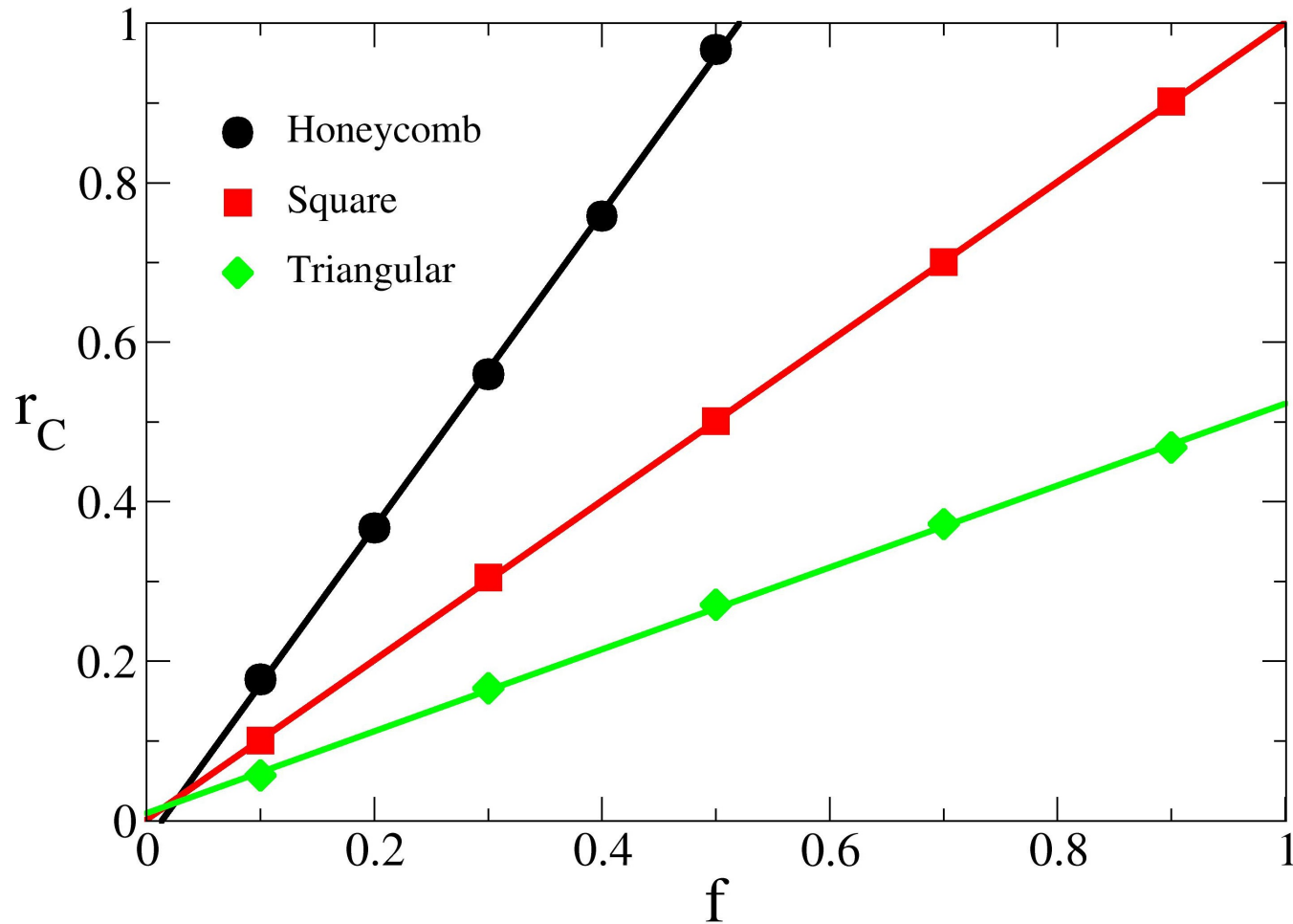
$$s_{\text{MAX}}/N = 6/8$$



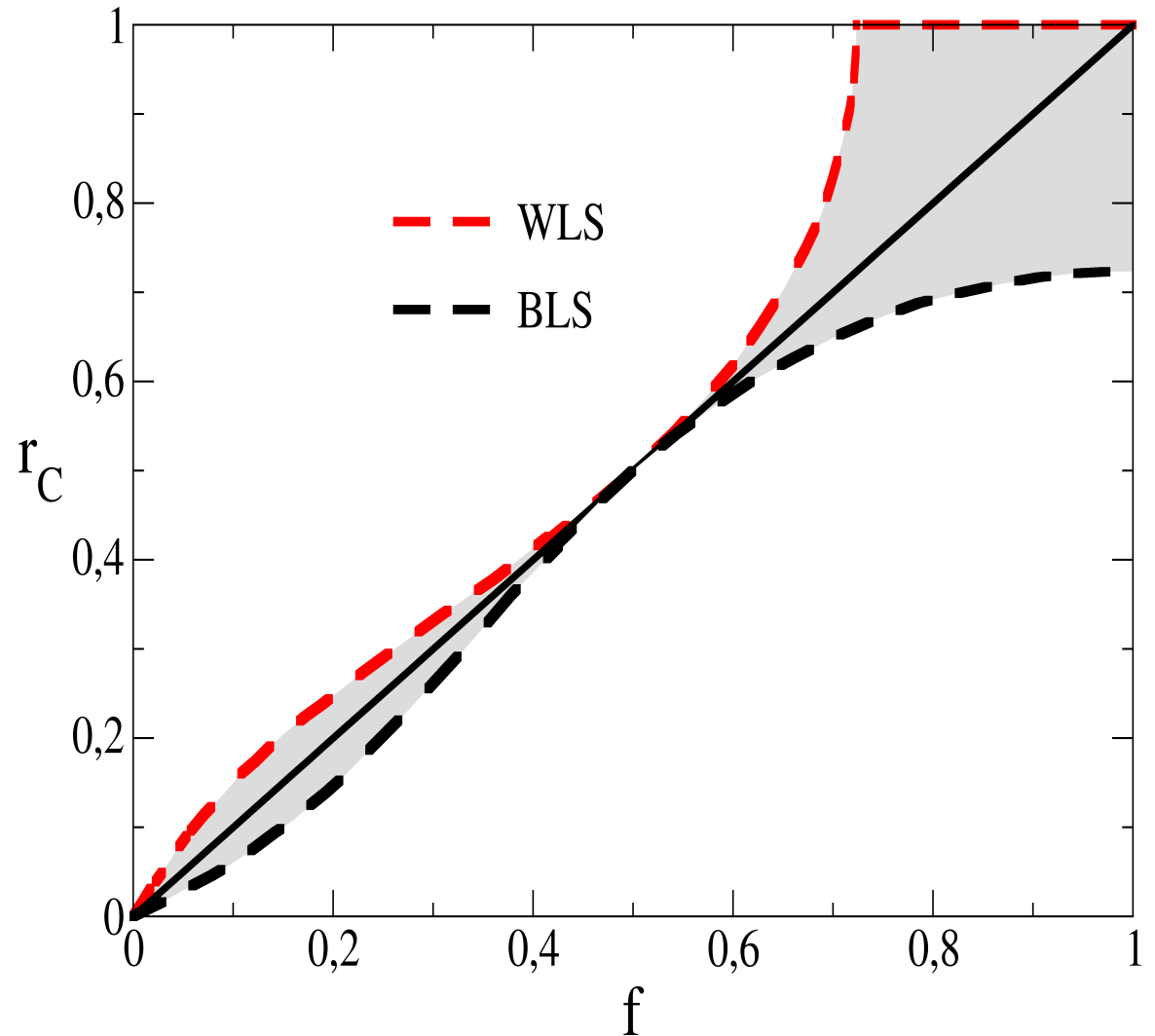
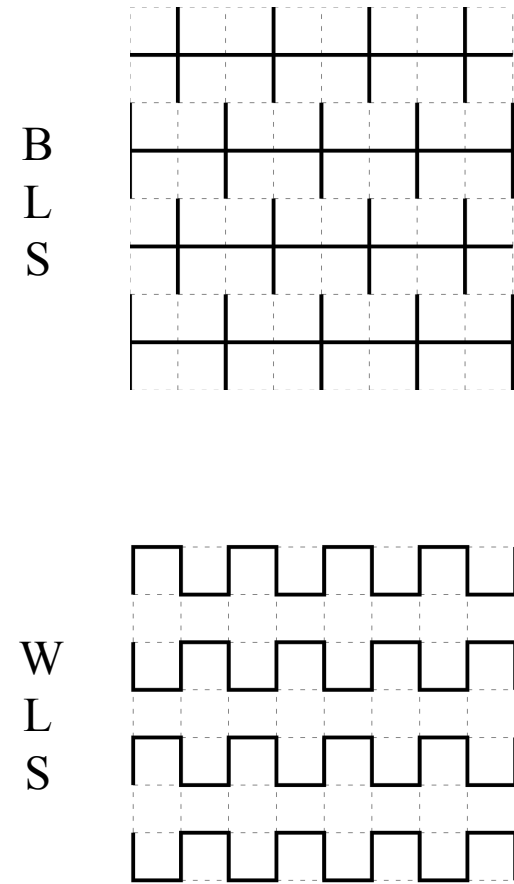
For random nets:

- Chain is the most fragile
- Star is the most robust

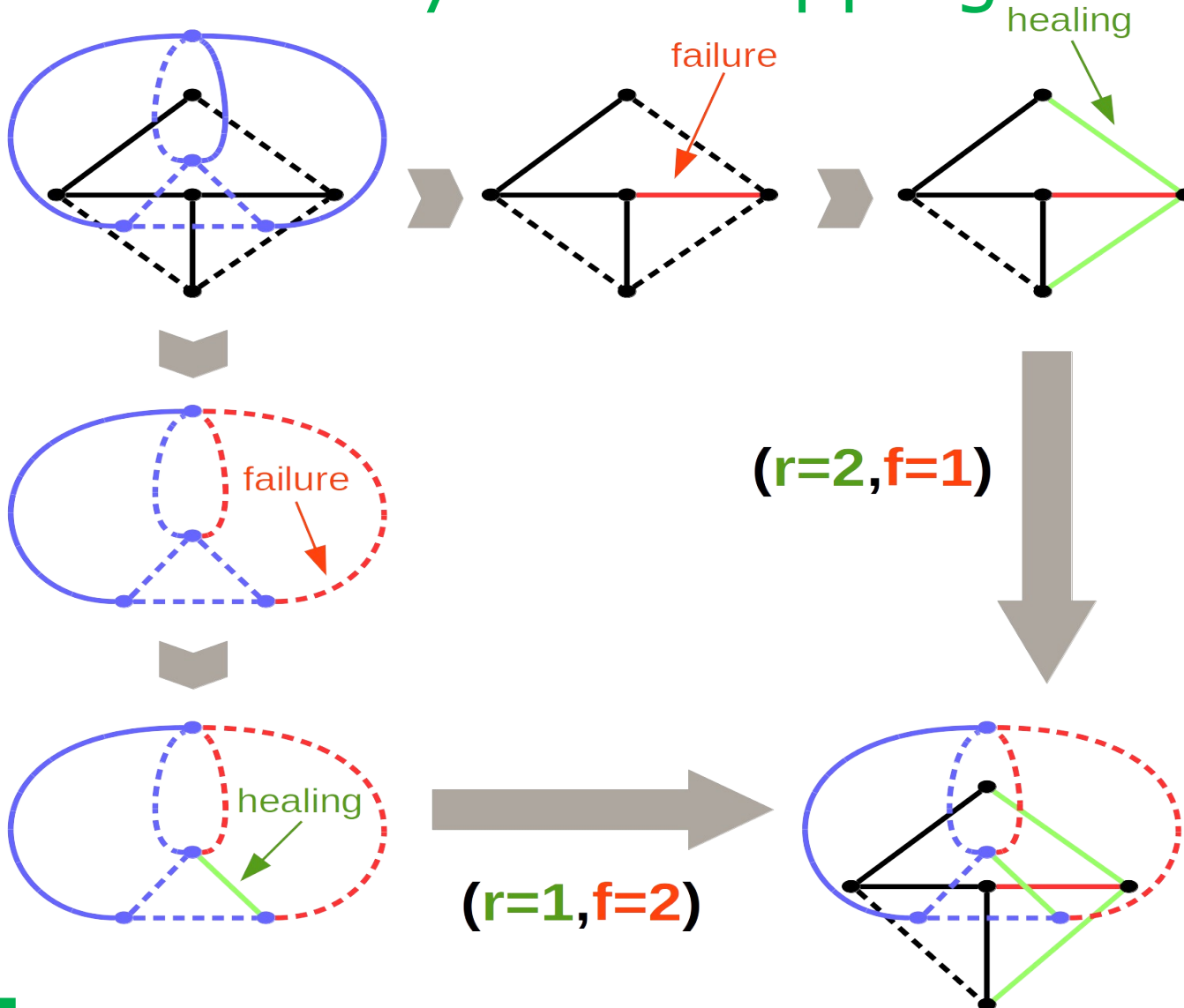
# Self-healing for planar lattices



# Optimal trees on SQ lattice



# Failure-recovery dual mapping





# Conclusions

- distribution networks have often built-in redundancies; “smartness” can be introduced and accomplished customizing “on the market” routing protocols

***smartness = stat-mech + distributed algorithms ?***

- Self-healing as a message passing problem
- Self-healing as a percolation problem

***much yet to do !!!***

- optimising the topology
- optimising the redundancies
- considering fluxes
- multiple/competing sources

THANKS TO:

- CNR-PNR National Project Crisis-Lab
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- EU HOME/CIPS project CI2C “Critical Infrastructures and Cloud Computing: understanding cross-sectorial criticalities and security practices”
- EU FP7 project DOLFINS “Distributed Global Financial Systems for Society”