The concept of coupling impedance in the plasma wake field excitation as a new tool for describing the self-consistent interaction of the driving beam with the surrounding plasma

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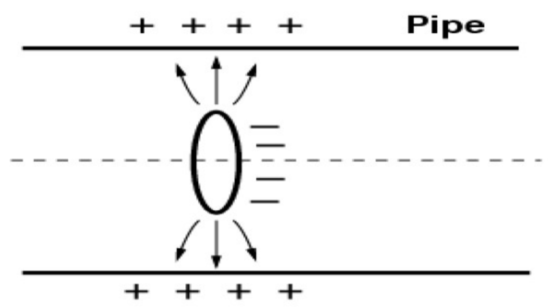
Introduction

THE COUPLING IMPEDANCE IN CONVENTIONAL ACCELERATORS

- **In a conventional particle accelerator, the *coupling impedance* schematizes the interaction of a (relativistic) charged particle beam with the surrounding medium.**

- This interaction involves the ***wake fields*** that are produced by each charged particle of the beam and therefore is a macroscopic collective manifestation of the beam in the surroundings.

- **A very effective way to describe such an interaction makes use of the concept of both *image charges* and *image currents*. They are produced, for instance, on the metallic walls of the vacuum chamber, by the charged particles of the beam. They produce electric and magnetic fields capable, in principle, to affect the particles of the beam itself.**



Introduction

THE COUPLING IMPEDANCE IN CONVENTIONAL ACCELERATORS

- The beam particles experience the effects of the fields that the particles themself have produced (*self-interaction*).

- In general, the wake fields produced by a sufficiently short bunch affect the particle of another bunch moving behind.

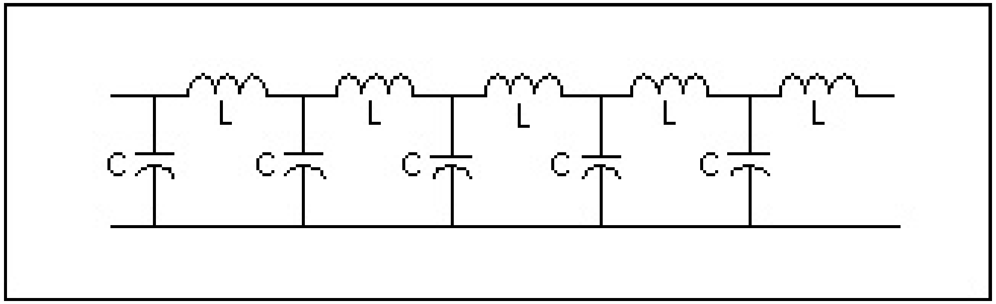
- **Due to the nature of the interaction between the beam and the surroundings, some *reactive* (*capacitive* as well as *inductive*) energy related to the beam space charge and current is involved in the system.**

- In addition, **the possible resistive character of the metallic walls experienced by the image currents, involves some *resistive* (i.e., *ohmic*) energy, as well.**

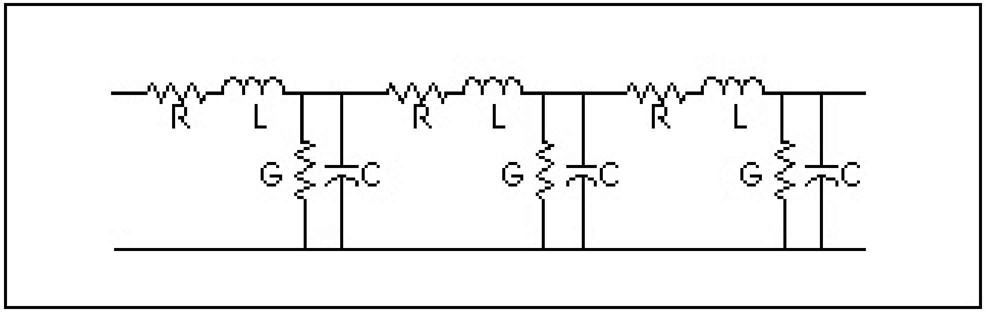
Introduction

THE COUPLING IMPEDANCE IN CONVENTIONAL ACCELERATORS

- Therefore, in the frequency and wave number domain, **the interaction of the beam with the surroundings can be effectively represented by a sequence of elements of an electric transmission line**. Each of these elements accounts for an equivalent impedance per unitary length which is constituted by an equivalent capacitance, inductance and resistance per unitary length.



Lossless Transmission Line Model



Lossy Transmission Line Model

Introduction

a.m. Sessler

Andrew M. Sessler
1928 - 2014

Introduction

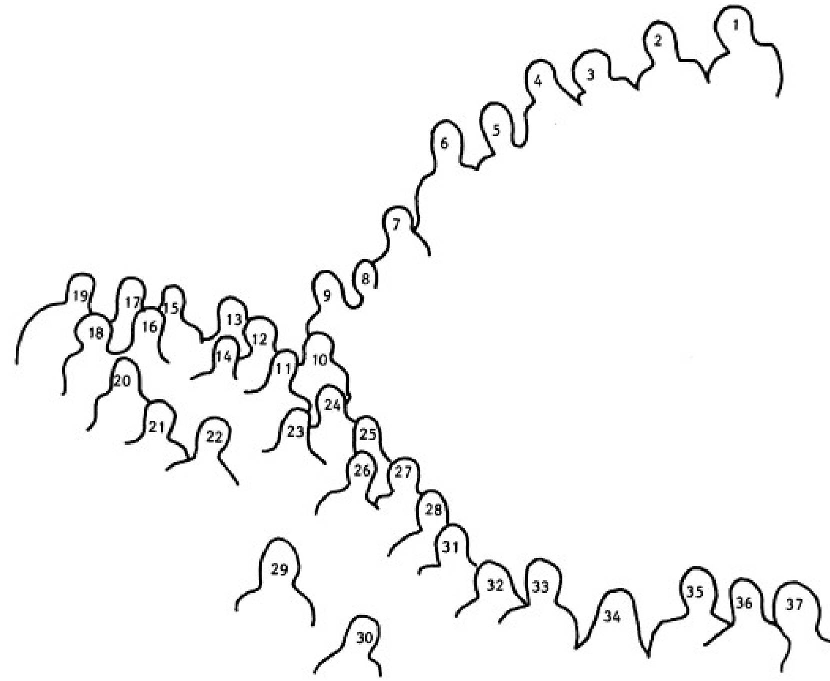
In 1959, Sessler went on to study dynamical instabilities. From his explorations, with Carl Nielsen and Keith Symon, of the negative mass instability emerged the first realization that particle beams could have dynamical instabilities due to their space charge. The researchers invoked Landau damping as a cure for these instabilities, and they developed techniques that have been used to study the other kinds of instabilities discovered.[A.M. Sessler, C. E. Nielsen and K. R. Symon. Longitudinal instabilities in intense relativistic beams. In *Proceedings of the international conference on high-energy accelerators and instruments*. Geneva: CERN, 239–252.].

Introduction

In subsequent studies with Ernest Courant, Sessler became interested in single bunches rather than a continuous beam, and he realized that wall resistance is only one aspect of the general concept of impedance [L. J. Laslett, V. K. Neil and A.M. Sessler, Transverse resistive instabilities of intense coasting beams in particle accelerators. *Rev. Sci. Instrum*. 36:436–448; E.D. Courant and A.M. Sessler]—later to be developed with Vittorio Vaccaro [A.M. Sessler and V.G. Vaccaro].

Sessler had a high opinion of Vaccaro, whose ideas had not been so well received at CERN. **Nowadays, however, everyone uses their work to calculate, measure, and control impedance in order to limit instabilities.**

Introduction



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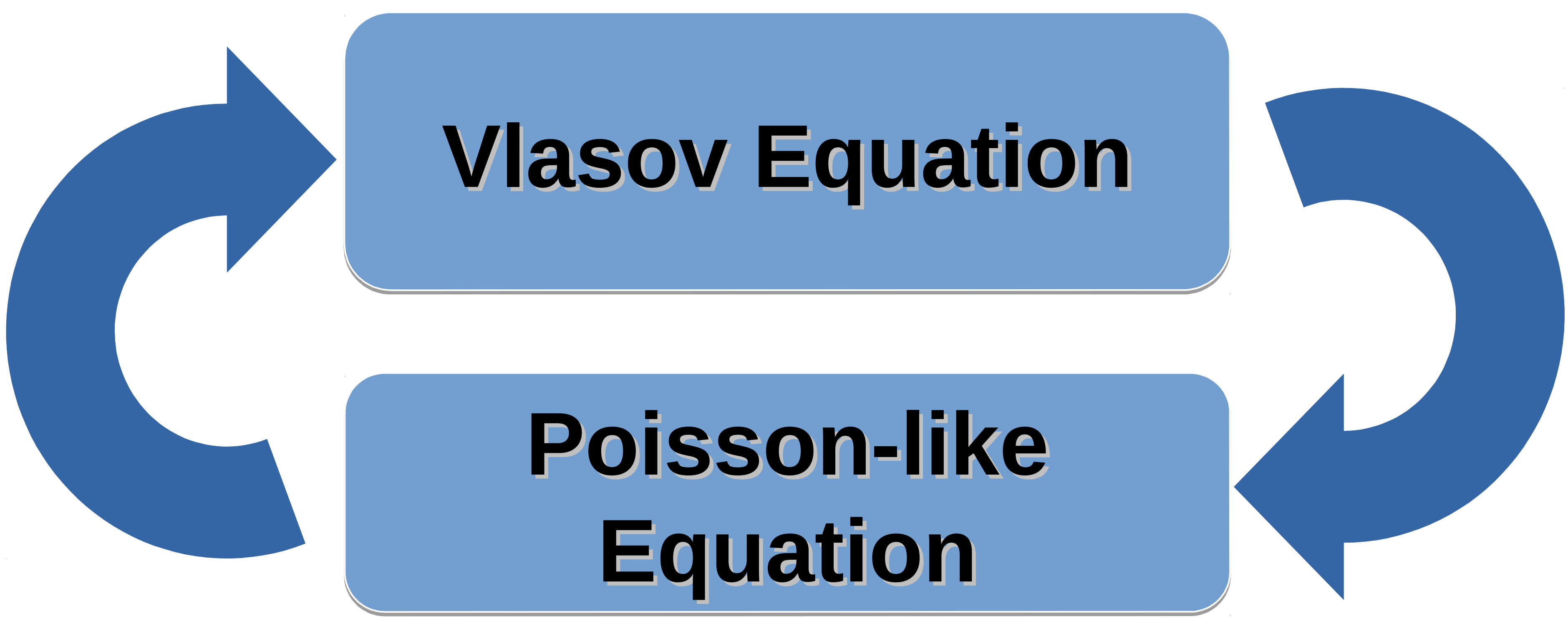
Coupling impedance in PWF interaction

In the self-consistent PWF excitation the beam particles experience the effects of the fields that the particle themself have produced (*self-interaction*).

Nonlinear and collective dynamics



Coupling impedance in PWF interaction

- **Vlasov equation**: kinetic equation (in comoving frame) governing the spatiotemporal evolution of the one-particle distribution function f(**r**,**p**,t) in the Boltzmann phase space (i.e., μ-space); it provides the kinetic description of the charged-particle beam while interacting with the surrounding plasma

- **Poisson-like equation**: differential equation which relates (in the comoving frame) the beam density, i.e., $n_b(\mathbf{r},t)$ to the wake potential, i.e., $\Omega(\mathbf{r},t)$; it also provides the relation between $\Omega(\mathbf{r},t)$ and f(**r**,**p**,t)

- The **concept of coupling impedance in beam-plasma interaction** ruled by the PWF excitation can be introduced after linearizing the Vlasov-Poisson-like system (around an unperturbed state) and taking the Fourier transform of the resulting equations

EXAMPLES OF VLASOV-POISSON-LIKE PAIR OF EQUATIONS

Example 1 - for simplicity we confine our attention to the ***purely longitudinal* beam dynamics** in a ***collisionless beam-plasma system***.

- Linearized Vlasov equation after making the coordinate transformation: $\xi = z - \beta t$, $\tau = t$ $\quad (\beta \simeq 1)$

$$\frac{1}{c}\frac{\partial f_1(\xi,\mathcal{P}_{1z},\tau)}{\partial \tau} + \mathcal{P}_{1z}\frac{\partial f_1(\xi,\mathcal{P}_{1z},\tau)}{\partial \xi} + \frac{q}{m_0\gamma_0 c^2}\frac{\partial \Omega_1}{\partial \xi}\frac{\partial f_0(z,\mathcal{P}_{1z},t)}{\partial \mathcal{P}_{1z}} = 0$$

- Linearized Poisson-like equation after making the coordinate transformation: $\xi = z - \beta t$, $\tau = t$ and imposing the quasi-stationary conditions $\frac{\partial}{\partial \tau} = 0$

$$\left(\frac{d^2}{d\xi^2} + k_p^2\right)\Omega_1 = k_p^2 \frac{m_0 c}{e^2 n_0}\frac{I_{b1}}{\beta\pi\sigma_\perp^2}$$

$$I_{b1} = q\beta c\pi\sigma_\perp^2 n_{b1}$$

Longitudinal coupling impedance in PWF interaction

$$\begin{cases} \tilde{f}_1 = -\dfrac{(q/m_0\gamma_0 c^2)k\tilde{\Omega}_1 f_0'}{k\mathcal{P} - \frac{\omega}{k}} \\ (-k^2 + k_p^2)\tilde{\Omega}_1 = k_p^2 \dfrac{m_0 c}{e^2 n_0} \dfrac{\tilde{I}_{b1}}{\beta\pi\sigma_\perp^2} \end{cases}$$

$$\int \tilde{f}_1 d\mathcal{P} = \tilde{n}_{b1} = \tilde{I}_1/q\beta c\pi\sigma_\perp^2$$

$$\boxed{1 = -\frac{q^2\beta\pi\sigma_\perp^2}{m_0\gamma_0 c}\frac{\tilde{\Omega}_1}{\tilde{I}_1}\int \frac{\hat{f}_0' dp_z}{p_z - \frac{\omega}{ck}}.}$$

Coupling impedance in PWF interaction

HEURISTIC DEFINITION OF THE COUPLING IMPEDANCE

For simplicity we confine our attention to the ***longitudinal coupling impedance*** (it can be easily generalized to the *transverse coupling impedance*)

$$\frac{Z_L(k,\omega)}{k} = i\frac{\tilde{\Omega}_1(k,\omega)}{\tilde{I}_1(k,\omega)}$$

Longitudinal coupling impedance and dispersion relation

$$\frac{Z_L}{k} = -\frac{K_0 k_p^2}{(k^2 - k_p^2)} \qquad 1 = i\eta\left(\frac{Z_L}{k}\right)\int \frac{\hat{f}_0' d\mathcal{P}}{\mathcal{P} - \frac{\omega}{k}}$$

$$\hat{f}_0 = f_0/n_{b0}, \; \eta = (k_p^2 \beta c \sigma_\perp^2 \, q^2 \, n_{b0})/4e^2 n_0 \gamma_0$$

$$K_0 = m_0 c/n_0 e^2 \beta \pi \sigma_\perp^2$$

Longitudinal coupling impedance in PWF interaction

$$Z_L = Z_R + iZ_I$$

$$\begin{cases} \dfrac{Z_I}{k} = -\dfrac{K_0 k_p^2}{(k^2 - k_p^2)} \\[2ex] Z_R = 0 \end{cases}$$

$$1 = -\eta \left(\frac{Z_I}{k} \right) \int \frac{\hat{f}_0' d\mathcal{P}}{\mathcal{P} - \frac{\omega}{k}}$$

EXAMPLES OF VLASOV-POISSON-LIKE PAIR OF EQUATIONS

Example 2 - for simplicity we confine our attention to the *purely longitudinal* beam dynamics in a *collisional beam-plasma*.
We assume that the collision frequency between the beam particles and the plasma electrons are not negligible:
- *collisional Vlasov equation for the beam*
- *collisional Lorentz-Maxwell system*

- Linearized Vlasov equation:

$$\int \tilde{f}_1 dw = n_{b1} = -\frac{q}{m_0\gamma_0 c^2}\tilde{\Omega}_1 \int \frac{f_0'\, d\mathcal{P}_z}{\mathcal{P}_z - \frac{\omega}{ck} - \frac{i}{ck\tau_c}}$$

- Linearized Poisson-like equation

$$\left[-k^2\left(1 + i\frac{b/m_0}{k\beta c}\right) + k_p^2\right]\tilde{\Omega}_1 = 4\pi q\left(1 + i\frac{b/m_0}{k\beta c}\right)\frac{\tilde{I}_1}{q\beta c\pi\sigma_\perp^2}$$

$$b = \frac{1}{\tau_c}$$

Longitudinal coupling impedance and dispersion relation

$$1 = \underbrace{-\frac{c^2}{\gamma_0}\frac{k_p^2}{\left[k_p^2 - \left(1 + i\frac{b/m_0}{k\beta c}\right)k^2\right]}\left(1 + i\frac{b/m_0}{k\beta c}\right)}_{i\eta\left(\frac{Z_L}{k}\right)}\int \frac{f_0'\, d\mathcal{P}_z}{\mathcal{P}_z - \frac{\omega}{ck} - \frac{i\nu}{ck}}$$

$$\nu = 1/\tau_c$$

$$\boxed{1 = -\frac{q^2\beta\pi\sigma_\perp^2}{m_0\gamma_0 c}\frac{\tilde{\Omega}_1}{\tilde{I}_1}\int \frac{f_0'\, dp}{p - \frac{\omega}{ck} - \frac{i\nu}{ck}}}$$

$$1 = i\eta\left(\frac{Z_L}{k}\right)\int \frac{f_0'\, d\mathcal{P}_z}{\mathcal{P}_z - \frac{\omega}{ck} - \frac{i\nu}{ck}}$$

$$\Rightarrow \quad \boxed{Z_R \neq 0}$$

The presence of the collision frequency in the Landau integral is an important difference with respect to the conventional accelerators

Longitudinal coupling impedance and dispersion relation

$$V_R + iV_I \equiv \eta \left(\frac{Z_R}{k} + i\frac{Z_I}{k} \right) = -\left[i \int_{PV} \frac{f_0'(p)}{p - \beta_{ph}} + \pi f_0'(\beta_{ph}) \right]^{-1}$$

$$\beta_{ph} = \frac{\omega}{k} \qquad \omega = \omega_R + i\omega_I$$

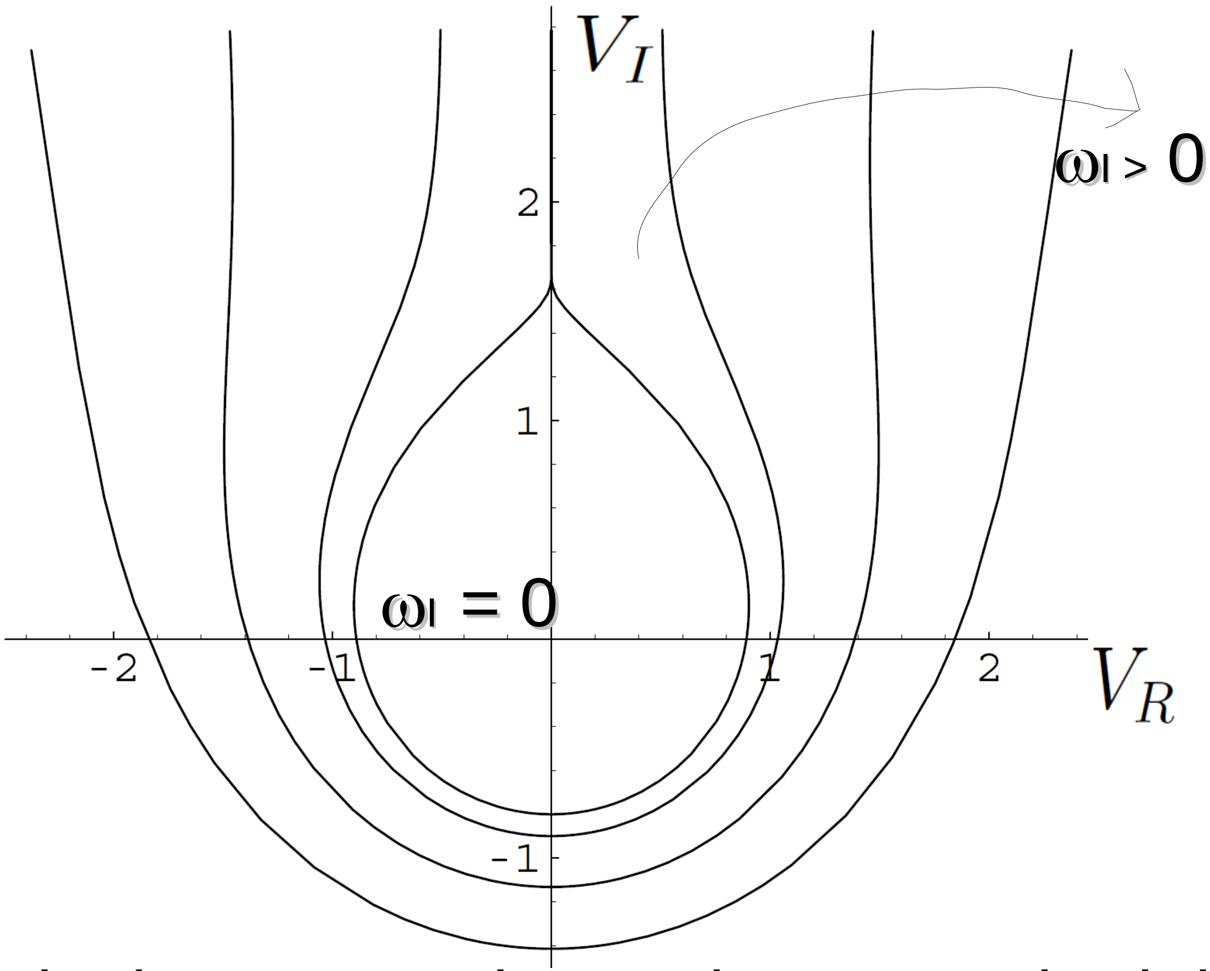
- **Weak Landau damping**:

$$\omega_I \propto f_0'({}_R/k)$$

- **Interplay between instability and stabilizing effect of Landau damping**:

- **Stability/instability analysis**: **universal Nyquist-like charts**. Curves that are mapping Z_R vs Z_i. They are plotted for fixed values of ω_I. They depend on the initial distribution profile and delimitate the stability regions (ω_I = 0) as well as the instability ones where $\omega_I \neq 0$ (growth rate of the instability)

Stability/Instability charts



arbitrary units (non monochromatic unperturbed distribution)

$$f_0(p) \propto \left(1 - p^2\right)^2$$

$f_0(p) \propto \delta(p)$: monochromatic beams. **What about them ?**

More general approach to longitudinal and transverse

- From the linearized Vlasov- Maxwell system (several species of plasma components + beam):

$$f_{s1} = \frac{iq_s}{\omega} \frac{(\omega - \mathbf{k}\cdot\mathbf{v})\hat{I} + \mathbf{k}\mathbf{v}}{\mathbf{k}\cdot\mathbf{v} - \omega} \cdot \mathbf{E}_1 \cdot \nabla_p f_{s0}$$

$$f_{b1} = \frac{iq_b}{\omega} \frac{(\omega - \mathbf{k}\cdot\mathbf{v})\hat{I} + \mathbf{k}\mathbf{v}}{\mathbf{k}\cdot\mathbf{v} - \omega} \cdot \mathbf{E}_1 \cdot \nabla_p f_{b0}$$

- Assuming formally the microscopic Ohm laws for each plasma component and for the beam

$$\hat{\sigma}_p(\mathbf{k},\omega) = \sum \frac{iq_s^2}{\omega} \int \mathrm{v} \frac{(\omega - \mathbf{k}\cdot\mathbf{v})\hat{I} + \mathbf{k}\mathbf{v}}{\mathbf{k}\cdot\mathbf{v} - \omega} \cdot \nabla_p f_{s0} d^3p$$

$$\hat{\sigma}_b(\mathbf{k},\omega) = \frac{iq_b^2}{\omega} \int \mathrm{v} \frac{(\omega - \mathbf{k}\cdot\mathbf{v})\hat{I} + \mathbf{k}\mathbf{v}}{\mathbf{k}\cdot\mathbf{v} - \omega} \cdot \nabla_p f_{b0} d^3p$$

More general approach to longitudinal and transverse impedance

- Plasma dielectric tensor and beam conductivity tensor

$$\hat{\epsilon}(\mathbf{k},\omega) = \hat{I} - \sum \frac{\omega_{ps}^2}{\omega^2} \int \frac{\mathbf{p}}{\gamma_s} \frac{(\omega - \mathbf{k}\cdot\mathbf{v})\hat{I} + \mathbf{k}\mathbf{v}}{\mathbf{k}\cdot\mathbf{v} - \omega}$$

$$\hat{\sigma}_b(\mathbf{k},\omega) = \frac{iq_b^2}{\omega} \int \mathbf{v} \frac{(\omega - \mathbf{k}\cdot\mathbf{v})\hat{I} + \mathbf{k}\mathbf{v}}{\mathbf{k}\cdot\mathbf{v} - \omega} \cdot \nabla_p f_{b0} d^3p$$

$$\hat{\epsilon}_{p\mu\nu} = \epsilon_p^L \frac{k_\mu k_\nu}{k^2} + \epsilon_p^T \frac{1}{2}(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})$$

$$\hat{\sigma}_{b\mu\nu} = \sigma_b^L \frac{k_\mu k_\nu}{k^2} + \sigma_b^T \frac{1}{2}(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})$$

$$(\hat{\epsilon}_{p\alpha\beta}) = \begin{pmatrix} \epsilon^T & 0 & 0 \\ 0 & \epsilon^T & 0 \\ 0 & 0 & \epsilon_p^L \end{pmatrix}$$

$$(\hat{\sigma}_{b\alpha\beta}) = \begin{pmatrix} \sigma_b^T & 0 & 0 \\ 0 & \sigma_b^T & 0 \\ 0 & 0 & \sigma_b^L \end{pmatrix}$$

- The longitudinal dielectric constant and the longitudinal beam conductivity

$$\epsilon_p^L = 1 + \sum \frac{\omega_{ps}^2 m_s}{\omega^2} \int \frac{v_\mu k_\mu k_\alpha}{k^2} \frac{\partial \hat{f}_{s0}}{\partial p_\alpha} d^3p - \sum \frac{\omega_{ps}^2 m_s}{\omega^2} \int \frac{v_\mu k_\mu v_\nu k_\nu k_\alpha}{k^2(\mathbf{k}\cdot\mathbf{v}-\omega)} \frac{\partial \hat{f}_{s0}}{\partial p_\alpha} d^3p$$

$$\sigma_b^L = -\frac{i\omega}{4\pi} \frac{\omega_b^2 m_b}{\omega^2} \int \frac{\mathbf{k}\cdot\mathbf{v}}{k^2} \mathbf{k}\cdot\nabla_p \hat{f}_{b0} d^3p + \frac{i\omega}{4\pi} \frac{\omega_b^2 m_b}{\omega^2} \int \frac{(\mathbf{k}\cdot\mathbf{v})^2}{k^2(\mathbf{k}\cdot\mathbf{v}-\omega)} \mathbf{k}\cdot\nabla_p \hat{f}_{b0} d^3p$$

For $\mathbf{v} = \beta c \hat{z}$

$$\sigma_b^L = \frac{i m_b \beta c \omega_b^2}{4\pi} \left(\frac{1}{k}\right) \int \frac{\hat{f}_{b0}' dp}{v_z - \frac{\omega}{k}}$$

- The specific longitudinal impedance

$$z_L \equiv \frac{1}{\sigma_b^L}$$

$$\boxed{1 = i \frac{m_b \beta c \omega_b^2}{4\pi} \frac{z_L}{k} \int \frac{\hat{f}_{b0}' dp}{v_z - \frac{\omega}{k}}}$$

- In an analogous way we can introduce the transverse specific impedance

- The dispersion relation for *beam* + *plasma* system

$$\mathcal{D}_{\alpha\beta}(\mathbf{k},\omega)E_{1\beta}(\mathbf{k},\omega)=0$$

$$\mathcal{D}_{\alpha\beta}=\frac{c^2}{\omega^2}k_\alpha k_\beta-\frac{c^2k^2}{\omega^2}\delta_{\alpha\beta}+\hat{\epsilon}_{p\alpha\beta}+\frac{i4\pi}{\omega}\hat{\sigma}_{b\alpha\beta}$$

$$(\hat{\mathcal{D}}_{\alpha\beta})=\begin{pmatrix}-\frac{c^2k^2}{\omega^2}+\epsilon_p^T+\frac{i4\pi}{\omega}\sigma_b^T & 0 & 0\\ 0 & -\frac{c^2k^2}{\omega^2}+\epsilon_p^T+\frac{i4\pi}{\omega}\sigma_b^T & 0\\ 0 & 0 & \epsilon_p^L+\frac{i4\pi}{\omega}\sigma_b^L\end{pmatrix}$$

- More reach dynamics by coupling e.m. radiation and beam
- More reach dynamics by introducing the collisional terms

Thank you!