Modeling, estimation and control of ring laser gyroscopes for the accurate estimate of the Earth rotation

Davide Cuccato, DEI-INFN. November, 18th 2014.

DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

Presentation Outline

- **Introduction**

- **Ring Laser Dynamics:**
  1. Model
  2. Rotational frequency estimation
  3. Results

- **Optical Cavity Geometry:**
  1. Beams position computation
  2. Pose & Shape decomposition
  3. Simulation results

- **RLG simulator:**
  1. Simulator overview
  2. GP2 case study

- **Conclusions**

Introduction

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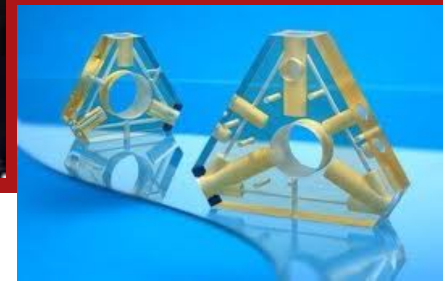
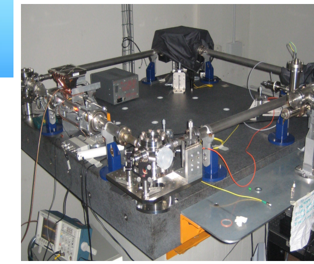
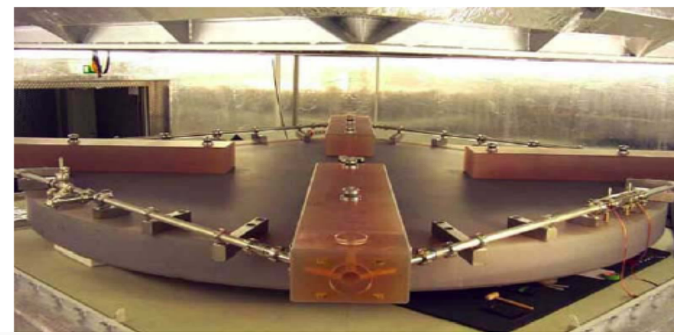
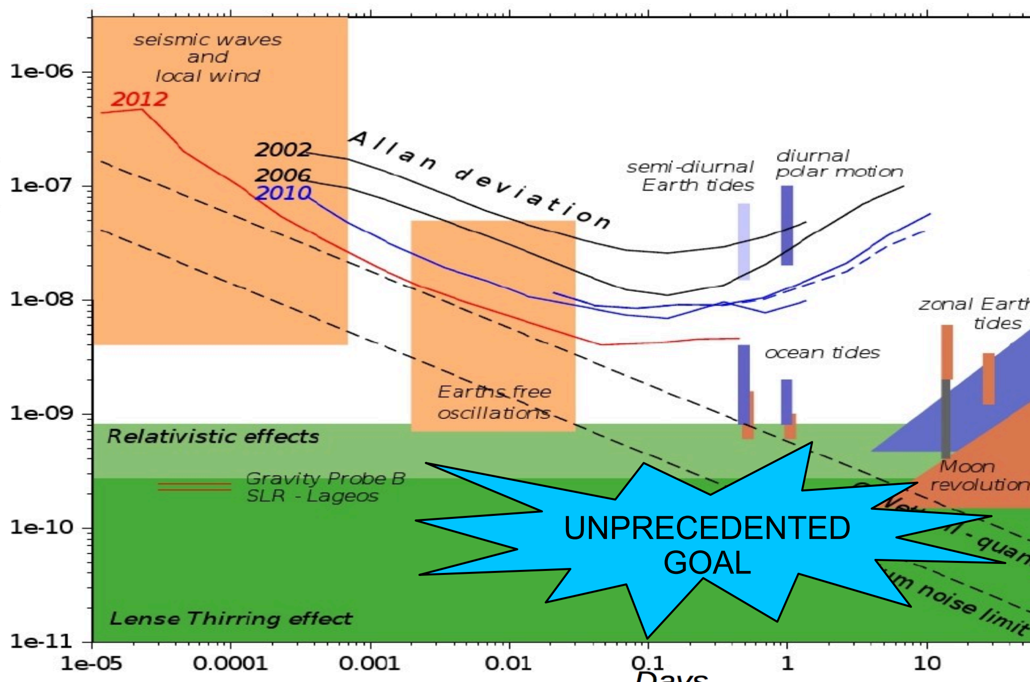
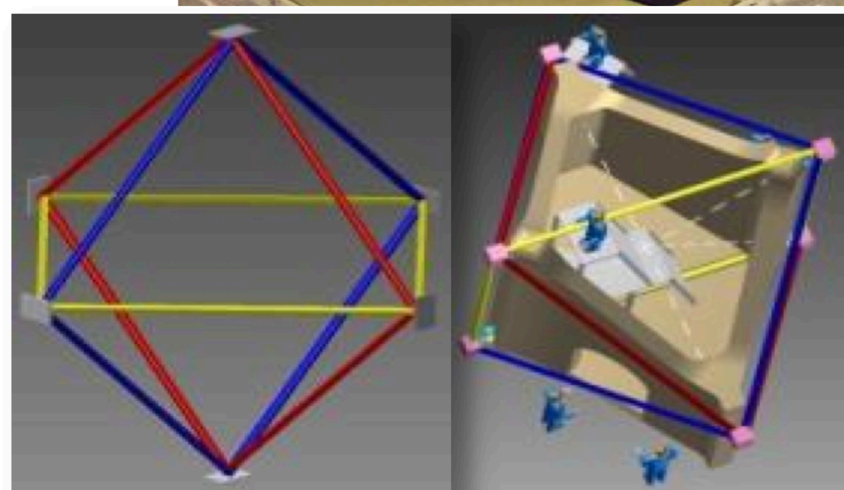




- Small Size: (5-50 cm) Inertial Guidance
- Medium Size: (1-5 m) Geophysics, Seismology, Metrology
- Large Size: (5-10 m) Geodesy, Geophysics
- GINGER: Gyroscope IN GEneral Relativity







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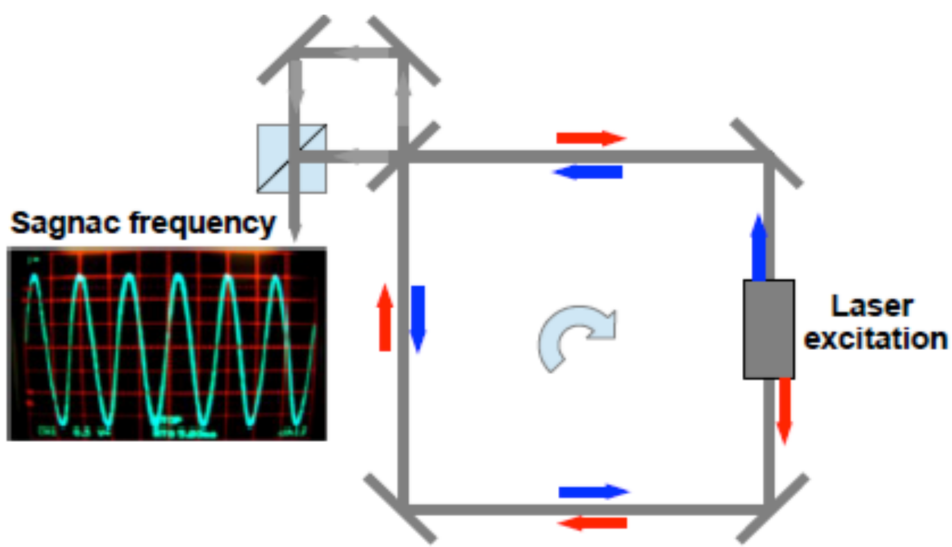
Introduction: Active Sagnac Interferometry

- Laser light excited inside a polygonal optical cavity
- Two electromagnetic waves travelling in opposite directions
- The frequency split between opposite travelling waves is mainly due to rotation.

$$\nu_s = \left(\frac{4A}{\lambda L} + \Delta\nu_{SF} \right) \mathbf{n} \cdot \boldsymbol{\Omega} + \Delta\nu_0 + \Delta\nu_{BS} + \eta$$

Optical cavity geometry

Laser dynamics



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Ring Laser dynamics: Model

$$\dot{E}_1 = (\alpha_1 + i\omega_s) E_1 + r_2 e^{i\epsilon} E_2 - f_1(I_1, I_2) E_1$$

$$\dot{E}_2 = (\alpha_2 - i\omega_s) E_2 + r_1 e^{i\epsilon} E_1 - f_2(I_1, I_2) E_2$$

Where

$$\begin{cases} I_{1,2} = |E_{1,2}^2| \quad S = |E_1 + E_2|^2 \\ f_{1,2}(I_1, I_2) = \beta I_{1,2} + (\theta + i\tau) I_{2,1} \end{cases}$$

$E_{1,2}(t)$ are linearly coupled through $r_{1,2}$ ϵ

And non-linearly coupled through θ τ

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Ring Laser dynamics: Model

The Ring Laser dynamics in compact from:

$$\dot{\mathbf{E}} = \left[\mathbf{A} - \mathscr{D}(\mathbf{E}).\mathbf{B}.\mathscr{D}(\mathbf{E}^{*})\right]\mathbf{E}$$

The matrices $\mathbf{A} \equiv \frac{c}{L}\mathbf{P}^{(0)} - \mathbf{M}$ and $\mathbf{B} \equiv \frac{c}{L}\mathbf{P}^{(2)}$ are given by

- **Atomic Polarization:**
  - Active medium= He-Ne isotopic mixture.
  - Non Linear Coupling.
  - Can be computed using QED.
- **Dissipative Effects:**
  - Related to cavity mirrors transmission,absorption and scattering.
  - Linear Coupling.
  - Sagnac effect.

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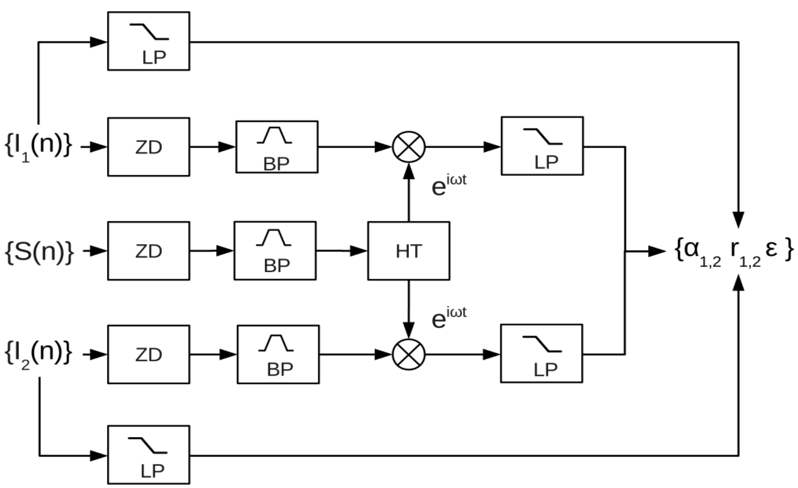
Ring Laser dynamics: Rotational frequency estimation

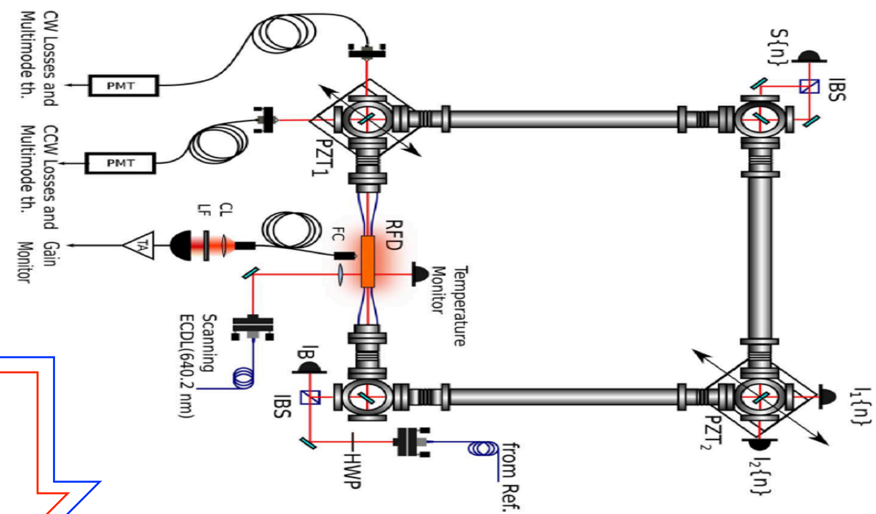
Dissipative parameters identification:

- Perturbative solutions of the RL dynamics
- Pre-filtering for electronic noise rejection



Active parameters estimation:

- Polarization computation with *Lamb* model
- Additional spectroscopic diagnostics



Extended Kalman Filter for laser systematic subtraction and rotational frequency estimation.

Ring Laser dynamics: Results

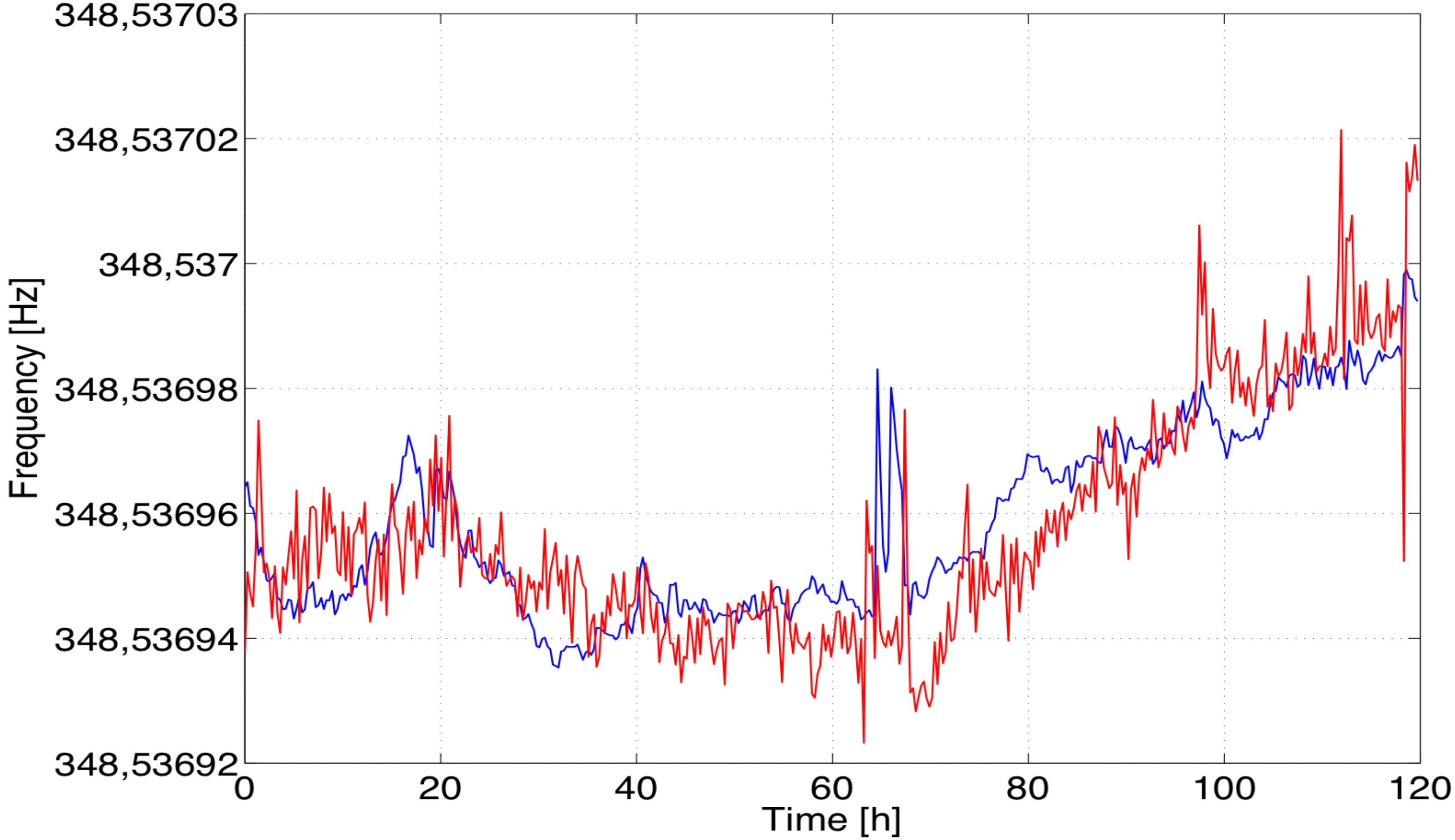


Fig. 5 Comparison of the backscattering (blue) estimated from the intensity channels with the residuals of the Sagnac frequency (red) estimated from the interferogram channel. Note that they correlate on the micro-hertz scale.

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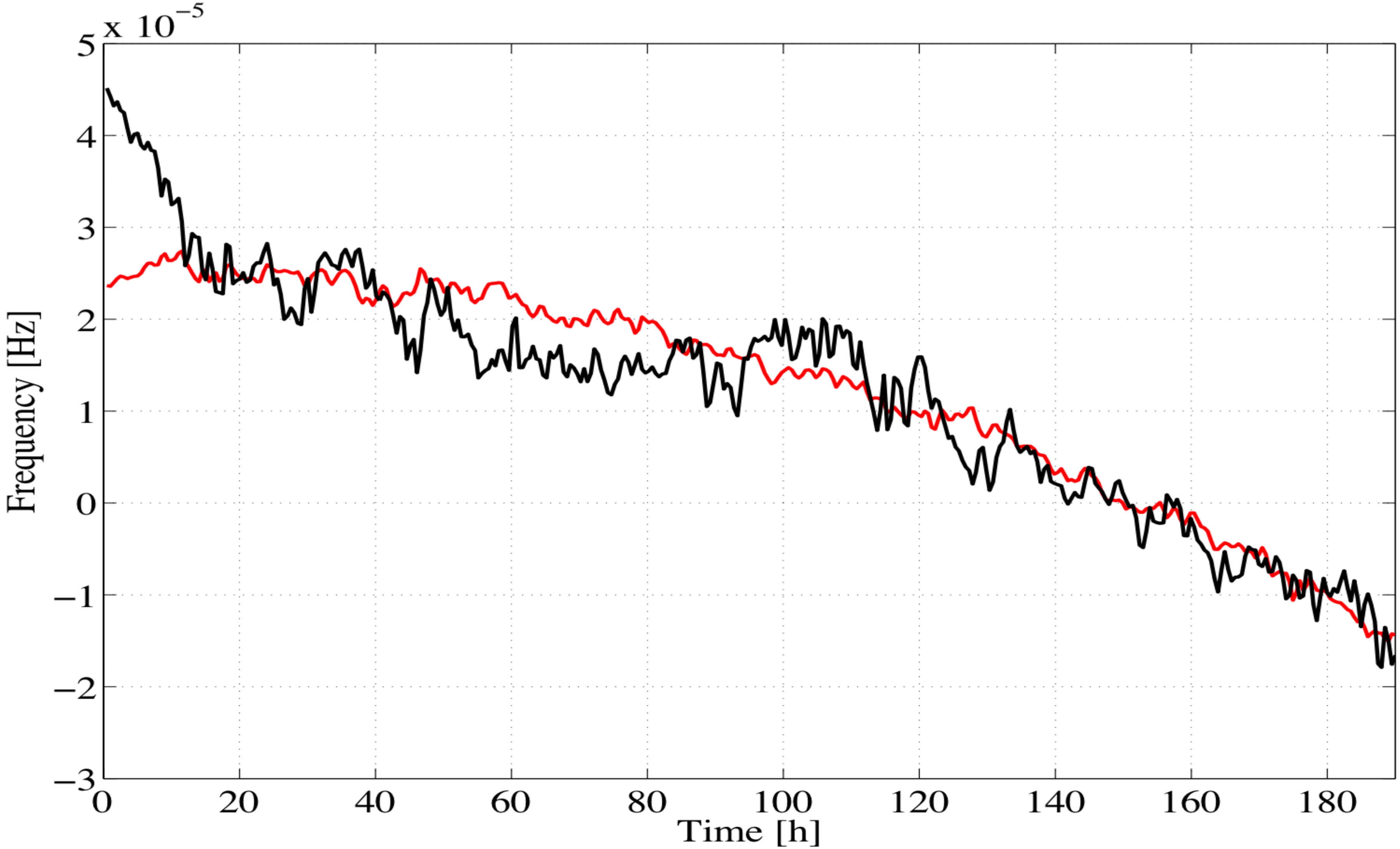
Ring Laser dynamics: Results



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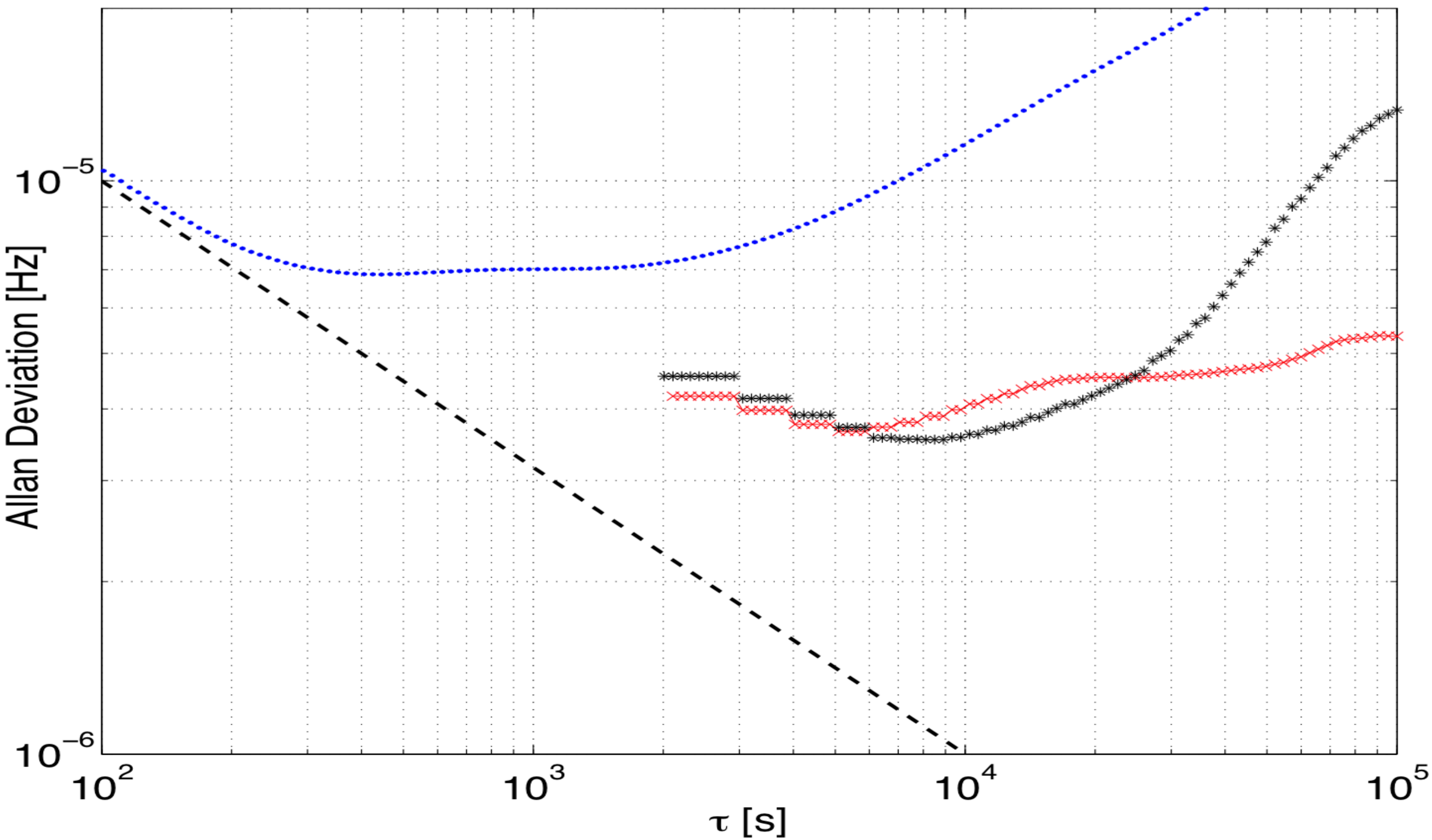
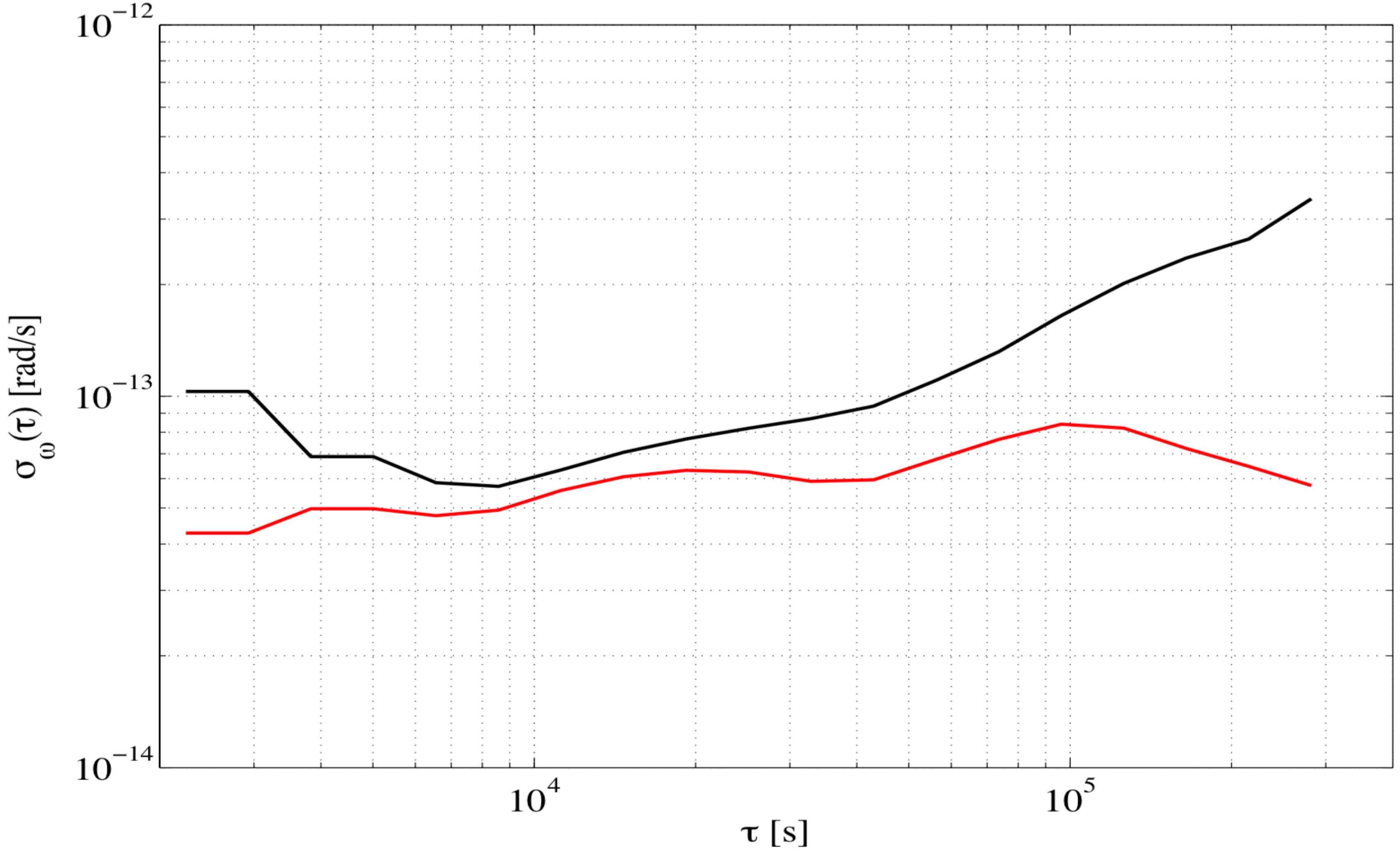
Ring Laser dynamics: Results



Ring Laser dynamics: Results



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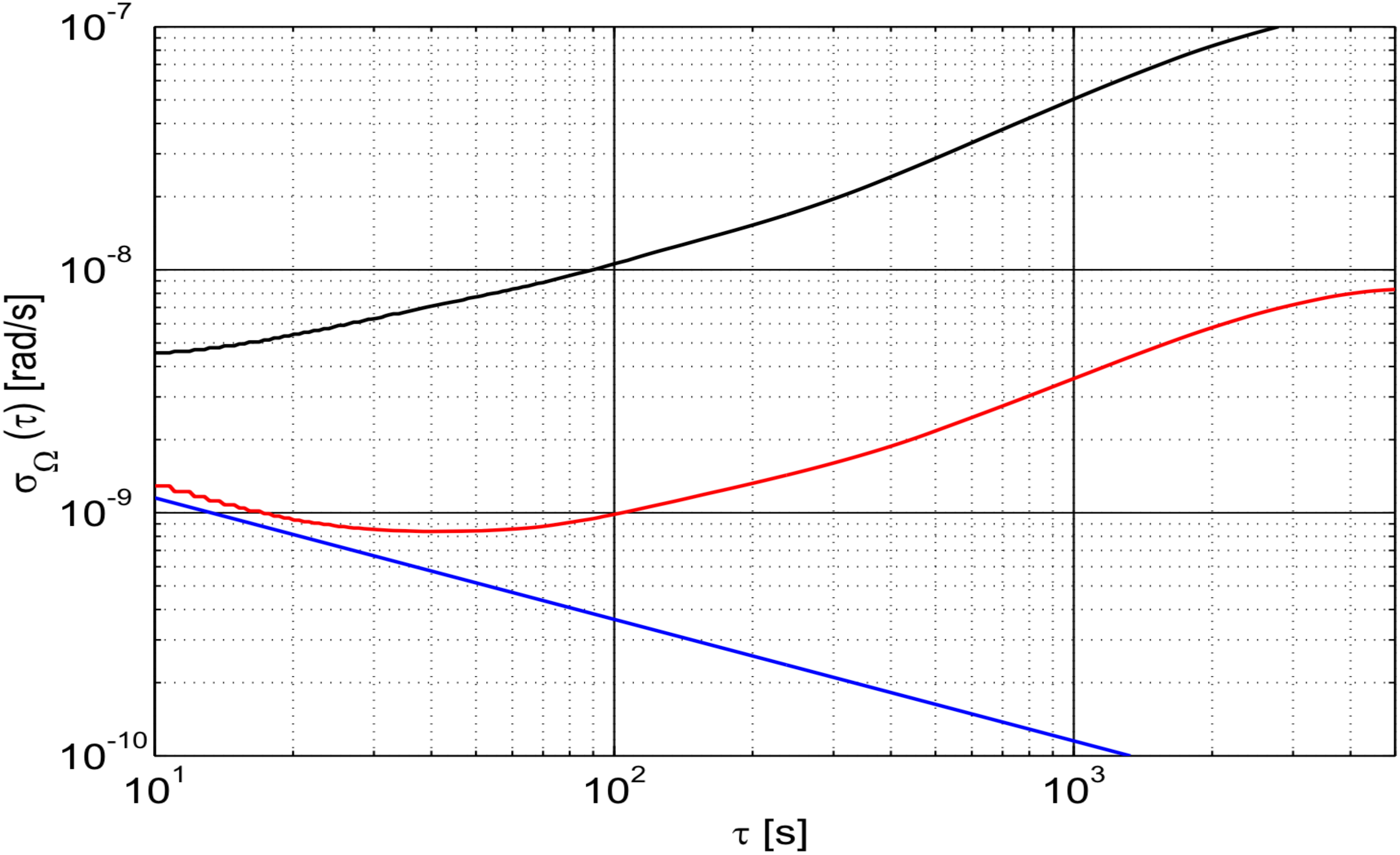
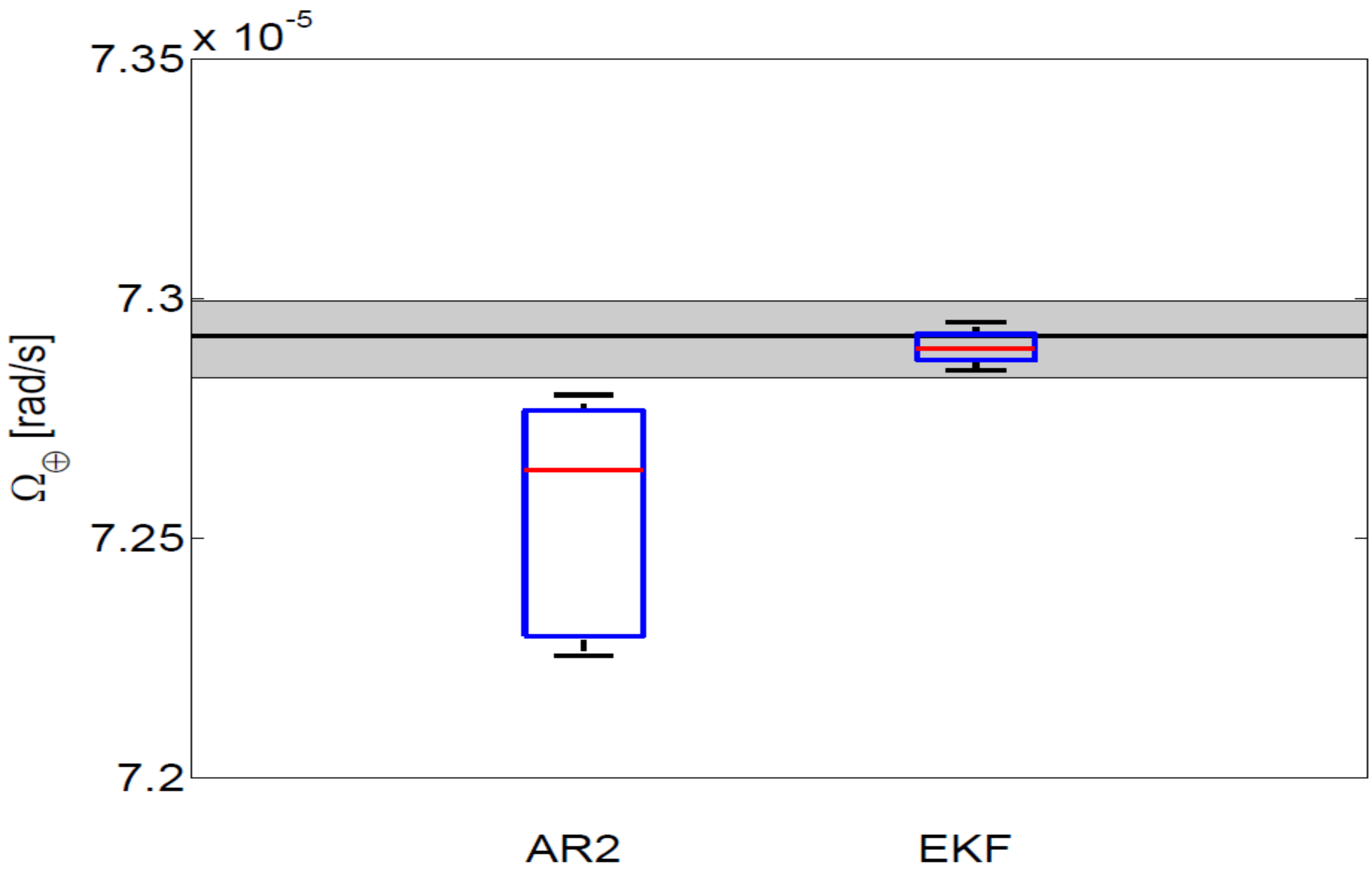
Ring Laser dynamics: Results



Ring Laser dynamics: Results



Optical Cavity geometry: Beams position computation

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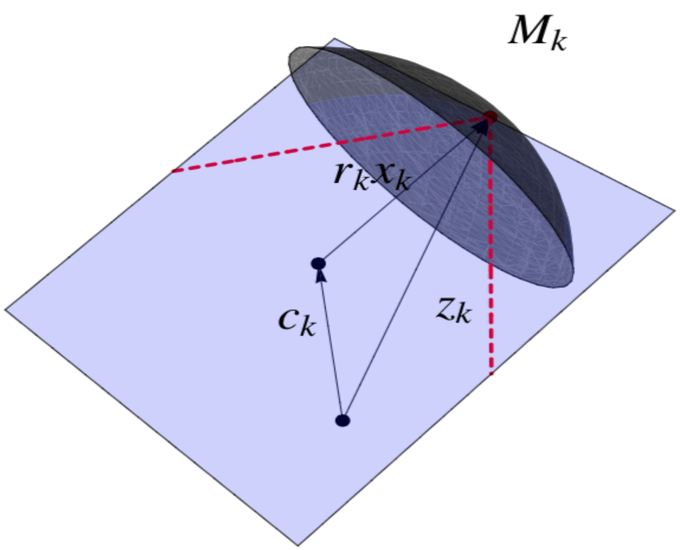
Task: Find the laser beams position for a given cavity configuration



Formalism: Geometric Optics

Problem Data:
- 4 Points in $\mathbb{R}^3$, The spherical mirrors C.O.C. $\mathbf{c}_k$
- 4 positive scalars in $\mathbb{R}$, the spherical mirrors R.O.C. r_k

Problem variables:
- 4 Points on the Unit Sphere $\mathbb{S}^2$, i.e. a point of the Oblique Manifold 2x4.

$$X = (\mathbf{x}_1, \ldots, \mathbf{x}_4) \in \mathcal{OB}(2,4)$$

To find the beams position, the Fermat's Principle (stationarity of the optical path length) is used:

$$\text{grad } p(X; C, R) = 0$$

Laser spot virtual positions: $\mathbf{z}_k = \mathbf{c}_k + r_k \mathbf{x}_k$

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Optical Cavity geometry: beams position computation

Geometric Newton Equation

$$\begin{cases} \text{Hess}\, f(x)[\eta_x] = & -\text{grad}\, f(x) \\ \eta_x \in T_x\mathcal{M} \end{cases}$$

Retraction

$$R : T_x\mathcal{M} \to \mathcal{M},$$

$$R(0_{T_x\mathcal{M}}) = x$$

$$DR(0_{T_x\mathcal{M}})[\xi_x] = \xi_x$$

Algorithm 1 *Geometric Newton Alghoritm.*

Input: $x_0 \in \mathcal{M}$, real valued function f on $\mathcal{M}$, Armijo parameters (α, β, σ), iterations maximum number n, target value ε for the gradient norm.
Output: Sequence of iterates $x_1, \ldots x_n$

1. *While* $k < n$ *or* $\|\text{grad}\, f(x_k)\| > \varepsilon$
2. Solve (4.3) in η_{x_k}.
3. Set $x_{k+1} = R_x(t_k \eta_{x_k})$, where t_k is the Armijo step size for the function h for given (α, β, σ).
4. *end While.*

Armijo line search

Algorithm 2 Armijo backtracking search procedure.

Input: $x \in \mathcal{M}$, real valued function h on $\mathcal{M}$, $\eta_x \in T_x\mathcal{M}$, Armijo parameters $\alpha > 0$, $(\beta, \sigma) \in (0, 1)$, iterations maximum number n.
Output: $y \in \mathcal{M}$
While $k < n$ *or*

$$h(x) - h\left(R(\alpha\beta^k\eta_x)\right) < -\sigma Dh(x)[\alpha\beta^k\eta_x] \quad (4.10)$$

$k = k + 1$.
end While.

Riemannian Gradient

$$\forall x \in \mathcal{M},\ \text{grad}\ f(x) = P_x\left(\partial f(\bar{x})\right)$$

Riemannian Hessian

$$\forall x \in \mathcal{M},\ \text{Hess}\, f(x)[\eta] = P_x\left(D\,\text{grad}\,\bar{f}(x)[\eta]\right)$$

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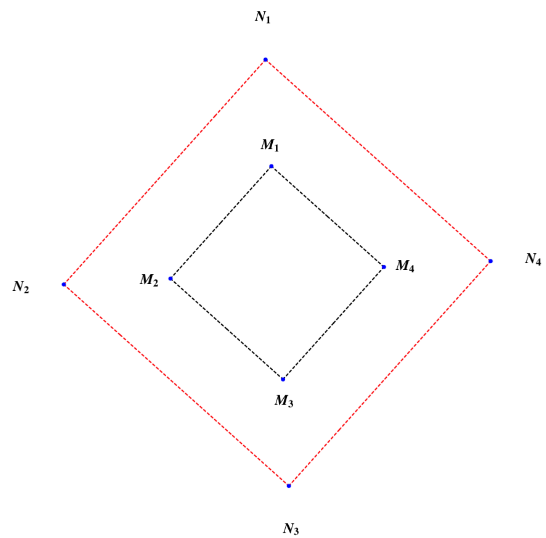
Optical Cavity geometry: Pose & Shape Decomposition

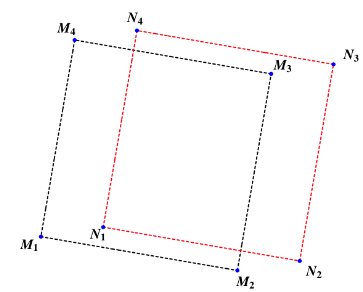
The matrix C accounts for both the pose and the shape of the mirrors

The optical cavity deformations are only induced by shape changes

Shape change



Pose change



Pose & Shape Theorem:

Regularity hypotesis on mirrors centers:

$$\mathcal{P} = \left\{ M \in \mathbb{R}^{3\times 4} | \, M_i \wedge M_{i+1} \neq 0, \, \overline{M} = 0_{3\times 1} \right\}$$

Decomposition:

$$\mathcal{P} = SO(3) \times \bar{\mathcal{T}} \times \mathcal{V}$$

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Optical Cavity geometry: Pose & Shape Decomposition

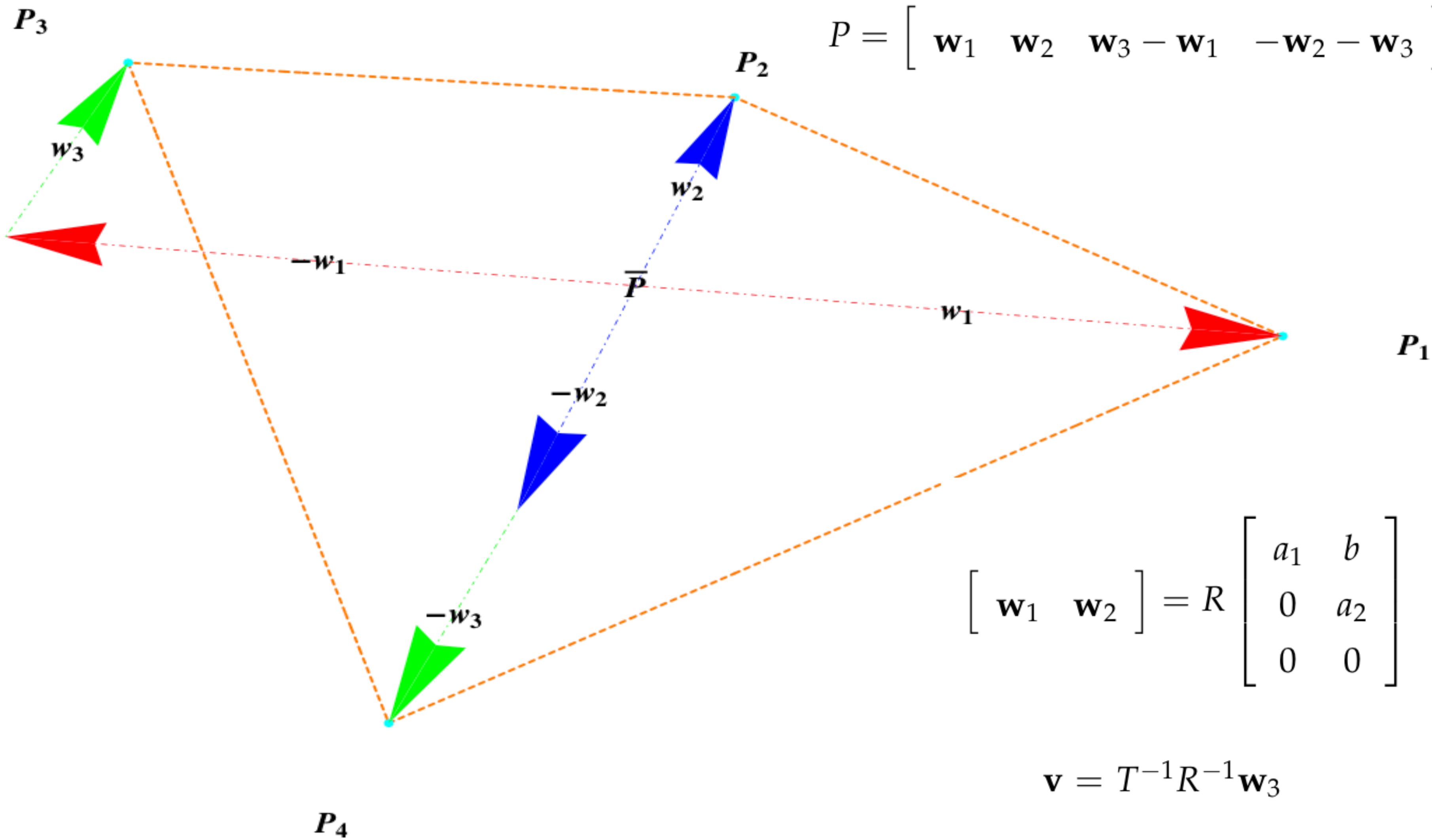
The isosceles trapezoids:

$$\mathcal{T} = \left\{ \begin{bmatrix} a_1 & b & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}, a_1, a_2 \in \mathbb{R}^+, b \in \mathbb{R} \right\}$$

The irregular quadrilaterals:

$$\mathcal{V} = \mathbb{R}^3 \setminus \left\{ \mathrm{e}_1 + \alpha \mathrm{e}_2, -\mathrm{e}_2 + \beta \mathrm{e}_1, \frac{\mathrm{e}_1 - \mathrm{e}_2}{2} + \gamma \left(\mathrm{e}_1 + \mathrm{e}_2 \right), \alpha, \beta, \gamma \in \mathbb{R} \right\},$$

Optical Cavity geometry: Pose & Shape Decomposition



$$P = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 - \mathbf{w}_1 & -\mathbf{w}_2 - \mathbf{w}_3 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 \end{bmatrix} = R \begin{bmatrix} a_1 & b \\ 0 & a_2 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{v} = T^{-1} R^{-1} \mathbf{w}_3$$

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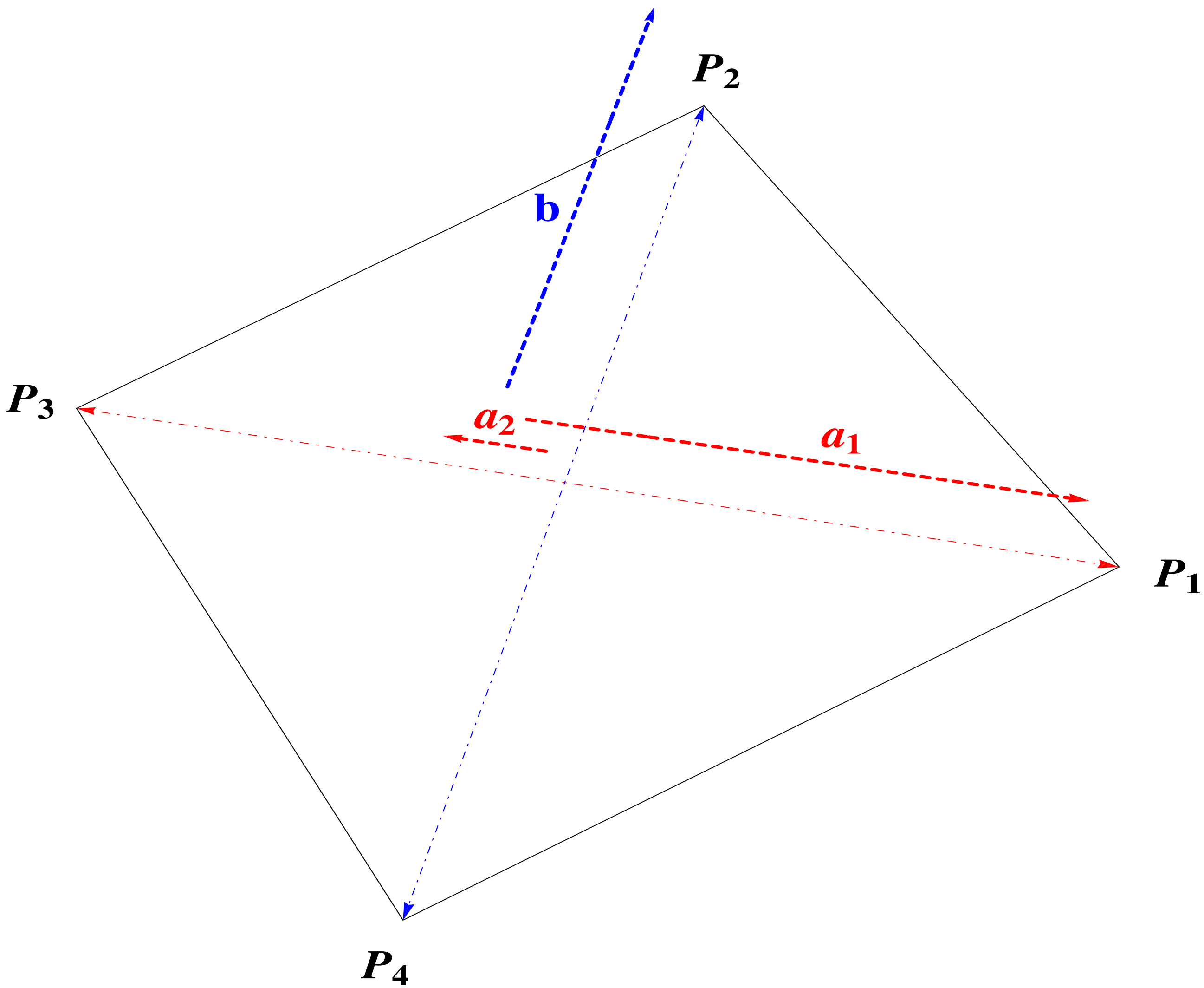
Optical Cavity geometry: Pose & Shape Decomposition

$\mathcal{T}$



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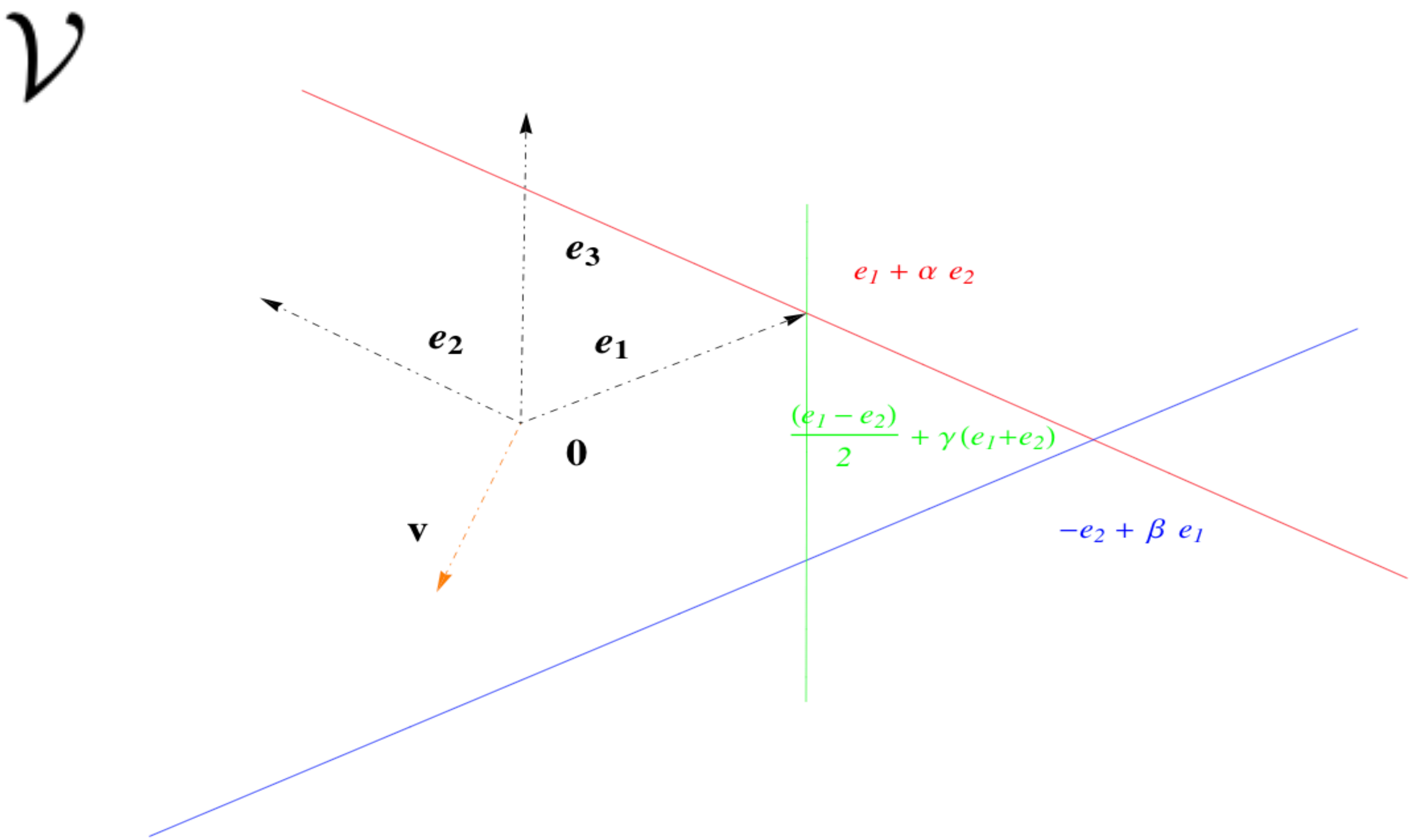
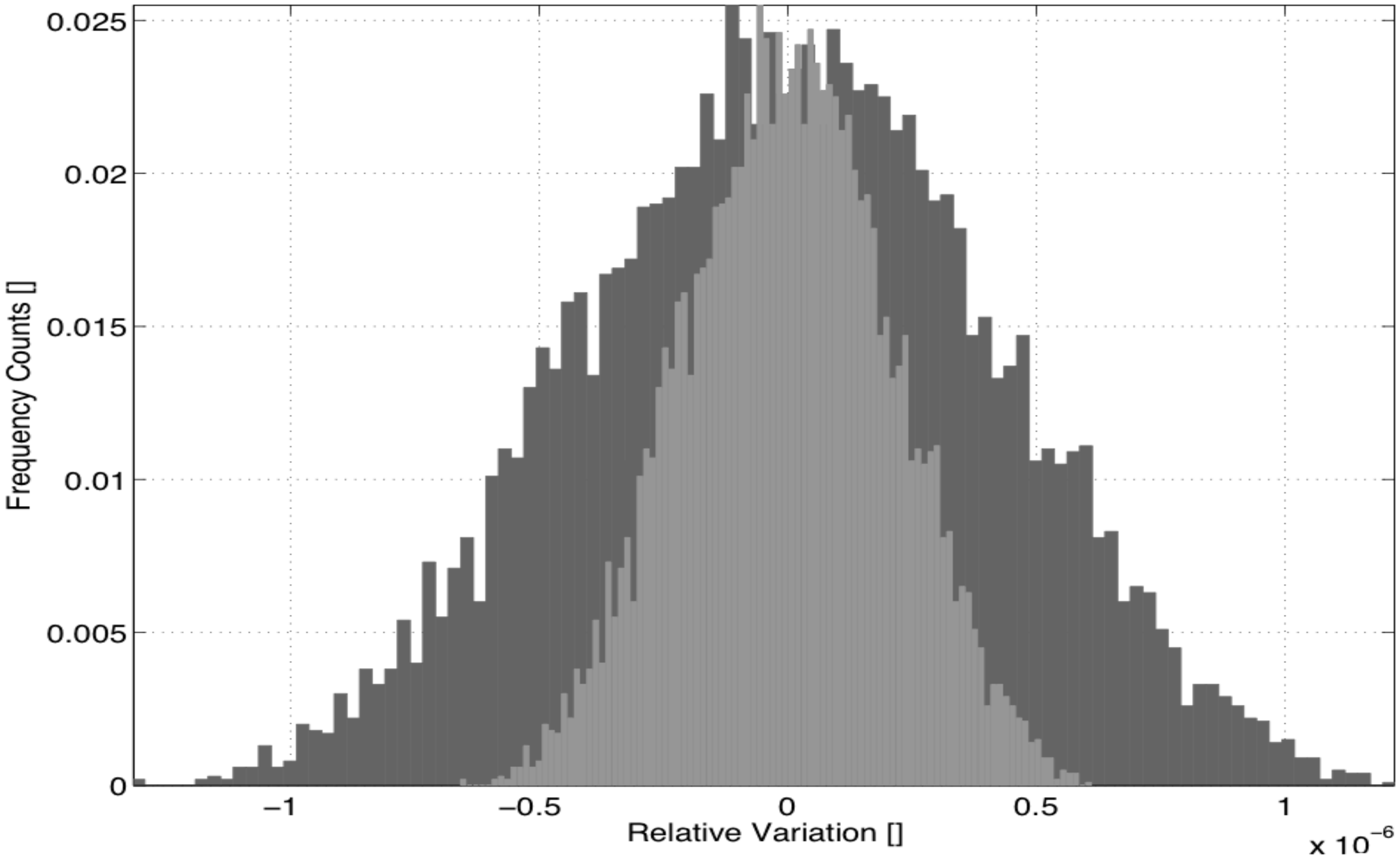
Optical Cavity geometry: Pose & Shape Decomposition



Optical Cavity geometry: Results



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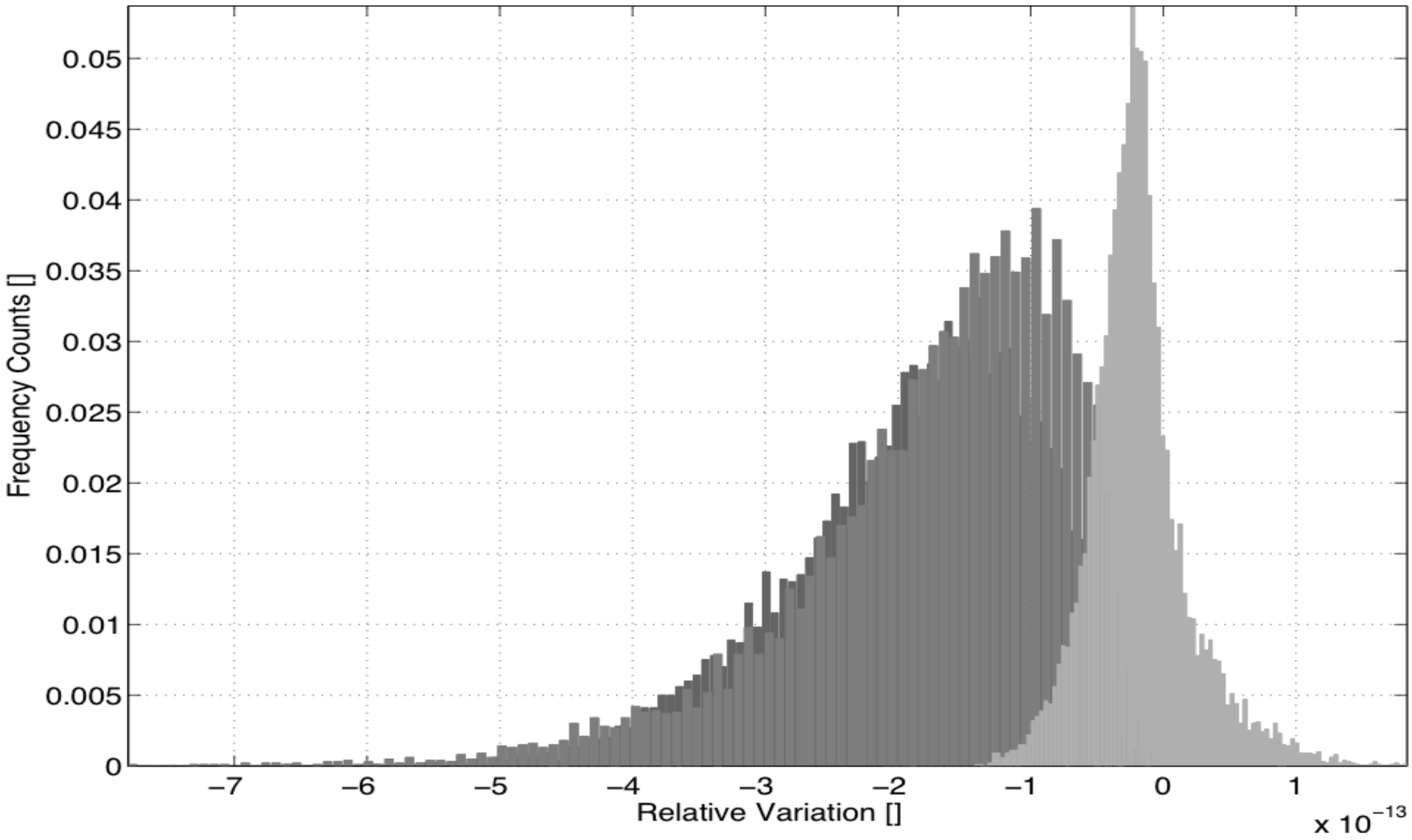

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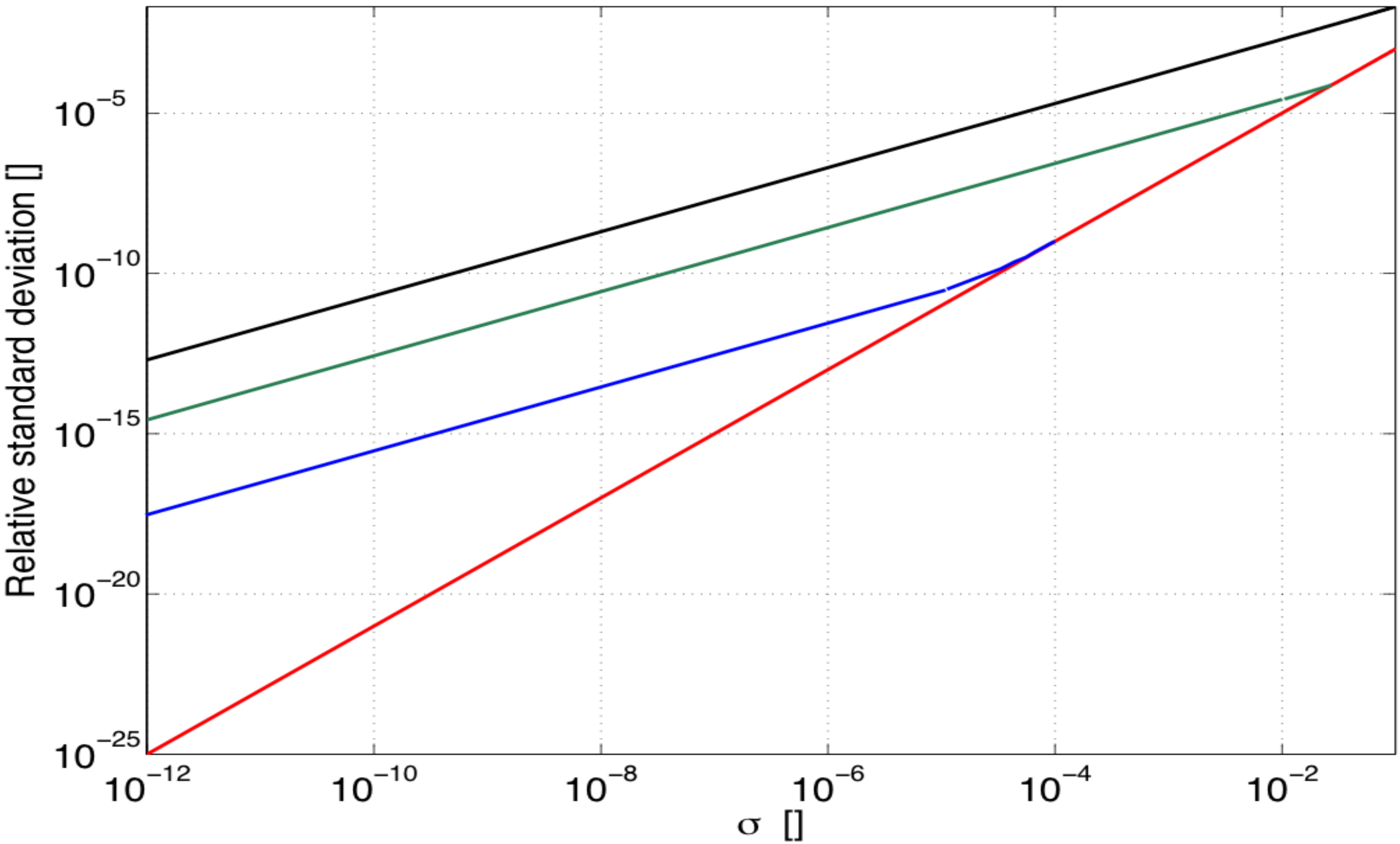

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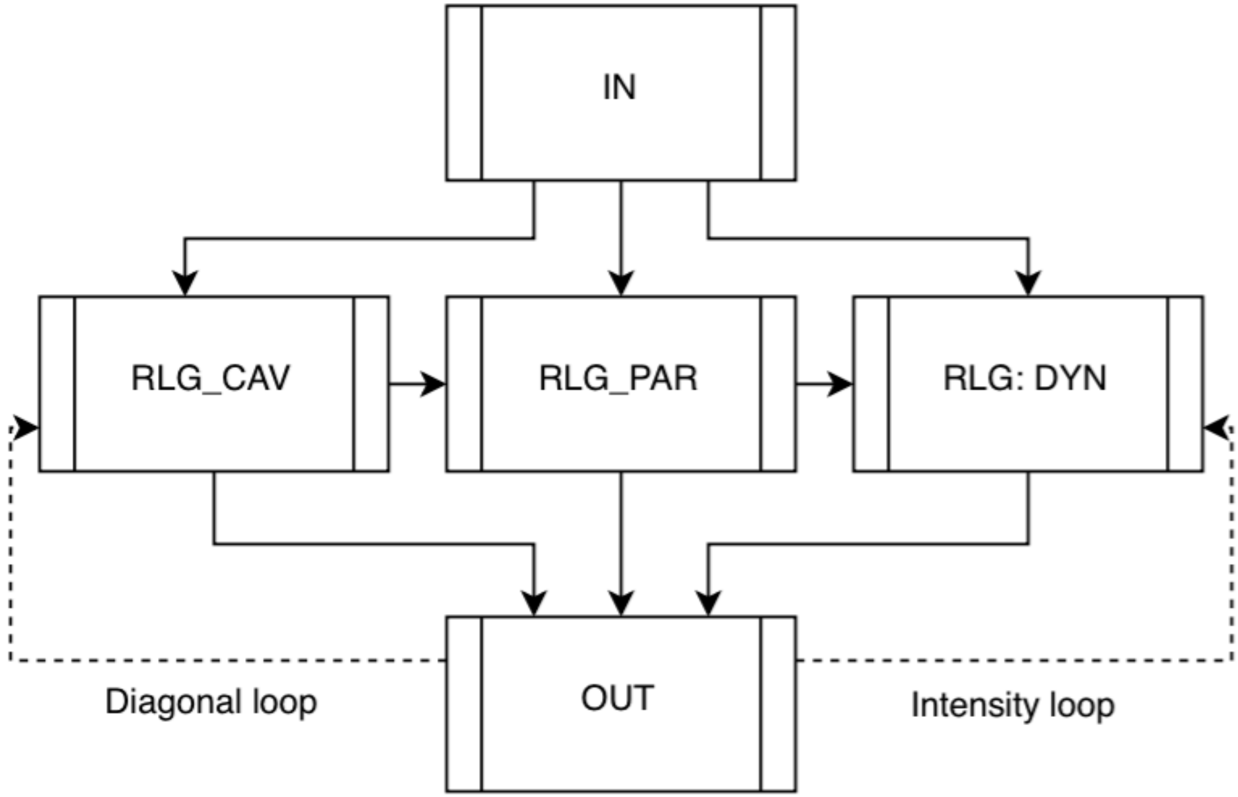
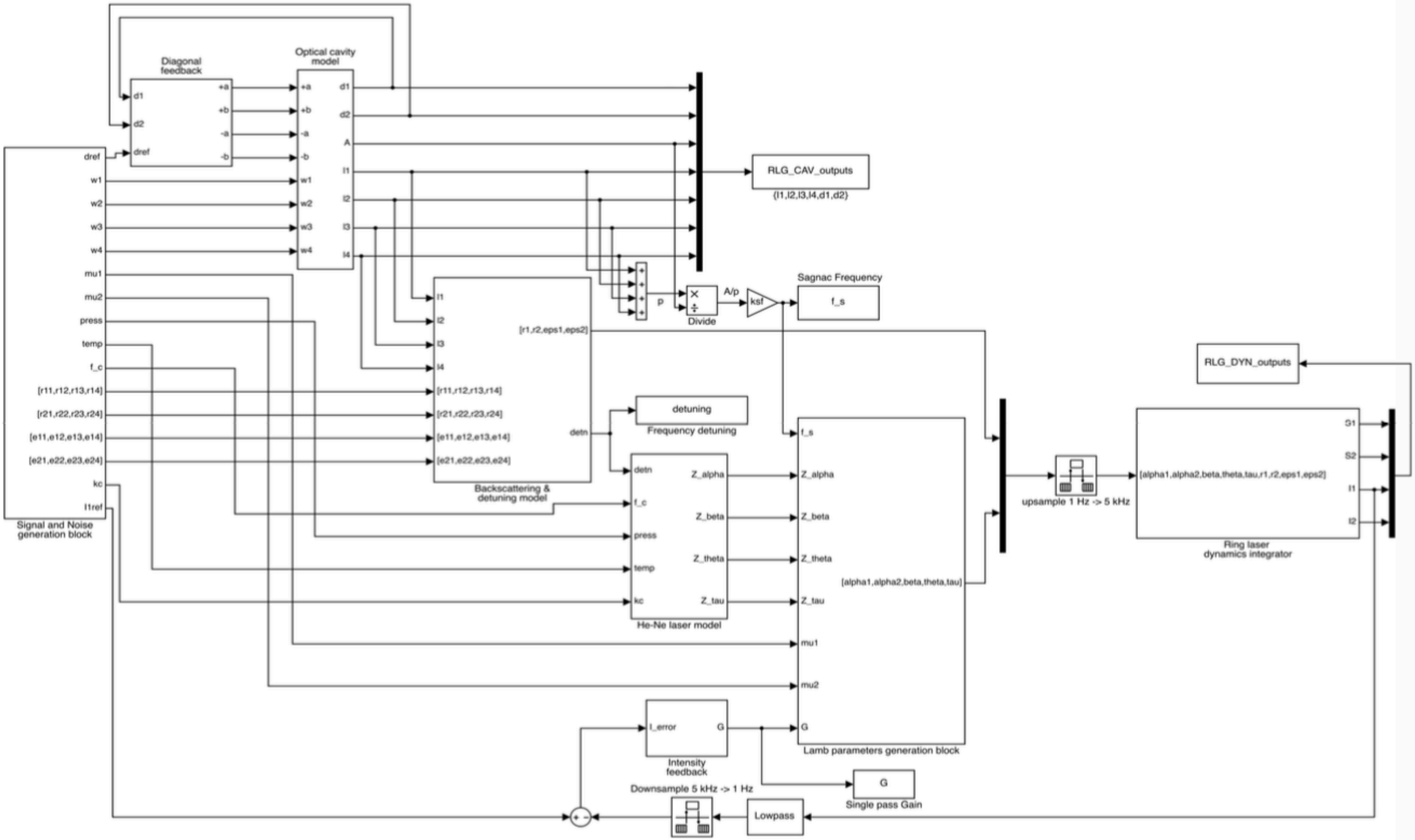
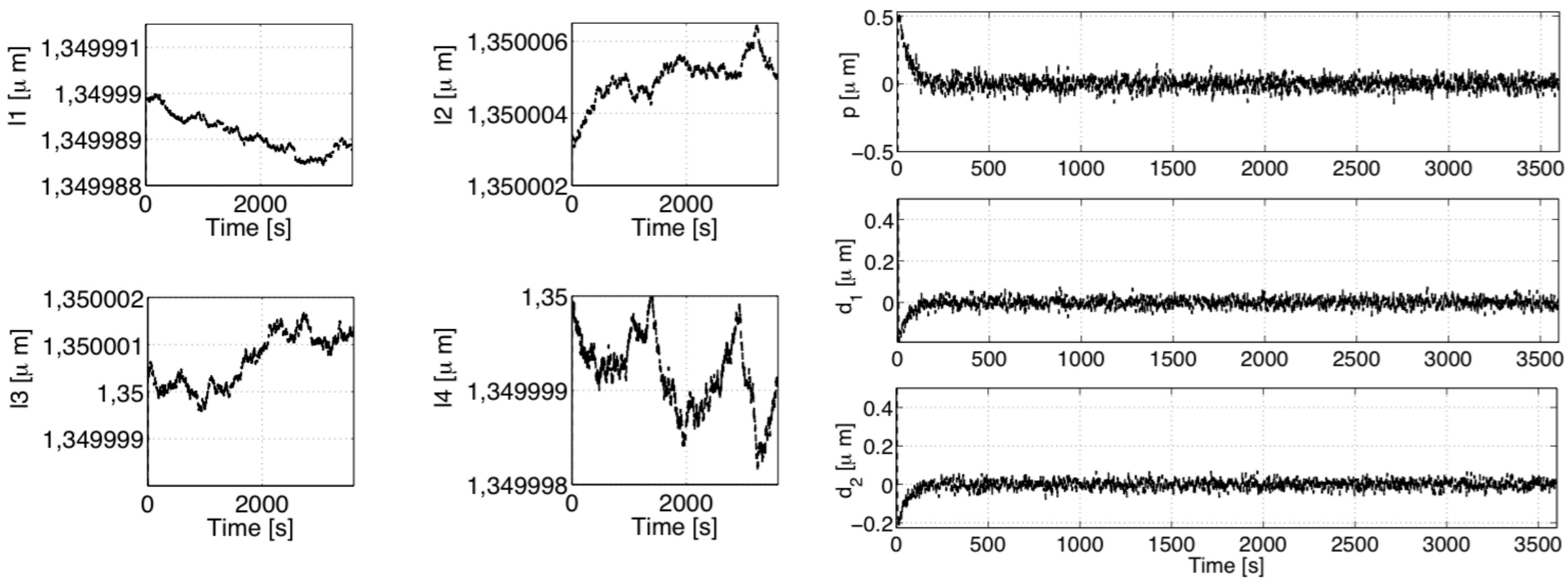
RLG Simulator: Overview



RLG Simulator: Overview



RLG Simulator: GP2 case study



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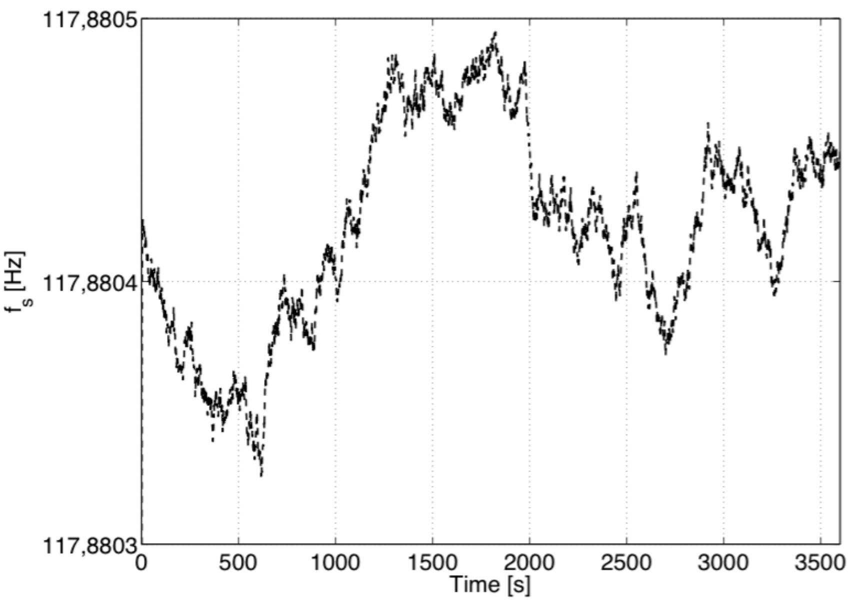
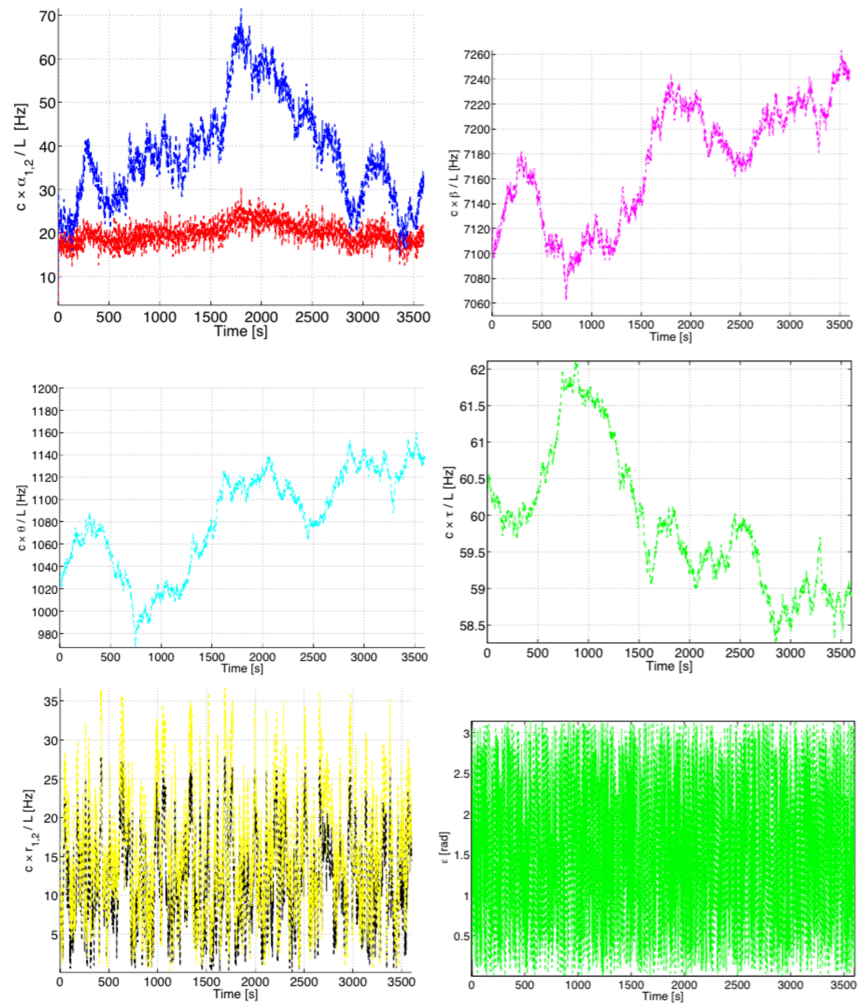
RLG Simulator: GP2 case study





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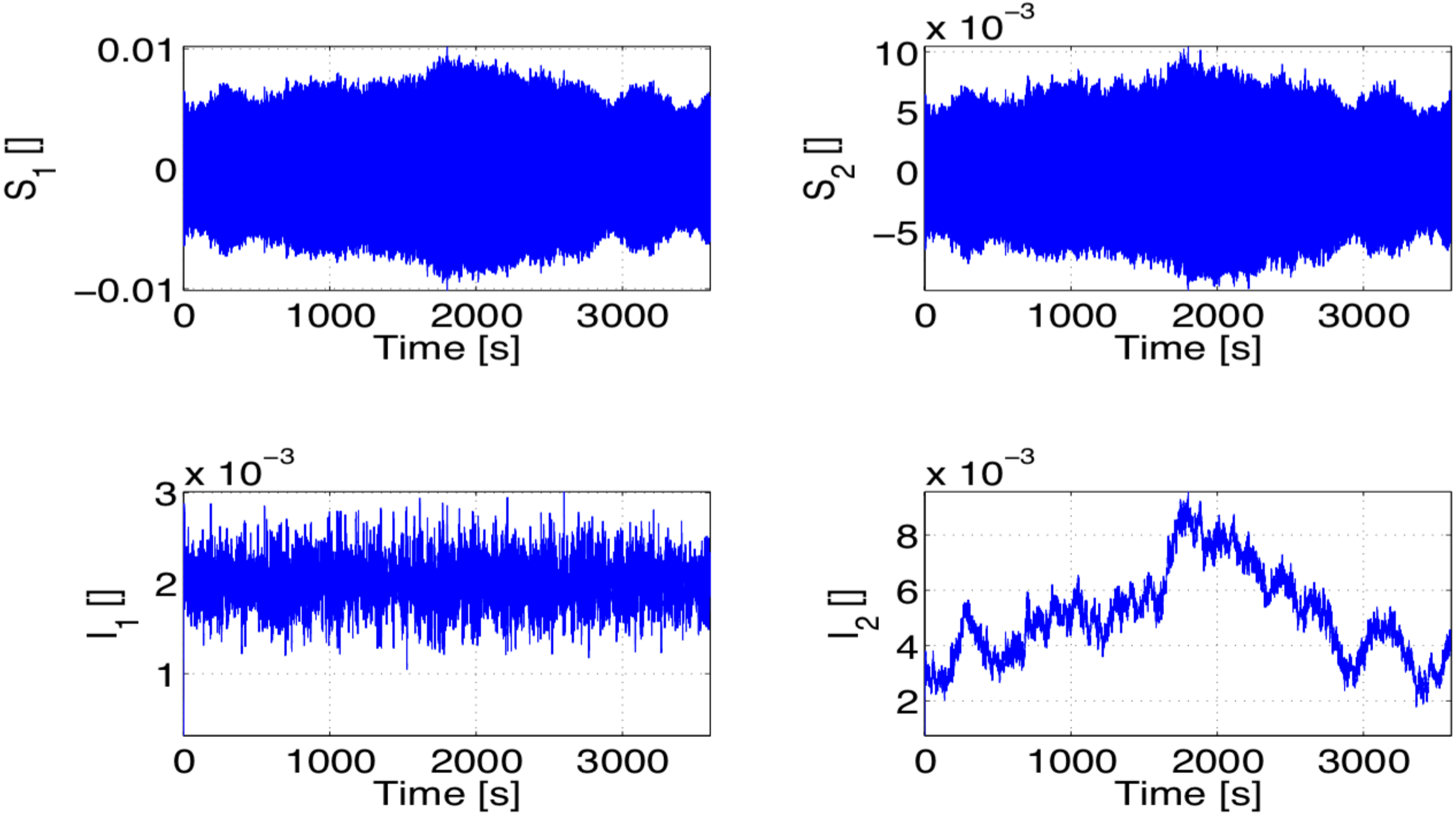
RLG Simulator: GP2 case study



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Conclusions

- Ring laser dynamics effects on the accuracy rotational frequency estimation **reviewed**
- Offline procedure for the subtraction of laser systematics **designed and demonstrated**
- Geometric Newton algorithm for the computation of the beams position in the optical cavity **designed and demonstrated**
- Pose & Shape decomposition of a square optical cavity **proposed**
- RLG Simulator of all the relevant processes involved in the Ring Laser operation **developed**

Collaboration

DEI	Alessandro Beghi Davide Cuccato Alberto Donazzan Giampiero Naletto	University of Canterbury New Zealand	Robert Hurst Geoff Stedman Robert Thirkettle Jon-Paul Wells
INFN (PI, LNL)	Jacopo Belfi Angela Di Virgilio Antonello Ortolan	LMU München Germany	Celine Hadziioannou Heiner Igel Maria Nader Joachim Wassermann
University of Pisa	Nicolo Beverini Giorgio Carelli Enrico Maccioni Rosa Santagata	TUM, BKG (Wettzell) Germany	Andre Gebauer Thomas Klügel Ulrich Schreiber Alexander Velikoseltsev
CNR (PD, NA)	Maria G. Pellizzo Alberto Porzio	TUE (Eindhoven) Holland	Alessandro Saccon
Politecnico (Torino)	Matteo L. Ruggero Angelo Tartaglia		

… and many more

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The End

Thanks for the Attention !!!