Transport signatures of strong Rashba spin-orbit coupling

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Purpose of the talk

clarify whether and how spin-orbit coupling affects DC charge transport

Outline

- Motivations
- Model and regimes
- Results
  - Single-particle properties
  - Conductivity and mobility
- Perspectives

Simple analytical expressions !

Spin-orbit (SO) coupling in solids

- weak anti-localization
- anomalous Hall effect
- spin Hall effect, spin relaxation
- topological phases
- Majorana fermions
-...

SPINTRONICS = spin transport electronics ⇒ "understand and control the transport of spin-polarized currents and to eventually apply this knowledge in information technologies"

D. Awshalom, Physics (2009)

Intense efforts to engineer structures and materials with strong spin-orbit coupling

Types of spin-orbit coupling:

- Intrinsic
  - **Rashba** ⇐ structural inversion asymmetry or external gates
  - Dresselhaus ⇐ bulk inversion asymmetry
- Extrinsic → Due e.g to scattering with impurities...

In this talk: Rashba spin-orbit coupling in charge transport

Emerging new materials …

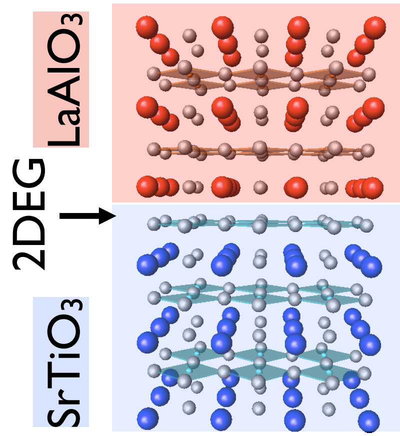
- Surface alloys

C. Ast *et al.*, *Phys. Rev. Lett. (2007); K. Yaji et al., Nature Comms. (2009);*

Tunability by changing stoichiometry

Spin-orbit coupling up to 200 meV

- Oxides heterostructures

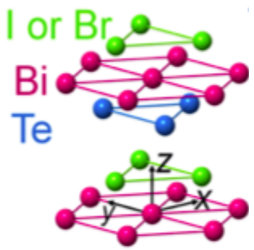


A. Ohtomo & . Huang, Nature (2004); A. Caviglia *et al., Nature* (2008); …

Gate tunable

Spin-orbit coupling estimates range from 5 to 20 meV;

- Surfaces of BiTeX, X=Cl,I,..



Spin-orbit coupling up to 100 meV

After Sakano et al. Phys. Rev. Lett. (2013); Eremeev et al.ibid. (2012); A. Crepaldi ibid. (2012);…

- Other systems:
  - HgTe quantum wells
  - Organometal compunds
  - Ferroelectric oxides
  - …

…with strong (*tunable*) *Rashba* coupling

Common features are:

- 2-dimensional
- strong spin-orbit coupling, E_0
- tunable carrier density, small E_F

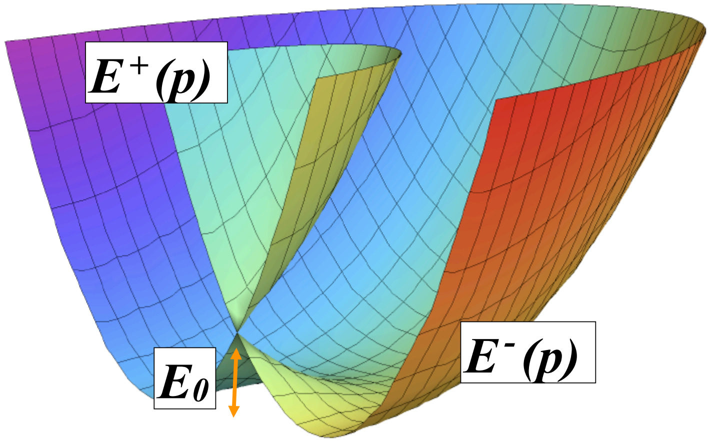
Very different from traditional III-V semicondutors where SO coupling is a small perturbation!

Need for a theoretical description of transport non-perturbative in E_0/E_F

Hamiltonian

$$\mathcal{H} = \int d\mathbf{r}\, \Psi^\dagger(\mathbf{r}) \left(\frac{p^2}{2m} + \alpha(p_x\sigma_y - p_y\sigma_x) + V_{imp}(\mathbf{r}) \right) \Psi(\mathbf{r})$$

Rashba model + Disorder



- Gaussian random disorder with "white noise" correlations

$$\langle V_{\rm imp}(\mathbf{r}) V_{\rm imp}(\mathbf{r}')\rangle_{\rm imp} = n_i v_{\rm imp}^2 \delta(\mathbf{r} - \mathbf{r}')$$

- Inelastic scattering (phonons, e-e) negligible

Two chiral sub-bands with opposite helicities

$$E^\pm(p) = \frac{1}{2m}(p \pm p_0)^2 - E_0$$

$$p_0 = m\alpha$$

Helicity operator: $\hat{s} = [\hat{p} \times \boldsymbol{\sigma}]_z$

Disorder characterized by $\Gamma_0 = \frac{m n_{imp} v_0^2}{2}$

elastic scattering at zero of spin-orbit

Spin-orbit coupling strength $E_0 = \frac{1}{2} m\alpha^2$

E.I. Rashba, *Sov. Phys. Usp. (1965)*

Three regimes

Relevant energy scales: Γ, E_0 and E_F

Diluted impurities ⇨ $\Gamma << E_F$

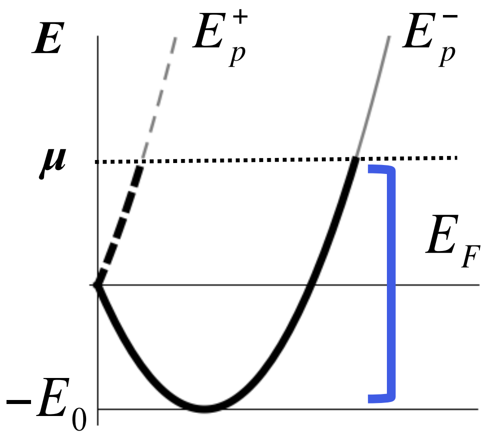
(i) Weak SO regime : $E_0 << \Gamma$

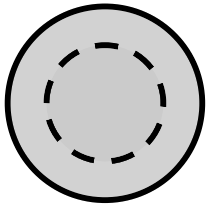
(Dyakonv-Perel' regime)

Chiral bands heavily broadened by disorder

$$E_F = \mu + E_0$$

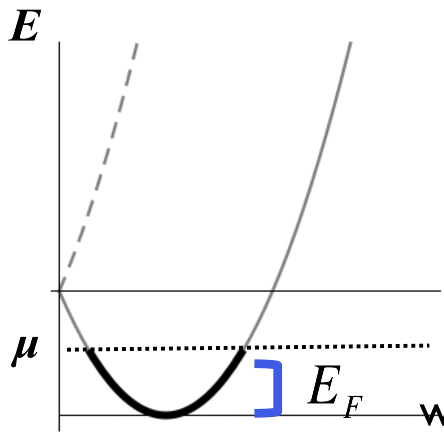
(ii) Strong SO regime : $\Gamma << E_0 < E_F$

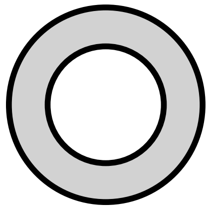




Fermi surface (FS) = two circles with opposite helicities

(iii) Ultrastrong SO regime: $\Gamma << E_F < E_0$

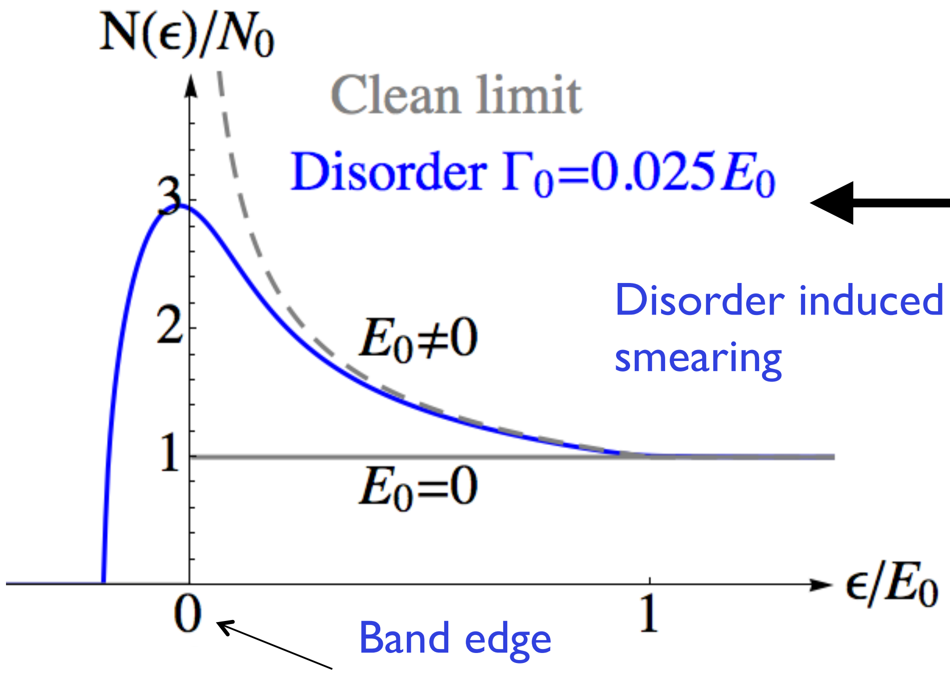




FS = two circles with equal helicities

Single-particle properties

Density of states (DOS)

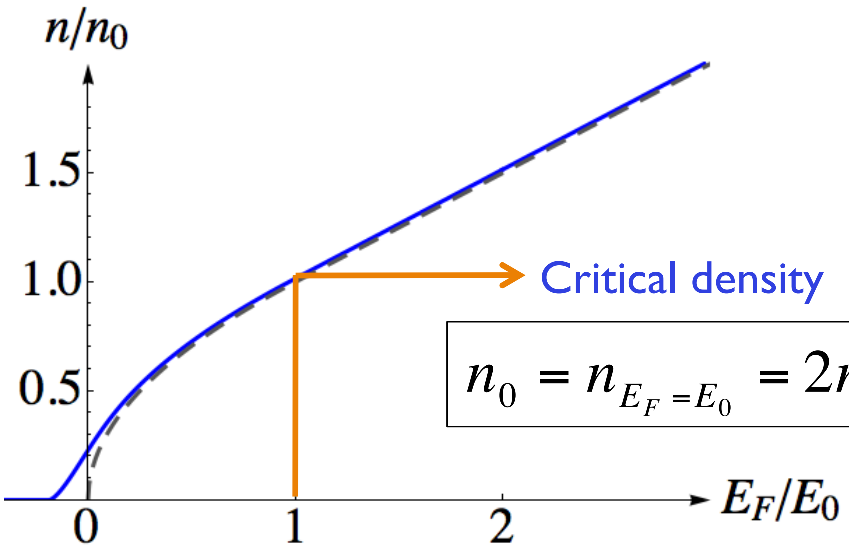


Van Hove singularity due to Rashba coupling

$$N(\varepsilon) = \begin{cases} N_0 & E_F > E_0 \\ N_0\sqrt{E_0/\varepsilon} & E_F < E_0 \end{cases}$$

$$N_0 = m/\pi$$

Charge density



$$n_0 = n_{E_F = E_0} = 2mE_0/\pi$$

$$n = \begin{cases} N_0(E_F + E_0) & E_F > E_0 \\ 2N_0\sqrt{E_F E_0} & E_F < E_0 \end{cases}$$

Analytical formulae in clean limit !

Green's function

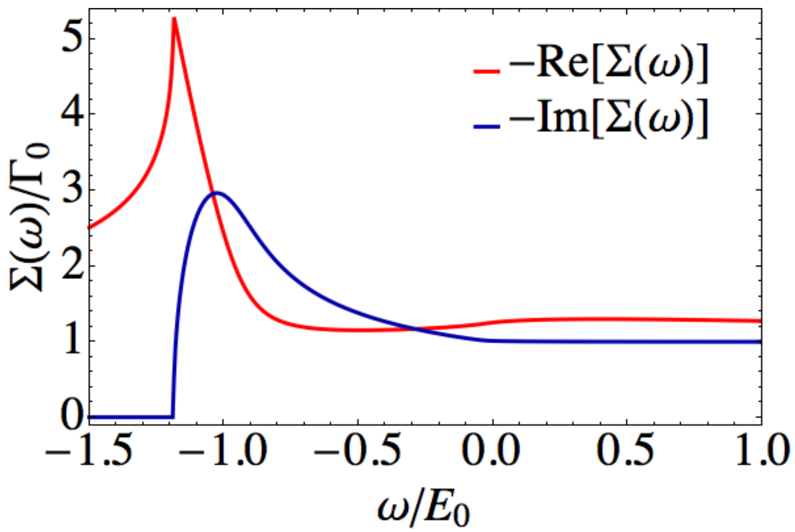
Diagonal matrix in the helicity basis

$$G^R(\mathbf{p},\omega) = \begin{pmatrix} g_+^R(p,\omega) & 0 \\ 0 & g_-^R(p,\omega) \end{pmatrix}$$

$$g_s^R(p,\omega) = \left[\omega - E_p^s + \mu - \Sigma^R(\omega)\right]^{-1}$$

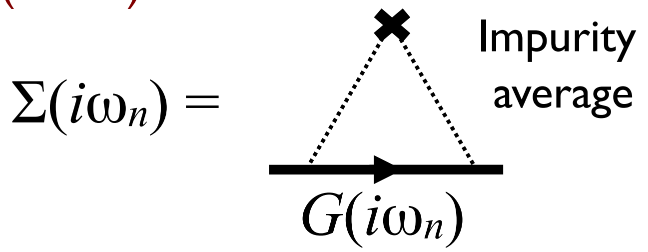
Spin-independent self-energy!

$$\Sigma^R(\omega) = \frac{n_i v_0^2}{\mathcal{V}} \sum_{\mathbf{p},s} g_s^R(p,\omega)$$



Diagrammatic perturbation theory in Matsubara frequencies

Self-consistent Born approximation (SCBA)



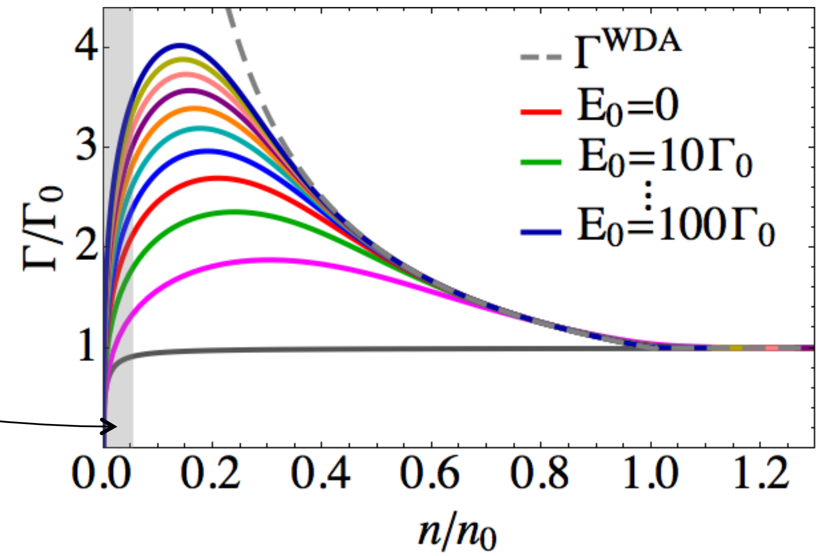
Lowest order in $\frac{\Gamma}{E_F}$

DOS and elastic scattering rate:

$$\left.\begin{matrix}\Gamma \\ N(E_F)\end{matrix}\right\} \propto -\mathrm{Im}\left[\Sigma^R(0)\right]$$

Elastic scattering rate within self-consistent Born approximation

Shaded region : corrections to SCBA important



⟺ **Weak-disorder approximation (WDA)** ⟺ Neglect broadening of spectral functions

$$|g_s^R(p,0)|^2 \simeq \delta(E_p^s - \mu)\pi/\Gamma$$

Quasi-particle approx.

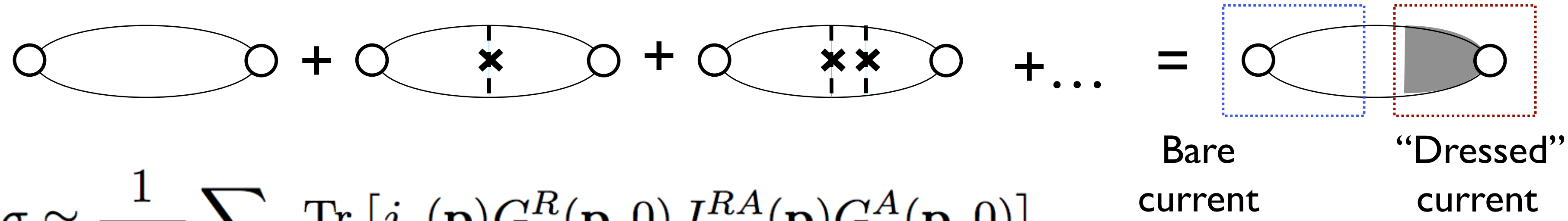
Elastic scattering rate and time in the WDA

- $E_F > E_0$ ⇒ $\Gamma \cong \Gamma_0$ ⇒ $\tau \simeq \tau_0$
- $E_F < E_0$ ⇒ $\Gamma \cong \Gamma_0 \dfrac{n_0}{n}$ ⇒ $\tau \simeq \tau_0 \dfrac{n}{n_0}$

Transport properties

Longitudinal DC conductivity within Born approximation

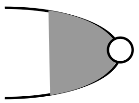
Matsubara frequencies diagrams



$$\sigma \simeq \frac{1}{2\pi\mathcal{V}} \sum_{\mathbf{p}} \mathrm{Tr}\left[j_x(\mathbf{p}) G^R(\mathbf{p},0) J_x^{RA}(\mathbf{p}) G^A(\mathbf{p},0) \right]$$

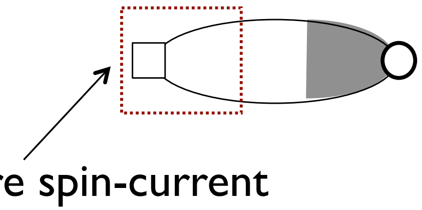
A or R = advanced or retarded argument

○ = $j_x(\mathbf{p}) = e(p_x/m + \alpha\sigma_y)$


= $J_x^{RA}(\mathbf{p}) = e(p_x/m + \tilde{\alpha}^{RA}\sigma_y)$

Same vertex also leads to vanishing spin-Hall effect !!!

Spin-Hall bubble



Disorder only affects the “anomalous” part of the current

Anomalous terms → • Momentum and current (velocity) NOT parallel

• $[J,H] \neq 0$

Conductivity

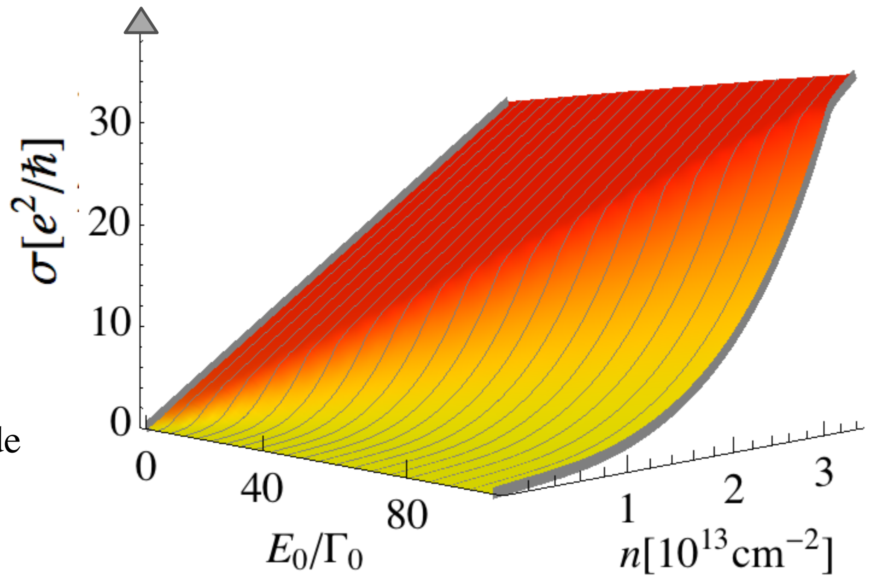
In the absence of spin-orbit coupling
within our assumptions:

$$\sigma = \sigma_{\text{Drude}} \equiv \frac{e^2 n \tau}{m}$$

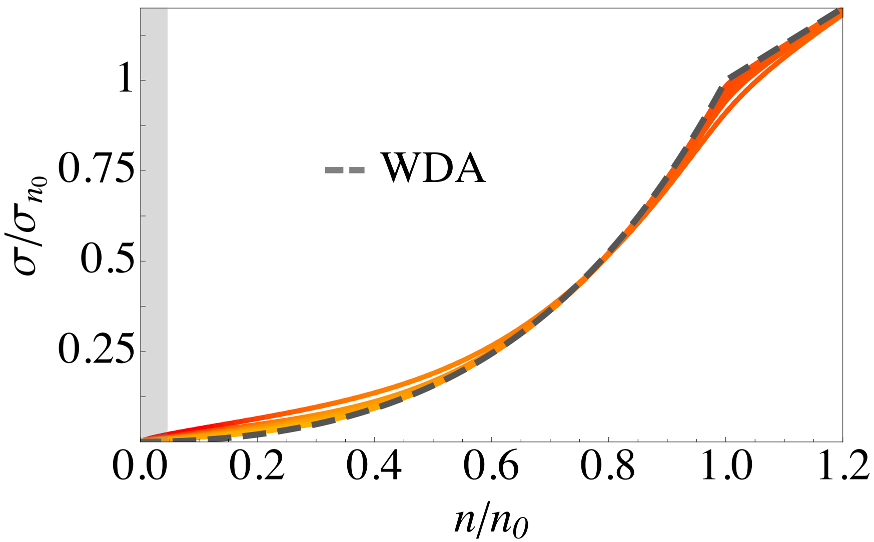
Conductivity as a function of density and Rashba coupling

$\sigma = 0$ $\quad$ $\sigma = \sigma_{\text{Drude}}$

$$\sigma_{\text{Drude}} \equiv \frac{e^2 n \tau}{m}$$



- If $E_0 > \Gamma_0$ at low-doping conductivity becomes sublinear and deviates from Drude law
- By appropriate rescaling universal behavior obtained



Two charge-transport regimes

Analytical results within WDA

$n > n_0$ ➡ $\sigma_{\mathrm{Drude}} \equiv \dfrac{e^2 n \tau}{m}$ $\qquad \tau \simeq \tau_0$

$n < n_0$ ➡ $\sigma_{\mathrm{DSO}} \equiv \dfrac{e^2 n_0 \tau_0}{2m}\left(\dfrac{n^2}{n_0^2} + \dfrac{n^4}{n_0^4}\right)$ $\qquad \tau \simeq \tau_0 \dfrac{n}{n_0}$

Remarkably simple formula!

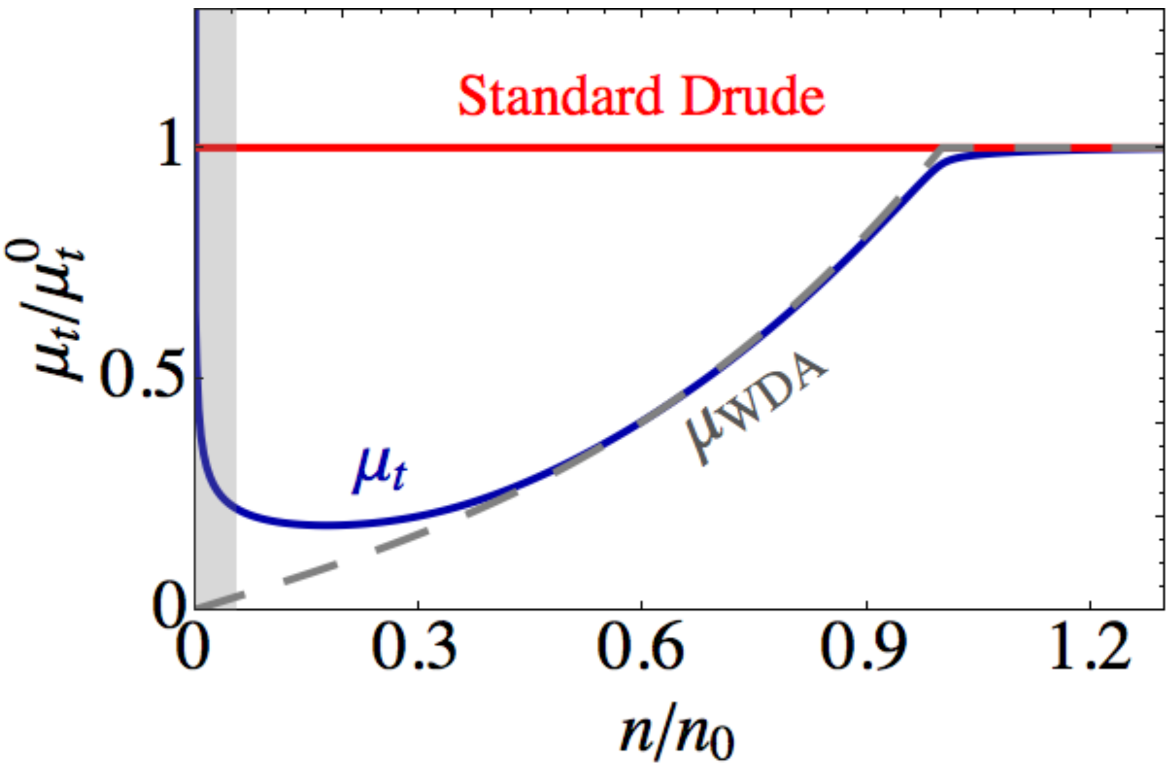
Decrease of the conductivity due to BOTH

- Increase of the scattering rate
- Non-zero anomalous vertex

$$\sigma_{\mathrm{DSO}} = \frac{1}{2}\left(1 + \frac{n^2}{n_0^2}\right)\sigma_{\mathrm{Drude}}$$

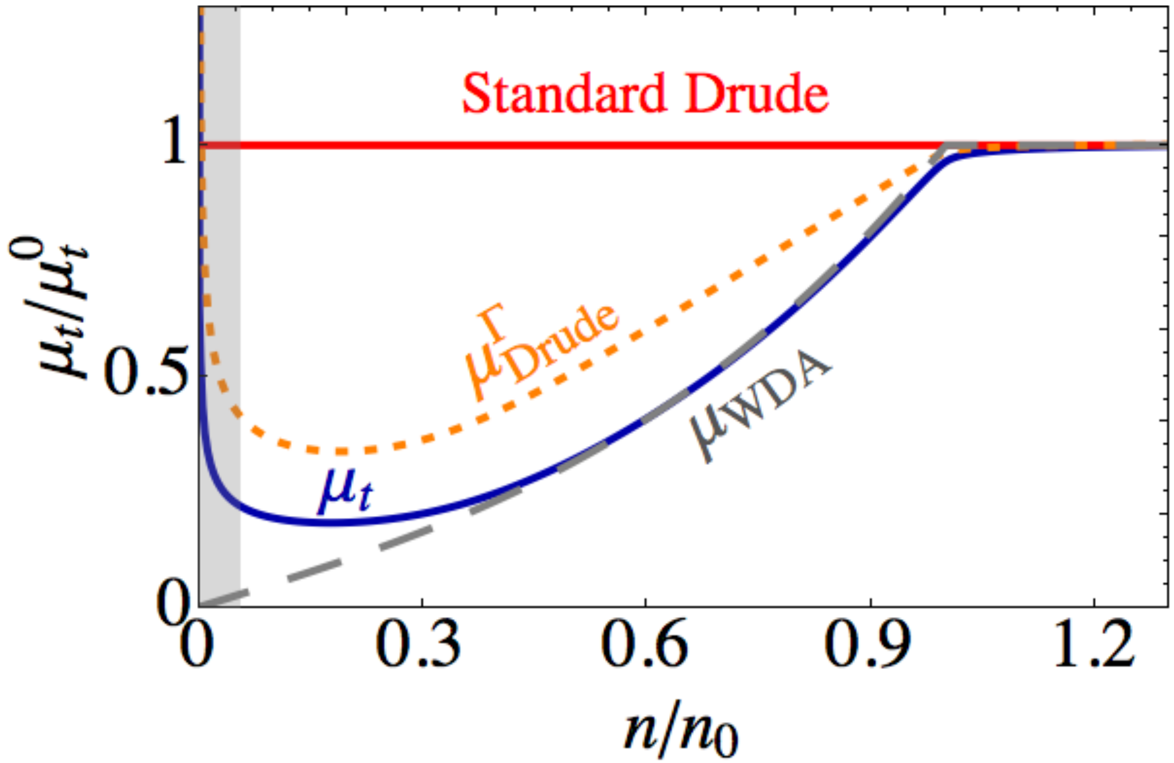
Mobility, μ_t Drift velocity per unit electric field $\mu_t = v_{drift}/E$

Related to the conductivity, *via* $\mu_t = \frac{\sigma}{en}$ Drude limit $\mu_t^0 = \frac{e\tau_0}{m}$



High accuracy of WDA !

Two-fold origin mobility modulation



$$\mu_{\mathrm{Drude}}^{\Gamma} = \frac{e\tau}{m}$$

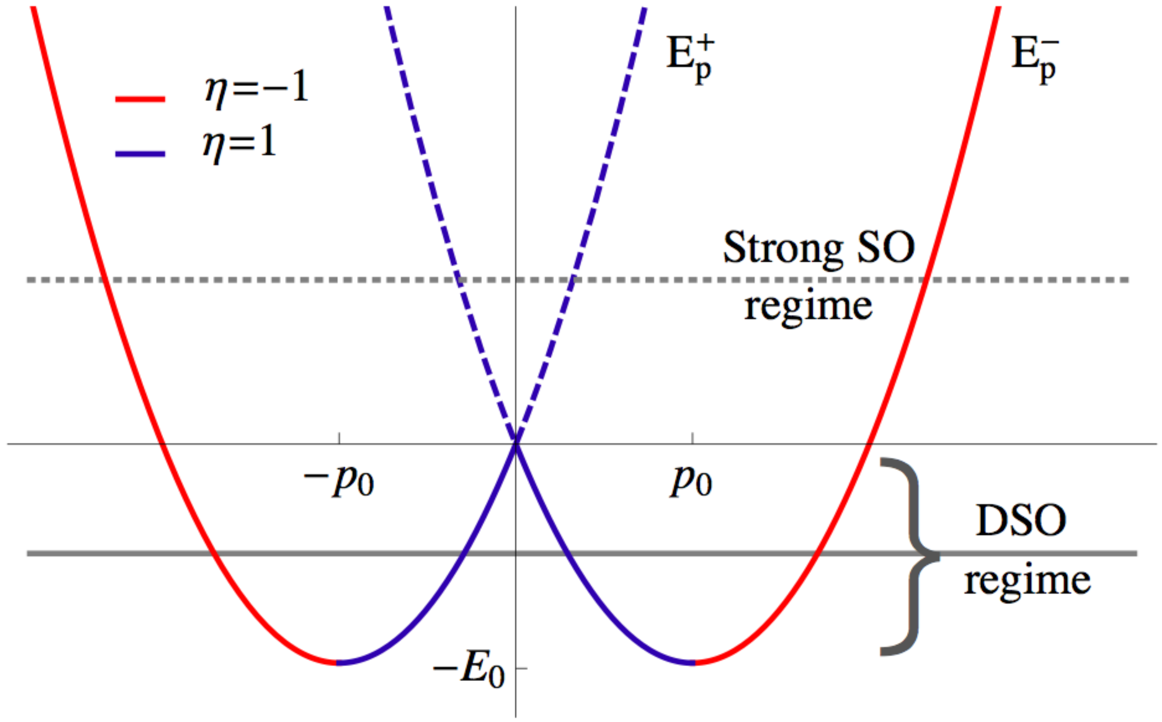
Strong dependence of the mobility on doping due to BOTH

- Increase of the scattering rate
- Non-zero anomalous vertex

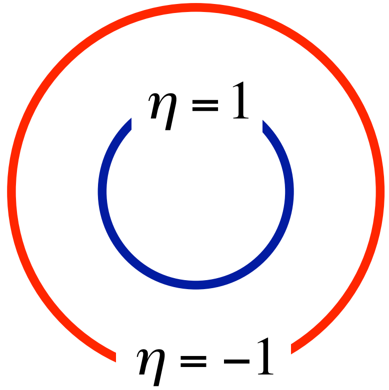
Different classification of the states in the strong and dominant SO coupling regimes

$$\eta = (\hat{v}_{\mathbf{p}s} \cdot \hat{p})\, s$$

Average velocity of helicity state *s*



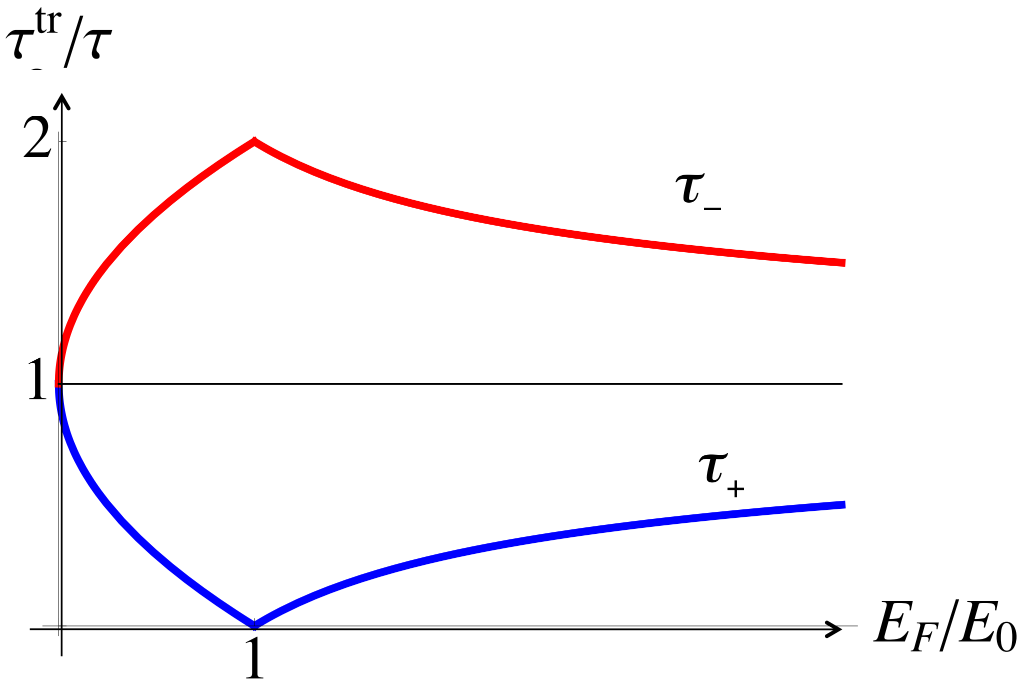
In both regimes we have:

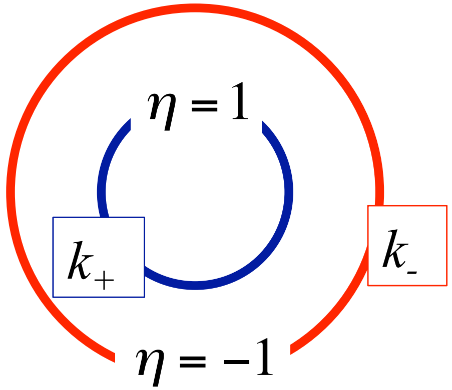


Boltzmann approach and relaxation time approximation

$$\begin{cases} \tau_\pm = \tau\, k_\pm/p_0 & n < n_0 \\ \tau_\pm = \tau\, k_\pm/(mv_F) & n > n_0 \end{cases} \quad\Longrightarrow\quad \begin{matrix} \sigma = \sigma_{WDA} \\ \sigma = \sigma_{\text{Drude}} \end{matrix}$$

recover Kubo results within WDA





$k_\pm$ = Fermi momenta

Back-scattering within the same band is suppressed

Conclusions

Two regimes of charge transport in the presence of Rahsba coupling

In the dominant SO-coupling regime:

- Enhancement of transport scattering
- Non-zero anomalous vertex

→ Reduction of the mobility and of the longitudinal conductivity

Analytical universal formulae to describe the conductivity which could be readily used to fit experiments

Perspectives

- New physics in ultrastrong spin-orbit regime
- Measure E_0 in a plain DC transport experiment!

arxiv: 1506.01944

Some numbers

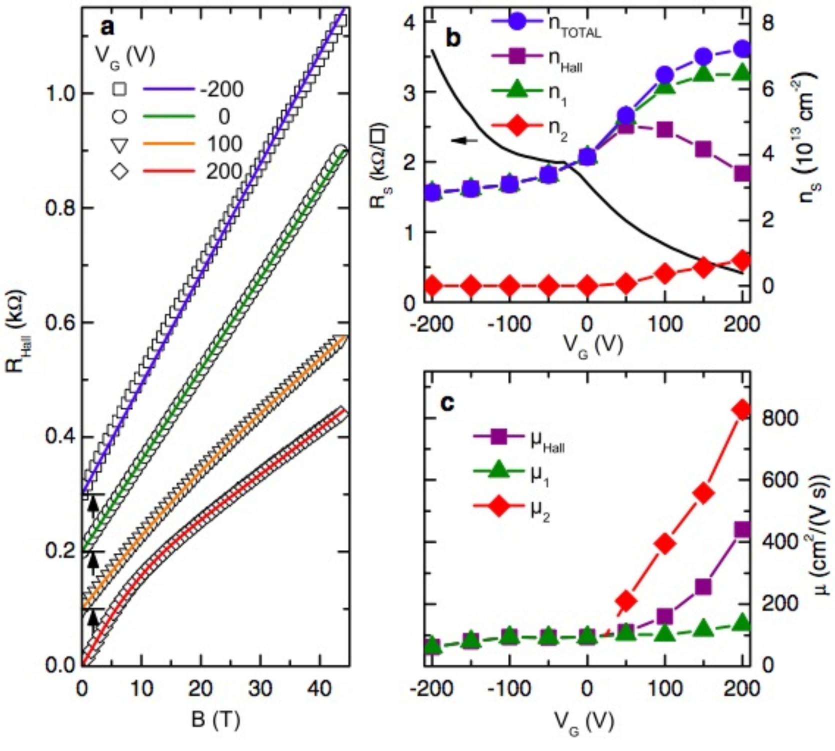
$$\sigma_{xx}^{\mathrm{B}} = \sum_{s\mathbf{k}} \delta(\mu - E_k^s) v_{sk}^2 \tau_{sk}^{\mathrm{tr}}$$

$$\frac{\tau_{sk}^{\mathrm{tr}}}{\tau} = 1 + \frac{n_i}{\mathcal{V}} \sum_{\mathbf{k}'\beta} W_{\mathbf{k}\mathbf{k}'}^{ss'} \frac{\vec{v}_{\mathbf{k}'s'} \cdot \vec{v}_{\mathbf{k}s}}{|\vec{v}_{\mathbf{k}s}|^2} \tau_{s'k'}^{\mathrm{tr}}$$

$$W_{\mathbf{k}\mathbf{k}'}^{ss'} = \pi v_0^2 (1 + ss' \hat{k} \cdot \hat{k}') \delta(E_k^s - E_{k'}^{s'})$$

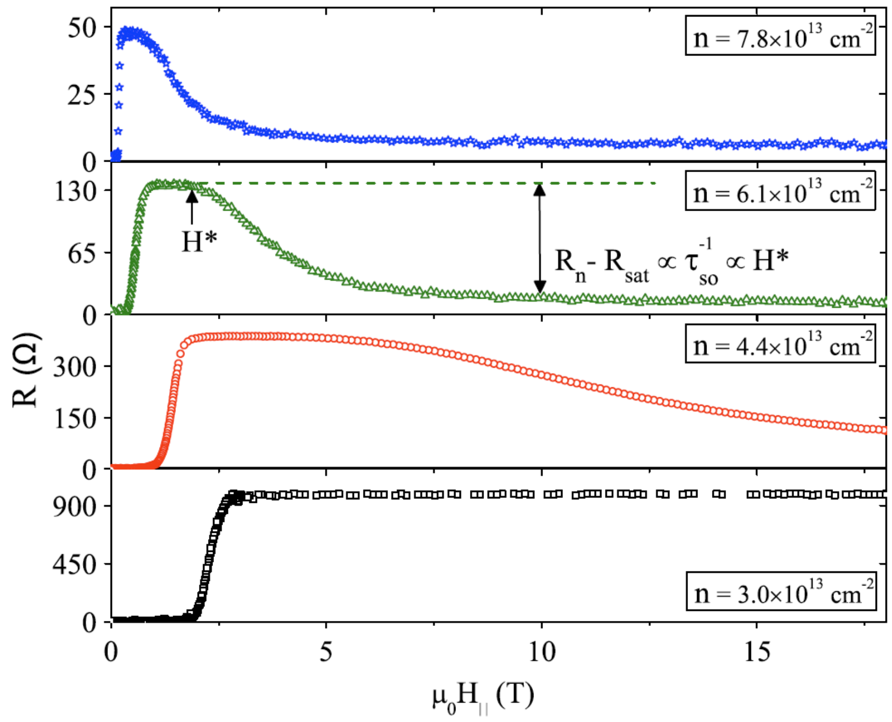
$$\sigma_{WDA} = \frac{n_+ \tau_+}{m_+} + \frac{n_- \tau_-}{m_-}$$

Anomalies in transport



After J. Biscaras et al. PRL (2012)

In-plane magnetic field



After M. Ben Shalom *et al.* PRL 2010

Density dependent mobility due to different species of carriers, different subbands ?

Decomposition of the current

Transverse and longitudinal components

$$j_x = e(\mathrm{v}_{\mathbf{p}}\hat{p}_x + \mathrm{v}_{\mathbf{t}}\hat{p}_y)$$

$$J_x = e(\mathrm{V}_{\mathbf{p}}^{LM}\hat{p}_x + \mathrm{V}_{\mathbf{t}}^{LM}\hat{p}_y)$$

$\mathrm{v}_{\mathbf{p}} = \mathbf{v}\cdot\hat{p}$ Diagonal in the chiral basis

$\mathrm{v}_{\mathbf{t}} = \mathbf{v}\cdot\hat{t}$ Proportional to $\alpha\sigma_y$ $\hat{t} = (\hat{p}_y, -\hat{p}_x)$

$$\Rightarrow \quad P_{xx}^{LM} = P_{\mathrm{intra}}^{LM} + P_{\mathrm{inter}}^{LM}$$

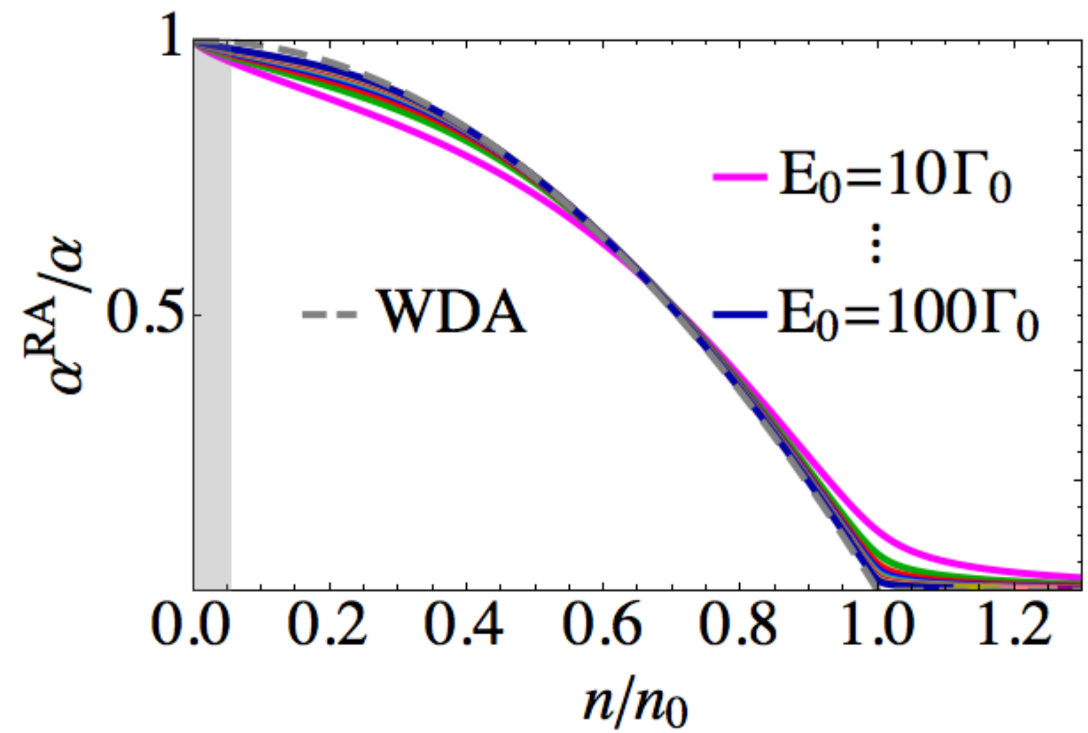
$$P_{\mathrm{intra}}^{LM} = \frac{e^2}{2\mathcal{V}}\sum_{\mathbf{p}s}\mathrm{v}_{\mathbf{p},s}\mathrm{V}_{\mathbf{p},s}^{LM}g_s^L(p,0)g_s^M(p,0)$$

$$P_{\mathrm{inter}}^{LM} = \frac{e^2}{2\mathcal{V}}\alpha\,\tilde{\alpha}^{LM}\sum_{\mathbf{p}\,s\neq s'}g_s^L(p,0)g_{s'}^M(p,0)$$

Always negligible contribution due to negligible overlap of spectral functions

At strong spin-orbit only intraband contributions to the conductivity

Dressed anomalous vertex



Universal analytical results within WDA

$n < n_0$ ⇨ $\alpha^{RA} \approx \alpha\left(1 - \frac{n^2}{n_0^2}\right)$

$n > n_0$ ⇨ $\alpha^{RA} = 0$

Vertex corrections are relevant in both regimes!