Thresholds, localization and centrality in epidemic spreading on networks

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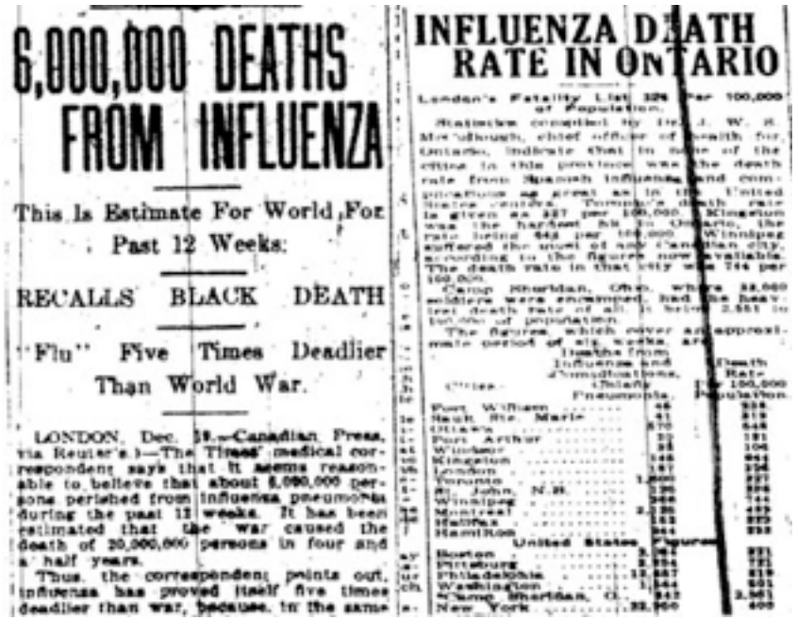
Epidemics

Disease epidemics have been a constant threat to mankind

- Black death epidemics killed one third of europeans between 1347 and 1353

- In 1918 spanish flu killed 40-50 million people, many more than world war I


6,000,000 DEATHS FROM INFLUENZA

This Is Estimate For World For Past 12 Weeks:

RECALLS BLACK DEATH

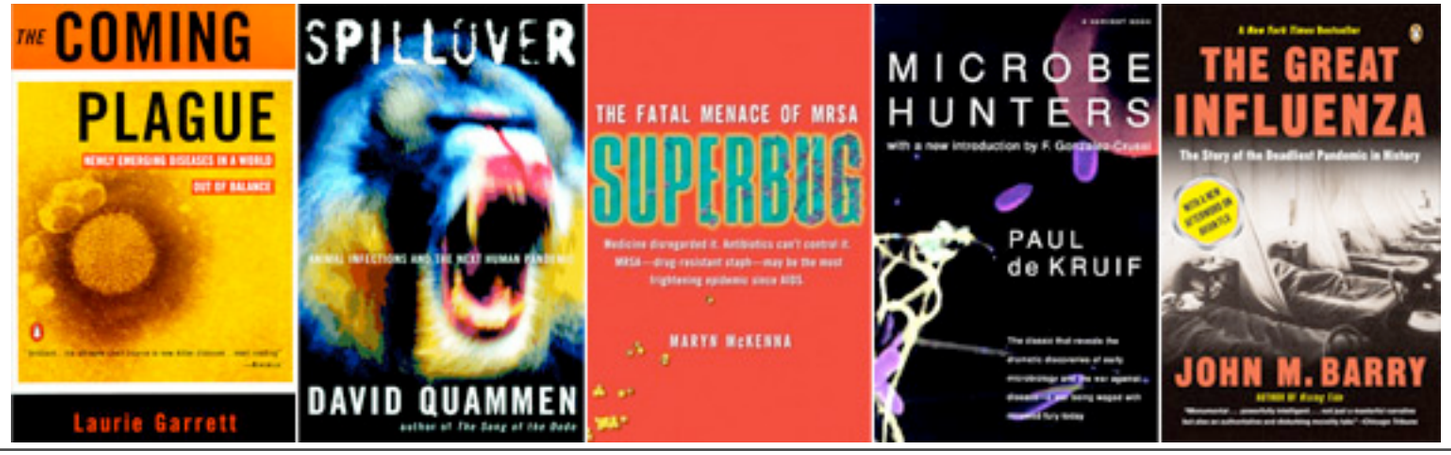
"Flu" Five Times Deadlier Than World War.

LONDON, Dec. 18.—Canadian Press, via Reuter's.)—The Times' medical correspondent says that it seems reasonable to believe that about 6,000,000 persons perished from influenza pneumonia during the past 12 weeks. It has been estimated that the war caused the death of 20,000,000 persons in four and a half years.

Thus, the correspondent points out, influenza has proved itself five times deadlier than war, because, in the same

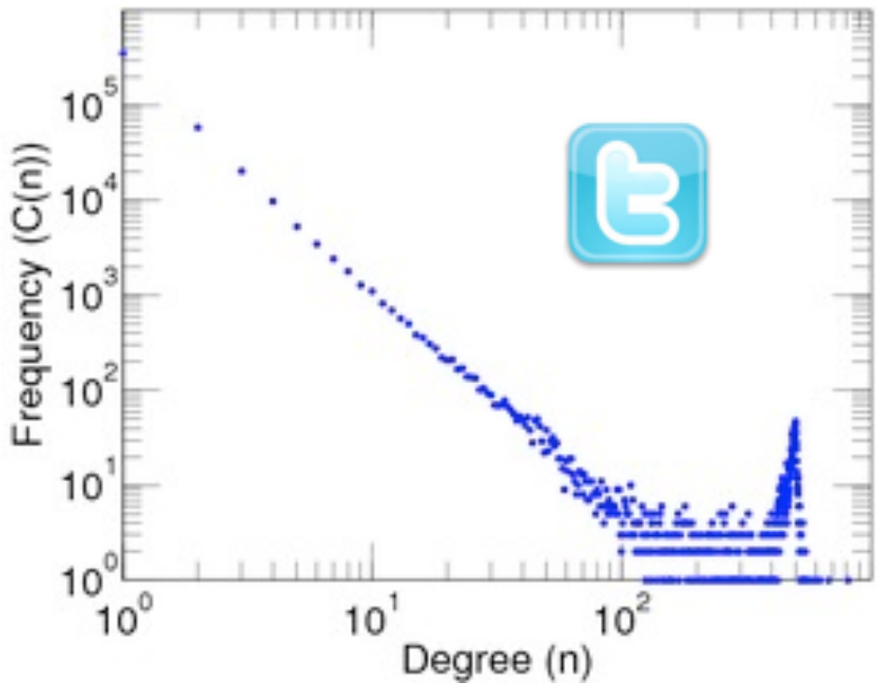
INFLUENZA DEATH RATE IN ONTARIO

- New epidemics constantly appear (HIV, SARS, Ebola...)



Networks are relevant

At the scale of individuals
interaction patterns are not regular



Heterogeneous networks are relevant

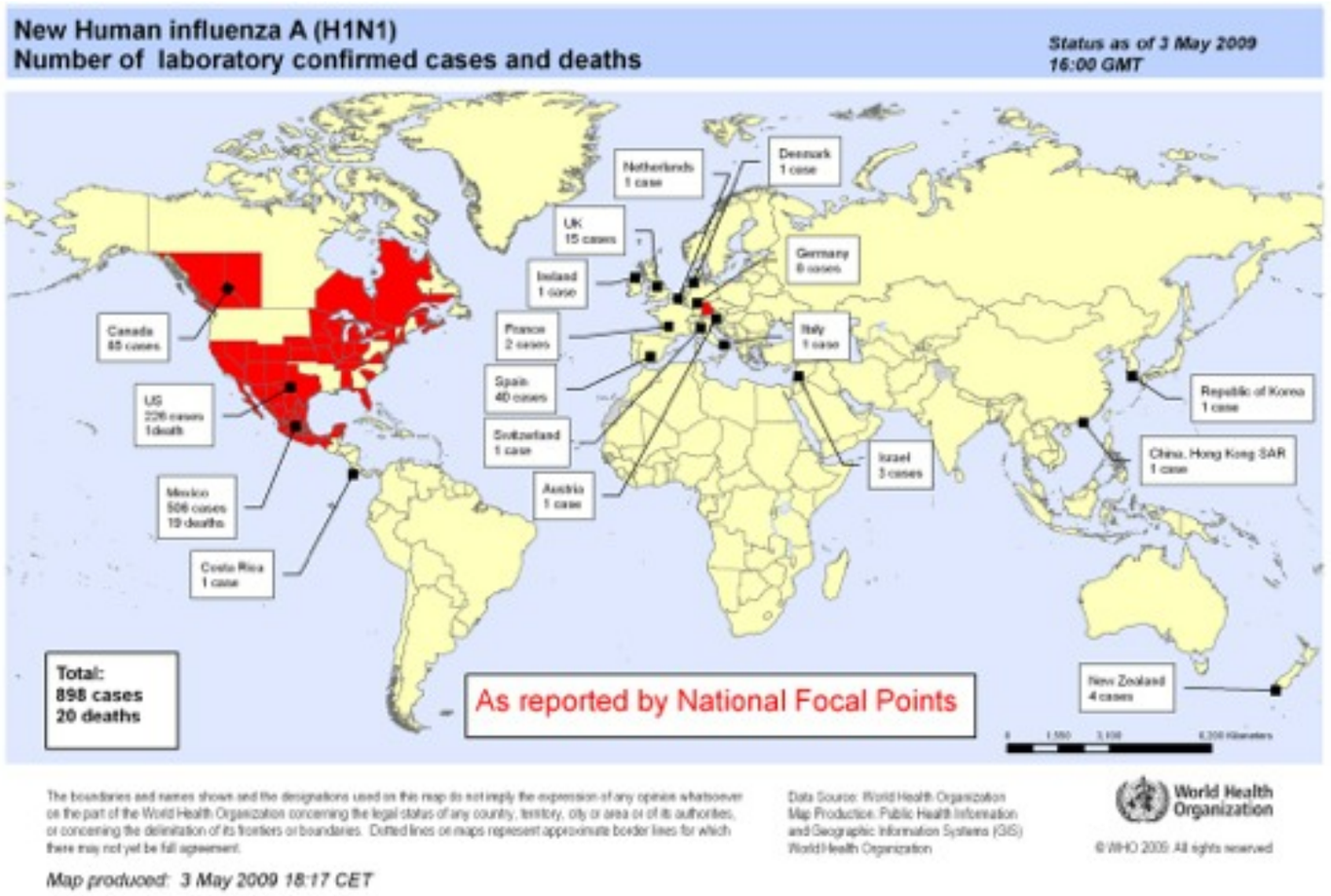
HIV "patient zero" infected 40 of the 284 first cases of AIDS in the USA

Epidemics and networks

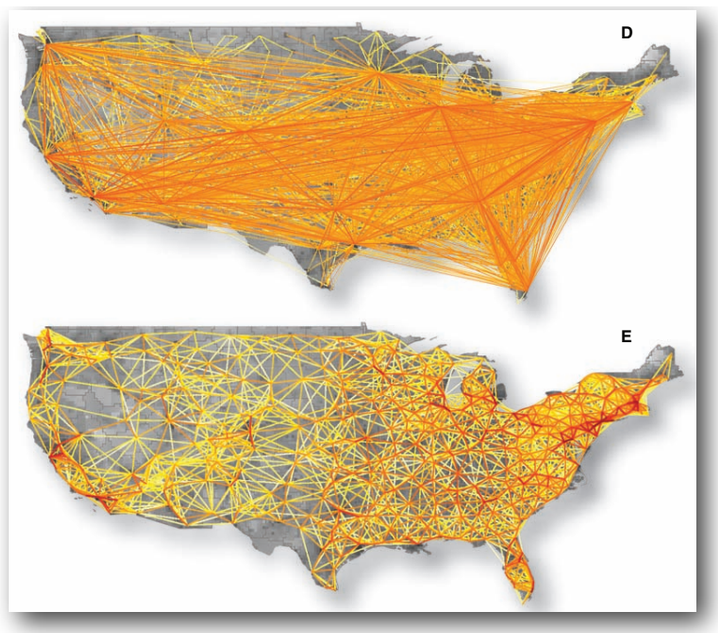
Networks are relevant

Black death

H1N1 2009 pandemics



Fast and long-range travel is crucial



Large-scale heterogeneous transportation networks are relevant

Modeling epidemics on networks

- Practical interest

  Crucial problem throughout human history

- Theoretical interest

  Nontrivial dynamics (percolation, branching processes, absorbing phase transitions)

- What is the value of the epidemic threshold?
- How does the prevalence varies?
- Which immunization protocols control the epidemics?
- Which spreaders are most influential?
- How can the origin of an outbreak be reconstructed?

Classes of epidemics

SIR class

- Permanent immunity
- Individuals are infected at most once
- Outbreaks have finite duration

SIS class

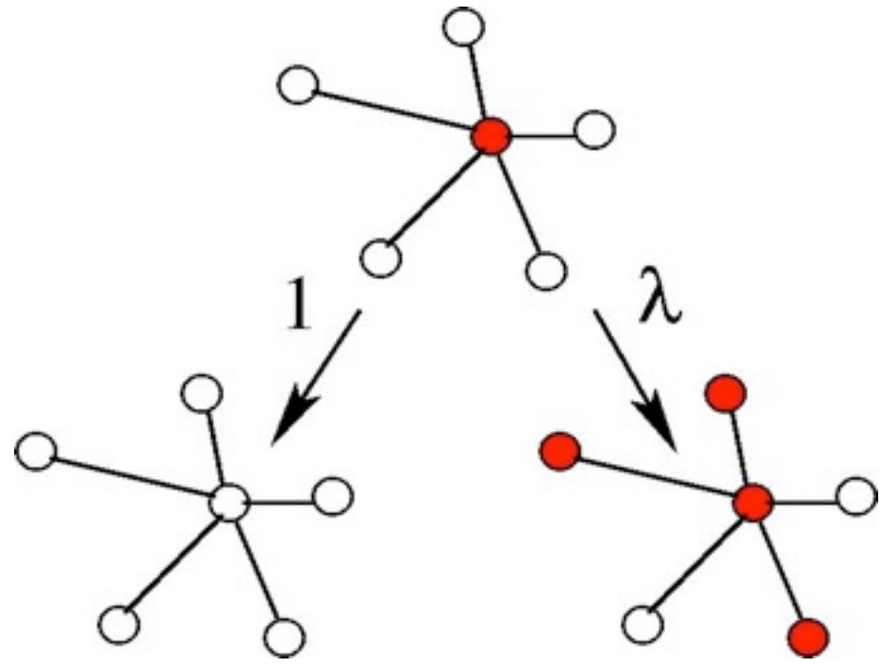
- Temporal/no immunity
- Individuals can be infected many times
- Outbreaks can persist forever

Susceptible-Infected-Susceptible (SIS) model

- Two possible states:

  susceptible (S)

  infected (I)

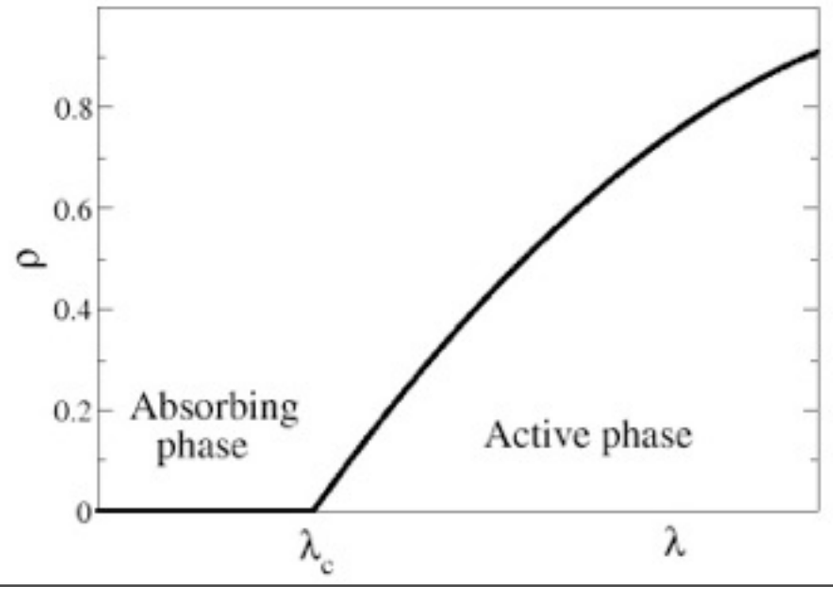


- Two possible events for infected nodes:

  Recovery I → S (rate μ=1)

  Infection to neighbors S+I → I+I (rate λ)

- Order parameter
  ρ = fraction of infected nodes
  in the stationary state



Heterogeneous Mean-Field theory (HMF) for SIS

Pastor-Satorras and Vespignani (Phys. Rev. Lett., 2001)

- Standard MF theory: $\dot{\rho} = -\rho + \lambda k \rho(1-\rho)$ $\qquad \lambda_c = \frac{1}{k}$
- What happens for heterogeneous networks ($P(k) \sim k^{-\gamma}$)?
- Assumption: degree determines the state of the node
- ρ_k = fraction of infected nodes of degree k

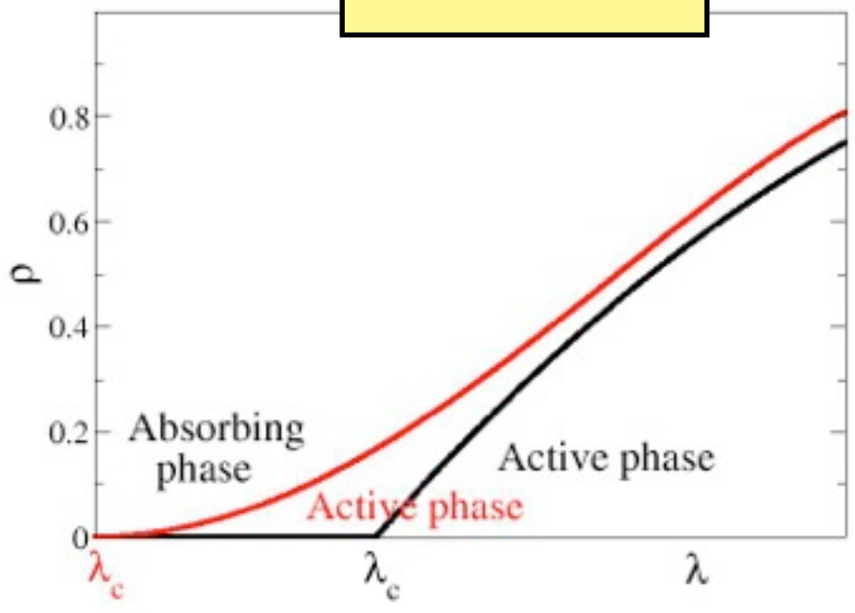
$$\dot{\rho}_k = -\rho_k + \lambda k[1-\rho_k] \sum_{k'} P(k'|k)\rho_{k'}$$

$$\lambda_c = \frac{\langle k \rangle}{\langle k^2 \rangle}$$

- In the limit of large system size

$$\lambda_c \to \begin{cases} 0 & \gamma \leq 3 \\ \text{finite} & \gamma > 3 \end{cases}$$

Zero threshold for scale-free networks



Beyond Heterogeneous Mean Field

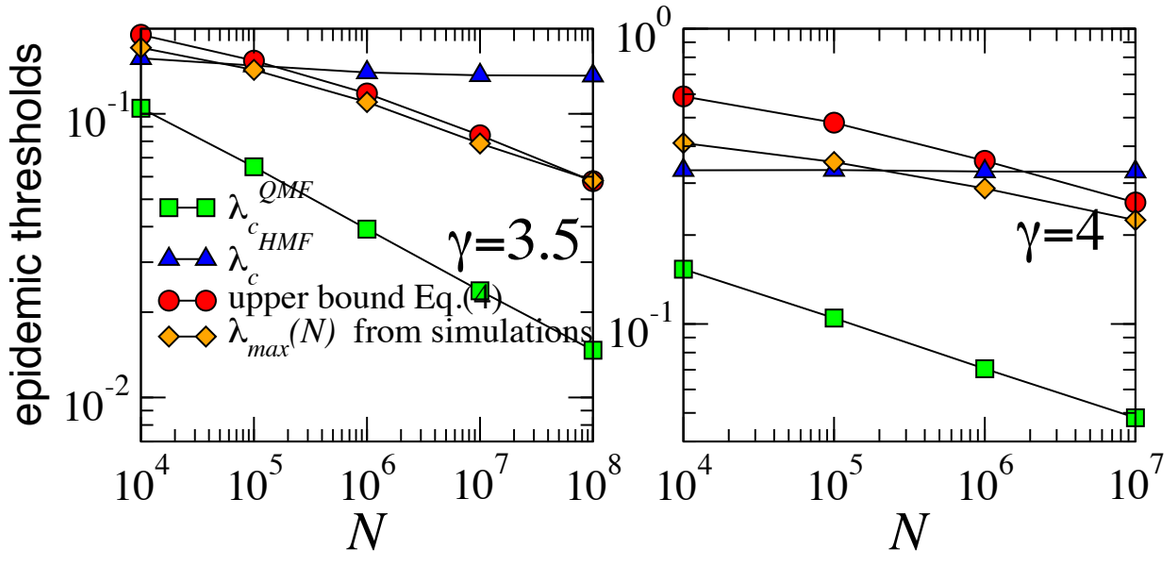
- Inclusion of detailed structure of the network: Quenched Mean Field

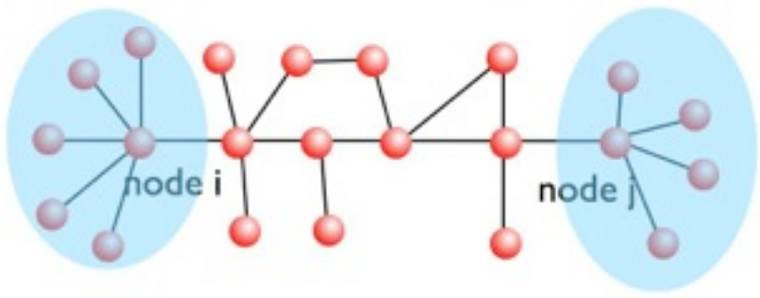
$$\lambda_c = \frac{1}{\Lambda_N}$$

Λ_N = Largest eigenvalue of adjacency matrix

- Eigenvector localization for $\gamma > 5/2$: global activity slightly decays over time (Griffiths phase)?

- Inclusion of dynamical correlations between distant neighbors (reinfection among distant hubs)

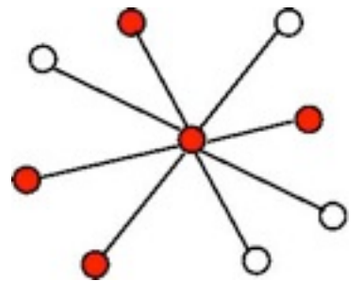




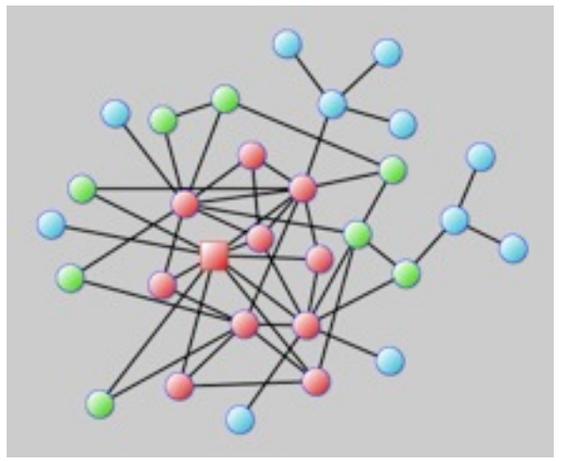
The epidemic threshold goes to zero for any γ

Distinct triggering mechanisms

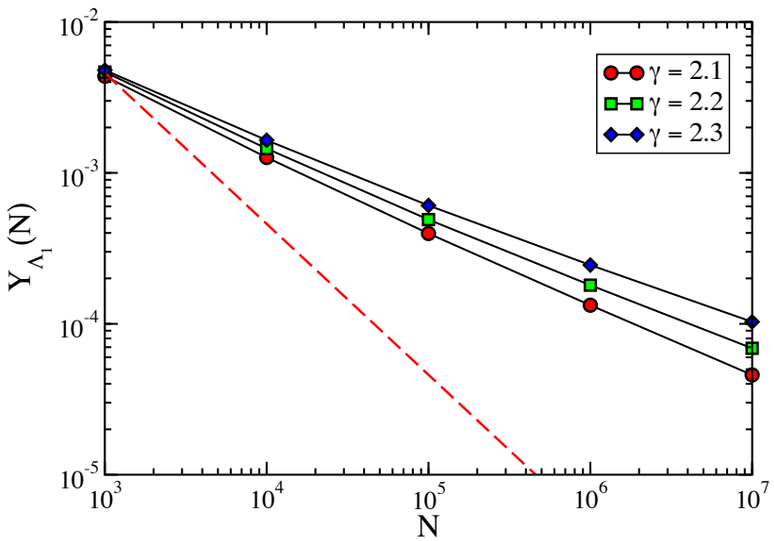
- γ > 5/2: single hub triggers the epidemics

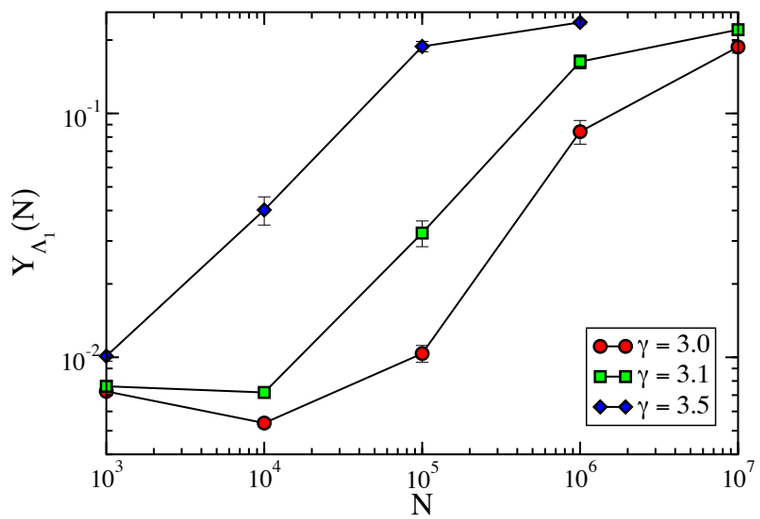


- γ < 5/2: mesoscopic subgraph of densely connected nodes triggers the epidemics



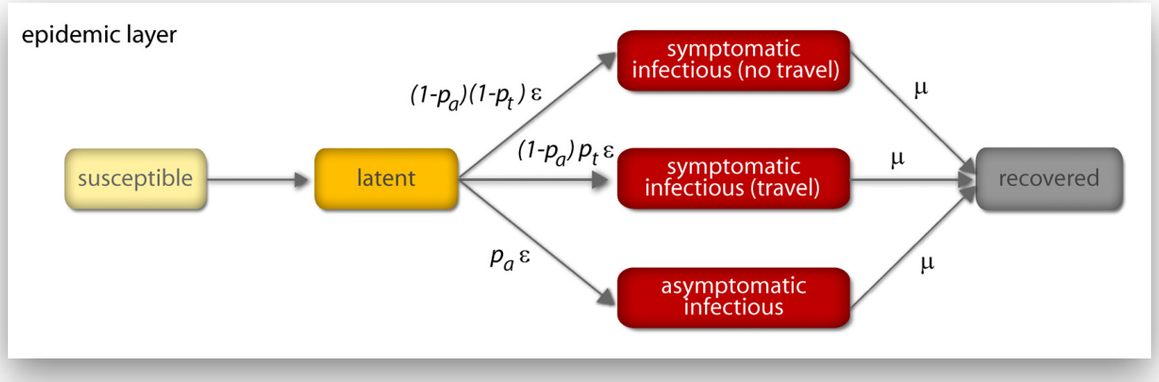
- Distinct triggering mechanisms correspond to distinct eigenvector localizations in networks

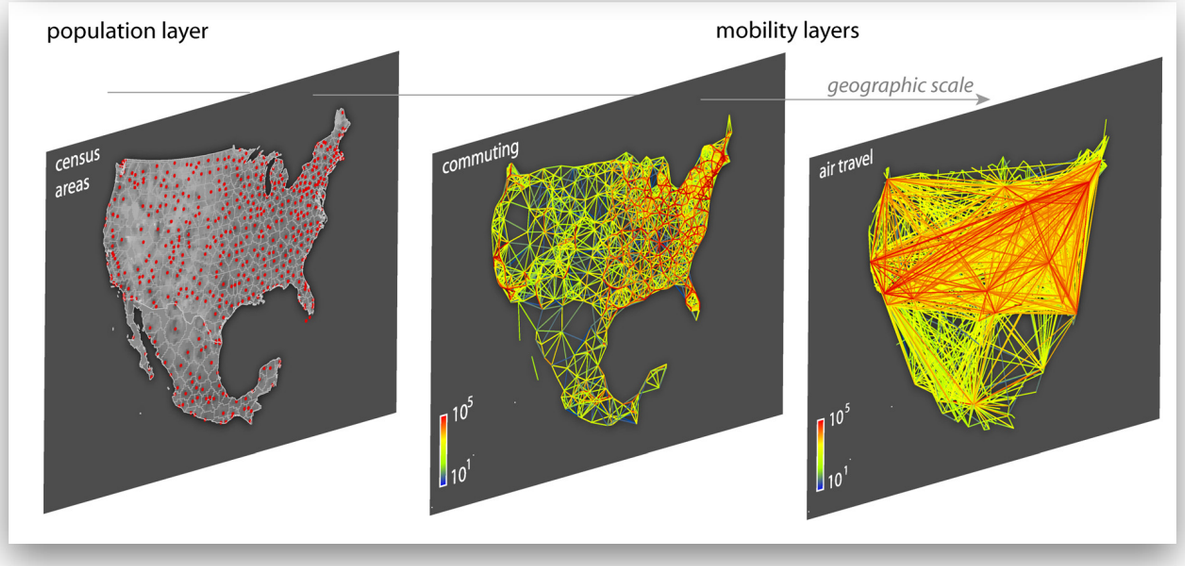


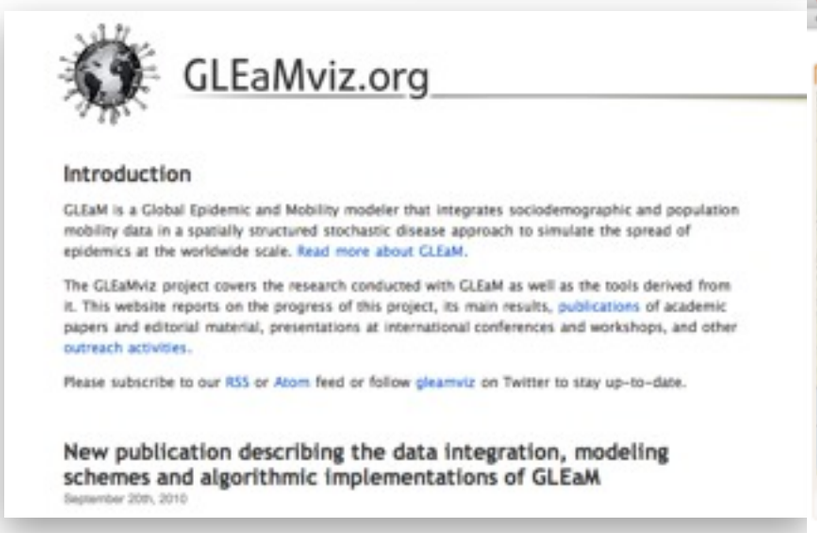


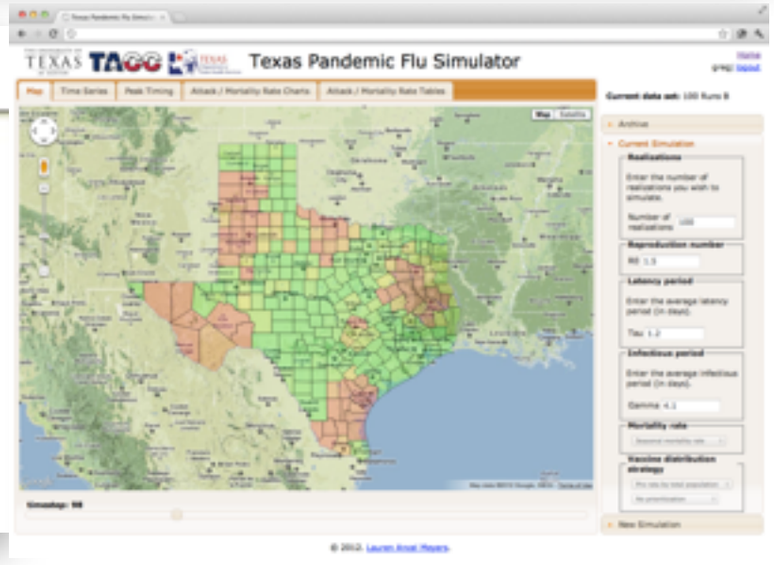
Predicting pandemics

- Forecasting simulation frameworks are nontrivial systems, implementing:
  - Realistic models of disease propagation, based on real epidemiological data
  - Different layers of social, population and mobility data
  - Computational power









Social contagion

Epidemic-like phenomena are ubiquitous

- Computer viruses
- Information diffusion
- Rumor spreading
- Adoption of innovations
- Fashion
- Behavioral contagion
- .....

Conclusions

- SIS epidemic threshold always vanishes in the large size limit
- Mean-field approaches capture only part of the picture
- Depending on heterogeneity (value of γ)
  - Different mechanisms trigger the epidemic transition
  - Different types of eigenvector centrality localization may occur
- Networks with $\gamma < 5/2$ are much different from those with $\gamma > 5/2$

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