Padova - 2/2/2015

Giroscopi Laser

Applicazione alla metrologia angolare

Nicolò Beverini

RLG project

Ring Laser Gyroscope for accurate angle metrology and as demonstrator of self-calibration principle for Lense-Thirring effect measurements

INRIM

- Marco Pisani
- Milena Astrua

INFN – Sezione di Pisa

- Angela Di Virgilio
- Jacopo Belfi

INFN – LNL

- Antonello Ortolan
- Davide Cuccato

Dipartimento di Fisica – Università di Pisa

- Nicolò Beverini
- Giorgio Carelli
- Enrico Maccioni

Progetto premiale finanziato dal MIUR

The goal

- **Realization of a mid-scale RL (50 - 90 cm), with a lightweight and stiff structure which can be placed on a precision rotating table -in combination with a precision autocollimator**

- 1) Implementation of an extremely accurate transportable rotational standard for the calibrating the best angular measurement instrument, which resolution today is well beyond the traceability capabilities of most NMIs;
- 2) Realization of a very sensitive gyroscope for the measurement of seismic effects (S-wave phase velocity, Co-seismic rotations), and for the measurements of test mass acceleration in the next generation Earth based gravitational antennas (e.g. Virgo)
- 3) The demonstration of a self-calibration concept leading to the design of a larger rotating RL for geodetic and relativistic experiments, free from the need of extremely difficult dimensional measurements

Filatov's goniometer

Quantum Electronics 30 (2) 141 – 146 (2000)

PACS numbers: 42.62.Eh; 42.55.Lt; 42.60.Da; 06.30.Bp
DOI: 10.1070/QE2000v030n02ABEH001675

Development of new methods and means of dynamic laser goniometry

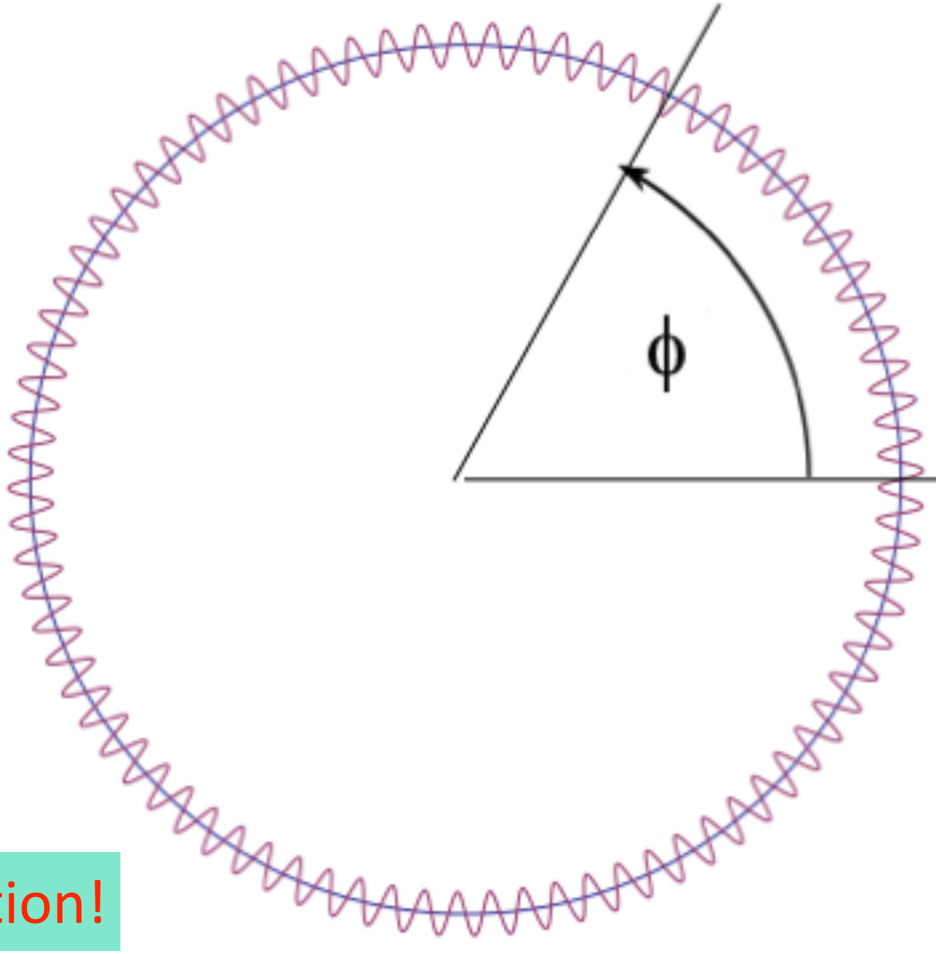
M N Burnashev, D P Luk'yanov, P A Pavlov, Yu V Filatov

Gyrolaser goniometer

$$f(t) = 4\frac{A}{\lambda p}\Omega(t)\cos\theta = K\Omega(t)$$

$$N_\phi = \frac{1}{2\pi}\int_{t_0}^{t_1} f(t)dt = \frac{K}{2\pi}\int_{t_0}^{t_1}\Omega(t)dt = \frac{K}{2\pi}\phi$$

$$N_{2\pi} = \frac{1}{2\pi}\int_{t_0}^{t_0+T} f(t)dt = \frac{K}{2\pi}\int_{t_0}^{t_0+T}\Omega(t)dt = K$$

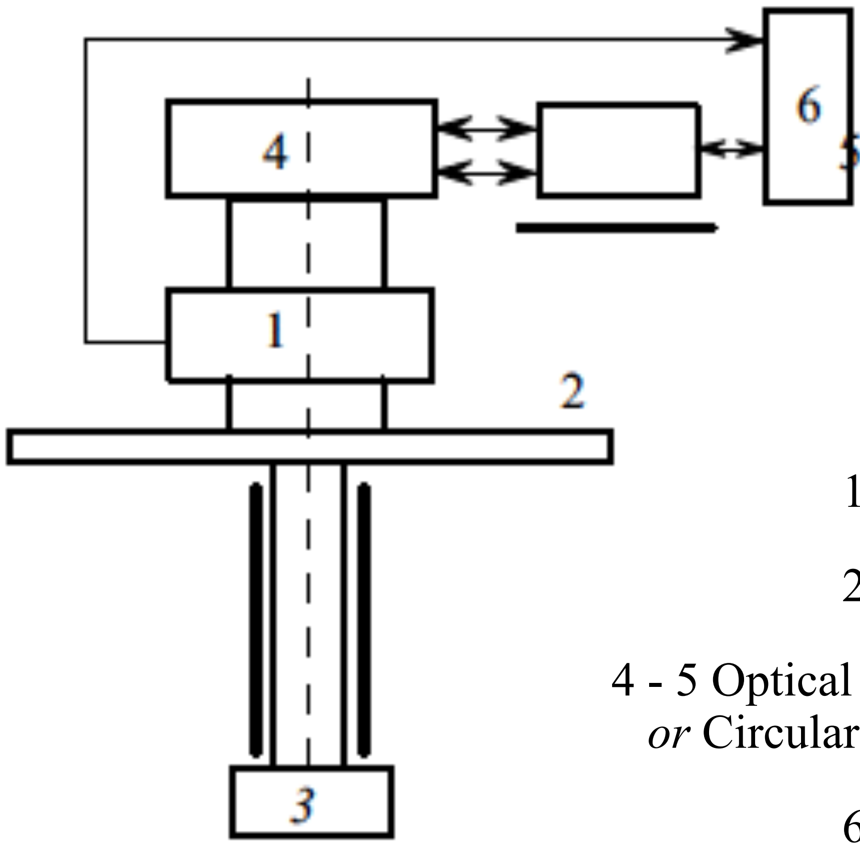


$$\phi = 2\pi\frac{N_\phi}{N_{2\pi}}$$

Self-calibration!

If K could be considered as a constant, the angle should be known with a precision limited only by the noise of the detector

Goniometro laser



1 - Ring Laser

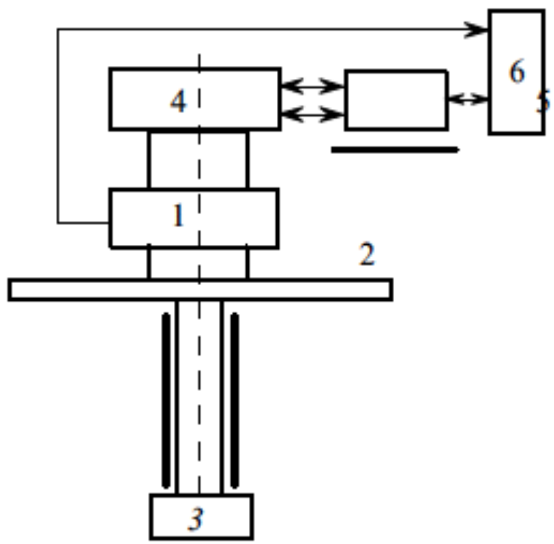
2 - Rotatory table

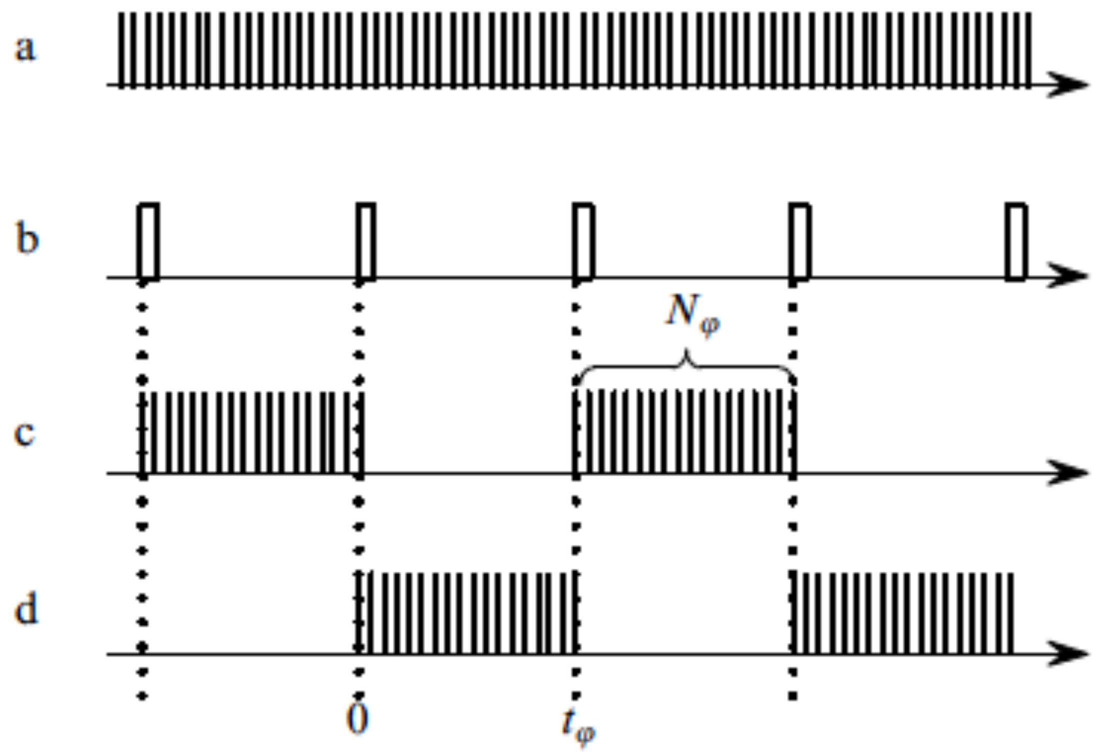
4 - 5 Optical polygon + autocollimator
or Circular encoder disc + reading head

6 - Counter

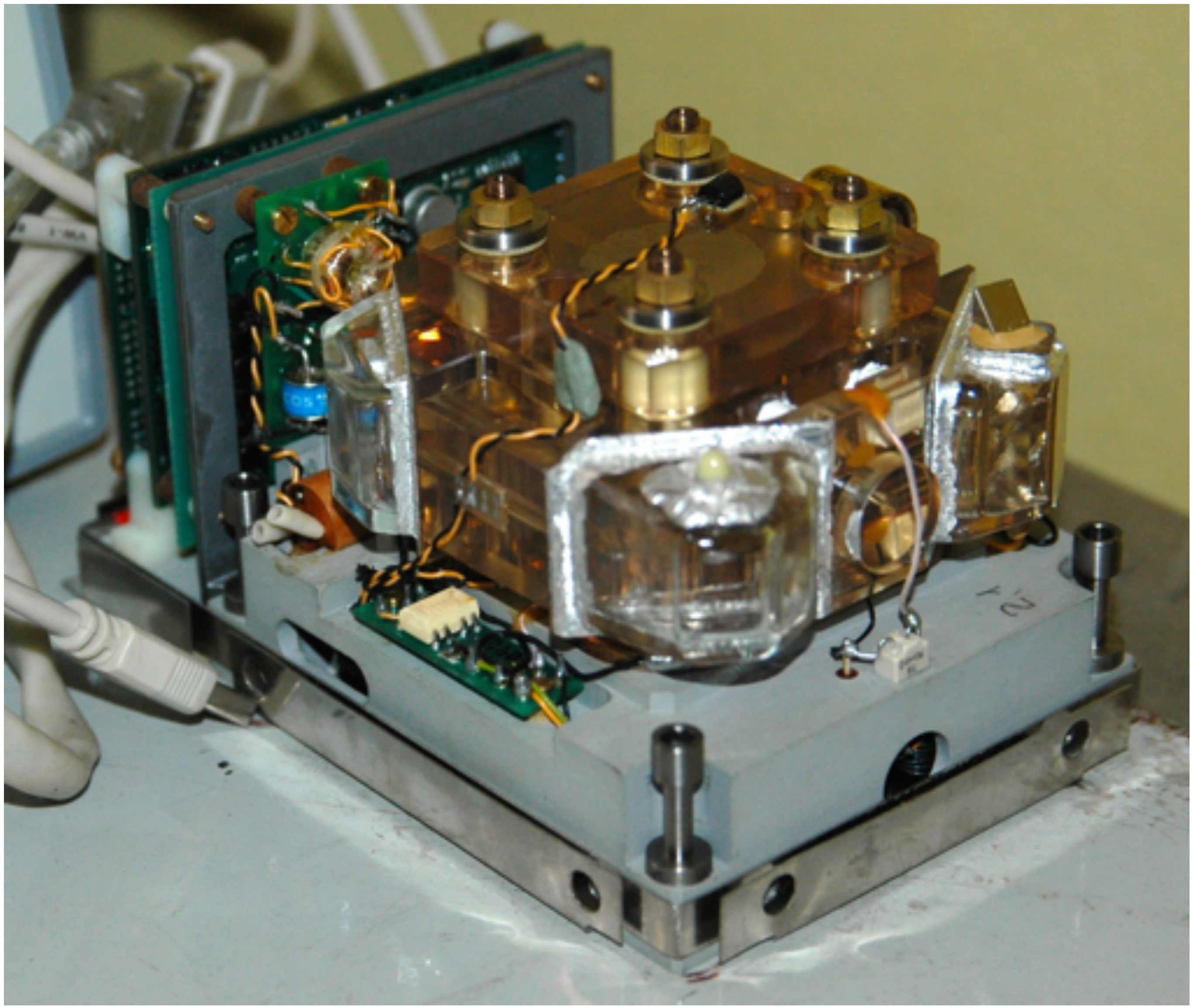
Filatov's goniometer

Goniometro laser





Gyrolaser a prismi



He-Ne Laser gyros

- Small gyroscopes for navigation
  - 3- or 4-mirror ring cavity design, with an optical path of the order of 10 – 40 cm
  - Large FSR (comparable with Doppler width)
    ➪ *robust transversal single-mode operation*
  - They need of a bias rotation or a dithering to avoid self-locking

- Large apparatus integral to the ground
  - Optical path larger than 4 m up to 100 m and more
  - Biased by Earth rotation
  - Small FSR ➪ *difficult single-mode operation*
  - Seismology, geodesy, length of the day, fundamental physics

Ring laser accuracy

$$f_{Sagnac} = 4\frac{A}{\lambda p}\left(1+k_A\right)\hat{n}\cdot\left(\vec{\Omega}+\vec{\Omega}_{\oplus}\right)+\Delta f_0+\Delta f_{bs}$$

$$= k_s\left(1+k_A\right)\left(\Omega\cos\theta+\Omega_{\oplus}\sin\gamma\right)+\Delta f_0+\Delta f_{bs}$$

geometrical factor (k_s)

atomic factor (k_A)

Earth rotation ($\Omega_{\oplus}$)

zero shift (non-reciprocal factor (Δf_0)

backscattering (Δf_{bs})

$$N_{\phi} = \frac{1}{2\pi}\int_{t_0}^{t_1} f(t)\,dt = \frac{1}{2\pi}K\bar{\Omega}+\delta_{\phi}$$

$$N_{2\pi} = \frac{1}{2\pi}\int_{t_0}^{t_0+T} f(t)\,dt = \frac{1}{2\pi}K\bar{\Omega}+\delta_{2\pi}$$

Linear term: Geometrical factor

$$f = k_s(1+k_A)(\Omega + \Omega_\oplus \sin\gamma) + \Delta f_0 + \Delta f_{bs}$$

- The geometrical factor $A/p\cos\vartheta$ must be kept stable better than 10^{-8} over the measuring time.

  - *Linear change smaller than 10^{-8}*
    - For a dilation coefficient of the structure ≈ 10^{-6}/K, a temperature stability of 10^{-2} K is required
  - *Wobble of the table lower than 10^{-4}*

Linear term: Atomic factor

$$f = k_s(1 + k_A)(\Omega + \Omega_\oplus \sin\gamma) + \Delta f_0 + \Delta f_{bs}$$

- The laser action is sensitive to changes of the discharge properties that can produce variation in the laser gain and in the plasma dispersion function.
  - Evolution of the discharge gas composition
  - Stability of the discharge power

➪ Control of the lab temperature, pressure, humidity

➪ Good stabilization of the discharge power and of the laser wavelength

➪ Control of wall outgassing through getter pumps

➪ *Can be quite easily controlled on measurement time ≲ 100 s*

Null-shift

$$f = k_s\left(1+k_A\right)\left(\Omega+\Omega_{\oplus}\sin\gamma\right)+\Delta f_0+\Delta f_{bs}$$

Non-reciprocity in the optical cavity

- *rf* discharge makes negligible asymmetries in the active medium.
- For an ideal ring laser the two counter-propagating laser beams would be identical both in size and in intensity.
- In practice, there have been found quite large differences in the intensities
- Large effect in total reflective prism cavities (*magnetic field effects due to the Verdet coefficient of the prism glass*)
- Also super-mirror cavities exhibit some minute effects (anisotropy in the reflection or birefringence in the super-mirror coating)

Null-shit measured in G-Pisa (1.35 m of side): 10^{-2} Hz

➪ *This null-shift effect can be modeled and taken in account.*

➪ *Diagnostic: relative intensity of the two beams*

Back-scattering

$$f = k_s\left(1+k_A\right)\left(\Omega+\Omega_{\oplus}\sin\gamma\right)+\Delta f_0+\Delta f_{bs}$$

Mirror backscattering couples together the two counter-propagating laser beams through a coupling parameter *r* : $r=\sum_i r_i \exp(2ikz)$

➪ *frequency pulling*: $f=\sqrt{f_0^2-l^2}$ for $f_0>l$

$l=r/\pi$ lock-in threshold frequency.

- By geometry $|r_i| \propto 1/L$
- Moreover, its effectiveness is directly related to the ratio between cavity line-width ($\propto 1/L$) and inversely related to the beams frequency difference ($\propto L$) .

r value can vary between $-\sum_i |r_i|$ and $\sum_i |r_i|$ for small perturbations in the ring geometry of the order of λ

If $\Omega \gg \Omega_L$ $\left(f_0 \gg l\right)$,

➪ $f=\sqrt{f_0^2-l^2}\approx f_0\left(1-l^2/f_0^2\right)=f_0-\dfrac{l^2}{K\Omega}$

$$l \approx \frac{cs\lambda}{\pi dp} \approx \frac{100}{p}\,\mathrm{Hz}$$

Accuracy

$$f = K_1\Omega + K_0 + \frac{K_{-1}}{\Omega}$$

Primalov and Filatov, Sov. J. QE **7** 802 (1977)
Yu.Filatov, *et al*. NATO RTO AG-339 (1999)

K_1 scale factor
K_0 null-shift
K_{-1} back scattering

For p = 2 m e Ω = 2π · 0.1 Hz:

$$K_1 = 4\frac{A}{\lambda p}\cos\theta \approx \frac{L}{\lambda} \approx 8\times 10^5$$

$$K_{-1} \approx \frac{l^2}{K_1^0} \approx \frac{(100\ \mathrm{Hz}/2)^2}{8\times 10^5} = 3\times 10^{-3}\mathrm{Hz}^2$$

$$f^0 \approx 350\ kHz$$

$$\delta f_{bs} \approx \frac{3\times 10^{-3}}{2\pi\cdot 10^{-1}}\mathrm{Hz} \approx 5\times 10^{-3}\mathrm{Hz}$$

$$\frac{\delta f_{bs}}{f_0} \sim \frac{5\times 10^{-3}}{350\times 10^3} \approx 1.5\times 10^{-8}$$

Earth rotation effect

$$f = K_1\left(\Omega + \Omega_\oplus \sin\gamma\right) + K_0 + \frac{K_{-1}}{\left(\Omega + \Omega_\oplus \sin\gamma\right)}$$

Correction in K_{-1} is a small correction of a small term
Negligible!

$\Omega_\oplus$ is well known with high accuracy, but if Ω is not uniform:

$$\delta N_\phi = \frac{1}{2\pi}\left(K_1^0 \frac{\left(\Omega_\oplus \sin\gamma + K_0\right)}{\Omega^0} + \frac{K_0^0}{\Omega^0} + 2\frac{K_{-1}^0}{\left(\Omega^0\right)^2}\right)\int_t^{t+\Delta t} \delta\Omega(t')\,dt'$$

$$= \frac{1}{2\pi}\left(K_1^0 \frac{\Omega_\oplus \sin\gamma}{\Omega^0} + \frac{K_0^0}{\Omega^0} + 2\frac{K_{-1}^0}{\left(\Omega^0\right)^2}\right)\langle\delta\Omega\rangle\frac{\phi}{\Omega^0}$$

First term is the dominant one:

With L=50 cm, $K_1 \Omega_\oplus \sin\gamma = 40$ Hz, while $K_0 \ll 1$ Hz

$$\delta\phi = 2\pi\frac{\delta N_\phi}{N_{2\pi}} \approx \frac{\Omega_\oplus \sin\gamma}{\Omega^0}\frac{\langle\delta\Omega\rangle}{\Omega^0}\phi$$

For $\Omega_0 = 2\pi \cdot 0.1$ Hz , an angular accuracy of 10^{-8} rad (0.002”) requires $\langle\delta\Omega\rangle/\Omega < 10^{-4}$

Quantum noise

$$\delta\Omega_{shot\ noise} = \frac{c(4L)}{4L^2 Q}\sqrt{\frac{h\nu}{P\tau}} = \frac{c}{LQ}\sqrt{\frac{h\nu}{P\tau}}$$

P output power
τ measurement time
Q optical cavity quality factor

$$\delta\phi_{shot\ noise} = \frac{c}{LQ}\sqrt{\frac{h\nu\,\tau}{P}} \approx 12\ \text{nrad}\ (0.0025'')$$

L = 50 cm
τ = 5 s
Q = 6,20×10^{11}
(*cavity losses ≈ 32 ppm*)
P = 10 nW

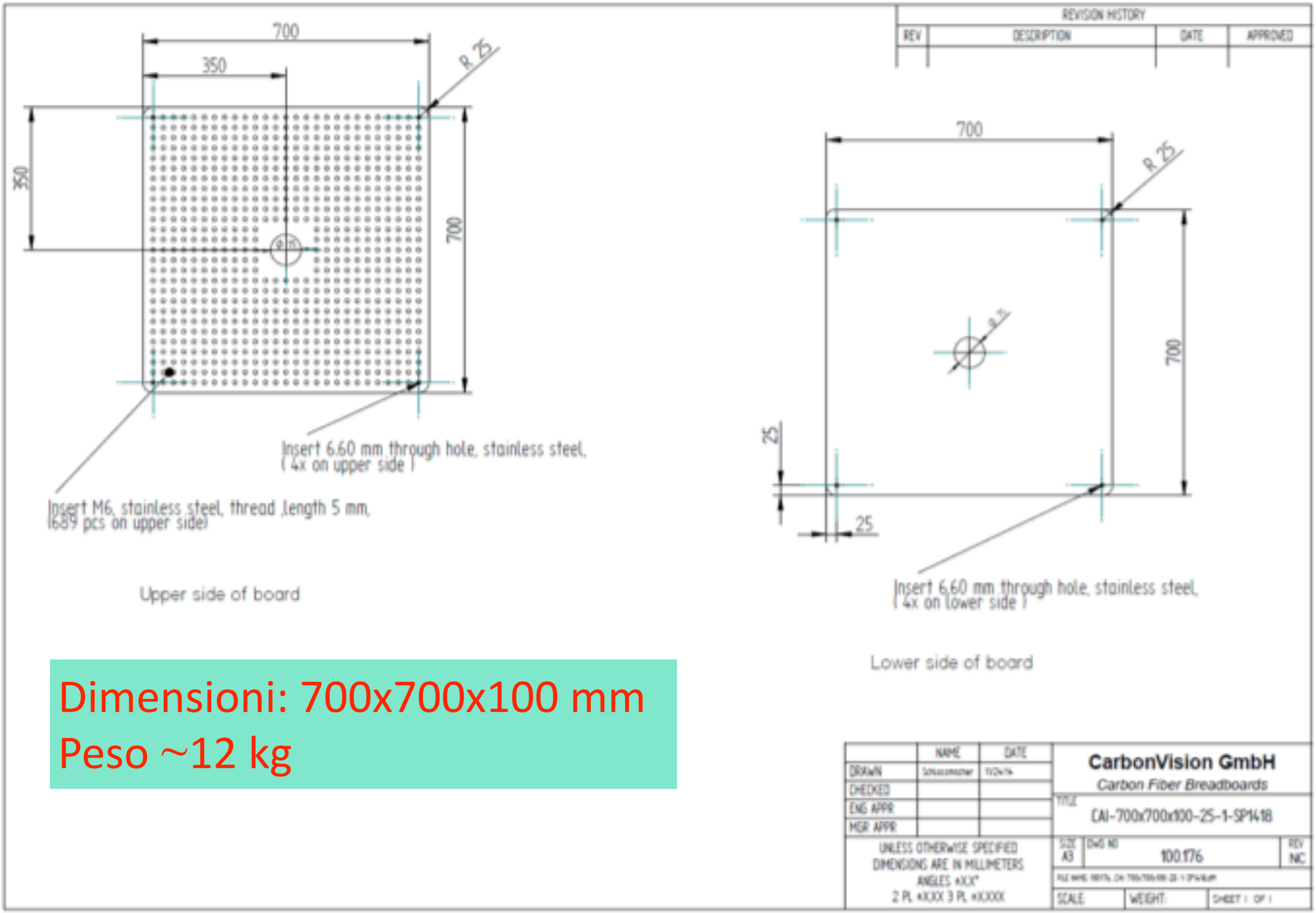
AEROTECH®
Dedicated to the Science of Motion

ABRS Air-Bearing Direct-Drive Rotary Stage

Resolution[1]		0.174 µrad (0.036 arc sec)
Fundamental Encoder Resolution		18,000 lines/rev
Max Speed[2]		500 rpm
Accuracy[3]		<1 arc sec
Bidirectional Repeatability		<1 arc sec
Max Load[4]	Axial	97 kg
	Radial	51 kg
	Tilt	45 N-m
Axial Error Motion (Synchronous)		<100 nm
Radial Error Motion (Synchronous)		<250 nm
Tilt Error Motion (Synchronous)		<2.4 µrad (<0.5 arc sec)
Axial Error Motion (Asynchronous)		<20 nm
Radial Error Motion (Asynchronous)		<20 nm
Tilt Error Motion (Asynchronous)		<0.2 µrad (<0.04 arc sec)

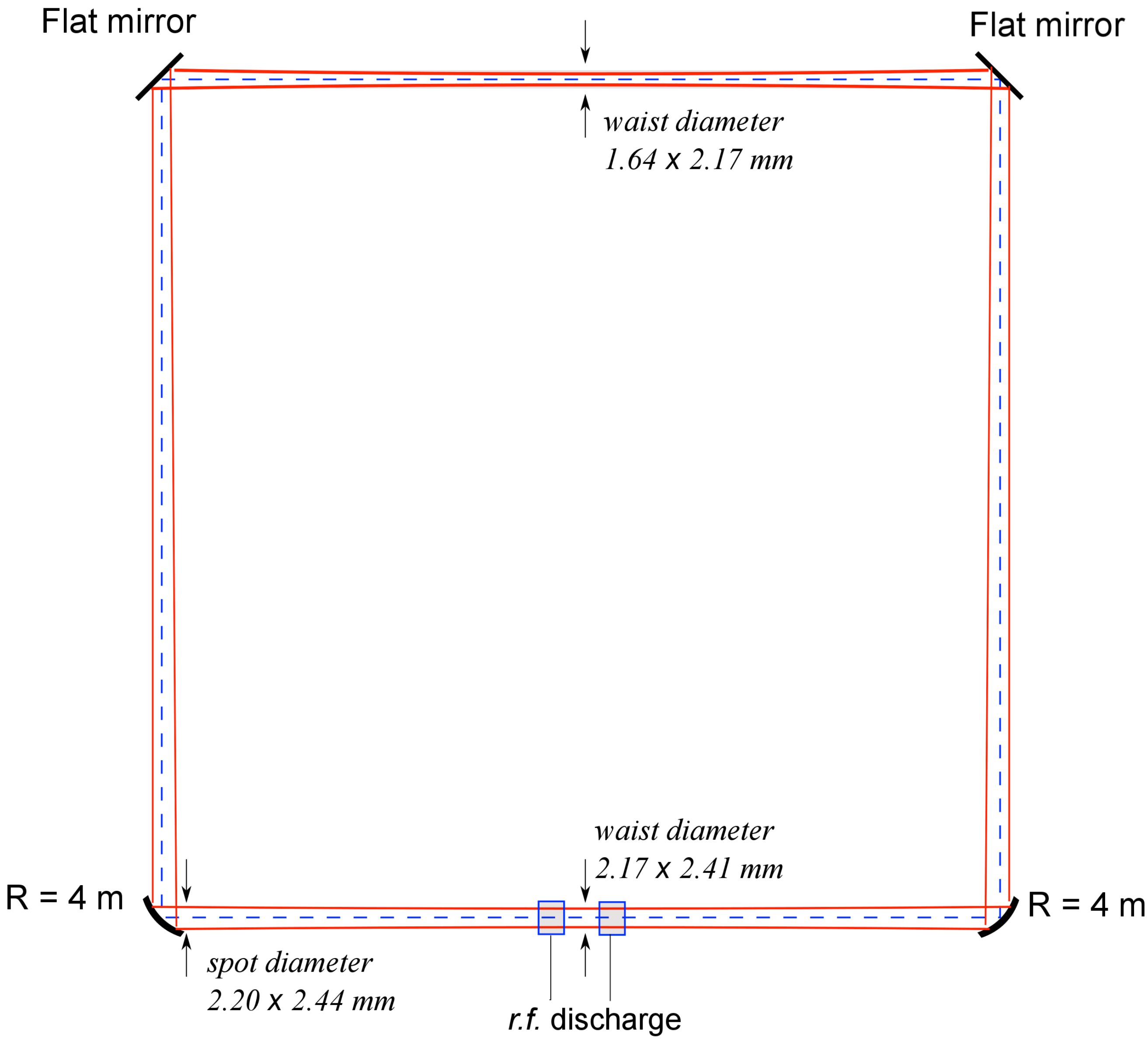
Width	300 mm
Tabletop Diameter	278.1 mm
Height	110 mm

Basamento in fibra di carbonio

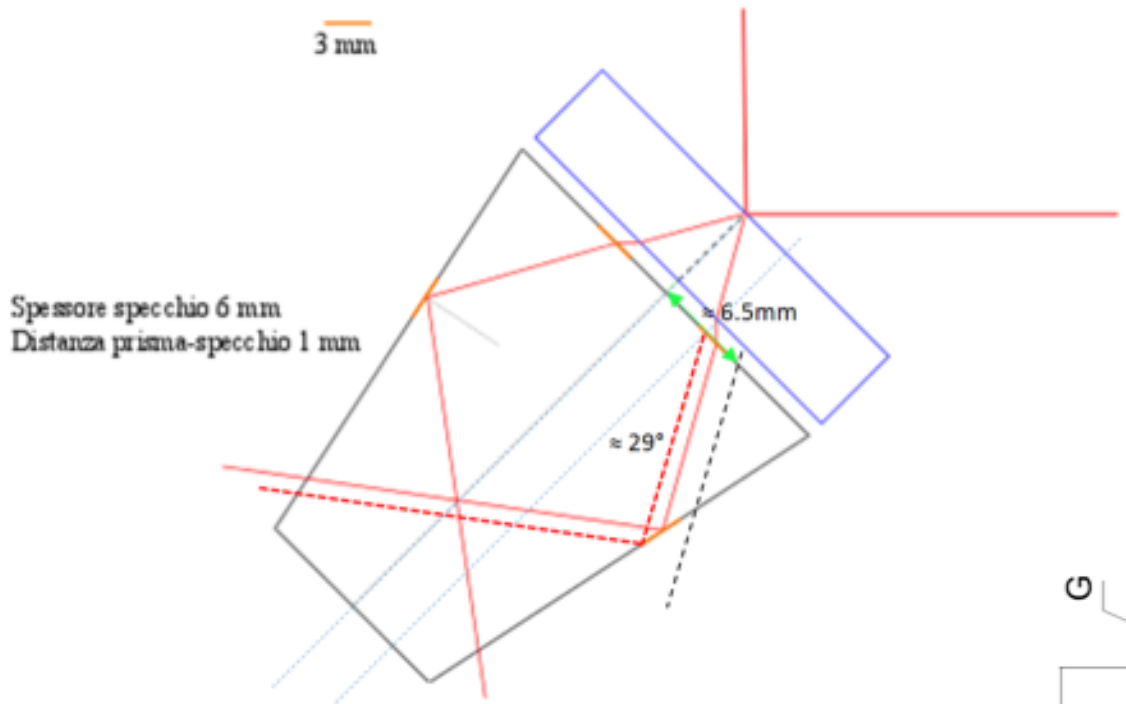


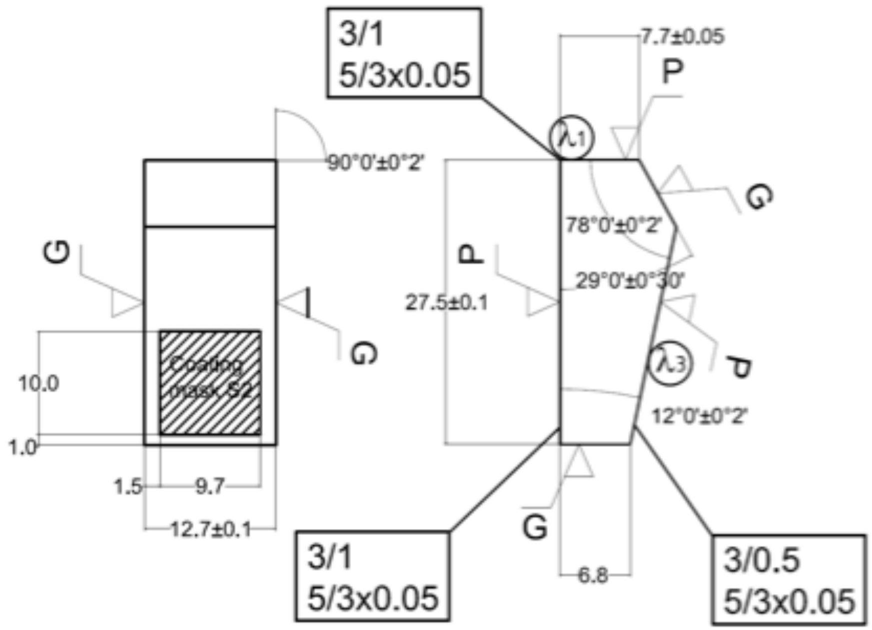
Dimensioni: 700x700x100 mm
Peso ~12 kg

Optical cavity design



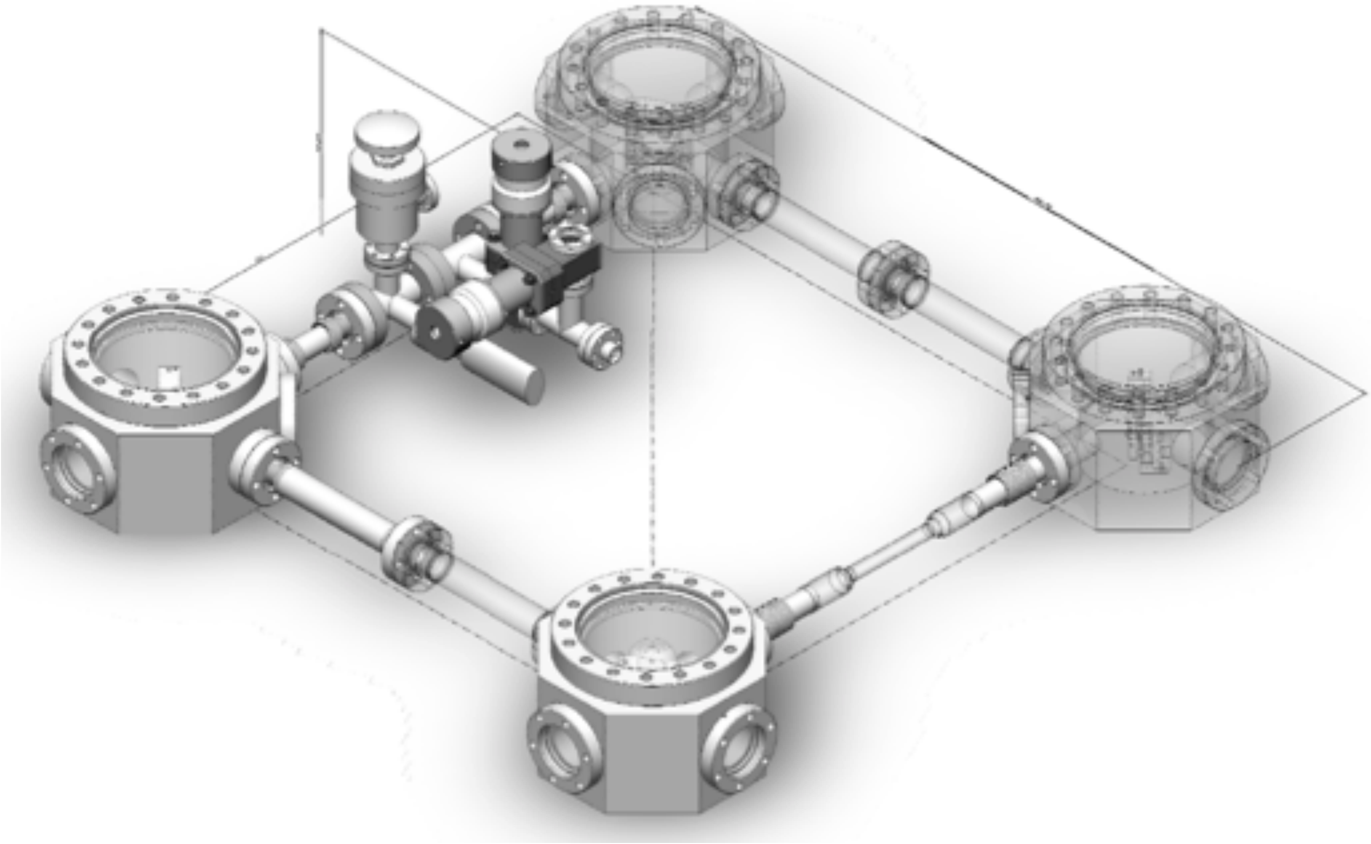
Prisma di ricombinazione

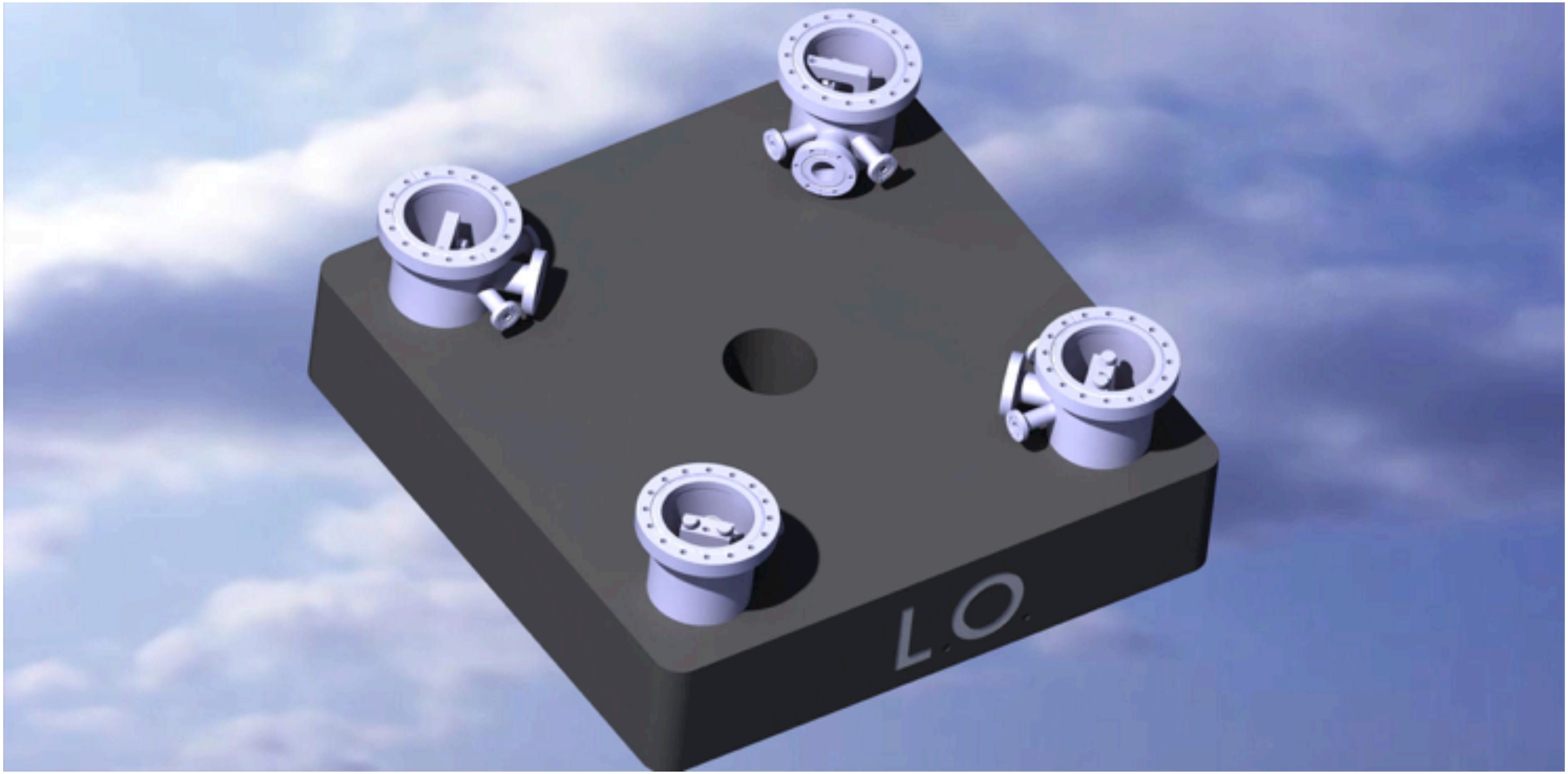


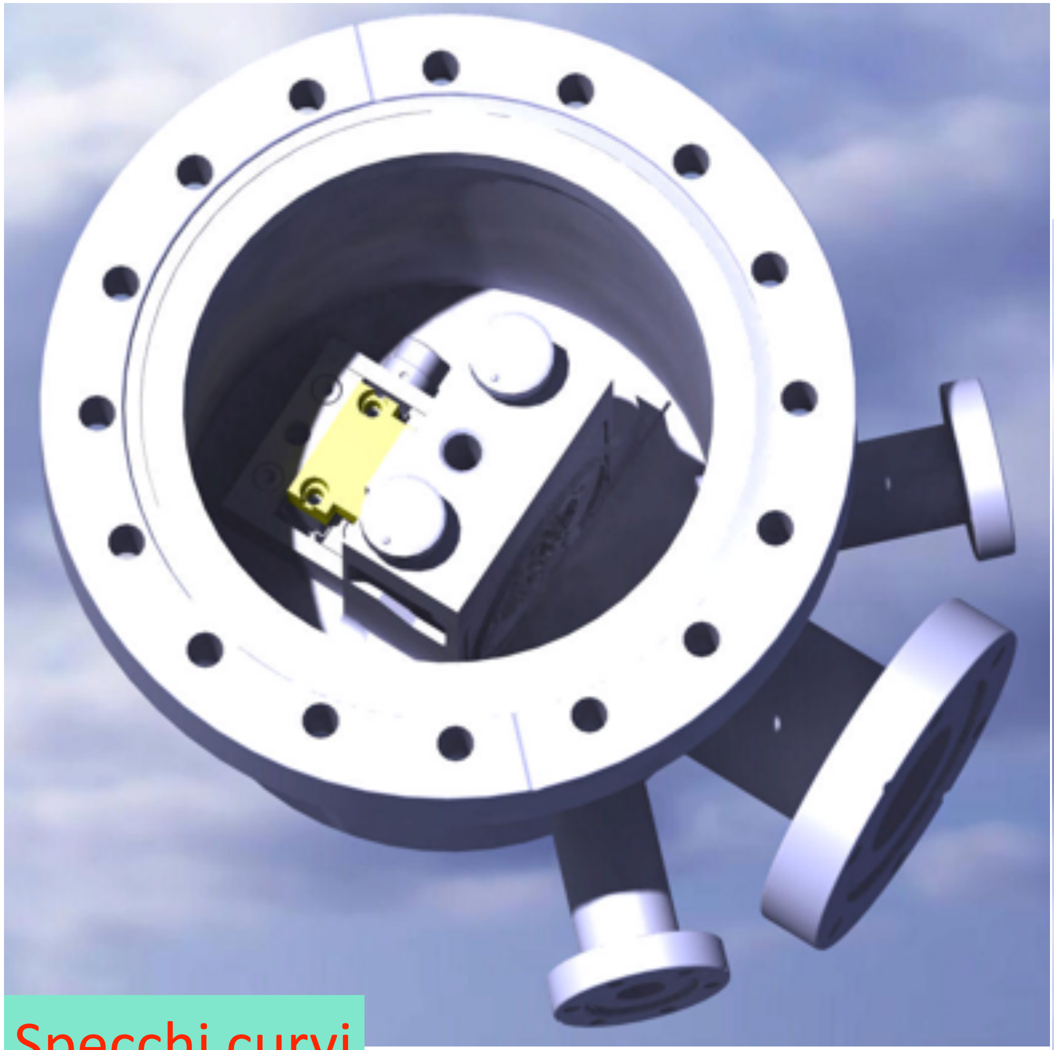


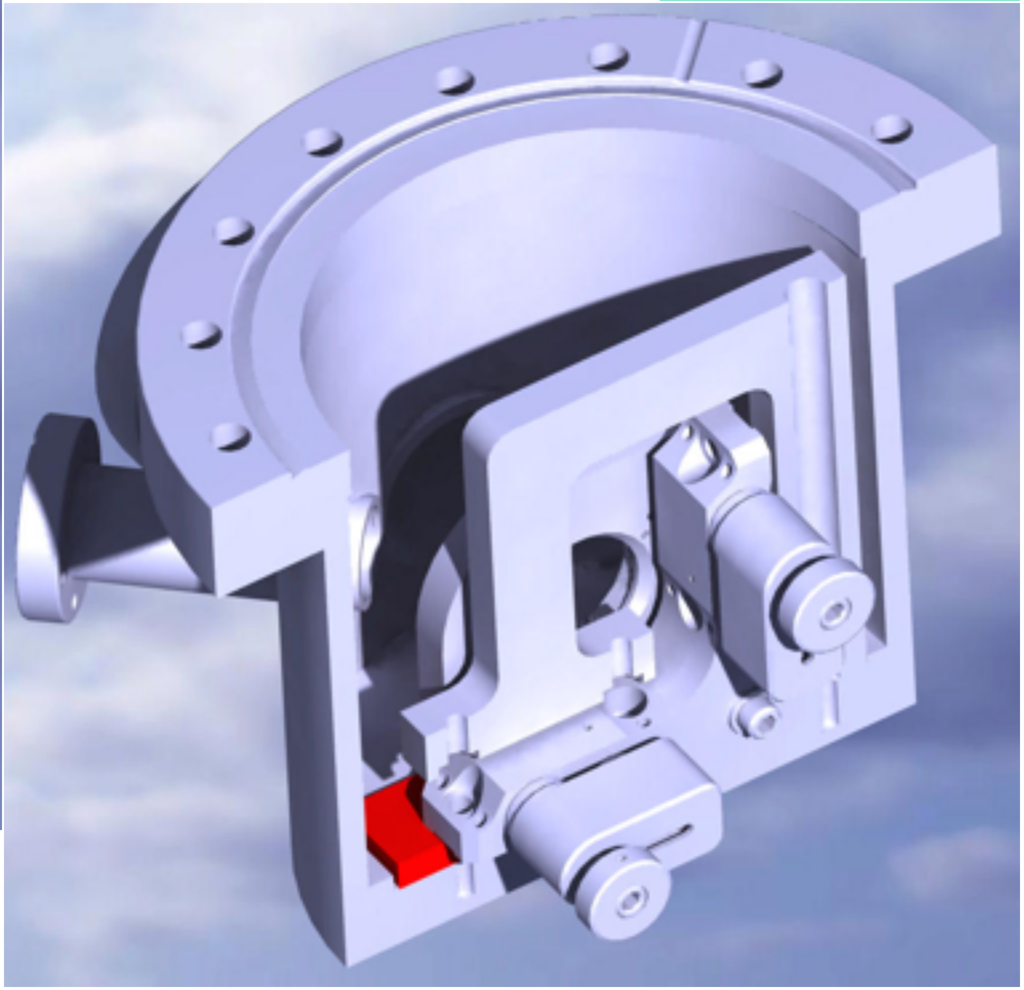
L m	*f* @ .1 Hz kHz	*f Earth** Hz	FSR MHz	optical cavity Q^	q. noise nrad $\sqrt{\tau}$$	$\Delta\phi$ *b. s.* nrad
0,11	77,3	9,0	682	$1{,}36\times10^{11}$	268	8650
0,50	351,3	40,8	150	$6{,}20\times10^{11}$	12,1	20
0,90	632,4	73,4	83	$1{,}12\times10^{12}$	5,42	1,9
1,35	948,6	110,1	56	$1{,}68\times10^{12}$	1,65	0,04

* Calculated for the latitude of Turin (45°04')
^ Estimated total cavity losses 32 *ppm*
$ $\tau = 5$ s




L.O.


Specchi curvi


Specchi piani