Betatron Radiation in Capillaries for Plasma Acceleration Experiments

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Summary

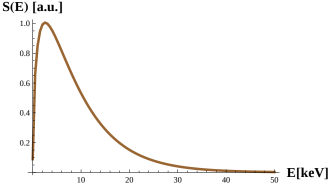
1. Betatron Radiation
   - Quasi-linear wakefield regime
2. Electron acceleration in capillaries
   - Wakefield modes in capillaries
   - Betatron Radiation from capillaries
3. Laser coupling in capillary tubes with dielectric walls
   - Instrumentation and Setup
   - Measurements
4. Conclusions

Betatron Radiation Spectra: X-rays

$I_0 \sim \times 10^{18} W/cm^2, \ \gamma_{max} \sim 1000, \ n_e = 5 \times 10^{17}/cm^3$

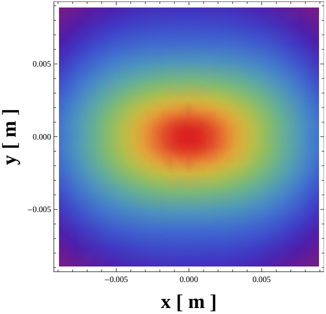
Acceleration length $\sim$ 5 *cm*

Betatron spectrum



Critical energy $E_c \sim 10$ *keV*

Spatial distribution of the radiation

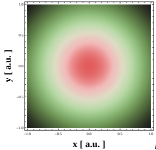


Radiation collected at 1 meter

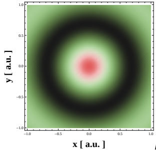
Divergence $\theta_\beta \sim \sigma k_\beta \sim 3$ *mrad*

Electromagnetic hybrid modes/1

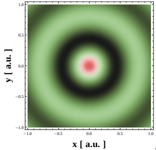
Energy density



EH_{11}



EH_{12}



EH_{13}

Electric field component of the mth mode:

$$E_{1m} = J_0(u_m \frac{r}{R_{cap}})e^{-k_m^I z}\cos[\omega_0 t - k_{zm} z]$$

Longitudinal wave number of the mth mode:

$$k_{zm} = \sqrt{k_0^2 - \frac{u_m^2}{R_{cap}^2}}$$

Damping coefficient of the mth mode:

$$k_m^I = \frac{u_m^2}{2k_{zm}^2 R_{cap}^3} \frac{1+\varepsilon_r}{\sqrt{\varepsilon_r - 1}}$$

Group velocity of the mth mode:

$$v_{g,m} = c\sqrt{1 - (\frac{u_m}{R_{cap} k_0})^2}$$

Total electric field inside the capillary:

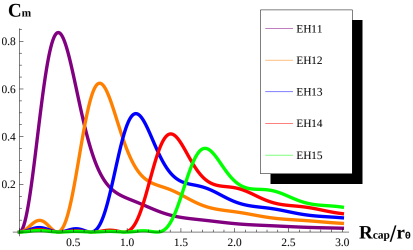
$$E_L = \sum_m A_m E_{1m}$$

Expansion coefficient for the mth mode of the total electric field:

$$A_m = 2\frac{\int_0^1 x E_L(x) J_0(u_m x)dx}{J_1^2(u_m)}$$

Electromagnetic hybrid modes/2

Coupling efficiency for a flat top laser profile



Laser electric field in the focus of a flat top laser profile:

$$E_L = E_{L0} \frac{J_1(\frac{\nu_3 r}{r_0})}{r}$$

Third zero of the Bessel J_1:
$\nu_3 = 10.174$

Coupling efficiency for the mth mode:

$$C_m = \frac{4}{J_1^2(u_m)} | \int_0^1 J_1(\frac{\nu_3 R_{cap} x}{r_0}) J_0(u_m x) dx|^2$$

Laser wakefields/1

For a matched flat top profile and linearly polarized laser pulse

Normalized laser vector potential:

$$a = \frac{0.91 a_0}{\sqrt{2}} e^{-\zeta^2/2\sigma_L^2} J_0(u_0 \frac{r}{R_{cap}})$$

The optimal coupling efficiency is obtained when:

$R_{cap}/r_0 \sim 0.35$

From the Poisson equation, the continuity equation, and the fluid momentum equation, in the linear ($a_0 < 1$) 3D regime, the scalar wakefield potential comes to be:

$$\Phi(r,\zeta) = -\frac{mc^2 k_p}{2} \int_\zeta^\infty d\zeta' \sin[k_p(\zeta - \zeta')] a^2(r,\zeta')$$

The corresponding longitudinal wakefield is:

$$E_z \sim 0.83 E_0 \frac{\sqrt{\pi}}{4} a_0^2 \sigma_L k_p J_0(u_1 \frac{r}{R_{cap}}) e^{-k_p^2 \sigma_L^2/4} \cos[k_p \zeta]$$

The corresponding transverse wakefield is:

$$E_r \sim -0.83 \frac{u_1}{R_{cap}} J_1(u_1 \frac{r}{R_{cap}}) E_0 \sqrt{\pi} a_0^2 \sigma_L e^{-k_p^2 \sigma_L^2/4} \sin[k_p \zeta]$$

Laser wakefields/2

Laser propagation equation in a plasma medium inside a capillary ($a_0 < 1$):

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\overrightarrow{A} \sim k_p^2\frac{n_e}{\gamma n_0}\overrightarrow{A}$$

By the assumption $\overrightarrow{A} \propto e^{i(kz-\omega t)}$(neglecting focusing effects):

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \left(\frac{u_m}{R_{cap}}\right)^2\right)\overrightarrow{A(r)} = k_p^2\frac{n_e(r)}{\gamma n_0}\overrightarrow{A(r)}$$

$\frac{u_m}{R_{cap}} \equiv k_{\perp m}$ u_m=mth zero of the Bessel J_0, the solution of the Schroedinger homogeneous equation

Definition of plasma operator: $\widehat{P} \equiv \frac{n_e}{\gamma n_0}$

First order correction to the transverse wavenumber for the mth mode:

$$k_{\perp m}^{2\prime} \sim k_{\perp m}^2 + k_p^2 < EH_{1m}|\widehat{P}|EH_{1m} >,$$ k_p = plasma wavenumber

Approximated expression of potential vector inside a capillary in a plasma medium:

$$|\overrightarrow{A} >= |\overrightarrow{EH_{11}} > + \frac{k_p^2}{k_{\perp 1}^2 - k_{\perp 2}^2} < \overrightarrow{EH_{12}}|\widehat{P}|\overrightarrow{EH_{11}} > |\overrightarrow{EH_{12}} >$$

Laser wakefields/3

Developing the normalized vector potential in the capillary modes:

$$a^2 = (\sum_m \overrightarrow{a_m})^2$$

We have for the first order wakefield:

$$\Phi(r,\zeta) = -\frac{mc^2 k_p}{2} \int_\zeta^\infty d\zeta' \sin[k_p(\zeta-\zeta')] a^2(r,\zeta') \sim$$

$$\sim -\frac{mc^2 k_p}{2} \int_\zeta^\infty d\zeta' \sin[k_p(\zeta-\zeta')] (a_1^2 + a_1 a_2)(r,\zeta') \sim$$

$$\sim \Phi_1 + \Phi_{12}$$

For a perfectly matched flat top laser profile $\Phi_{12} \sim 0.1\ \Phi_1$

Laser wakefields/4

The coupling term Φ_{12} is an oscillating beating term with wavenumber $\Delta k_{z2} = k_{z1} - k_{z2}$, corresponding for most of the real cases to wavelengths of the order of millimeters up to few centimeters. Therefore both the longitudinal and radial wakefields manifest long-range oscillations beside their natural one (that at the plasma wavelength).

Example:

$R_{cap} \sim 100\mu m$

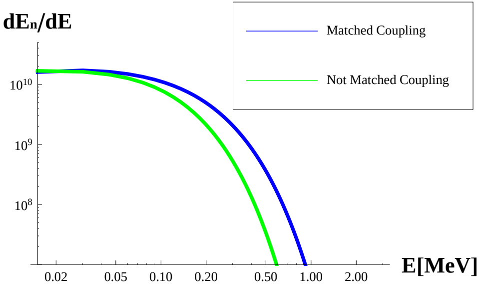
$\lambda_0 = 0.8\mu m$

$\Delta k_{z2} = 628.7\ m^{-1}$

Corresponding to a beating wavelength of $1mm$

Spectrum modification in case of coupling

$a_0 \sim 1,\ n_e = 1.8 \times 10^{17}/cm^3,\ L_{cap} = 2cm,\ R_{cap} = 100\mu m$

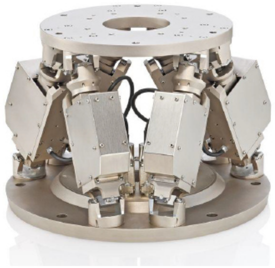


E_n is the irradiated energy. The Matched Coupling corresponds to $R_{cap}/r_0 \sim 0.35$ while the Not Matched Coupling corresponds to $R_{cap}/r_0 \sim 0.6$. The net effect of the coupling can be viewed as a red shift of the critical energy. The decrease of the critical energy in case of coupling is basically due to the decrease of the laser group velocity, namely of the plasma wave phase velocity. In this calculation the coupling with higher modes has been neglected considering a beating wavelength much shorter than the capillary length.

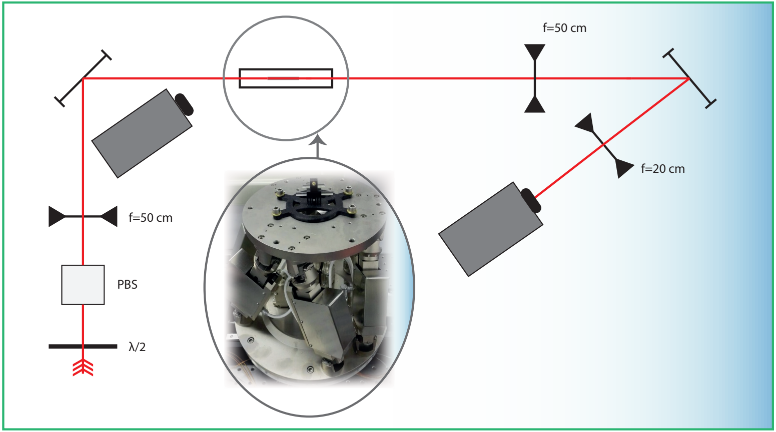
Hexapod PI

			Unit	Tolerance
	for higher resolution and load	for higher velocity		
Active axes	X, Y, Z, θ_X, θ_Y, θ_Z	X, Y, Z, θ_X, θ_Y, θ_Z		
Motion and positioning				
Travel range* X, Y	±22.5	±22.5	mm	
Travel range* Z	±12.5	±12.5	mm	
Travel range* θ_X, θ_Y	±7.5	±7.5	°	
Travel range* θ_Z	±12.5	±12.5	°	
Single- actuator design resolution	0.007	0.5	µm	
Min. incremental motion X, Y, Z	0.3	1	µm	typ.
Min. incremental motion θ_X, θ_Y, θ_Z	3.5	12	µrad	typ.
Backlash X, Y	3	1	µm	typ.
Backlash Z	1	1	µm	typ.
Backlash θ_X, θ_Y	20	15	µrad	typ.
Backlash θ_Z	25	25	µrad	typ.
Repeatability X, Y	±0.5	±0.5	µm	typ.
Repeatability Z	±0.1	±0.1	µm	typ.
Repeatability θ_X, θ_Y	±2	±2	µrad	typ.
Repeatability θ_Z	±2.5	±2.5	µrad	typ.
Max. velocity X, Y, Z	1	25	mm/ s	
Max. velocity θ_X, θ_Y, θ_Z	11	270	mrad/ s	
Typ. velocity X, Y, Z	0.5	10	mm/ s	
Typ. velocity θ_X, θ_Y, θ_Z	5.5	55	mrad/ s	
Mechanical properties				
Stiffness X, Y	1.7	1.7	N/ µm	
Stiffness Z	7	7	N/ µm	
Load (base plate horizontal / any orientation)	10 / 5	5 / 2.5	kg	max.
Holding force, de- energized (base plate horizontal / any orientation)	100 / 50	15 / 5	N	max.
Motor type	DC gear motor	DC motor		
Miscellaneous				
Operating temperature range	-10 to 50	-10 to 50	°C	
Material	Aluminum	Aluminum		
Mass	8	8	kg	±5 %
Cable length	3	3	m	±10 mm

Hexapod

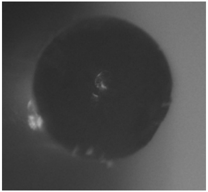


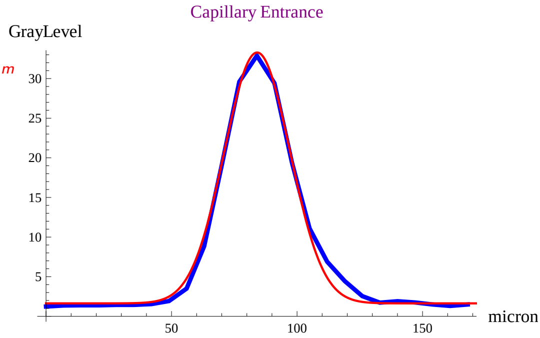
Experimental Setup



Laser Focus at the Entrance

- ⋆ Laser Power : $\sim 40 \pm 1\ mW$
- ⋆ Focus diameter : $2w_0 \sim (52.3 \pm 7)\ \mu m$

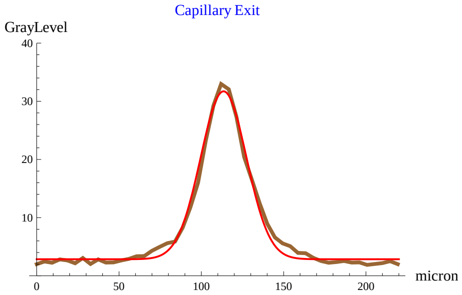




Fitting Curve: $a + be^{-(r-c)^2/w_0^2}$
$a = 1.63$
$b = 31.65$
$c = 84.11\mu m$
$w_0 = 26.15\mu m$

Laser Focus at the Exit

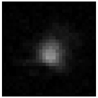
- ⋆ Laser Power : $\sim 35 \pm 1\ mW$
- ⋆ Focus diameter : $\sim (55.6 \pm 7)\ \mu m$
- ⋆ Magnification : ~ 1.5



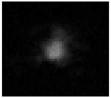
Fitting Curve: $a + be^{-(r-c)^2/w_0^2}$
$a = 2.85$
$b = 28.86$
$c = 113.54 \mu m$
$w_0 = 27.80 \mu m$

Laser-offset Test

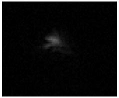
The offset dR is considered in the transverse plane with respect to the laser-capillary axis



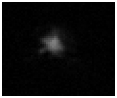
Best Coupling



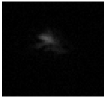
dR=10μm



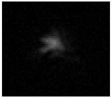
dR=40μm



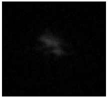
dR=20μm



dR=50μm



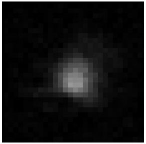
dR=30μm



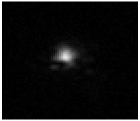
dR=60μm

Laser-misalignment Test

The misalignment $d\theta$ is considered in the propagation plane with respect to the laser-capillary axis

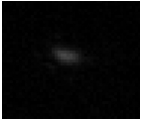


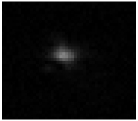
Best Coupling



$d\theta$=3.5 *mrad*

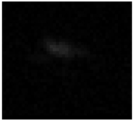
$d\theta$=7 *mrad*





$d\theta$=5.2 *mrad*

$d\theta$=8.7 *mrad*



Conclusions e perspectives

⋆The propagation of an ultra-short laser in a capillary waveguide has been considered from the point of view of the multimode wakefield structure and the modification in betatron radiation spectra.

⋆ We would like to test these methods during the forthcoming experiments of plasma acceleration at LNF.

⋆ We have tested the coupling of the laser inside a capillary with the help of the Hexapod PI. The coupling with the $EH11$ mode is significantly maintained for an offset of about 30 μm and a misalignment of about 5 $mrad$.