



Non-Symmetrized Hyperspherical Harmonics Method Applied to Light Hypernuclei

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Hypernuclei

- Hypernuclear chart
- Production
- Perspectives

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Our method

- Ab initio methods
- Bound states calculation
- Non Symmetrized Hyperspherical Harmonics method
- Goals

Nuclei with Strangeness

$$m_{\Lambda} = 1116 \text{ MeV}$$

$$m_{\Sigma^+} = 1189$$

$$m_{\Sigma^0} = 1193$$

$$m_{\Omega^-} = 1673$$



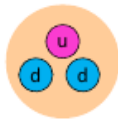
$$\tau_{\Lambda} = 263 \text{ ps}$$

$$\tau_{\Sigma^+} = 80 \text{ ps}$$

$$\tau_{\Sigma^0} = 7.4 \cdot 10^{-20} \text{ s}$$

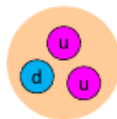
$$\tau_{\Omega^-} = 82 \text{ ps}$$

neutron



No charge

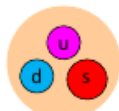
proton: 3 quarks



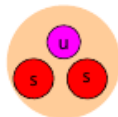
+charge

Mass: 938 MeV

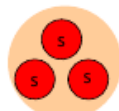
hyperon: including strangeness quark



Λ, Σ

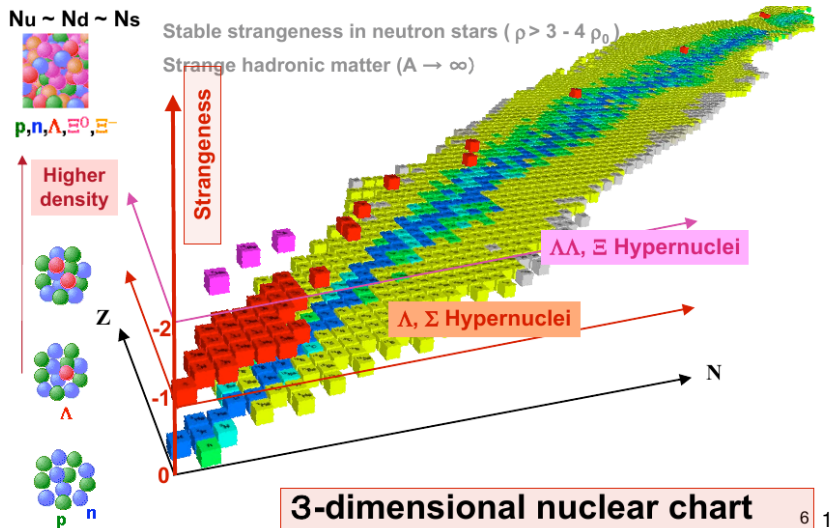


Ξ



Ω

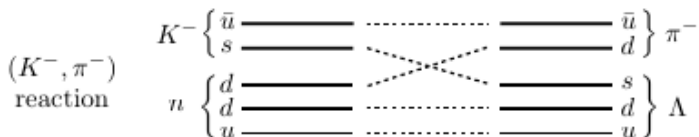
Hypernuclear Chart



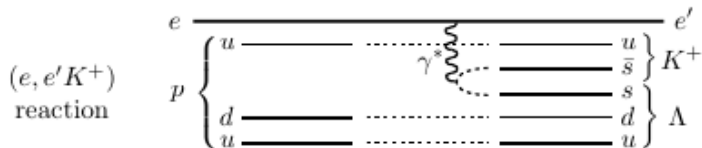
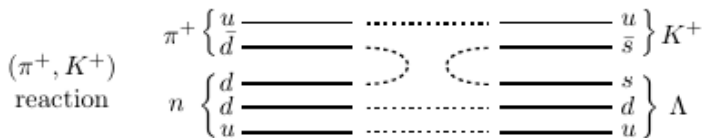
¹by M. Kaneta inspired by HYP06 conference poster

Production of Hypernuclei

Strangeness exchange reaction



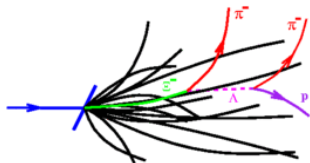
Associated production reaction



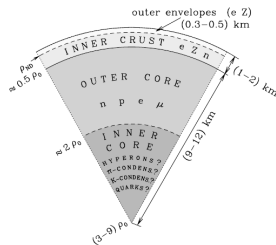
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²from the PhD thesis of D.Lonardoni, 'From Hypernuclei to Hypermatter...'

SUBNUCLEAR PHYSICS:
weak decay, quark structures ...



NUCLEAR ASTROPHYSICS:
Hyperon puzzle...



⇒ **LOW ENERGY NUCLEAR PHYSICS:** ⇐

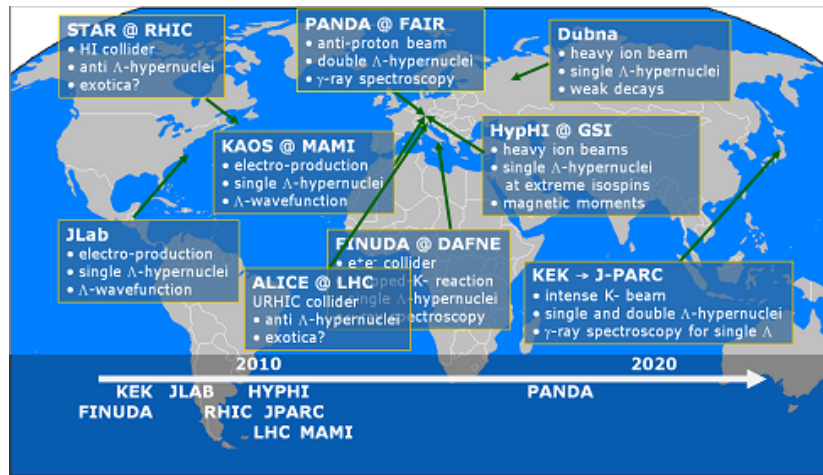
- study of N-Y **interaction** (and Y-Y);
- few-many body **nuclear models** (mainly for bound states);
- Experimental reference: **γ -ray spectroscopy**

Present and Future Perspectives

- Despite extensive investigations, single Λ hypernuclei knowledge is **far** from that of ordinary nuclei;
- Only **one** bound Σ -hypernucleus detected!
- **No** Ξ hypernuclei detected (some indications of weak attraction);
- **No** experimental information about Ω hypernuclei;
- **Four** $\Lambda\Lambda$ -hypernuclei energies measured (${}_{\Lambda\Lambda}^6\text{He}$, ${}_{\Lambda\Lambda}^{10}\text{Be}$, ${}_{\Lambda\Lambda}^{12}\text{Be}$, ${}_{\Lambda\Lambda}^{13}\text{B}$);

⇒ Main goal: **extension** of nuclear chart in all directions!

Experimental investigation in the world



We consider:

- **A-body** system;
- **A** particle **degrees of freedom**;
- **well-defined** microscopic Hamiltonian;
- internal relative motion treated **correctly**.

Then:

an **ab initio method** enables one to obtain the observable considered by solving the quantum mechanical many body equations, **without any uncontrolled approximation**.

⇒ We can focus on the input **interactions**.

Several **ab initio** methods:

- Monte Carlo methods;
- Coupled cluster model;
- No core shell model;
- Gaussian expansion method;
- \Rightarrow **Hyperspherical Harmonics method.**

The HH basis

We define **hyperspherical Jacobi coordinates**:

\Rightarrow *hyperradius* ρ + *hyperangles* ϕ_i + *angles*

The A-body kinetic operator is then:

$$\hat{T} = -\frac{1}{2m}\Delta_\rho + \hat{T}_K(\rho) \quad ; \quad \hat{T}_K = \frac{1}{2m} \frac{\hat{\mathbf{K}}_N^2}{\rho^2}$$

where $\hat{\mathbf{K}}_N^2$ is the **grand angular momentum** operator:

$$\hat{\mathbf{K}}_2^2 = -\frac{\partial^2}{\partial \phi_2^2} - 4\cot(2\phi_2) \frac{\partial}{\partial \phi_2} + \frac{1}{\cos^2 \phi_2} \hat{l}_1^2 + \frac{1}{\sin^2 \phi_2} \hat{l}_2^2$$

The HHs are the **eigenfunctions** of $\hat{\mathbf{K}}_N^2$:

$$\hat{\mathbf{K}}_N^2 \mathcal{Y}_{[K_N]}(\Omega_N) = K_N(K_N + 3N - 2) \mathcal{Y}_{[K_N]}(\Omega_N)$$

Non-symmetrized HH method

Problem: Selection of **antisymmetric** states (we deal with fermions):

⇒ We add to \hat{H} a special operator which **selects** 'by himself' the interesting states:

$$\hat{H}' = \hat{H} + \gamma \hat{C}(A)$$

where:

$$\hat{C}(A) = \sum_{i>j} \hat{P}_{ij}$$

⇒ No need for explicit symmetrization. In case of Hypernuclei:

$$\hat{C} = \hat{C}_N(A - N_Y) + \hat{C}_Y(N_Y)$$

We are dealing now with **different particles** → **extension** of the method.

Some results

$V_{NN} + V_{YN}$	System	NSHH	GEM
AV8'	${}^2\text{H}$	-2.22	
AV8'+S	${}^3_{\Lambda}\text{H}$	-2.41	
	B_{Λ}	0.19	0.19
AV8'	${}^2\text{H}$	-2.22	
AV8'+S'	${}^3_{\Lambda}\text{H}$	-2.55	
	B_{Λ}	0.35	0.36
AV8'	${}^2\text{H}$	-2.22	
AV8'+S''	${}^3_{\Lambda}\text{H}$	-2.91	
	B_{Λ}	0.69	0.72
AV8'	${}^3\text{H}$	-7.76	
AV8'+S	${}^4_{\Lambda}\text{H}$	-10.05	
	B_{Λ}	2.29	2.33

- 1 **Phenomenological** interactions;
 - 2-body potentials
 - 3-body potentials
- 2 Interactions from **effective field theory**
- 3 Study of $A > 5$ systems \rightarrow need for efficient **parallelization**
- 4 Study of **double** $\Lambda\Lambda$ hypernuclei?
- 5 Study of hypernuclei with **other hyperons**?

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