Non-paraxial non-diffracting beams in scale-free optics

F. Di Mei[1] , D. Pierangeli[1], J. Parravicini[1], A. J. Agranat[2], C. Conti[3], E. DelRe[1]

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[2]The Broide Center for innovative engineering and Computer Science,
Hebrew University of Jerusalem, Israel

[3]Istituto dei Sistemi Complessi, Consiglio Nazionale delle Ricerche

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SIF – Roma – 21st of September 2015

The Shackles of Diffraction

Diffraction-limited (details below ~ 200 nm are **lost**)

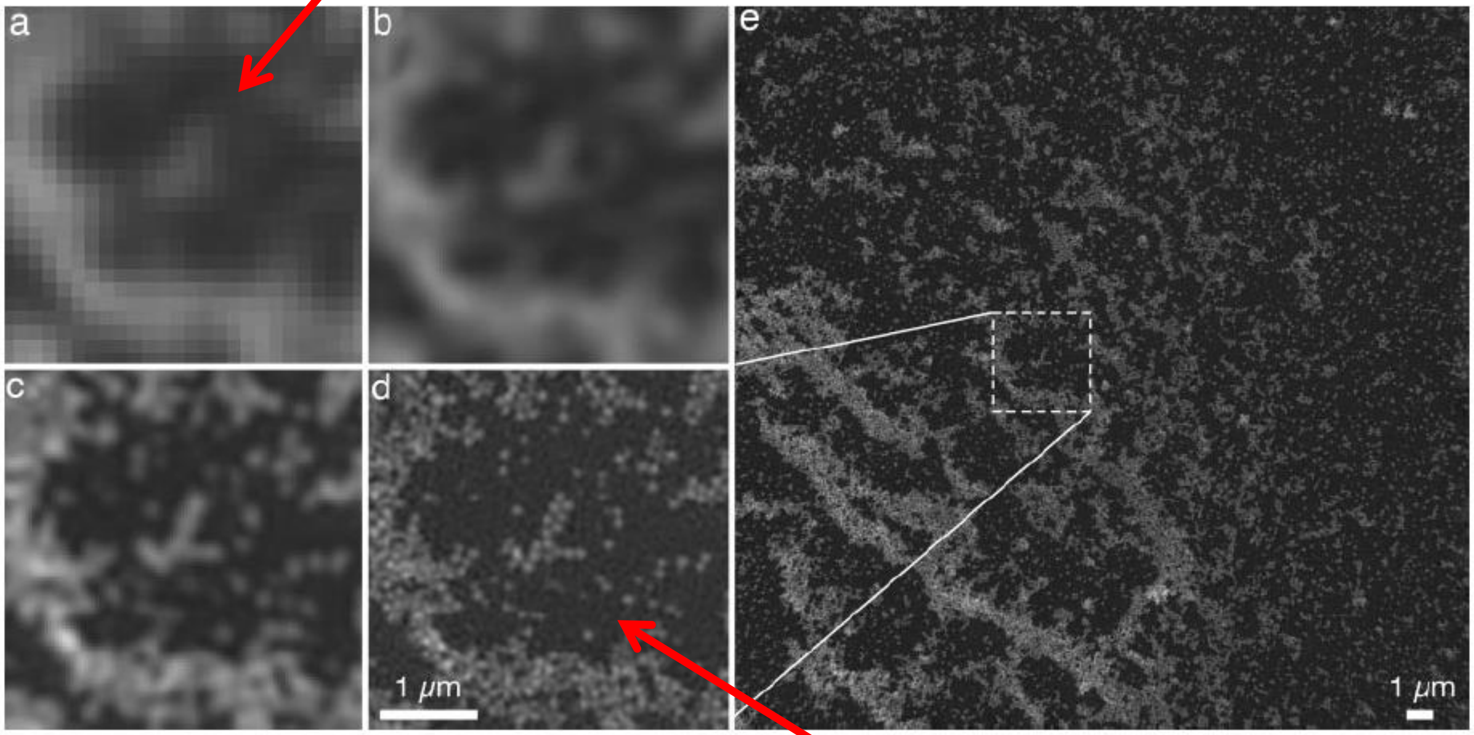
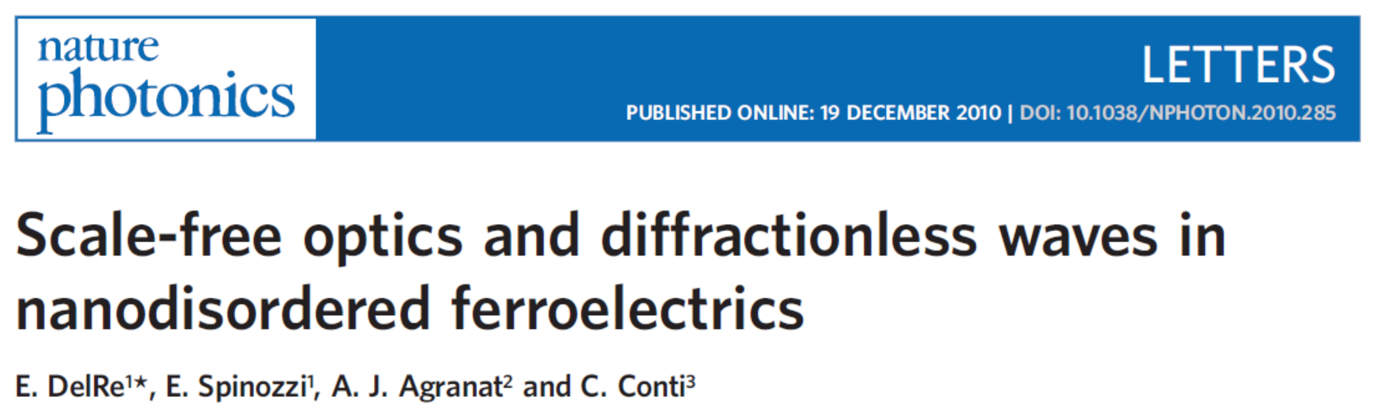


Fig. 6. A field of 50-nm fluorescent beads, imaged by conventional microscopy (*a*), conventional microscopy plus filtering (*b*), linear structured illumination (*c*), and saturated structured illumination using illumination pulses with 5.3 mJ/cm² energy density, taking into account three harmonic orders in the processing (*d*). Because no scanning is necessary, a wide field can be imaged simultaneously.

Super-resolution to below 50 nm (through **nonlinear microscopy**)

M. Gustafsson, PNAS 102, 13081 (2005)

Scale-free propagation



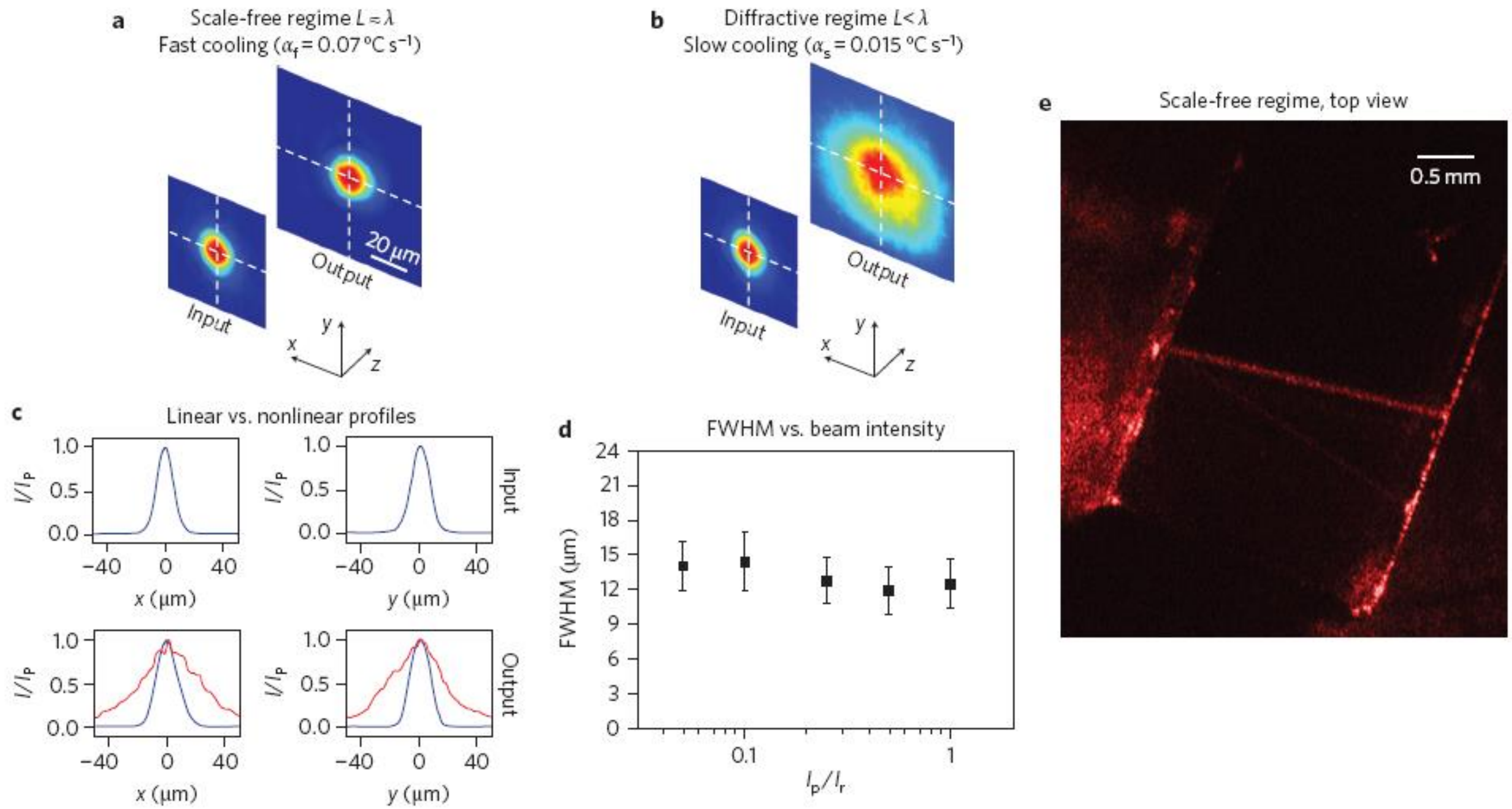
nature photonics

LETTERS

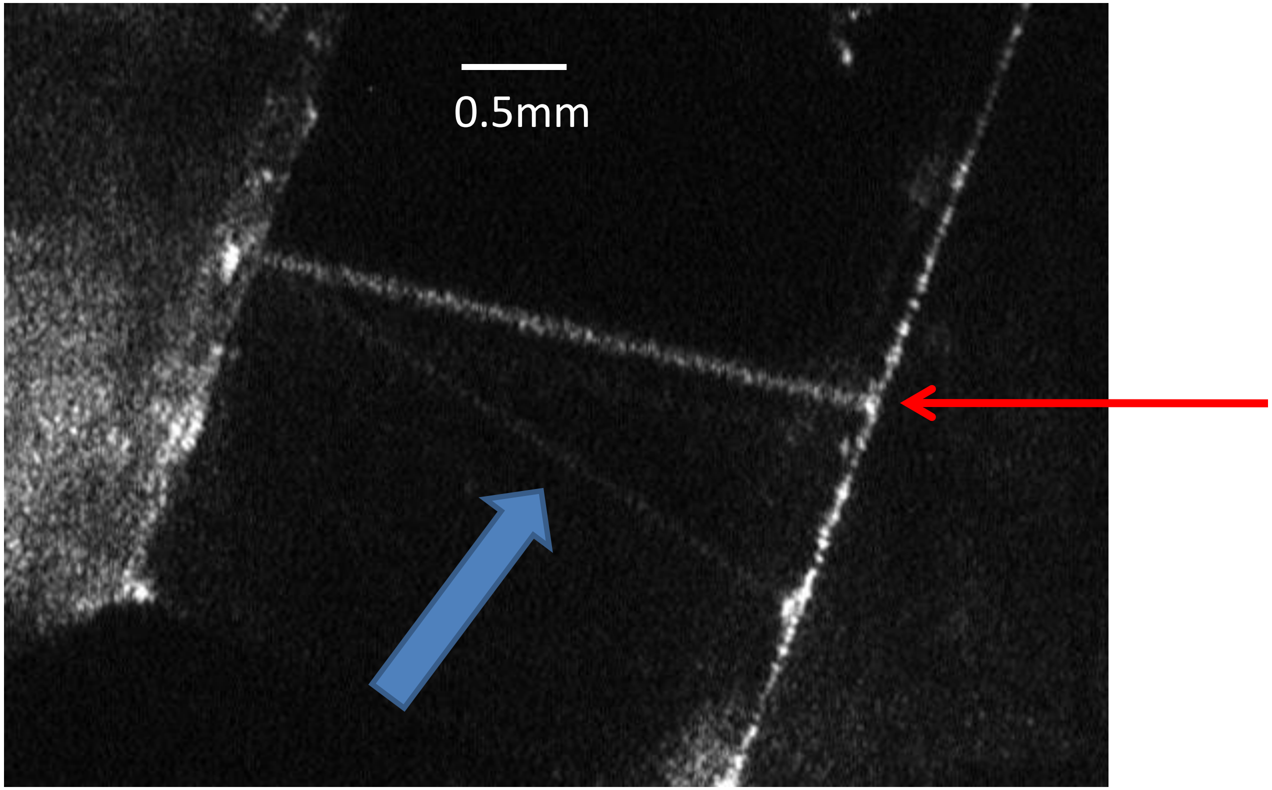
PUBLISHED ONLINE: 19 DECEMBER 2010 | DOI: 10.1038/NPHOTON.2010.285

Scale-free optics and diffractionless waves in nanodisordered ferroelectrics

E. DelRe[1]*, E. Spinozzi[1], A. J. Agranat[2] and C. Conti[3]



Fresnel reflection



Intensity independent

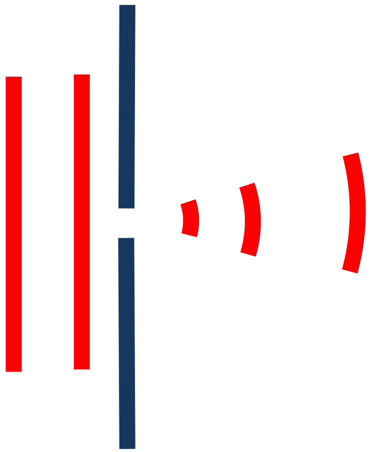
Challenge

Propagate a **non-paraxial subwavelength-sized beam in a volume along macroscopic distances** for imaging

Diffraction

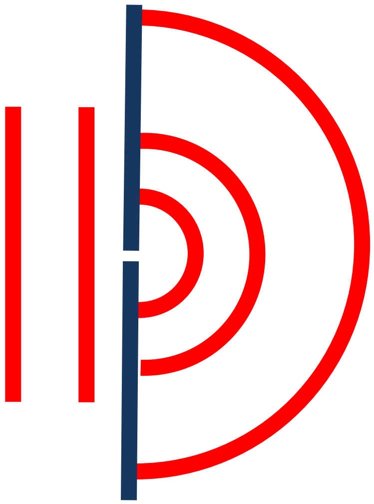
$d >> \lambda$

Small angular aperture



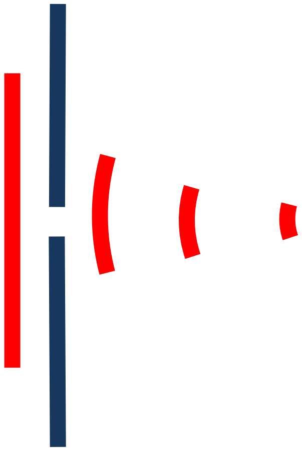
$d \approx \lambda$

Large angular aperture



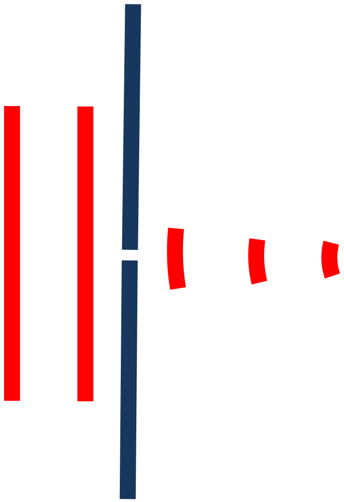
?iffraction

? angular aperture

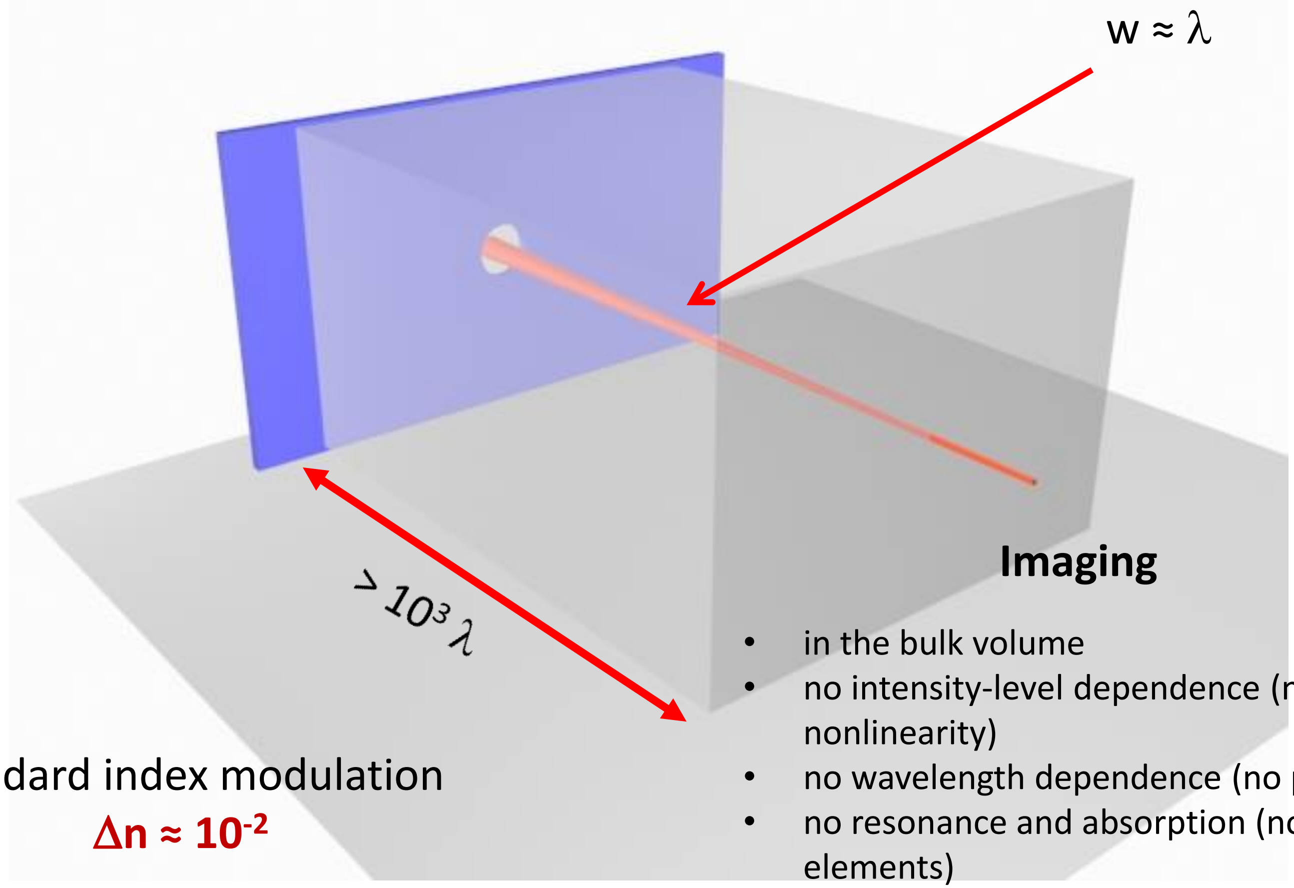


d >> λ

? angular aperture



d ≈ λ



Imaging

- in the bulk volume
- no intensity-level dependence (no standard nonlinearity)
- no wavelength dependence (no periodicity)
- no resonance and absorption (no conducting elements)
- must work beyond paraxial optics

Standard index modulation

$\Delta n \approx 10^{-2}$

Scale-free optics across all optical scales

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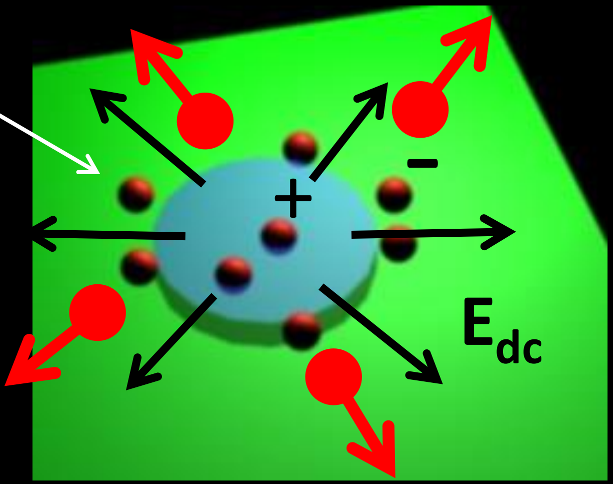
From HE to KGE behavior in nanodisordered ferroelectrics

Thermally-activated giant photorefraction



Diffusing room-temperature electrons

$$\mathbf{E}_{dc} = -(k_B T/q)\nabla I/I$$

Super-cold PNR electro-optic response

$$\Delta n = -(n_0^3/2)g\epsilon_0^2\chi_{PNR}^2|\mathbf{E}_{dc}|^2$$

$$\boxed{\nabla^2\mathbf{E} - (L/\lambda)^2(|\nabla|\mathbf{E}|^2|/2|\mathbf{E}|^2)^2\mathbf{E} + k^2\mathbf{E} = 0}$$

$$L = 4\pi n_0^2\epsilon_0\sqrt{g}\chi_{PNR}(k_B T/q) \qquad k = k_0 n_0$$

«Nonlinear» propagation equation

$$\boxed{L/\lambda}$$

B. Crosignani, A. Degasperis, E. DelRe, P. Di Porto, and A.J. Agranat, PRL 82, 1664 (1999)

E. DelRe, M. Tamburrini, M. Segev, R. Della Pergola, and A.J. Agranat, PRL 83, 1954 (1999)

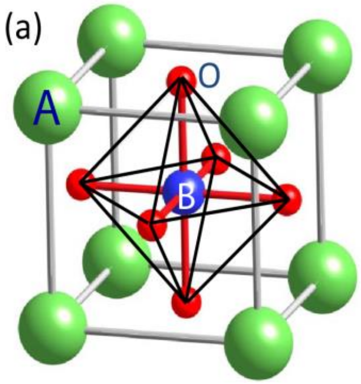
E. DelRe, E. Spinozzi, A.J. Agranat, and C. Conti, Nat. Photon. 5, 39-42 (2011)

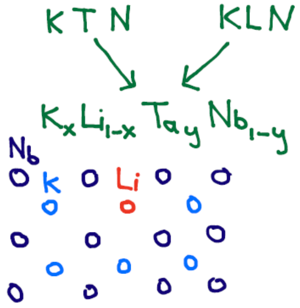
C. Conti, A.J. Agranat, and E. DelRe, PR A **84**, 043809 (2011)

E. DelRe and C. Conti, Scale-free optics, Springer Series in Optical Sciences 170, 207-230 (2012)

Material

Dipolar glass forming inside the KTN:Li crystal

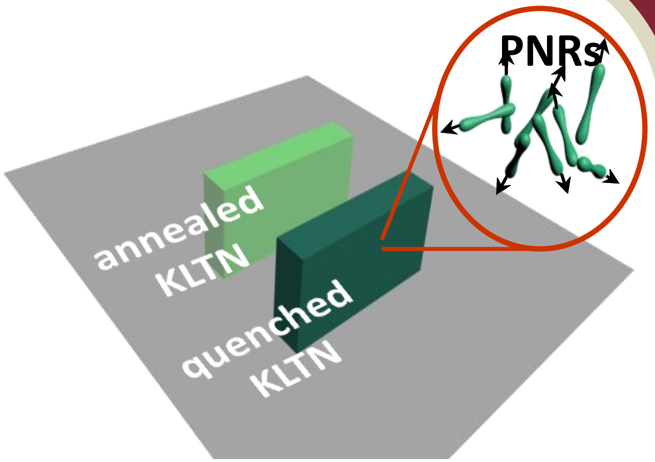




Compositional disorder

+

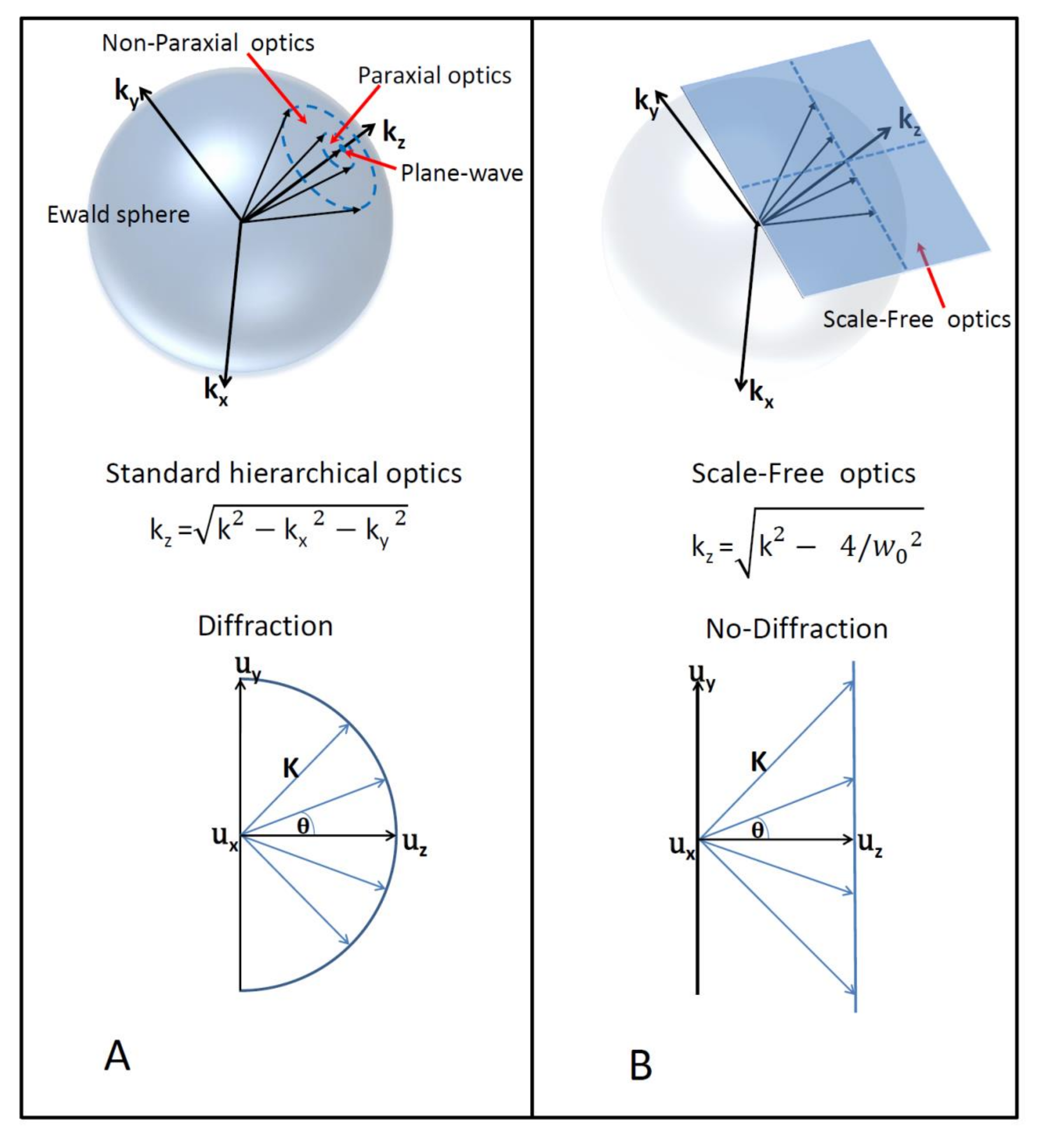
deep-in-band Cu impurities → Huge non-linear EO response



E. DelRe et al. Nat. Photon. **5**, 39 (2011)

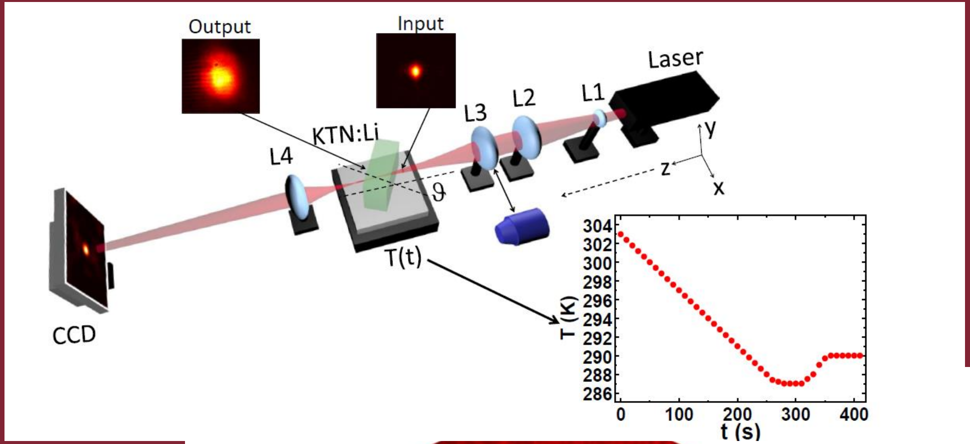
A.A. Bokov, Z. Ye Journal of Material Science 41 31-52 (2006)

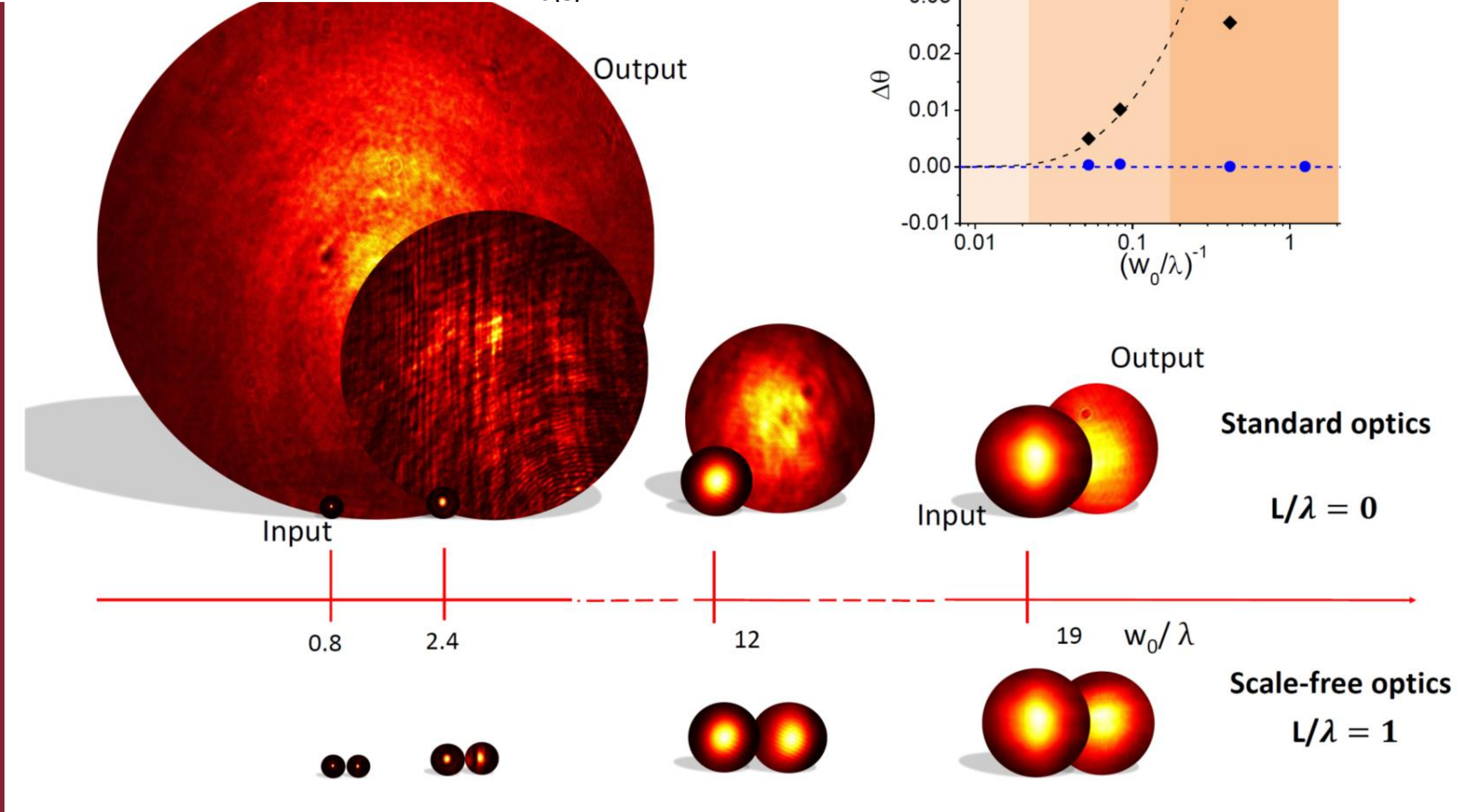
Comparing standard optics and scale-free optics



$$-\frac{\nabla^2 E}{E} + \left(\frac{L}{\lambda}\right)^2 \left(\frac{\nabla |E|^2}{2|E|^2}\right)^2 = k^2$$

Scale-invariance: the full picture





PHYSICAL REVIEW A **92**, 013835 (2015)

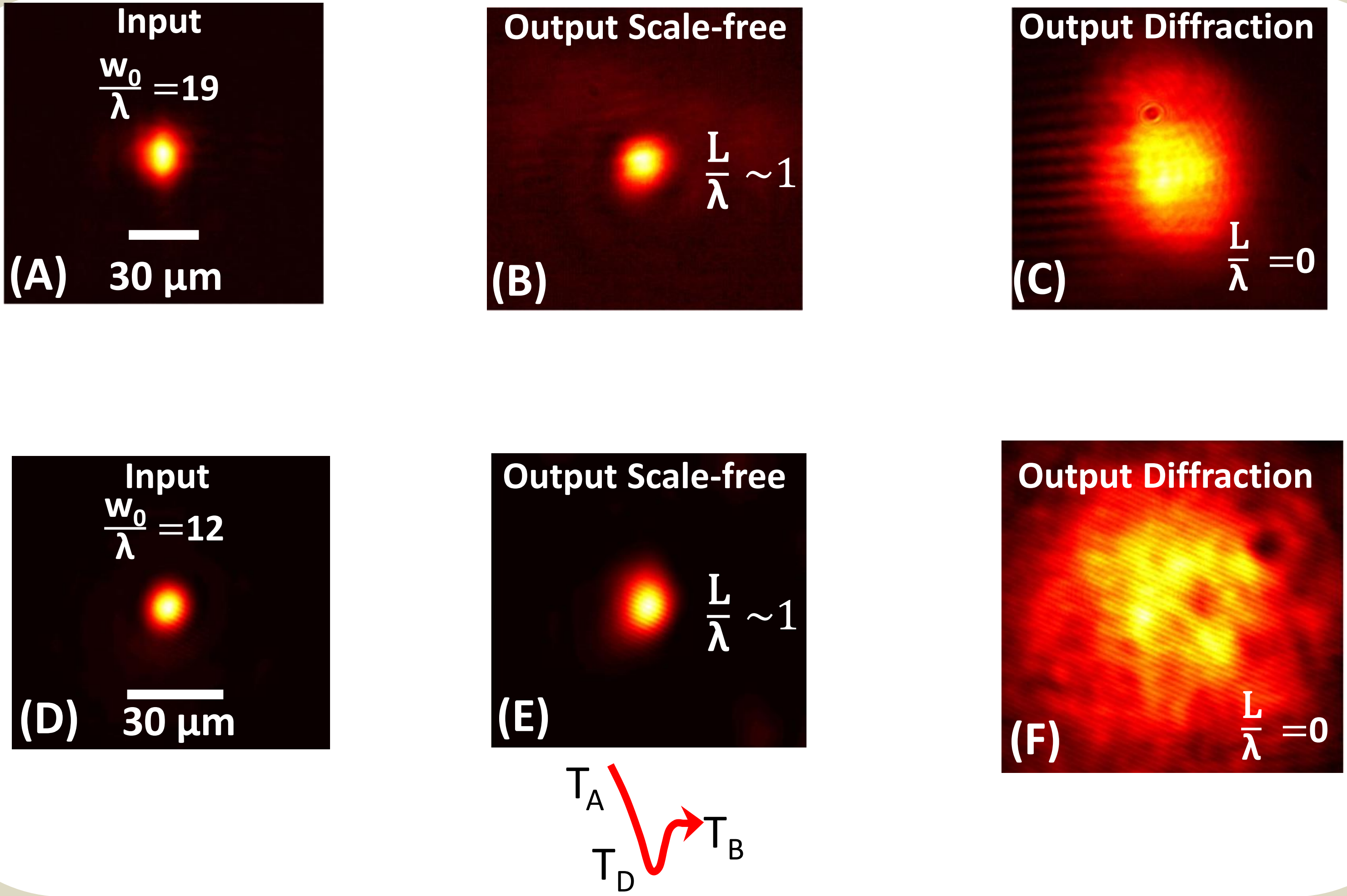
Observation of diffraction cancellation for nonparaxial beams in the scale-free-optics regime

F. Di Mei,[1,2] D. Pierangeli,[1] J. Parravicini,[1] C. Conti,[3,1] A. J. Agranat,[4] and E. DelRe[1,3,*]

Experiments and Results

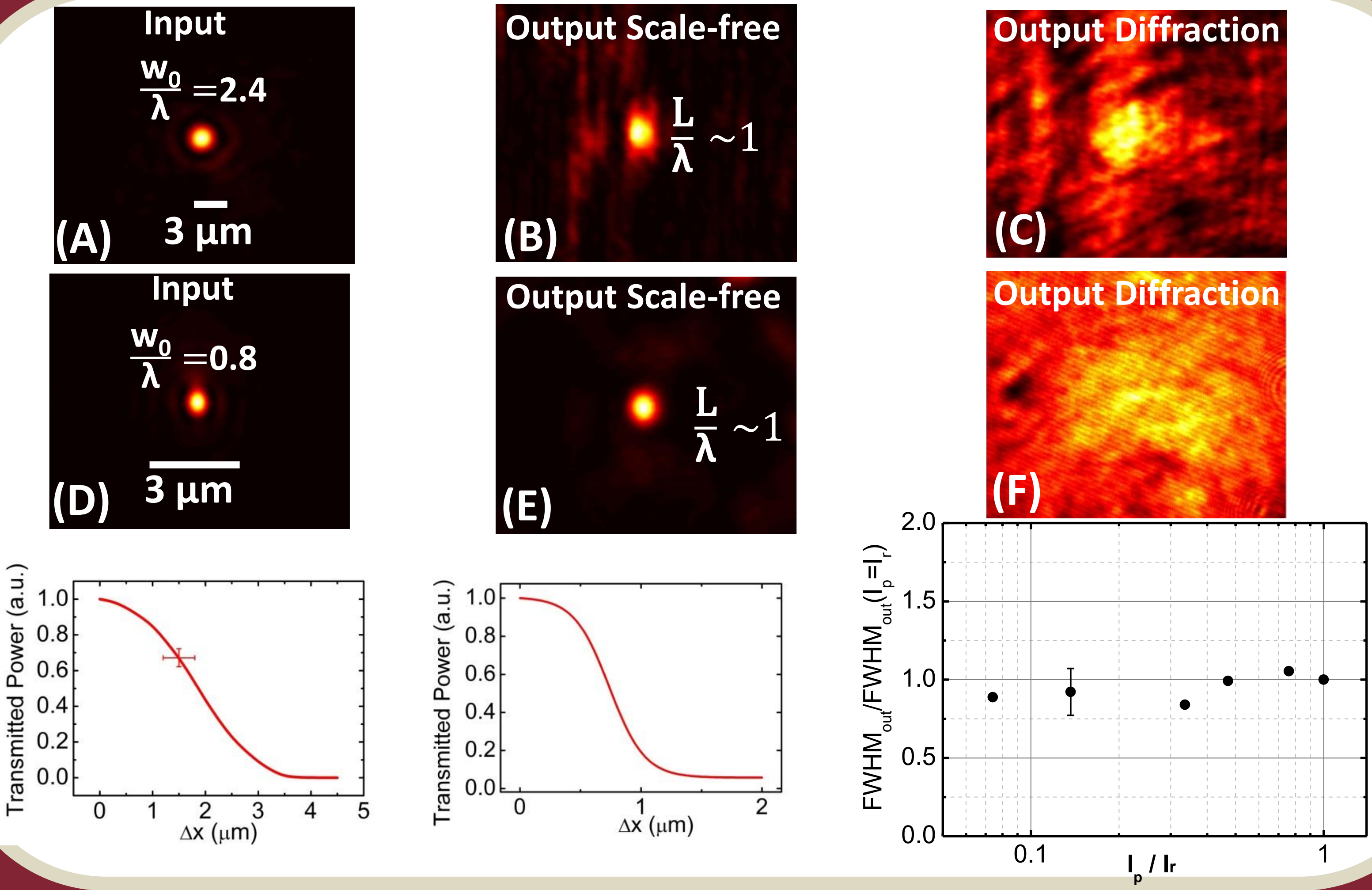
$$2ik\frac{\partial A}{\partial z} + \nabla_\perp^2 A - \frac{L^2}{\lambda^2}\left(\frac{\nabla_\perp|A|^2}{2|A|^2}\right)^2 A = 0$$

Paraxial regime



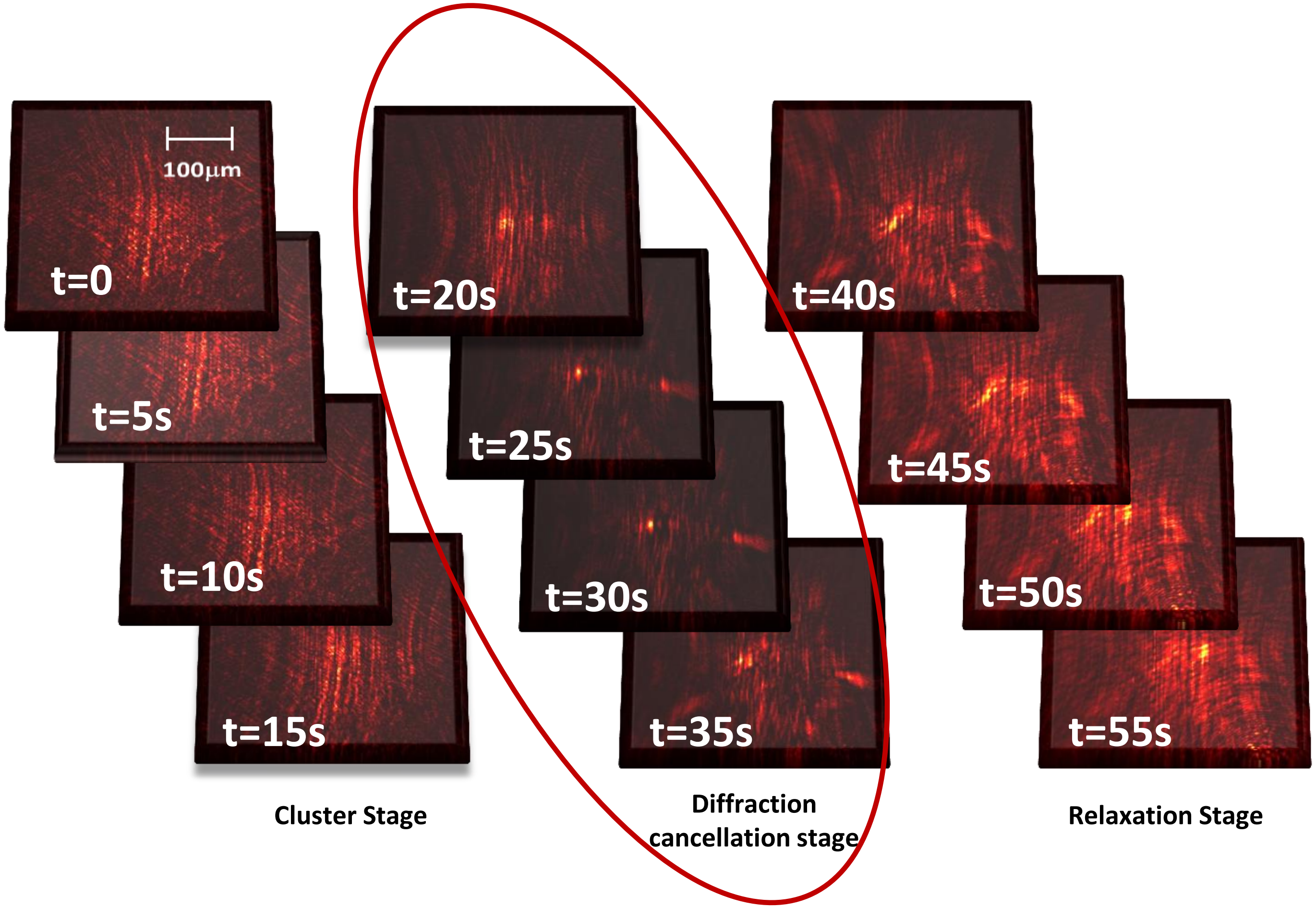
Experiments and Results

Non-Paraxial regime



Experiments and Results

Time sequence of output intensity distributions



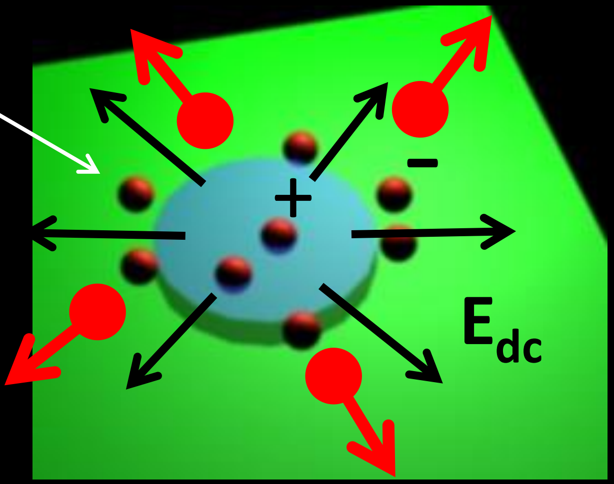
Cluster Stage

Diffraction cancellation stage

Relaxation Stage

From HE to KGE behavior in nanodisordered ferroelectrics

Thermally-activated giant photorefraction



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Super-cold PNR electro-optic response

$$\Delta n = -(n_0^3/2) g \epsilon_0^2 \chi_{PNR}^2 |\mathbf{E}_{dc}|^2$$

$$\boxed{\nabla^2 \mathbf{E} - (L/\lambda)^2 (|\nabla|\mathbf{E}|^2|/2|\mathbf{E}|^2)^2 \mathbf{E} + k^2 \mathbf{E} = 0}$$

$$L = 4\pi n_0^2 \epsilon_0 \sqrt{g} \chi_{PNR} (k_B T/q) \qquad k = k_0 n_0$$

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C. Conti, A.J. Agranat, and E. DelRe, PR A **84**, 043809 (2011)

E. DelRe and C. Conti, Scale-free optics, Springer Series in Optical Sciences 170, 207-230 (2012)

Idea: **non-perturbative** modification of wave-propagation laws

What if light had mass?

$$(\nabla^2 + n^2 k_0^2)\mathbf{E} = 0$$

$$m = 0$$

HE

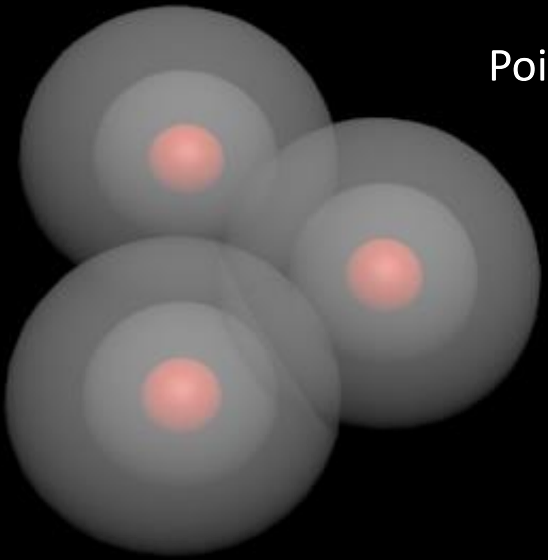
$$(\Box - n_{\mathrm{m}}^2 k_0^2)\mathbf{E} = 0$$

$$mc^2 = \hbar n_{\mathrm{m}} k_0 c$$

KGE

Point sources

$$\frac{e^{ikr}}{r}$$



Light with no mass

$$\frac{e^{-kr}}{r}$$

Light with mass

What if light had mass?

London Theory (1935)

Proca Lagrangian (1936)

Periodic systems

H.S. Eisenberg, Y. Silberberg, R. Morandotti, and J.A. Aitchison, PRL 85, 1863 (2000)

H. Kosaka, T. Kawashima, A. Tomita, M. Notomi, T. Tamamura, T. Sato, and S. Kawakami, APL 74, 1212 (1999)

K. Staliunas, R. Herrero, PRE 73, 016601 (2006)

Interacting systems

O. Firstenberg, P. London, M. Shuker, A. Ron, and N. Davidson, Nature Physics 5, 665 (2009)

T. Peyronel, O. Firstenberg, Q.-Y. Liang, S. Hofferberth, A. V. Gorshkov, T. Pohl, M. D. Lukin, V. Vuletic, Nature 488, 57 (2012)

O. Firstenberg, T. Peyronel, Q. Liang, A. V. Gorshkov, M. D. Lukin, V. Vuletic, Nature 502, 71 (2013)

Use massive light for high resolution (non-paraxial) **imaging**

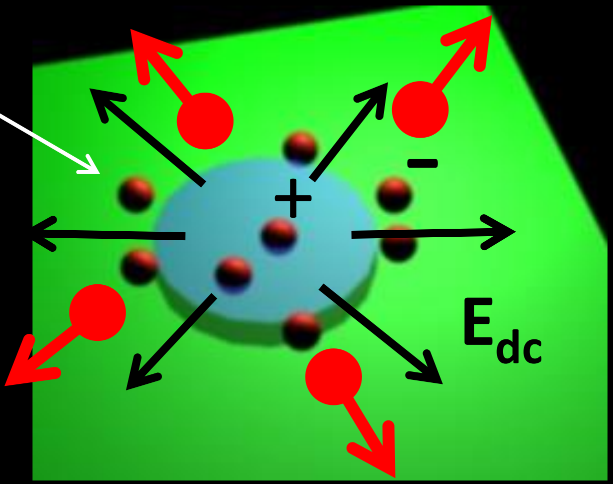
HE $$(\nabla^2 + n^2 k_0^2)\mathbf{E} = 0$$

$$\Big\downarrow$$

KGE $$(\Box - n_{\mathrm{m}}^2 k_0^2)\mathbf{E} = 0$$

From HE to KGE behavior in nanodisordered ferroelectrics

Thermally-activated giant photorefraction



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From HE to KGE behavior in nanodisordered ferroelectrics

Gaussian «bubbles»

$$E \propto \exp\left(-r^2/w_0^2\right)$$

$$(\nabla^2 - n_m^2 k_0^2)\mathbf{E} = 0 \qquad 1 < L/\lambda < (w_0 k/\sqrt{6})$$

$$n_{\mathrm{m}}^2(L) = n_0^2(1 - (L/\lambda)^2(6/k^2 w_0^2))/((L/\lambda)^2 - 1)$$

Gaussian «filaments»

$$E \propto \exp\left(-r_\perp^2/w_0^2\right)B(z)$$

$$-\partial^2_{z'z'} + \nabla_\perp^2 \mathbf{E} - ((L/\lambda)^2 - 1)^{-1}(k^2 - (L/\lambda)^2(4/w_0^2))\mathbf{E} = 0$$

$$z' \equiv z\sqrt{(L/\lambda)^2 - 1} \qquad \Box \equiv -\partial^2_{z'z'} + \nabla_\perp^2$$

$$(\Box - n_m^2 k_0^2)\mathbf{E} = 0 \qquad 1 < L/\lambda < (w_0 k/2)$$

$$n_{\mathrm{m}}^2(L) = n_0^2(1 - (L/\lambda)^2(4/k^2 w_0^2))/((L/\lambda)^2 - 1)$$

$$\Delta n \ll n_0$$

Use massive light for high resolution (non-paraxial) **imaging**

HE $$(\nabla^2 + n^2 k_0^2)\mathbf{E} = 0$$

↓

KGE $$(\Box - n_{\mathrm{m}}^2 k_0^2)\mathbf{E} = 0$$

$$1 < L/\lambda < (w_0 k/2)$$

$$L = 4\pi n_0^2 \epsilon_0 \sqrt{g}\chi_{PNR}(k_B T/q)$$

$$\boxed{\frac{L}{\lambda} > 1 \qquad \chi_{PNR} \sim 10^4 - 10^5}$$

Massive beam propagation (L>λ)

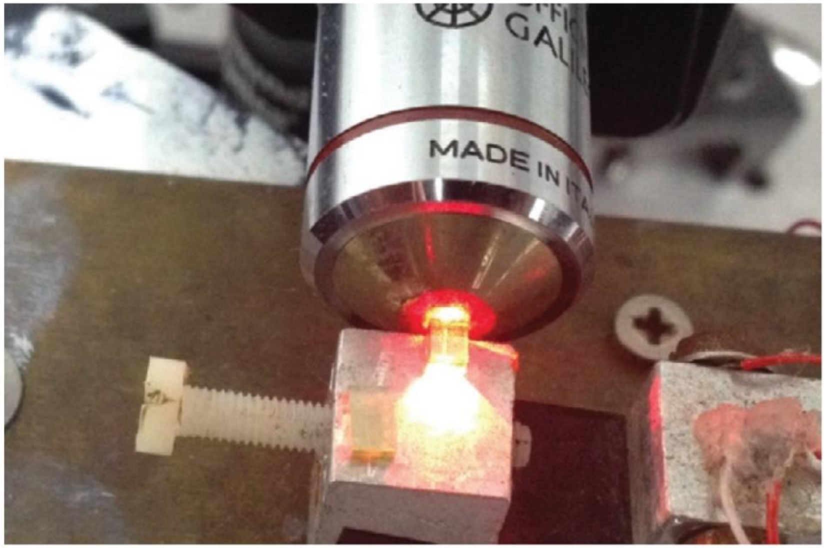
nature photonics

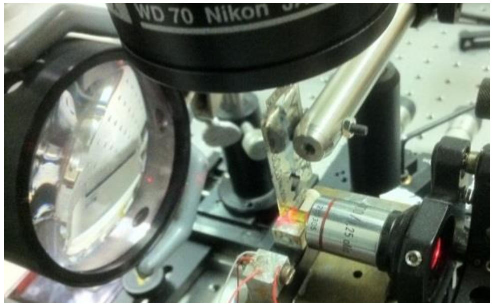
LETTERS

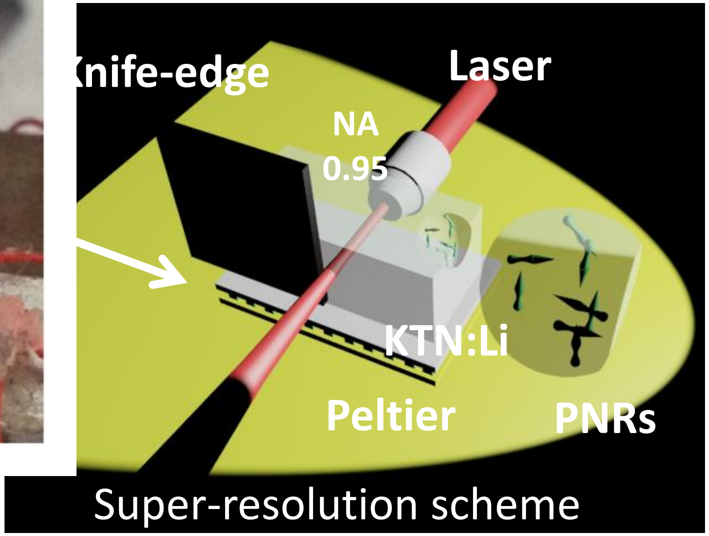
PUBLISHED ONLINE: 2 MARCH 2015 | DOI: 10.1038/NPHOTON.2015.21

Subwavelength anti-diffracting beams propagating over more than 1,000 Rayleigh lengths

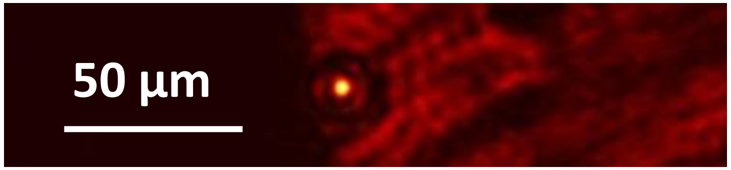
Eugenio DelRe[1]*, Fabrizio Di Mei[1,2], Jacopo Parravicini[1], Gianbattista Parravicini[3], Aharon J. Agranat[4] and Claudio Conti[1,5]



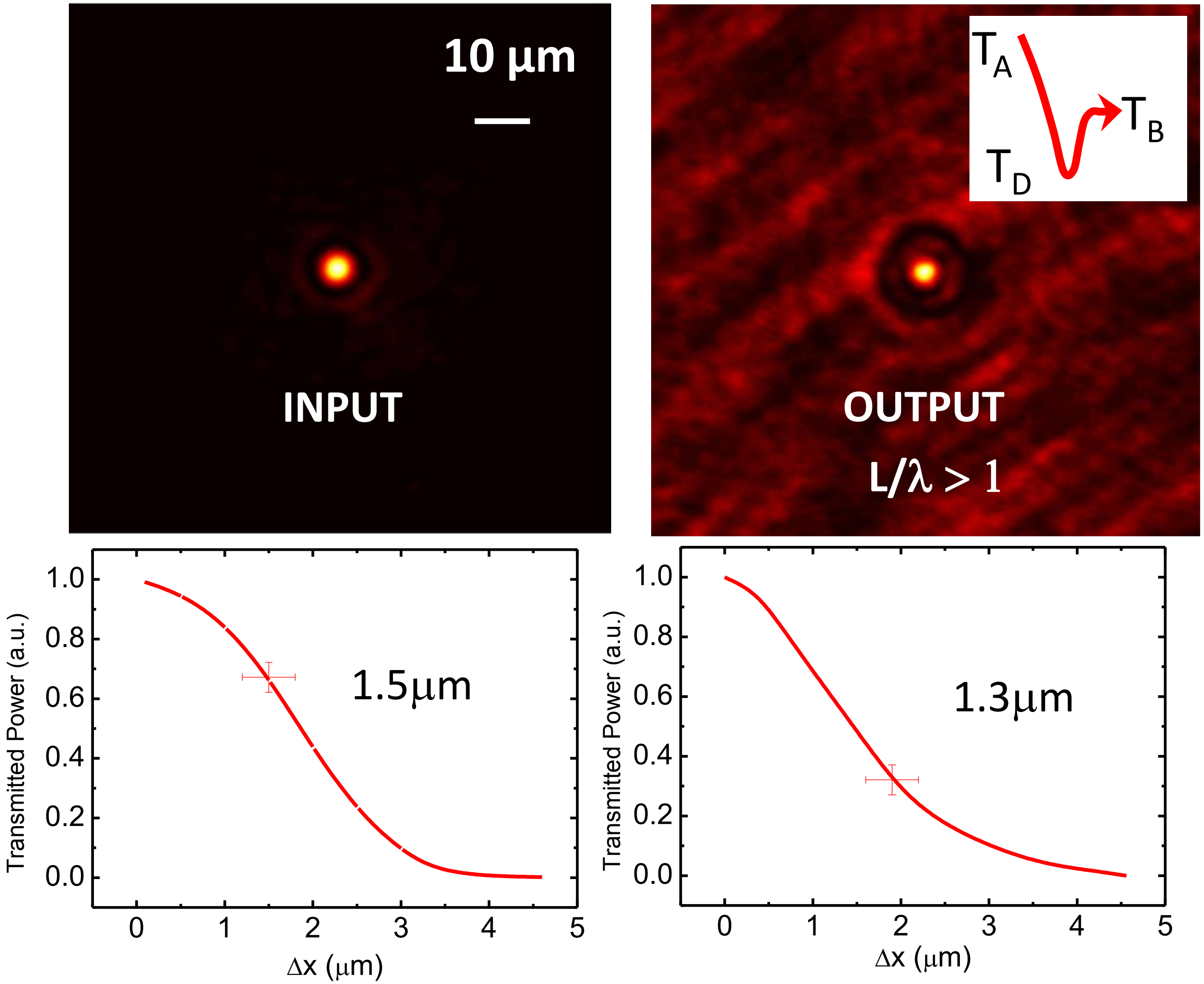




Super-resolution scheme



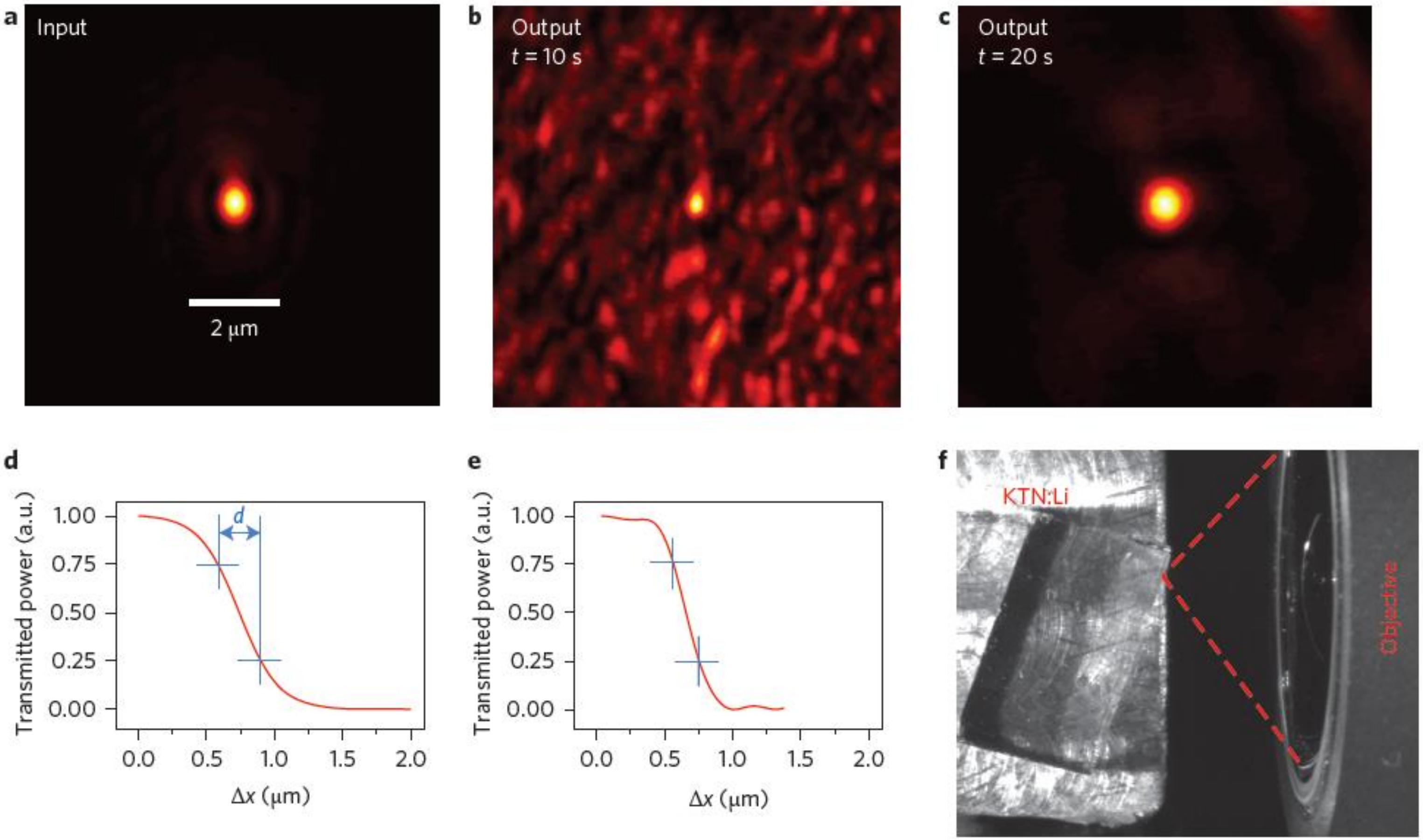
Massive beam propagation (L/λ=1.1)



134 Rayleigh lengths (L_z=2.6 mm)

Subwavelength massive beam propagation ($L/\lambda=1.1$)

0.28 micrometers...



1000 Rayleigh lengths (L_z=2.6 mm)

Anti-diffraction in the paraxial KGE regime

$$2ik\frac{\partial A}{\partial z}+\nabla_{\perp}^{2}A-\frac{L^2}{\lambda^2}\left(\frac{\nabla_{\perp}|A|^2}{2|A|^2}\right)^2 A=0$$

$$\frac{w(z)}{w_0}=\sqrt{1-\frac{\lambda^2}{\pi^2 n_m^2(L)w_0^4}z^2}$$

$$\frac{w(z)}{w_0}=\sqrt{1+\frac{\lambda^2}{\pi^2 n^2 w_0^4}z^2}$$

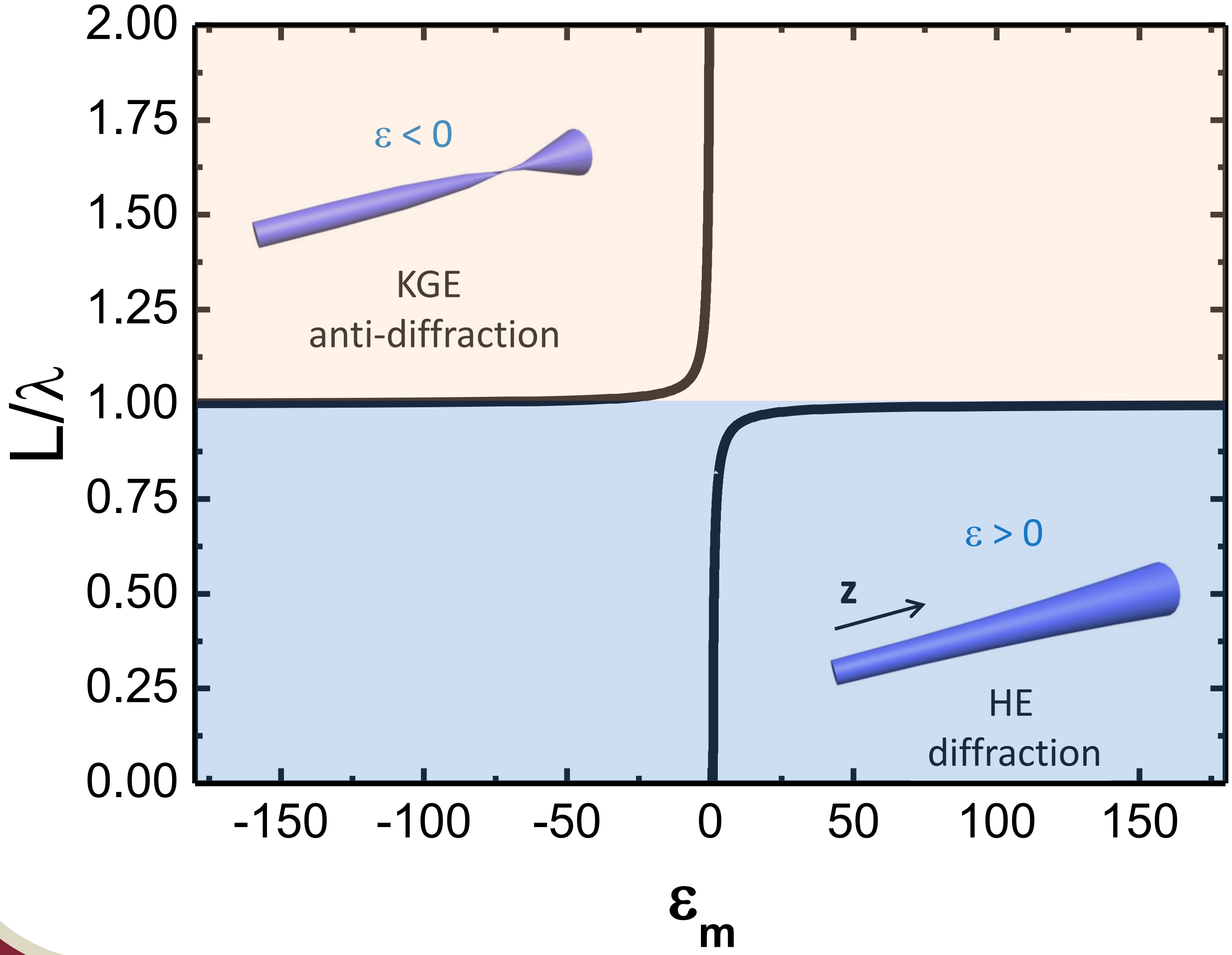
KGE

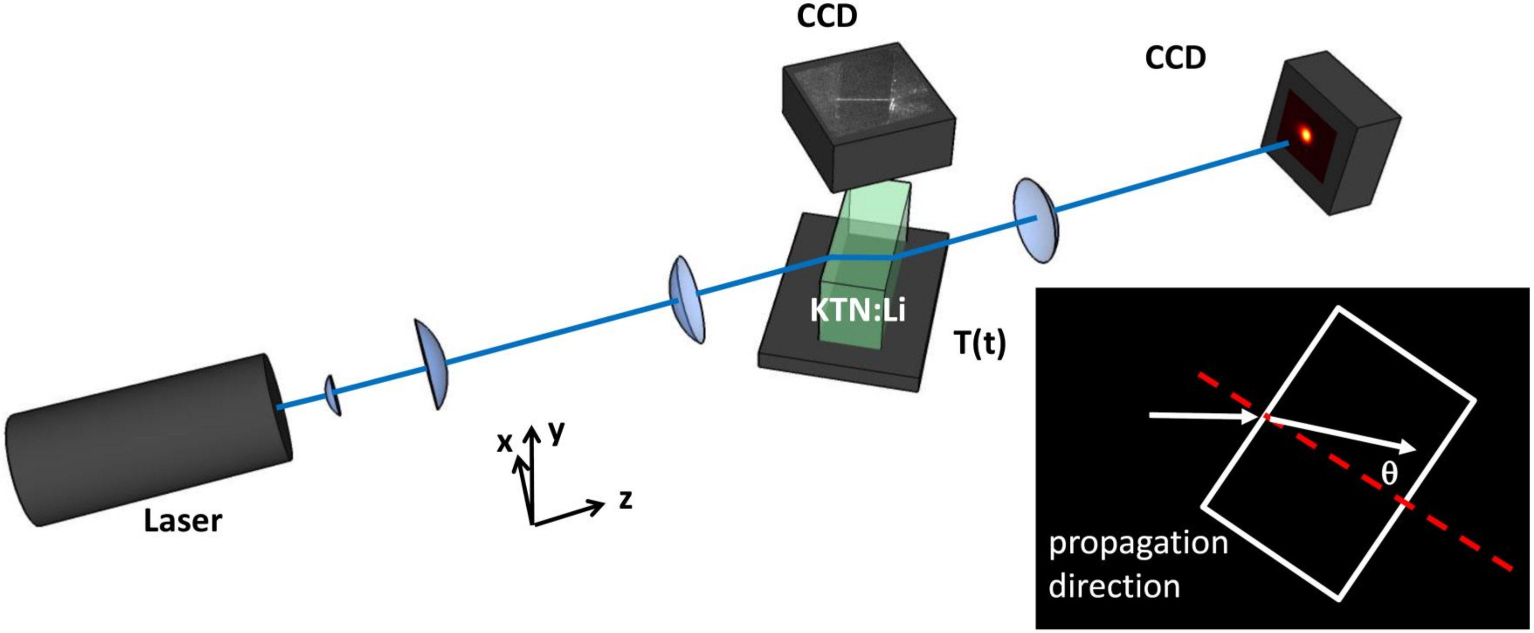
HE

$$w(z)=w_0\sqrt{1+\frac{4}{k^2w_0^4}\left[1-\left(\frac{L^2}{\lambda^2}\right)\right]z^2}$$

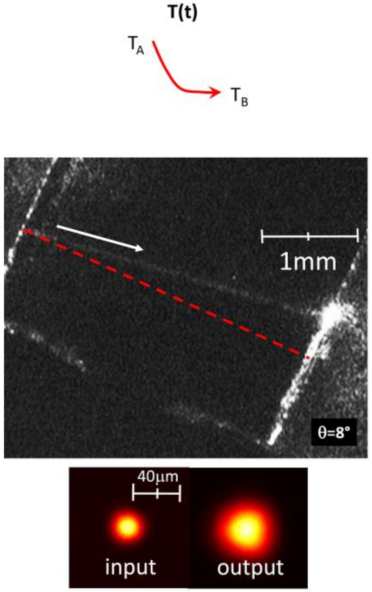
$$\epsilon_{\mathrm{m}}=\frac{\epsilon_r}{1-\left(\frac{L}{\lambda}\right)^2}$$

$$z_c=(n\pi/\lambda)w_0^2[(L/\lambda)^2-1]^{-1/2}$$


2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
L/λ
-150
-100
-50
0
50
100
150
ε_m
ε < 0
KGE
anti-diffraction
ε > 0
z
HE
diffraction

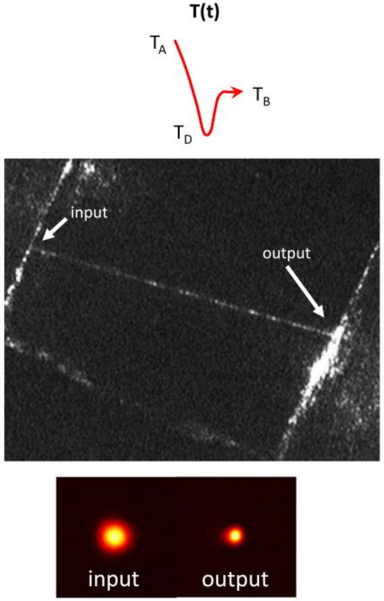


No T shock - Diffraction

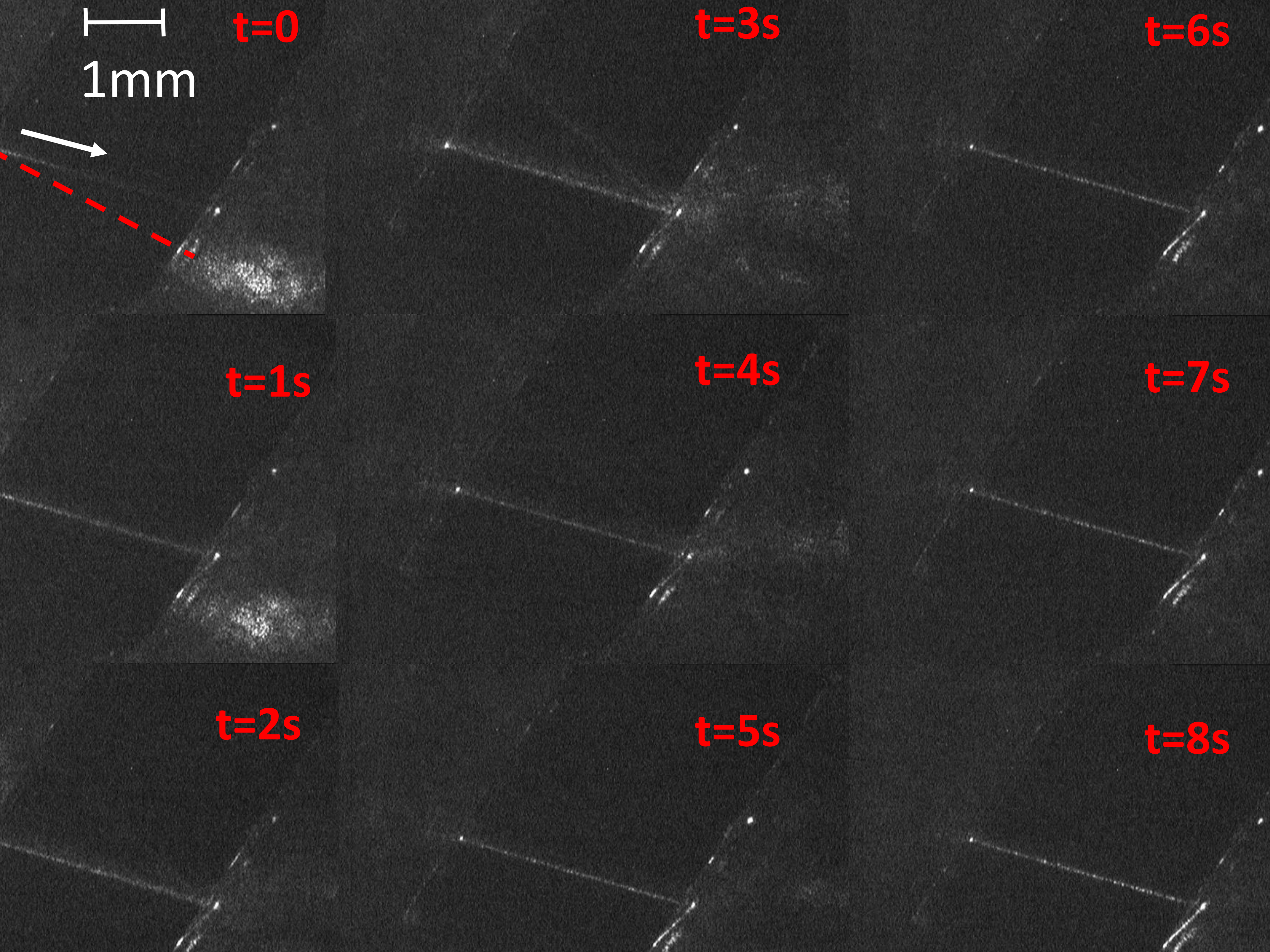


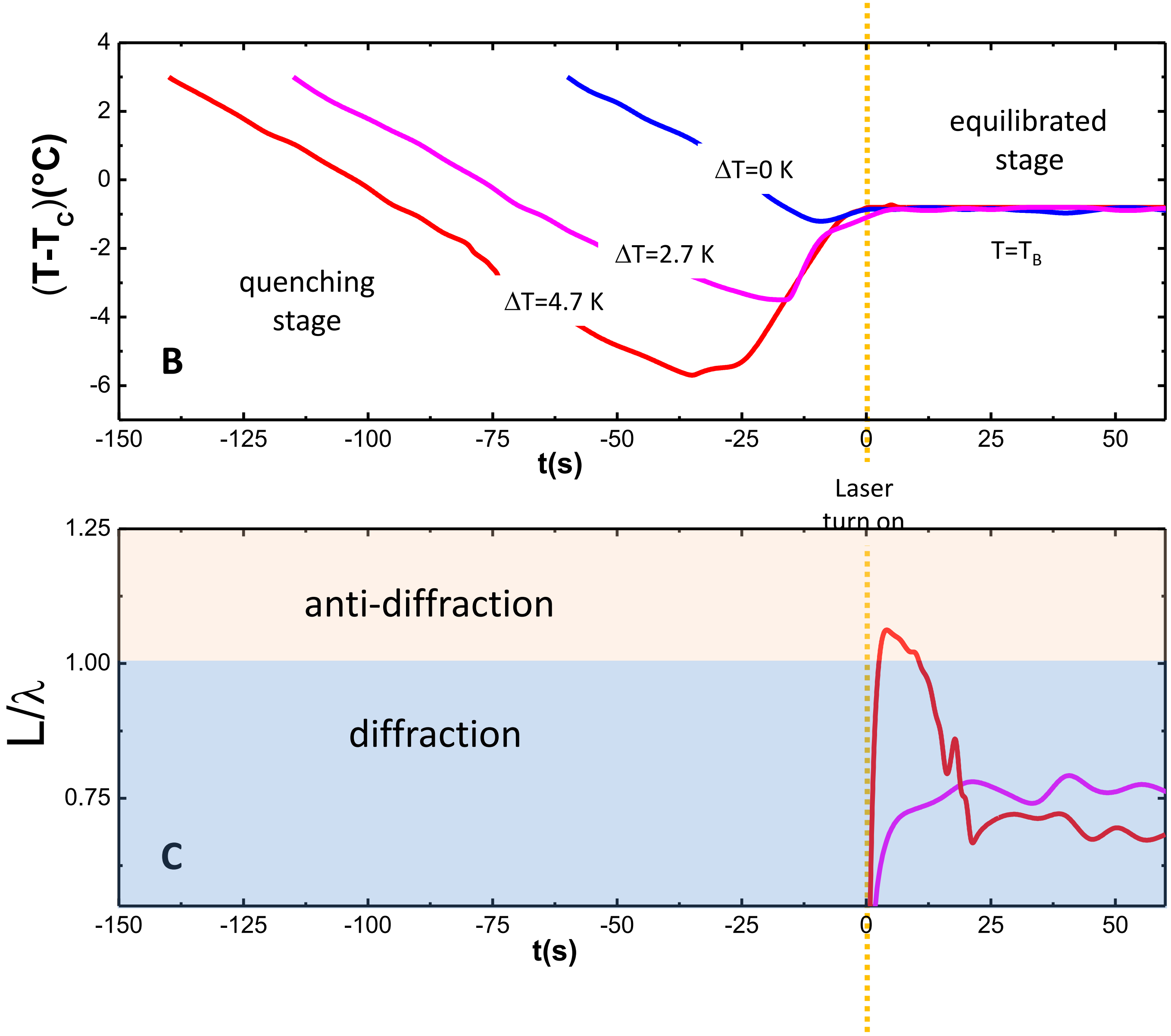
$L/\lambda \ll 1$

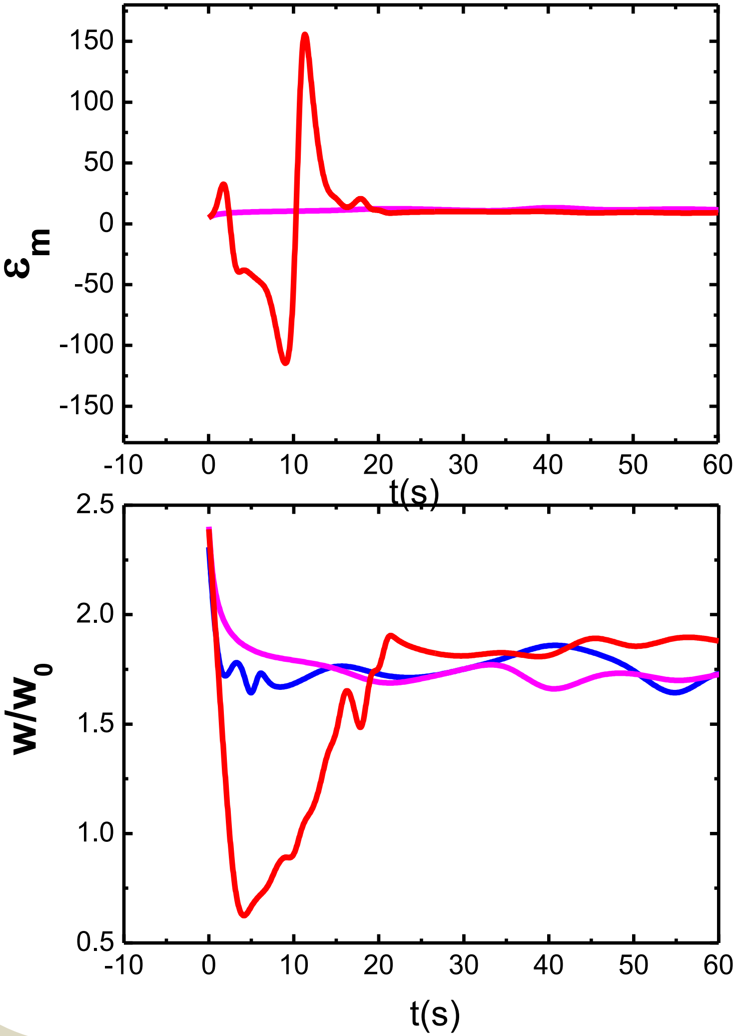
T shock - Anti-diffraction

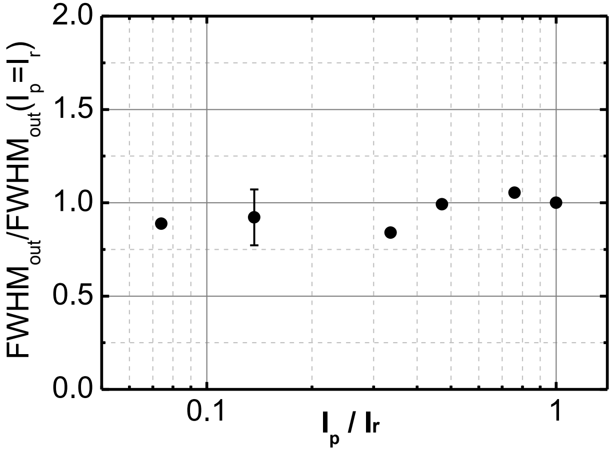


$L/\lambda \cong 1.1$


1mm
t=0
t=1s
t=2s
t=3s
t=4s
t=5s
t=6s
t=7s
t=8s


B
(T-T$_c$)(°C)
4
2
0
-2
-4
-6
quenching stage
ΔT=0 K
ΔT=2.7 K
ΔT=4.7 K
equilibrated stage
T=T$_B$
-150
-125
-100
-75
-50
-25
0
25
50
t(s)
Laser turn on
C
L/λ
1.25
1.00
0.75
anti-diffraction
diffraction
-150
-125
-100
-75
-50
-25
0
25
50
t(s)


150
100
50
0
-50
-100
-150
ε_m
-10
0
10
20
30
40
50
60
t(s)
2.5
2.0
1.5
1.0
0.5
w/w_0
-10
0
10
20
30
40
50
60
t(s)


2.0
1.5
1.0
0.5
0.0
$FWHM_{out}/FWHM_{out}(I_p=I_r)$
0.1
1
I_p / I_r