The Top Mass: Theoretical Interpretation and Uncertainties

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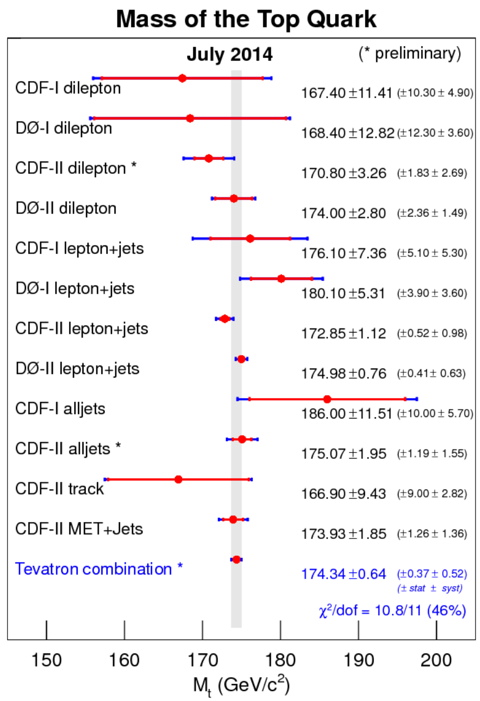
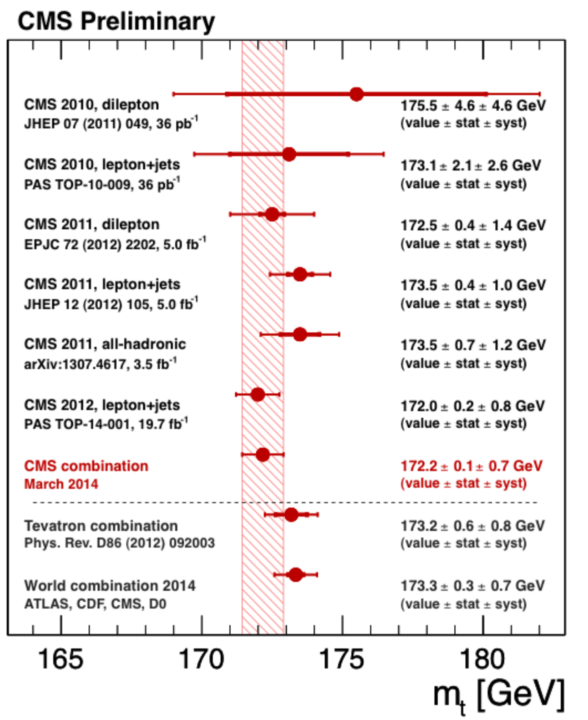
Motivation





→ Most precise mass from direct reconstruction: $m_t^{\mathrm{MC}} = 173.34 \pm 0.76\,\mathrm{GeV}$

→ m_t^{MC} cannot be used as direct input into NLO/NNLO calculations since it is not a field theoretic mass.

Outline

Part 1: $\rightarrow$ Theoretical thoughts on m_t^{MC}

- How m_t^{MC} is related to field theoretic masses.

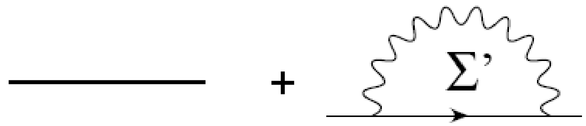
See: “The Top Mass: Interpretation and Theoretical Uncertainties”, arXiv:1412.3649

Same conclusions: AH, Stewart: arXive:0808.0222

Part 2: $\rightarrow$ Towards a determination of m_t^{MC}

- Variable Flavor Number Scheme for final state jets.
  Full massive event shape distribution
- Status & preliminary results

Top Quark Mass



$$= p - m^0 - \Sigma(p, m^0, \mu)$$

$$\Sigma(m^0, m^0, \mu) = m^0 \left[\frac{\alpha_s}{\pi\epsilon} + \ldots \right] + \boxed{\Sigma^{\rm fin}(m^0, m^0, \mu)}$$

$\overline{\text{MS}}$ scheme: $$m^0 = \overline{m}(\mu) \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right]$$

$\rightarrow$ $\overline{m}(\mu)$ is pure UV-object without IR-sensitivity

$\rightarrow$ Useful scheme for $\mu > m$

$\rightarrow$ Far away from a kinematic mass of the quark

Pole scheme: $$m^0 = m^{\rm pole} \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right] - \Sigma^{\rm fin}(m^{\rm pole}, m^{\rm pole}, \mu)$$

$\rightarrow$ Absorbes all self energy corrections into the mass parameter

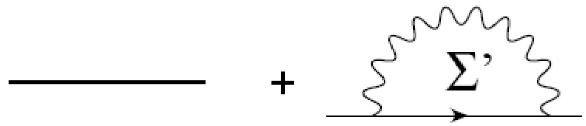
$\rightarrow$ Close to the notion of the quark rest mass (kinematic mass)

$\rightarrow$ Renormalon problem: infrared-sensitive contributions from < 1 GeV that cancel between self-energy and all other diagrams cannot cancel.

$\rightarrow$ $\Sigma^{\rm fin}$ has perturbative instabilities due to sensitivity to momenta < 1 GeV ($\Lambda_{\rm QCD}$)

Should not be used if uncertainties are below 1 GeV !

Heavy Quark Mass



$$= \not p - m^0 - \Sigma(\not p, m^0, \mu)$$

$$\Sigma(m^0, m^0, \mu) = m^0 \left[\frac{\alpha_s}{\pi\epsilon} + \ldots \right] + \boxed{\Sigma^{\rm fin}(m^0, m^0, \mu)}$$

$\overline{\rm MS}$ scheme: $\quad m^0 = \overline{m}(\mu) \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right]$

Pole scheme: $\quad m^0 = m^{\rm pole} \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right] - \Sigma^{\rm fin}(m^{\rm pole}, m^{\rm pole}, \mu)$

MSR scheme: $\quad m^{\rm MSR}(R) = m^{\rm pole} - \Sigma^{\rm fin}(R, R, \mu) \quad \text{for } R < m$ Jain, AH, Scimemi, Stewart (2008)

→ Like pole mass, but self-energy correction from < R are not absorbed into mass

→ Interpolates between MSbar and pole mass scheme

$$m_t^{\rm MSR}(R=0) = m^{\rm pole}$$

$$m_t^{\rm MSR}(R=\overline{m}(\overline{m})) = \overline{m}(\overline{m})$$

→ More stable in perturbation theory.

→ $m_t^{\rm MSR}(R = 1\,{\rm GeV})$ close to the notion of a kinematic mass, but without renormalon problem.

MSR Mass Definition

MSbar Scheme: $(\mu > \overline{m}(\overline{m}))$

$$\overline{m}(\overline{m}) - m^{\text{pole}} = -\overline{m}(\overline{m})\left[0.42441\,\alpha_s(\overline{m}) + 0.8345\,\alpha_s^2(\overline{m}) + 2.368\,\alpha_s^3(\overline{m}) + \ldots\right]$$

$\Downarrow$

MSR Scheme: $(R < \overline{m}(\overline{m}))$

$$m_{\text{MSR}}(R) - m^{\text{pole}} = -R\left[0.42441\,\alpha_s(R) + 0.8345\,\alpha_s^2(R) + 2.368\,\alpha_s^3(R) + \ldots\right]$$

$$m_{\text{MSR}}(m_{\text{MSR}}) = \overline{m}(\overline{m})$$

$\Longrightarrow$ $m_{\text{MSR}}(R)$ Short-distance mass that smoothly interpolates all R scales

- Excellent convergence of relation between MSR masses at different R values
- Excellent convergence of relation between MSR masses and other short-distance masses
- Smoothy interpolates to the MSbar mass.

MSR Mass Definition

- Mass definition must be close with the scale of the respective functions (→profile functions)

μ ≥ m: MSbar mass (n_l+1)

$$\bar{m}(\mu) = m_{\rm pole} - \bar{m}(\mu) \sum_{n=1}^{\infty} \sum_{k=0}^{n} a_{nk} \left(\frac{\alpha_s(\mu)}{4\pi}\right)^n \ln^k \frac{\mu}{\bar{m}}$$

→ usual MSbar RG-evolution

μ < m: R-scale short-distance mass (n_l)

→ power-like RG-evolution

→ stable evolution down to the Landau pole

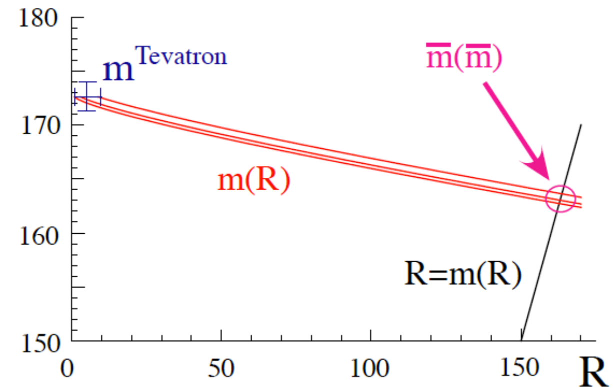
Jain, Scimemi, Stewart 08

Jain, Scimemi, Stewart, AH 08

$$m(R) = m_{\rm pole} - \delta m(R) \qquad \delta m(R) = R \sum_{n=1}^{\infty} \left(\frac{\alpha_s(R)}{4\pi}\right)^n a_n$$

$$R\frac{d}{dR}m(R) = -\frac{d}{d\ln R}\delta m(R) = R \sum_{n=0}^{\infty} \gamma_n^R \left[\frac{\alpha_s(R)}{4\pi}\right]^{n+1}$$

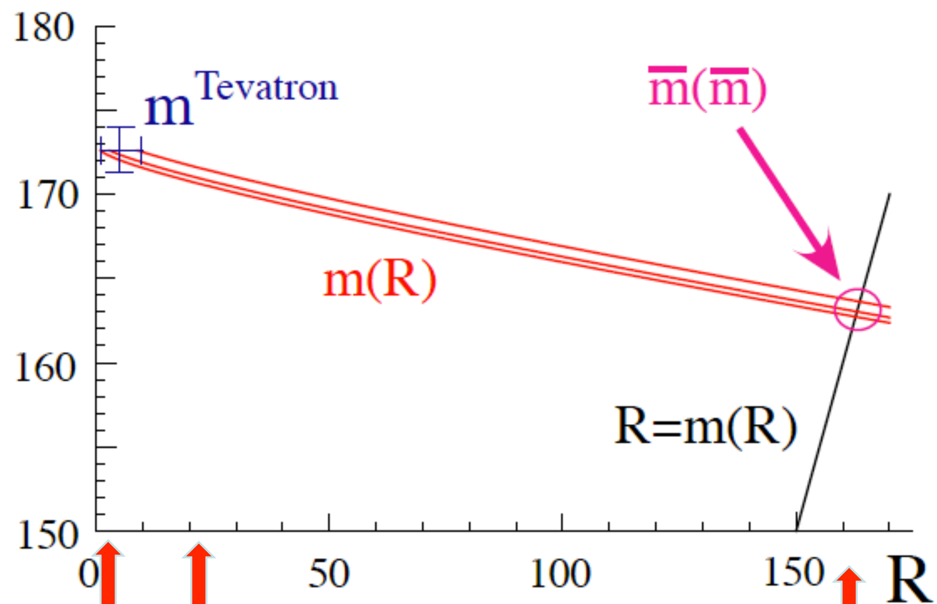
$$m(R_1) - m(R_0) = \int_{R_0}^{R_1} \frac{dR}{R} R\,\gamma^R[\alpha_s(R)]$$

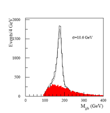


MSR Mass Definition

AH, Stewart: arXive:0808.0222

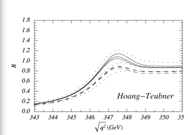
$$m_t^{\mathrm{MC}} = m_t^{\mathrm{MSR}}(3^{+6}_{-2}\ \mathrm{GeV}) = m_t^{\mathrm{MSR}}(3\ \mathrm{GeV})^{+0.6}_{-0.3}$$





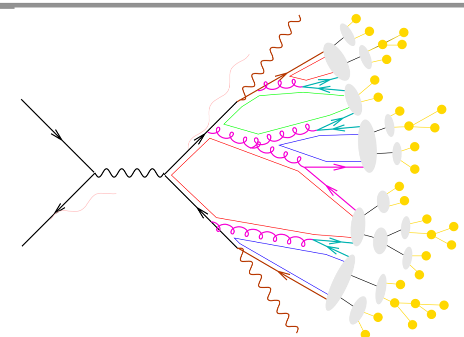
Peak of invariant mass distribution, endpoints

Top-antitop threshold at the ILC



Total cross section, e.w.precsion obs., Unification, MSbar mass

Heavy Quark Mass in the MC



Monte-Carlo event generator:

- Hard matrix element:

  Initial parton annihilation and top production plus additional hard partons from pQCD.

- Parton shower evolution:

  Splitting into higher-multiplicity partonic states (plus top decay) with subsequently lower virtualities until shower cut Λ_s. NO top self-energy contributions.

  Splitting probabilities from pQCD (approx LL accuracy, soft-collinear limit).

  Can be viewed as a way to sum dominant perturbative corrections down to Λ_s = 1 GeV.

- Hadronization model:

  Turns partons into hadrons.

  Tune strongly dependent on parton shower implementation.

  Description of data (frequently) much better than the conceptual (LL) precision of parton evolution part.

- MC mass:

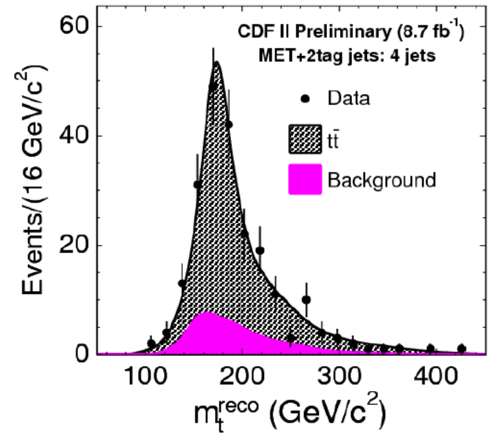
  Mass of top propagator prior to top decay.

→ Interpretation of m_t^{MC} dependent on view whether MC is more model or or more first principles QCD.

We have to assume this in order to go on.

Heavy Quark Mass in the MC

Let's take the reconstructed top invariant mass distribution as a concrete example to see how the MC components enter the templates and the MC mass fitting.



- Hard matrix element:

  Essentially only affects the norm

- MC mass:

  Determines overall location of mass range where distribution is peaked.

- Parton shower evolution + Hadronization model:

  Modify shape and distribution further.
  PS: perturbative part - <u>self-energy contributions absorbed into mass above</u> Λ_s
  HM: non-perturbative part below Λ_s

$$m_t^{\rm MC} = m_t^{\rm MSR}(R = 1\,{\rm GeV}) + \Delta_{t,{\rm MC}}(R = 1\,{\rm GeV})$$

$$\Delta_{t,{\rm MC}}(1\,{\rm GeV}) \simeq \mathcal{O}(1\,{\rm GeV})$$

Contains perturbative and non-perturbative contributions.
Conceptual reliability related to how precisely $\Delta_{t,{\rm MC}}$ can be determined.

Heavy Quark Mass in the MC

Analogy: Meson masses

$$m_B = m_b^{\text{MSR}}(1\,\text{GeV}) + \Delta_{b,B}(R = 1\,\text{GeV})$$

$$\Delta_{b,B}(1\,\text{GeV}) \simeq \mathcal{O}(1\,\text{GeV})$$

Table 1. Some B mesons masses, MSR masses $m_b^{\text{MSR}}(1\,\text{GeV})$ and $m_b^{\text{MSR}}(2\,\text{GeV})$ from $m_b^{1\text{S}} = 4780 \pm 66\,\text{MeV}$ [18], and corresponding values for $\Delta_{b,B}$. All in units of MeV, $\alpha_s(m_Z) = 0.1184$.

$m_b^{\text{MSR}}(1\,\text{GeV})$	$m_b^{\text{MSR}}(2\,\text{GeV})$	$m(B^0)$	$m(B^*)$	$m(B_1^0)$	$m(B_2^*)$
4795 ± 69	4571 ± 69	5279.58 ± 0.17	5325.2 ± 0.4	5724 ± 2	5743 ± 5
$\Delta_{b,B}(1\,\text{GeV})$		485 ± 69	530 ± 69	929 ± 69	948 ± 69
	$\Delta_{b,B}(2\,\text{GeV})$	709 ± 69	754 ± 69	1153 ± 69	1172 ± 69

Additional Comments

- Using NLO vs. LO matrix elements does not affect the interpretation of the MC mass
- Different parton evolution implies in principle a different MC mass.
- Relation of MC to MSR mass can be used to deal with mass dependent efficiencies for total cross section measurements.
- MC mass should be independent of the process and kinematic region used for fitting.

Theory Tools to Measure the MC mass

Part 2

The relation between MC mass and field theoretical mass can be made more precise by measuring the MC mass using a completely independent hadron level QCD prediction of a mass-dependent observable.

Need:

- Accurate analytic QCD predictions beyond LL/LO with full control over the quark mass dependence
- Theoretical description at the hadron level for comparison with MC at the hadron level

- Implementation of massive quarks into the SCET framework
- **VFNS for final state jets (with massive quarks)***

* In collaboration with: B. Dehnadi, V. Mateu, I. Stewart

arXiv:1302.4743 (PRD 88, 034021 (2013))
arXiv:1309.6251 (PRD 89, 014035 (2013))
arXiv:1405.4860 (PRD 90 114001 (2014))
More to come …

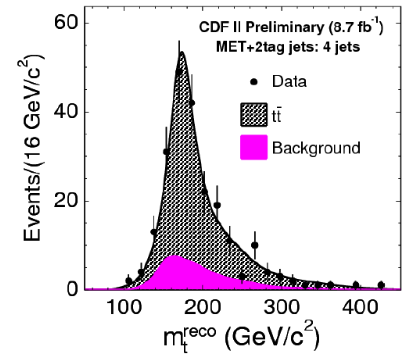
Theory Tools to Measure the MC mass

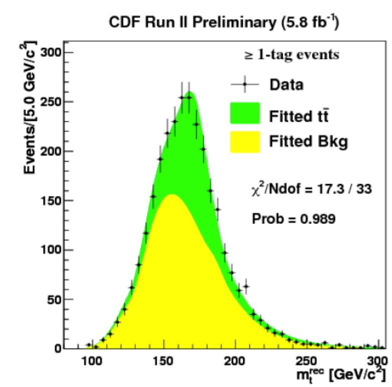
Observable: Thust in e+e-

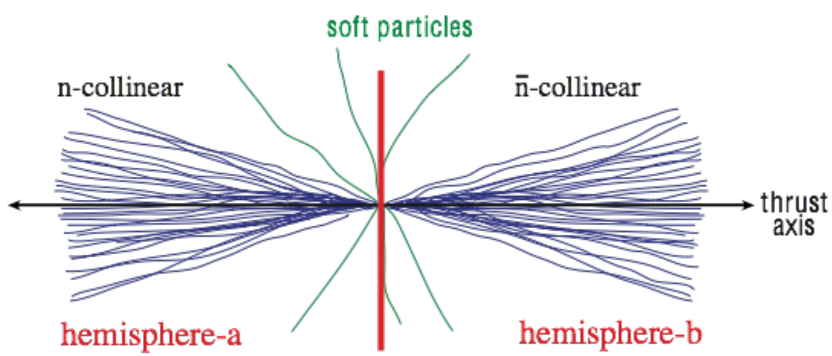
$$\tau \;=\; 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n}\cdot\vec{p}_i|}{Q}$$

$$\overset{\tau\to 0}{\approx} \; \frac{M_1^2 + M_2^2}{Q^2}$$

Invariant mass distribution in the resonance region !

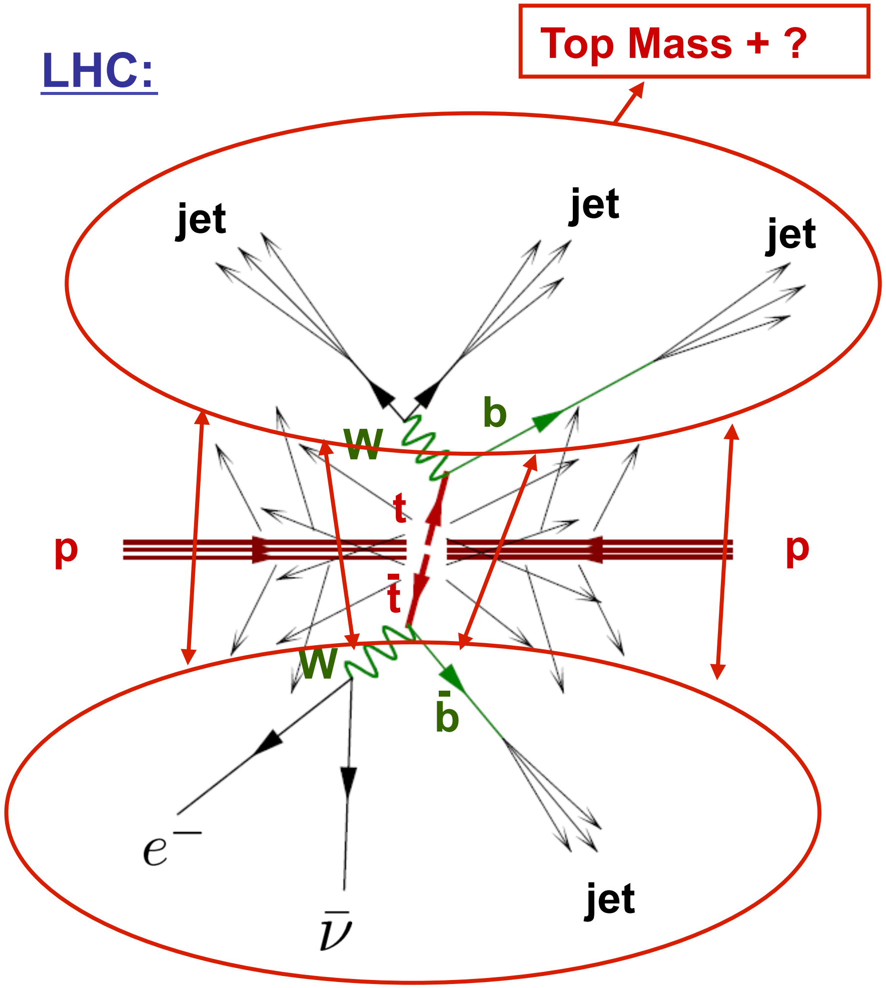






Description of Jets

LHC:



Principle of mass measurements:

Identification of the top decay products

$$\text{“}\; m_{\text{top}}^2 = p_t^2 = \left(\textstyle\sum_i p_i^\mu\right)^2 \;\text{”}$$

Problem is non-trivial !

- Measured object does not exist a priori, but only through the experimental prescription for the measurement. **Quantum effects !!**

The idea of a - by itself - well defined object having a well defined mass is incorrect !!

Details and uncertainties of the parton shower and the hadronization models in den MC's influence the measured top quark mass.

Top Mass vs. Peak Mass

Double differential invariant mass distribution (NLL):

$$Q = 5 \times 172\ \mathrm{GeV}$$

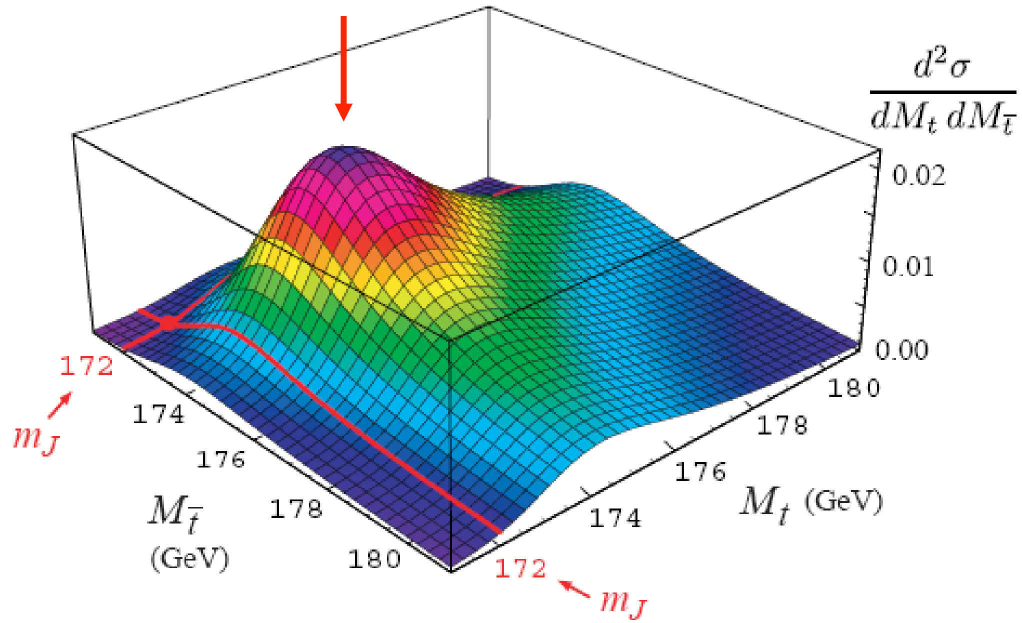
$$\Gamma = 1.43\ \mathrm{GeV}$$

$$m_J(2\,\mathrm{GeV}) = 172\ \mathrm{GeV}$$

$$\mu_\Gamma = 5\ \mathrm{GeV}$$

$$\mu_\Lambda = 1\ \mathrm{GeV}$$

$$a = 2.5\,, \quad b = -0.4$$
$$\Lambda = 0.55\ \mathrm{GeV}$$

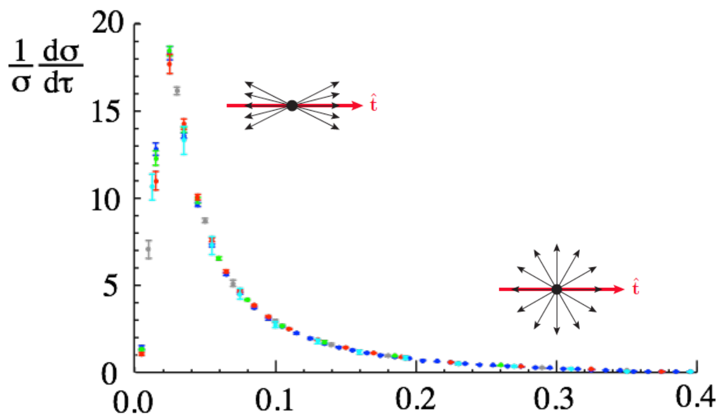


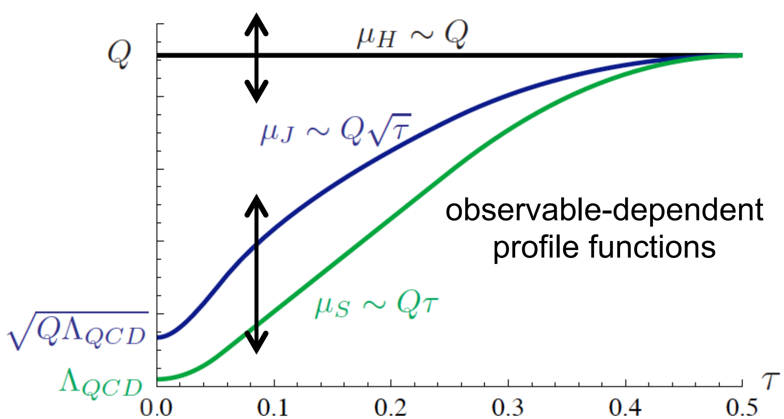
Non-perturbative effects shift the peak by <u>+2.4 GeV</u>
and broaden the distribution.

Factorization for Massless Quarks

$$\frac{d\sigma}{d\tau} = Q^2\sigma_0 H_0(Q,\mu)\int d\ell\, J_0(Q\ell,\mu)\, S_0\,(Q\tau-\ell,\mu)$$

Korshemski, Sterman
Schwartz
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman





Abbate, AH, Fickinger, Mateu, Stewart

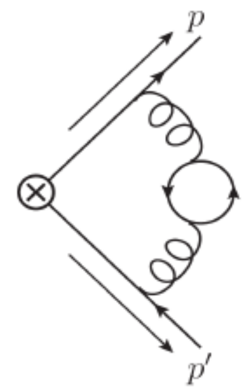
$$\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part} \sim \sigma_0\, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s)\int d\ell d\ell'\, U_J(Q\tau-\ell-\ell',\mu_Q,\mu_s)\, J_T(Q\ell',\mu_j)\, S_T(\ell-\Delta,\mu_s)$$

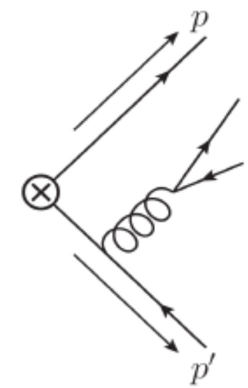
Primary Massless Quarks

- **Massless quarks**

  ✔ SCET: Full N³LL + 3-loop non-singular

  Becher, Schwartz
  Bauer, Fleming, Lee, Sterman
  Fleming, Hoang, Mantry, Stewart

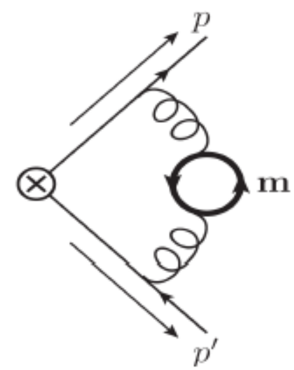


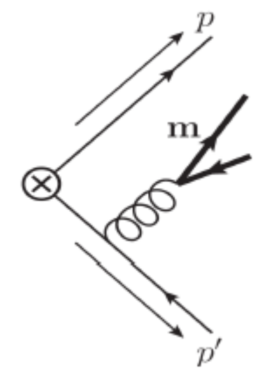


- **Secondary massive quarks**

  ✔ SCET: Full NNLL' / N³LL
  ✔ New degrees of freedom: **mass modes**
  ✔ Continuous description using **VFNS**

  Gritschacher, Hoang, Jemos,
  Pietrulewicz, Mateu (2013 + 2014)
  Presented by Piotr Pietrulewicz (SCET 2013/2014)



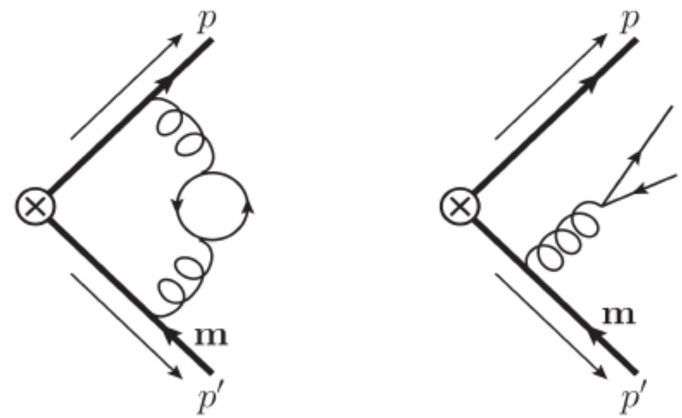


Primary Massive Quarks

- **Primary massive quarks**

  ✔ SCET with massive quarks NNLL
  ✔ bHQET: full NNLL' / N^3LL

  Fleming, Hoang, Mantry, Stewart
  Jain, Scimemi, Stewart

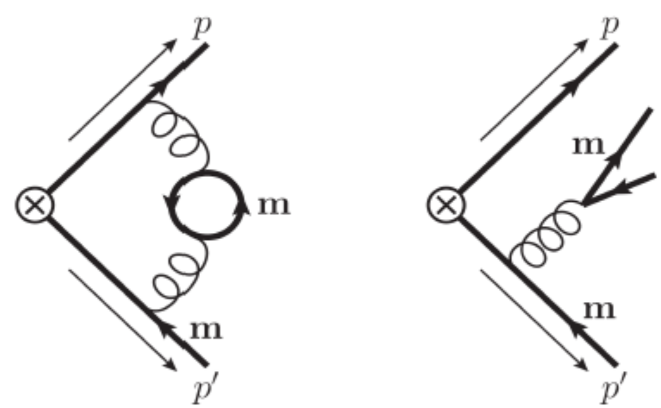


- **Fully massive (primary and secondary) quarks**

  ✔ **Complete and systematic description**

  Presented by Andre Hoang (SCET 2014)
  Presented by Aditya Pathak (SCET 2014)

  → Aim of this talk (status report)



VFN Scheme for Final State Jets

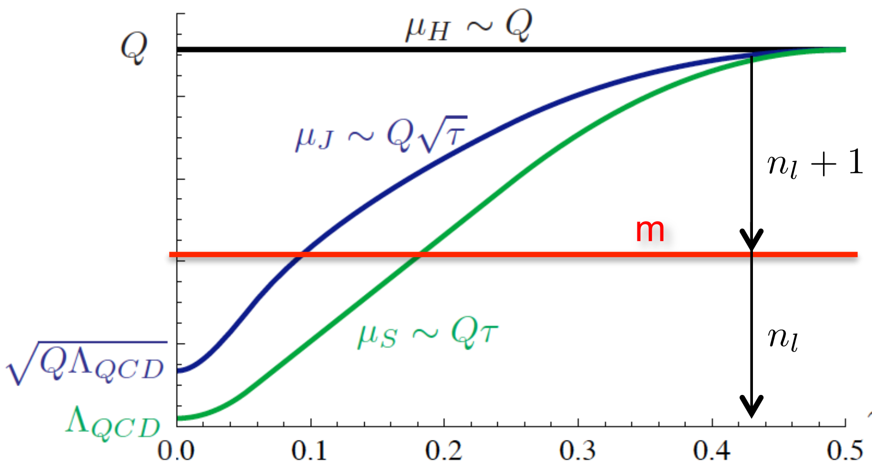
Gritschacher, AH, Jemos, Pietrulewicz

→ consider: dijet in e^+e^- annihilation, n_l light quarks ⊕ one massive quark

→ obvious: (n_l+1)-evolution for $\mu \gtrsim m$ and (n_l)-evolution for $\mu \lesssim m$

→ obvious: different EFT scenarios w.r. to mass vs. Q – J – S scales

"profile functions"



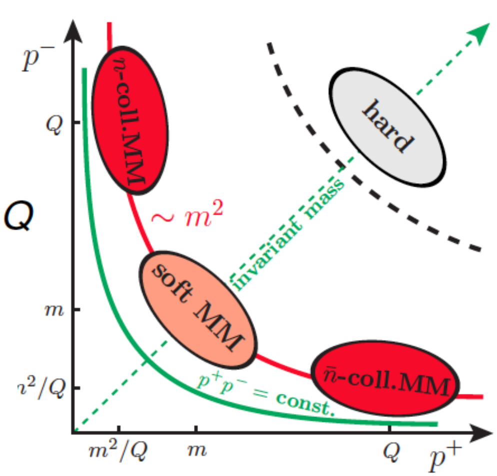
Aims:

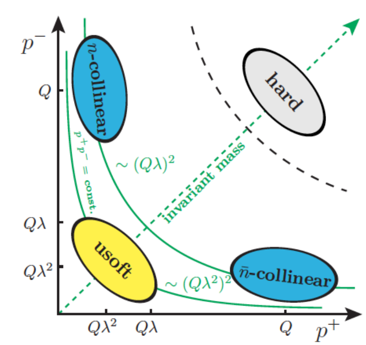
- Full mass dependence (little room for any strong hierarchies): decoupling, massless limit
- Smooth connections between different EFTs
- Determination of flavor matching for current-, jet- and soft-evolution
- Reconcile problem of $SCET_2$-type rapidity divergences

→ Deal with collinear and soft "mass modes"

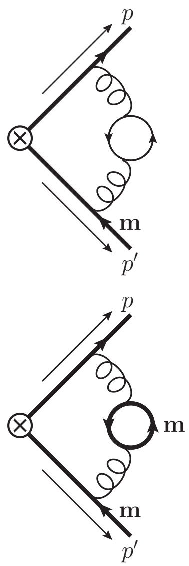
→ Additional power counting parameter $\lambda_m = m/Q$

mode	$p^\mu = (+, -, \perp)$	p^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2





VFN Scheme: Primary Massive Quarks

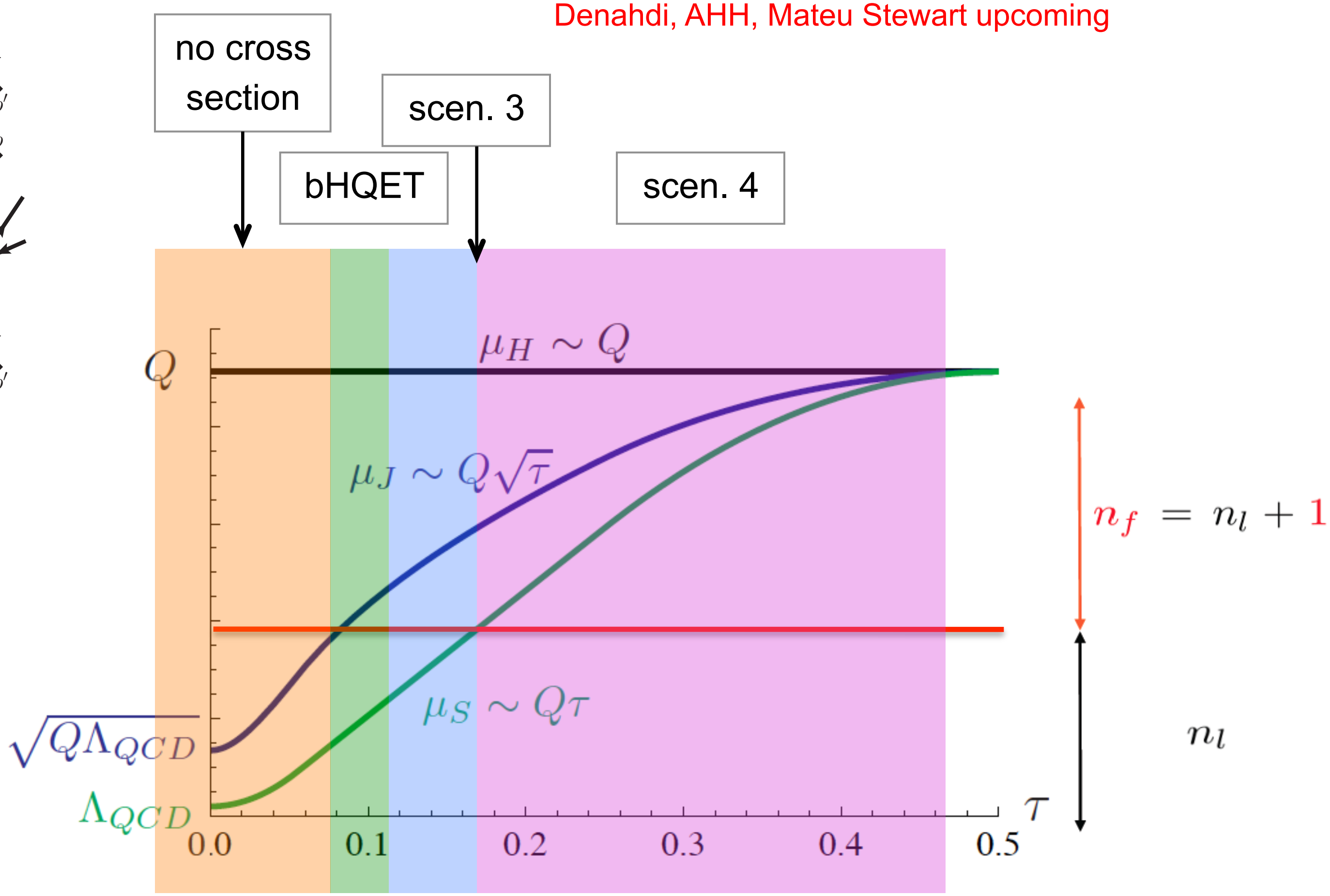


→ bHQET-type theory when
the jet scale approaches the quark mass

Fleming, AHH, Mantry, Stewart 2007

→ two SCET-type theories

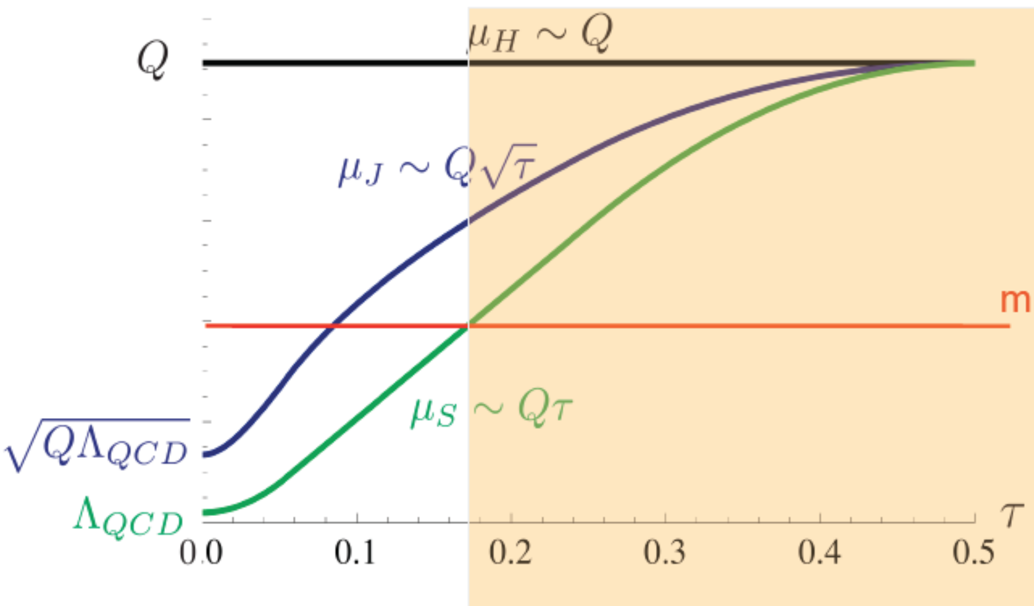
Denahdi, AHH, Mateu Stewart upcoming



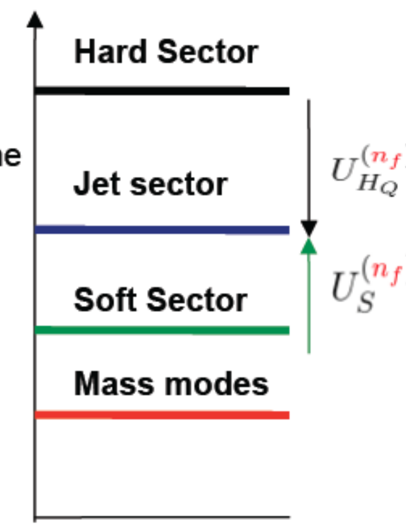
Scenario IV (SCET)

$$\left.\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{SCET-IV}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J)\int \mathrm{d}s \int \mathrm{d}k\, J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))$$
$$U_S^{(n_f)}(k,\mu_J,\mu_S)\, S_{\mathrm{part}}^{(n_f)}\left(Q\tau - Q\tau_{\min} - \frac{s}{Q} - k,\mu_S\right) \quad \textbf{+ (QCD) Non-Singular}$$

$$n_f = n_l + 1$$



mass modes enter in all the soft, jet and hard sectors.



- (QCD) No-singular → Non-singular + Sub-leading singular contributions

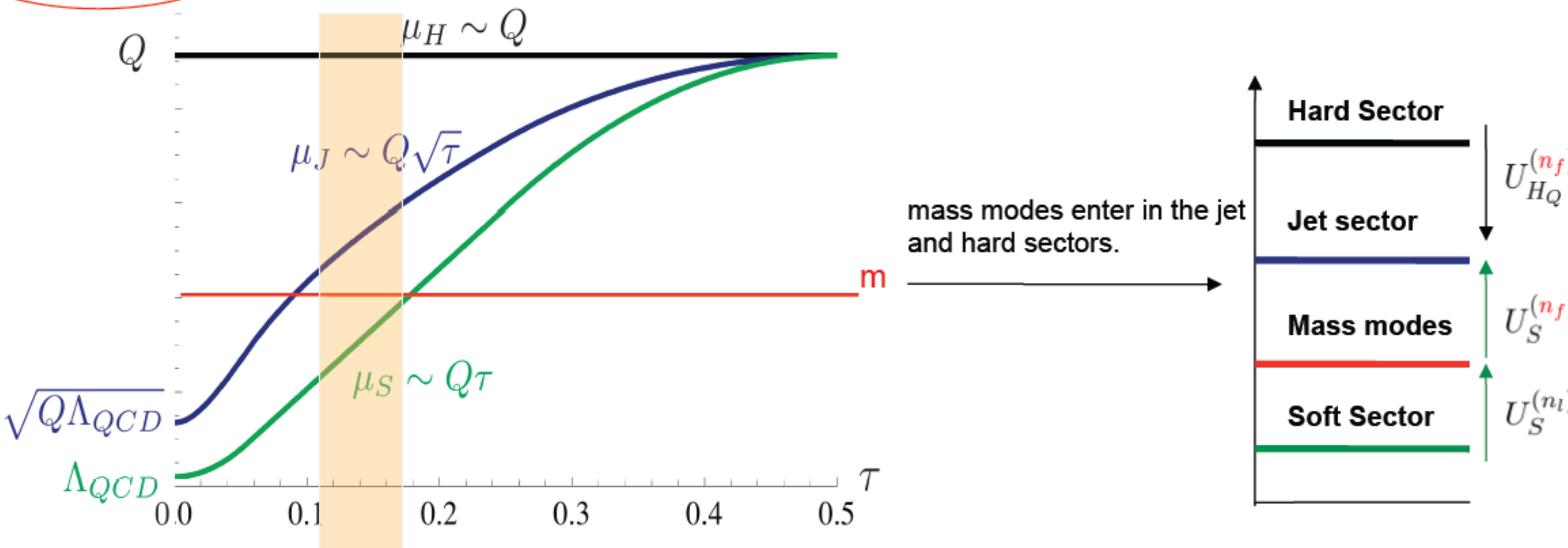
Scenario III (SCET)

$$\left|\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}(\tau)}{\mathrm{d}\tau}\right|^{\mathrm{SCET-III}} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_J) \int \mathrm{d}s \int \mathrm{d}k\,\mathrm{d}k'\,\mathrm{d}k''\, J^{(n_f)}(s,\mu_J,\overline{m}^{(n_f)}(\mu_J))\, U_S^{(n_f)}(k,\mu_J,\mu_m) \qquad n_f = n_l + 1$$

$$\boxed{\mathcal{M}_S^{(n_f)}(k'-k,\overline{m}^{(n_f)}(\mu_m),\mu_m,\mu_s)}\, U_S^{(n_l)}(k''-k',\mu_m,\mu_S)\, S_{\mathrm{part}}^{(n_l)}\left(Q\tau - Q\tau_{\min} - \frac{s}{Q} - k'',\mu_S\right)$$

large rapidity logs
$\alpha_s^2 \log \sim \alpha_s$

+ (QCD) Non-Singular



- Soft mass-mode matching: **integrating in** the **mass-mode (secondary)** effects in the **evolution of the soft function** (top-down resummation). $\mathcal{O}(\alpha_s^2)$

Rapidity Logarithms

- Secondary mass effects start at $O(\alpha_s^2)$
- Counting for rapidity logs: α_s Log ~ 1
- At $O(\alpha_s^2)$:
  - No resummation to all orders needed
  - Need terms at $O(\alpha_s^3\ \mathrm{Log})$ and $O(\alpha_s^4\ \mathrm{Log}^2)$

$O(\alpha_s)$

$O(\alpha_s^2)$

$$\mathcal{M}_H(Q,m,\mu_m) = 1 + \frac{\left(\alpha_s^{(n_l+1)}\right)^2 C_F T_F}{(4\pi)^2} \ln\left(-\frac{\mu_m^2}{Q^2}\right)\left\{\frac{4}{3}L_m^2 + \frac{40}{9}L_m + \frac{112}{27}\right\}$$

Explicit computation

$$+ \left[\frac{\left(\alpha_s^{(n_l+1)}\right)^2 C_F T_F}{(4\pi)^2}\left\{\frac{4}{9}L_m^3 + \frac{38}{9}L_m^2 + \left(\frac{242}{27} + \frac{2\pi^2}{3}\right)L_m - \frac{52}{9}\zeta(3) + \frac{875}{54} + \frac{5\pi^2}{9}\right\}\right.$$

Explicit computation

$$+ \frac{\left(\alpha_s^{(n_l+1)}\right)^3 C_F T_F}{(4\pi)^3}\ln\left(-\frac{\mu_m^2}{Q^2}\right)\left\{\mathcal{M}_H^{(3)} + \sum_{n=0}^{3} a_n\, L_m^n\right\}$$

Exponentiation

$$\left. + \frac{\left(\alpha_s^{(n_l+1)}\right)^4 C_F^2 T_F^2}{(4\pi)^4}\ln^2\left(-\frac{m^2}{Q^2}\right)\left\{\frac{8}{9}L_m^4 + \frac{160}{27}L_m^3 + \frac{416}{27}L_m^2 + \frac{4480}{243}L_m + \frac{6272}{729}\right\}\right]$$

$$L_M = \ln\left(\frac{m^2}{\mu_m^2}\right)$$

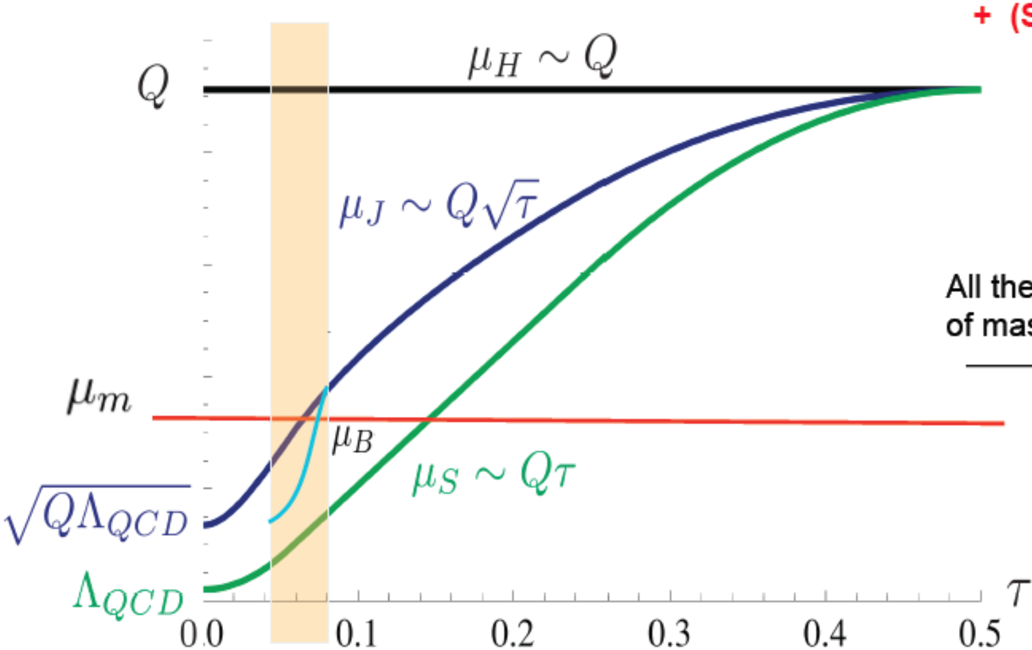
b(oosted)HQET

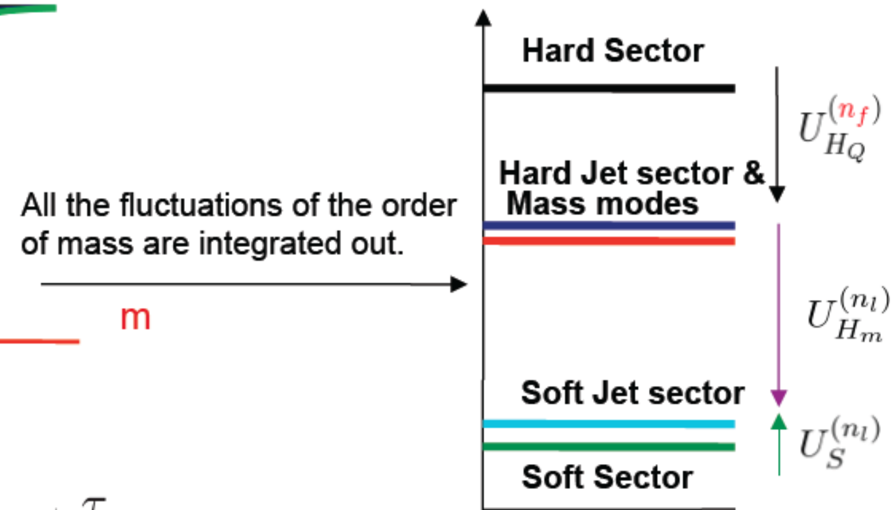
$$\left.\frac{1}{\sigma_0}\frac{\mathrm{d}\hat\sigma(\tau)}{\mathrm{d}\tau}\right|^{\rm bHQET} = Q\, H_Q^{(n_f)}(Q,\mu_Q)\, U_{H_Q}^{(n_f)}(Q,\mu_Q,\mu_m)\, \boxed{H_m^{(n_f)}(\overline{m}^{(n_f)},\mu_m)}\, U_{H_m}^{(n_l)}\left(\frac{Q}{\overline{m}^{(n_l)}},\mu_m,\mu_B\right)$$

$$\int \mathrm{d}s \int \mathrm{d}k\; B^{(n_l)}\left(\frac{s}{m_J^{(n_l)}},\mu_B, m_J^{(n_l)}\right) U_S^{(n_l)}(k,\mu_B,\mu_S)\, S_{\rm part}^{(n_l)}\left(Q\tau - Q\tau_{\rm MIN} - \frac{s}{Q} - k,\mu_S\right)$$

$n_f = n_l + 1$

+ (SCET) Non-Singular **+ (QCD) Non-Singular**



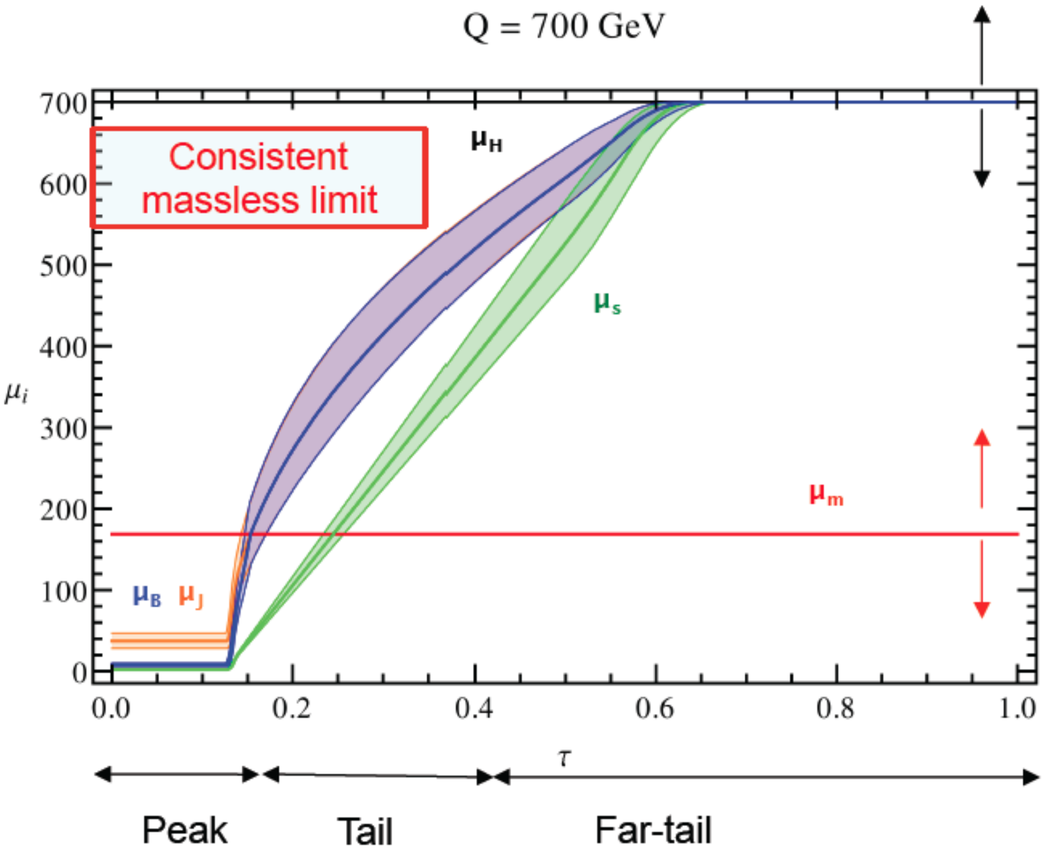


- Matching coefficient of SCET and bHQET have a large log from secondary corrections.

Profile Functions

Profile functions should sum up large logarithms and achieve smooth transition between the peak, tail and far-tail.

$$\log\left(\frac{Q}{\mu_H}\right) \quad \log\left(\frac{m_J}{\mu_m}\right) \quad \log\left(\frac{\mu_J^2}{Q\mu_s}\right) \quad \log\left(\frac{m_J\mu_B}{Q\mu_s}\right) \quad \log\left(\frac{Q(\tau-\tau_{\min})+2\Lambda_{\mathrm{QCD}}}{\mu_s}\right)$$



Scales Variation

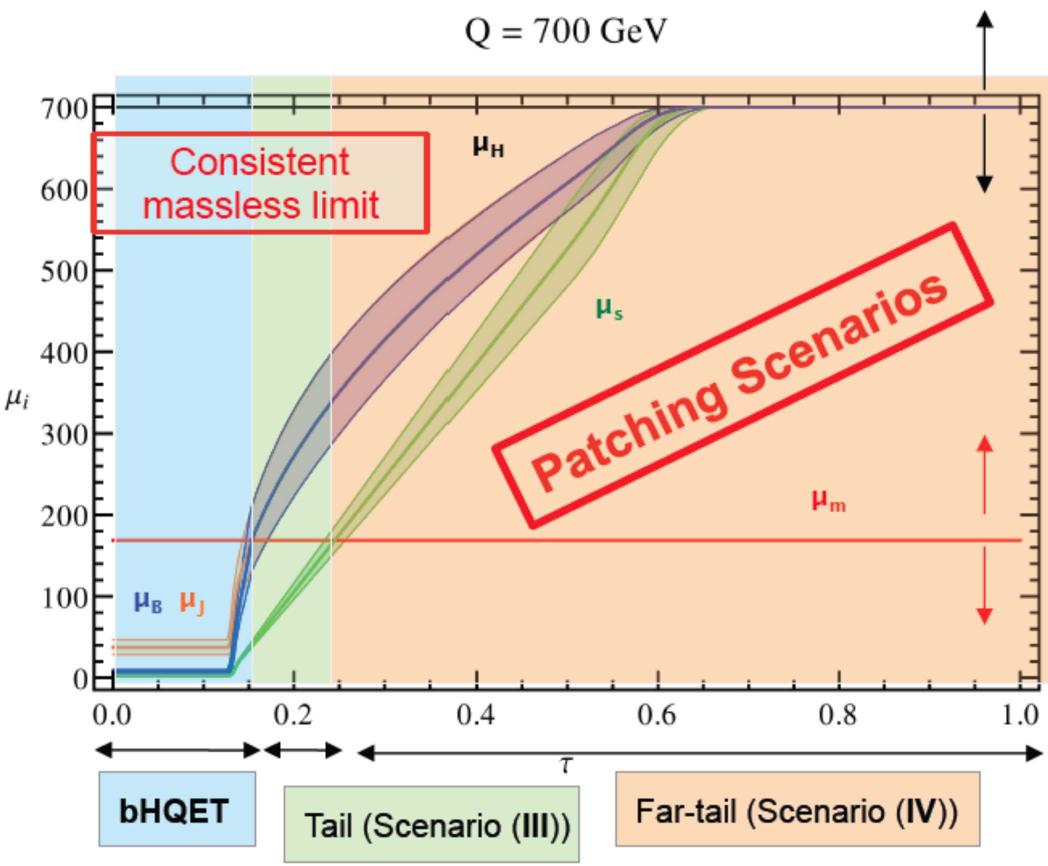
- ✔ **Generalized to arbitrary mass values**
- ✔ **Compatible with massless profiles**

Proper scale variations are essential in reliable estimation of missing higher order terms.

Profile Functions

Profile functions should sum up large logarithms and achieve smooth transition between the peak, tail and far-tail.

$$\log\left(\frac{Q}{\mu_H}\right) \quad \log\left(\frac{m_J}{\mu_m}\right) \quad \log\left(\frac{\mu_J^2}{Q\mu_s}\right) \quad \log\left(\frac{m_J\mu_B}{Q\mu_s}\right) \quad \log\left(\frac{Q(\tau-\tau_{\rm min})+2\Lambda_{\rm QCD}}{\mu_s}\right)$$



Scales Variation

- ✔ **Generalized to arbitrary mass values**
- ✔ **Compatible with massless profiles**

Proper scale variations are essential in reliable estimation of missing higher order terms.

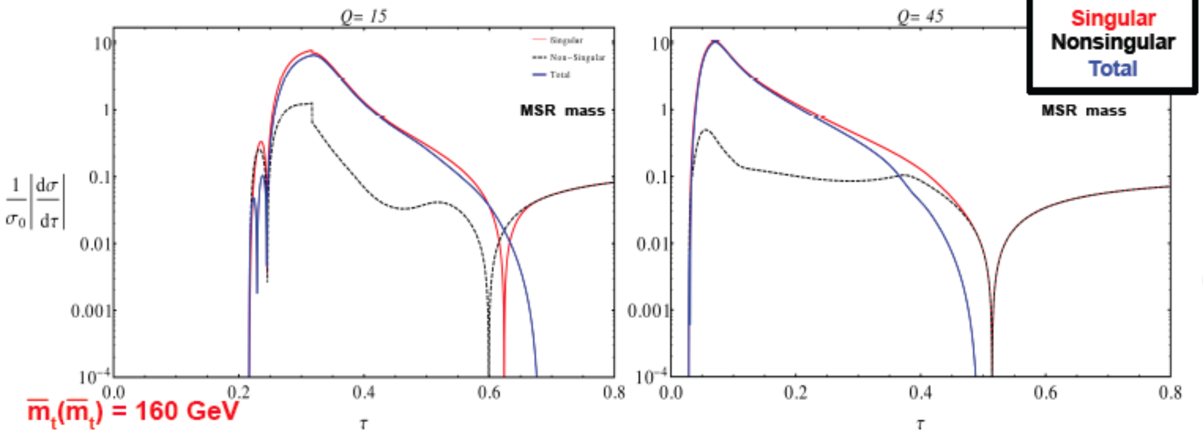
Thrust Components: Bottom and Top

NNLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

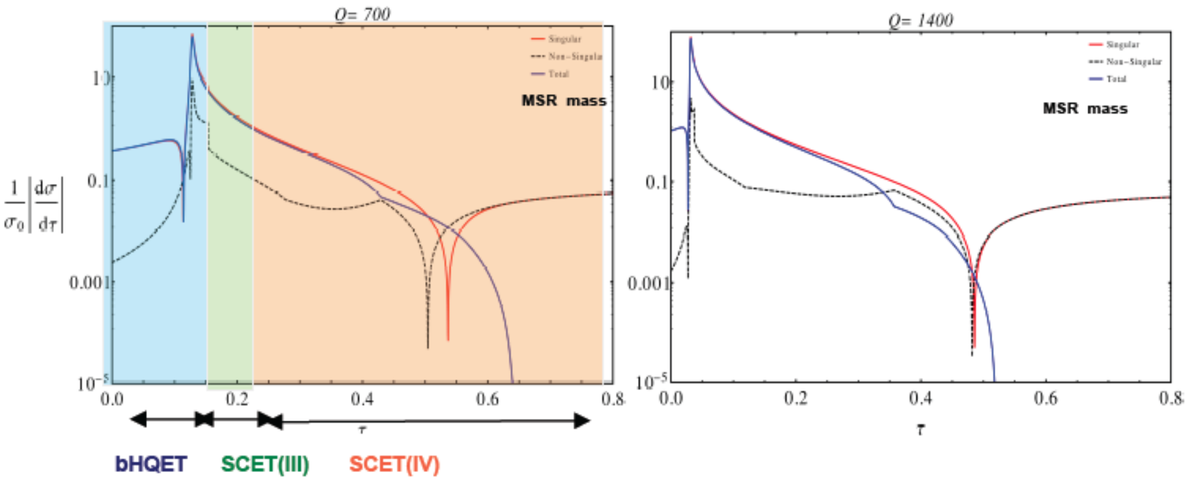
Singular vs Nonsingular

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart

$\overline{m}_b(\overline{m}_b)$ = 4.2 GeV



$\overline{m}_t(\overline{m}_t)$ = 160 GeV



- Cancellation of singular and nonsingular contributions at far-tail.
- Finite width effects turned off at tail.

$$\alpha_s(m_Z) = 0.1184$$
$$\Omega_1 = 0.5\,\mathrm{GeV}$$

Preliminary Results

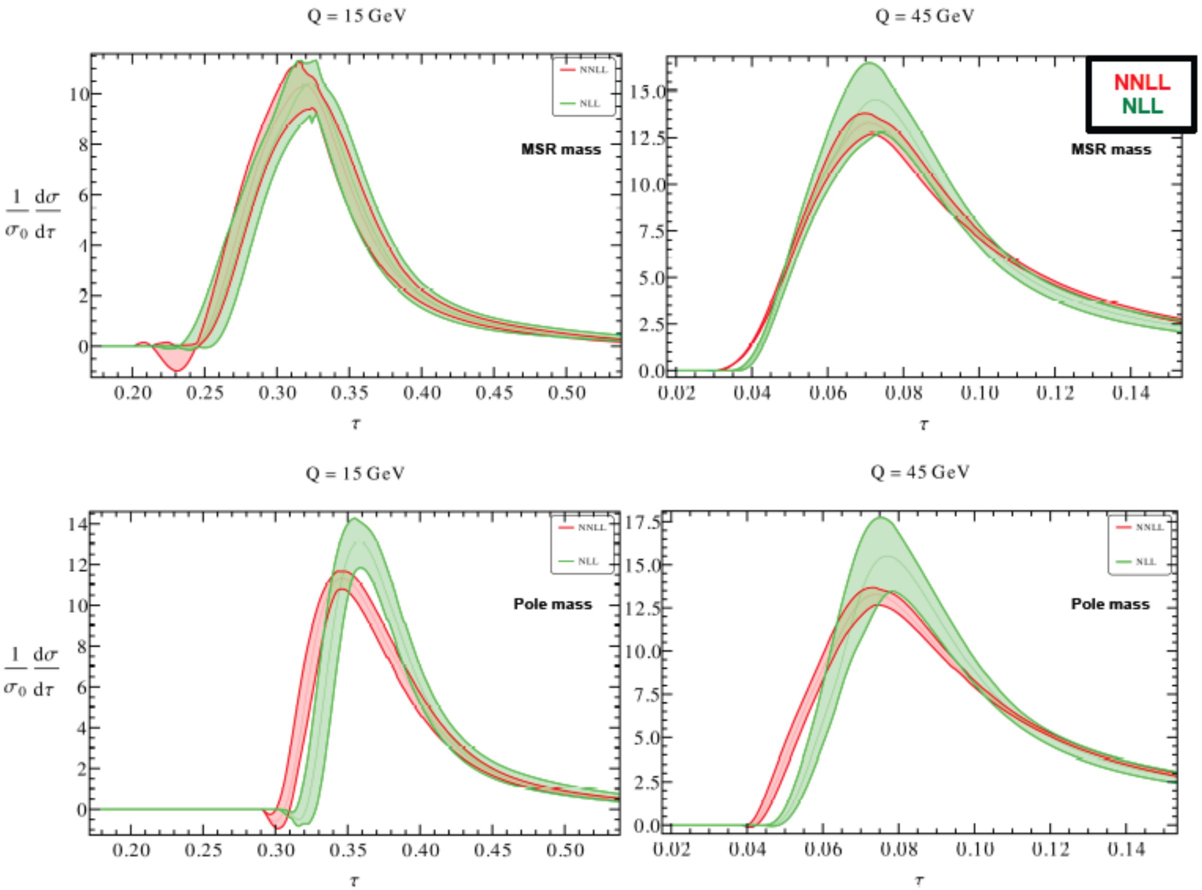
Thrust for Bottom Production

NNLL/NLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

$\overline{m}_b(\overline{m}_b) = 4.2$ GeV

Theory uncertainty (bottom)

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart



- Theory uncertainty is under control for bottom thrust distribution.
- Convergence of perturbation theory.
- The peak position in thrust is very sensitive to the mass.
- Stability of peak position with short distance mass scheme w.r.t. the pole mass scheme.

$$\alpha_s(m_Z) = 0.1184$$
$$\Omega_1 = 0.5\,\text{GeV}$$

Preliminary Results

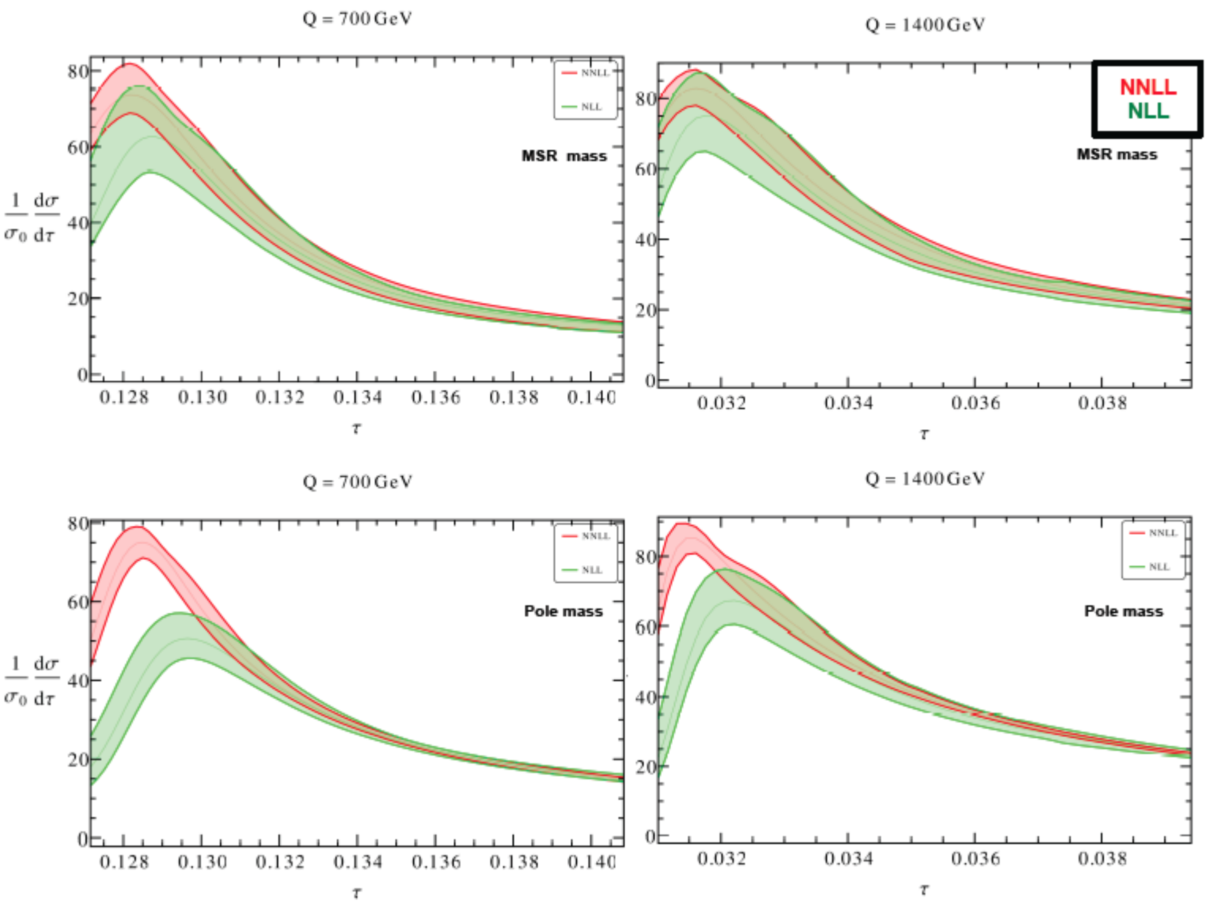
Thrust for Top Production

NNLL/NLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

$\overline{m}_t(\overline{m}_t)$ = 160 GeV

Theory uncertainty (top)

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart



- Theory uncertainty in peak + tail is under control for top thrust distribution
- Convergence of perturbation theory
- Stability of peak position with short distance mass scheme w.r t. the pole mass scheme

$$\alpha_s(m_Z) = 0.1184$$
$$\Omega_1 = 0.5\,\mathrm{GeV}$$

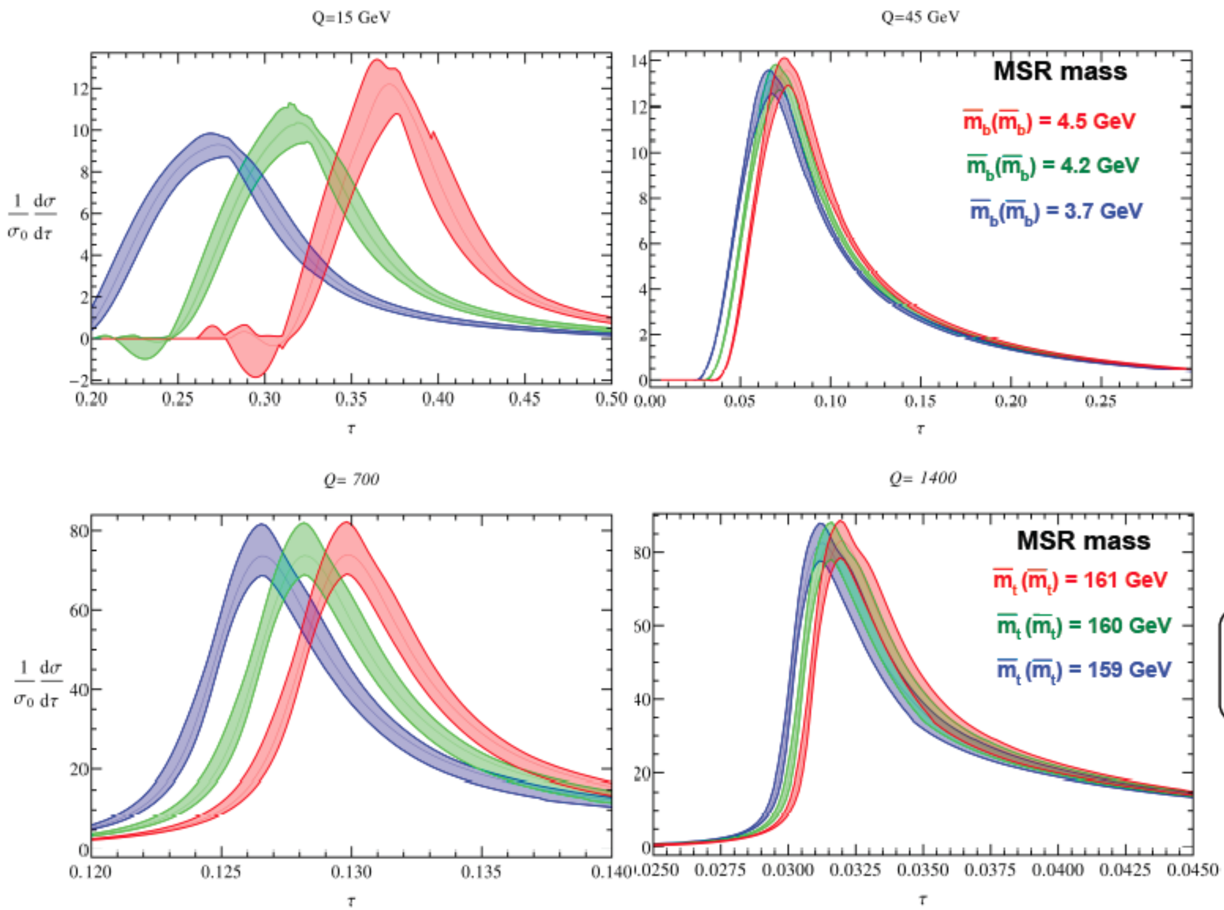
Preliminary Results

Thrust for Top Production

NNLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

Peak sensitivity to mass

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart



- **Peak position is more sensitive to the mass at low energies**
  - **bottom with better than 0.5 GeV**
  - **top with better than 1 GeV**

$$\alpha_s(m_Z) = 0.1184$$
$$\Omega_1 = 0.5\,\mathrm{GeV}$$

Preliminary Results

Sensitivities: Bottom and Top Production

NNLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

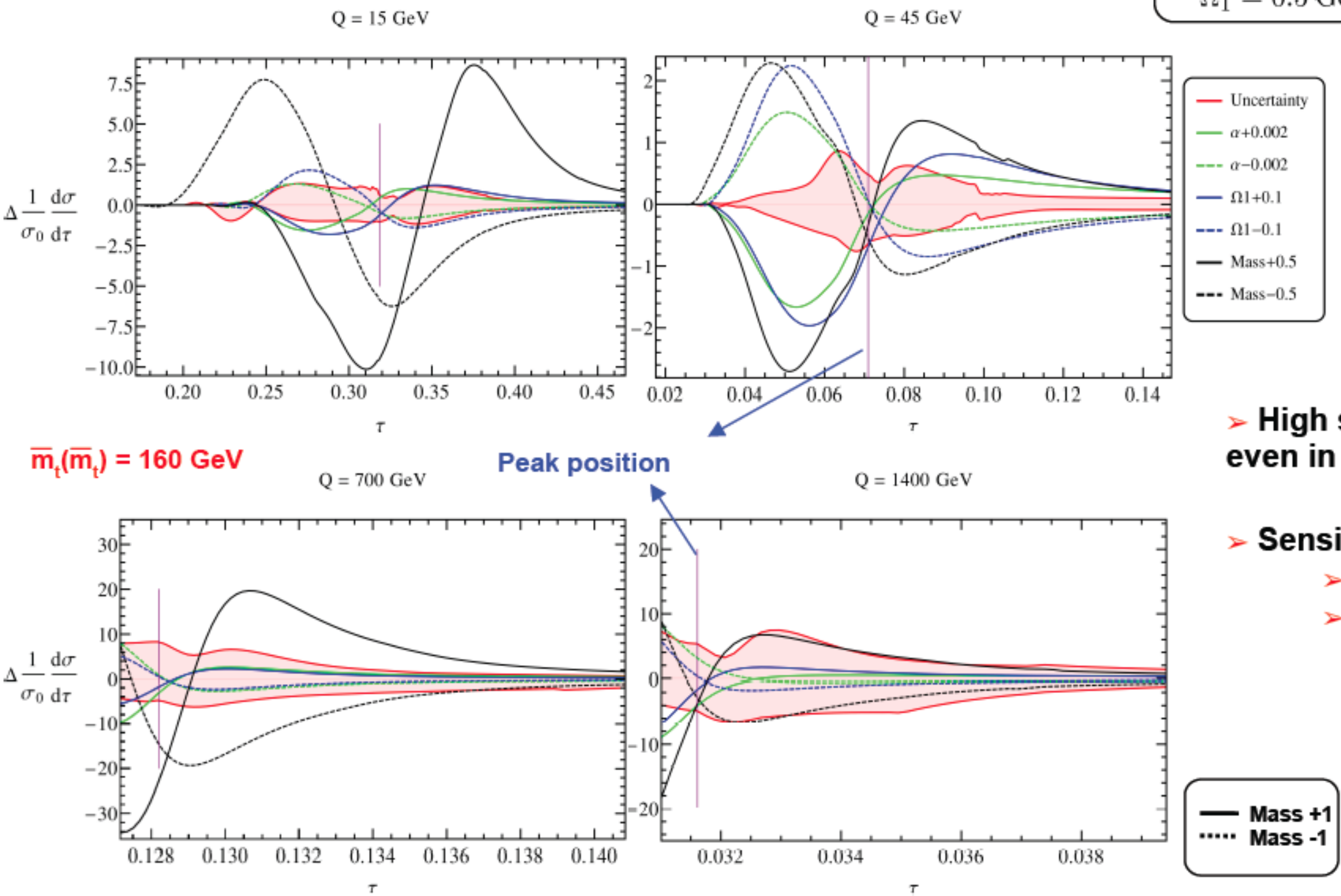
$\overline{m}_b(\overline{m}_b)$ = 4.2 GeV

Sensitivity of theory parameters

$\alpha_s(m_Z) = 0.1184$

$\Omega_1 = 0.5\,\text{GeV}$

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart



Comparison of the difference between the default cross section and the cross section varying only one parameter of theory w.r.t. to the theory uncertainties.

- **High sensitivity to (top & bottom) mass even in tail regions at low energies.**
- **Sensitivity to α_s and Ω_1**
  - **bottom → low sensitivity**
  - **top → negligible sensitivity (should be fixed externally)**

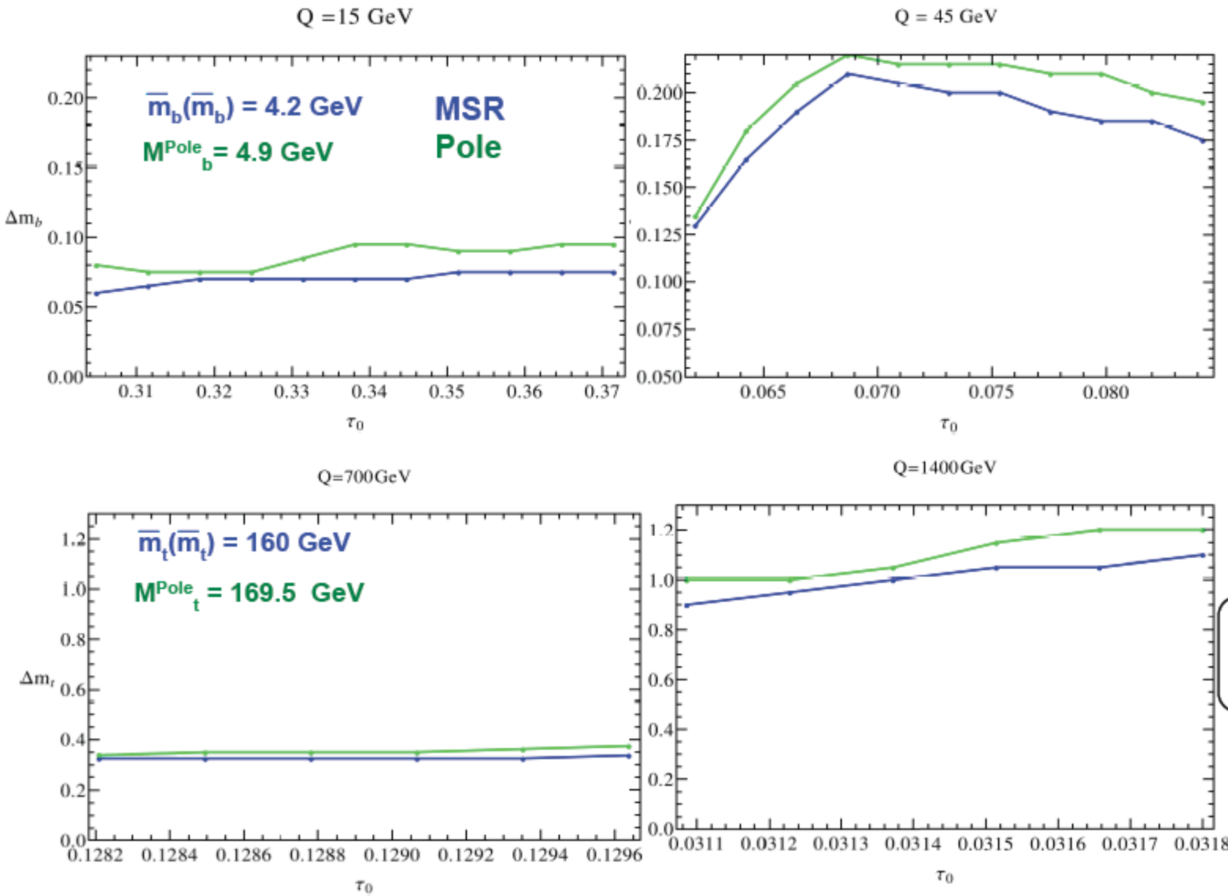
Preliminary Results

Theory Errors: Bottom and Top Mass

NNLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

Theory uncertainty in mass

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart



Error band method: Fitting the mass parameter of default cross section to the error band of default cross section.

- **bottom mass uncertainty < 0.5 GeV**
- **top mass uncertainty ~ 0.5 GeV**
- **Pole mass extractions are less precise.**

$$\alpha_s(m_Z) = 0.1184$$
$$\Omega_1 = 0.5\,\text{GeV}$$

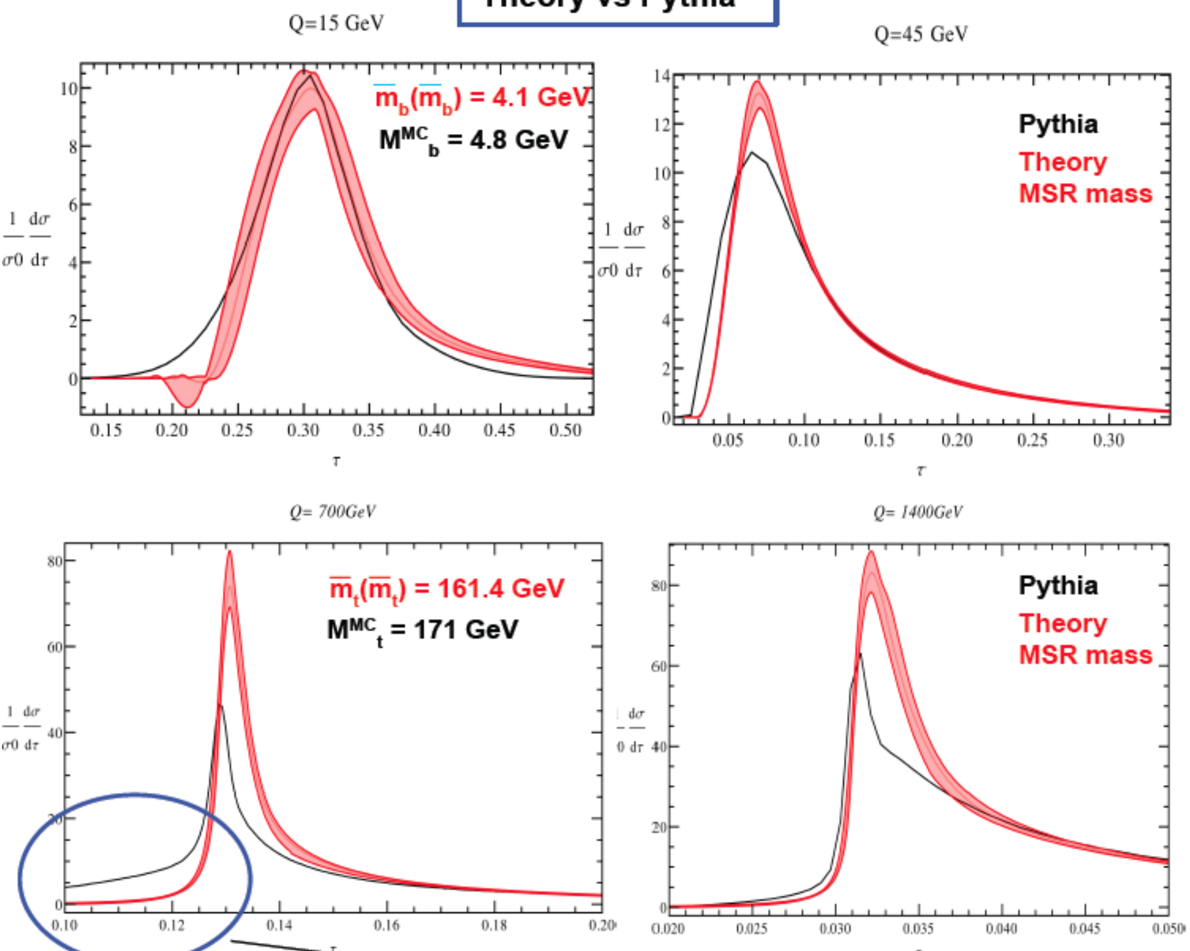
Preliminary Results

Theory vs. Pythia

NNLL (singular) + NLO (non-singular) + power correction and renormalon subtraction

Theory vs Pythia

A. H. Hoang, V. Mateu, BD,
M. Butenschoen & Iain W. Stewart



Shoulder region

- **Agreement of theory - Pythia :**
  - ✔ **Good for bottom**
  - ✔ **Some effect are likely missing (shoulder region) → off shell top + electroweak effects**

$$\alpha_s(m_Z) = 0.1184$$
$$\Omega_1 = 0.5\,\mathrm{GeV}$$

Preliminary Results

Conclusions

Conclusions

- Complete description of the entire thrust distribution for boosted heavy quarks achieved with the formalism of VFNS for final-state jets and a sequence of effective field theory setups.

- The peak position in thrust is very sensitive (particularly at low energies) to the mass.
- Estimating theory errors is challenging
  - ✔ Under control directly at peak and tail.
  - ✔ Below peak still under investigation.

- Our theory uncertainty for the mass extraction is reasonable and encouraging
  - ✔ Bottom $\rightarrow$ less than 0.5 GeV
  - ✔ Top $\rightarrow$ almost 0.5 GeV

- Simultaneous fit for α_s and Ω_1 is difficult, particularly for top $\rightarrow$ could be fixed externally
- Agreement between theory and Pythia:
  - ✔ Good for bottom
  - ✔ Some effects are likely missing for top (shoulder region) $\rightarrow$ off shell top + electroweak effects

Outlook

- Improving the precision to N^3LL seems mandatory.
- Off-shell top production + electroweak effects.

Backup Slides

Masses Loop-Theorists Like to use

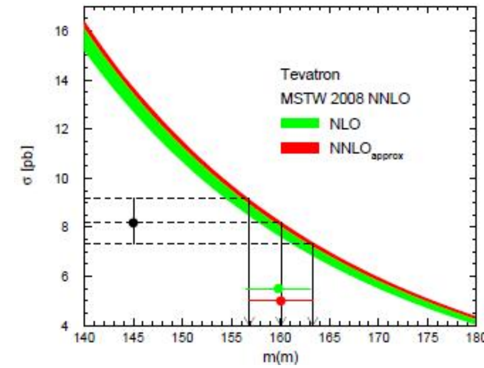
Total cross section (LHC/Tev):

$$m_t^{\rm MSR}(R=m_t)=\overline{m}_t(\overline{m}_t)$$

- more inclusive
- sensitive to top production mechanism (pdf, hard scale)
- indirect top mass sensitivity
- large scale radiative corrections

$$M_t \,=\, M_t^{(O)} \,+\, M_t(0)\alpha_s \,+\, \ldots$$

Langenfeld, Moch, Uwer



Threshold cross section (ILC):

$$m_t^{\rm MSR}(R\sim 20~{\rm GeV})\,,\;m_t^{\rm 1S}\,,\;m_t^{\rm PS}(R)$$

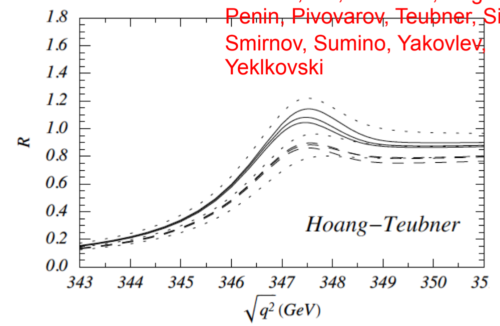
$$M_t \,=\, M_t^{(O)} \,+\, \langle p_{\rm Bohr}\rangle\alpha_s \,+\, \ldots$$

$$\langle p_{\rm Bohr}\rangle \,=\, 20\,{\rm GeV}$$

Mass schemes related to different computational methods

Relations computable in perturbation theory

Beneke, AH, Melnikov, Nagano, Penin, Pivovarov, Teubner, Signer, Smirnov, Sumino, Yakovlev, Yeklkovski



Inv. mass reconstruction (ILC/LHC):

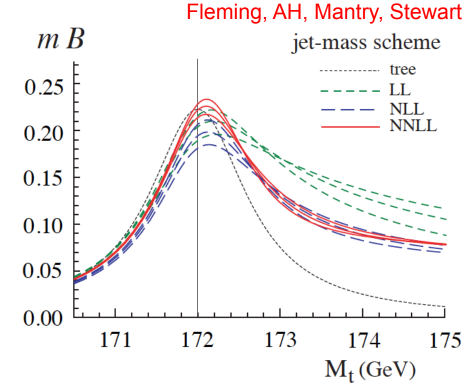
$$m_t^{\rm MSR}(R\sim \Gamma_t)\,,\;m_t^{\rm jet}(R)$$

$$M_t \,=\, M_t^{(O)} \,+\, \Gamma_t\alpha_s \,+\, \ldots$$

$$\Gamma_t \,=\, 1.3\,{\rm GeV}$$

- more exclusive
- sensitive to top final state interactions (low scale)
- direct top mass sensitivity
- small scale radiative corrections

Fleming, AH, Mantry, Stewart

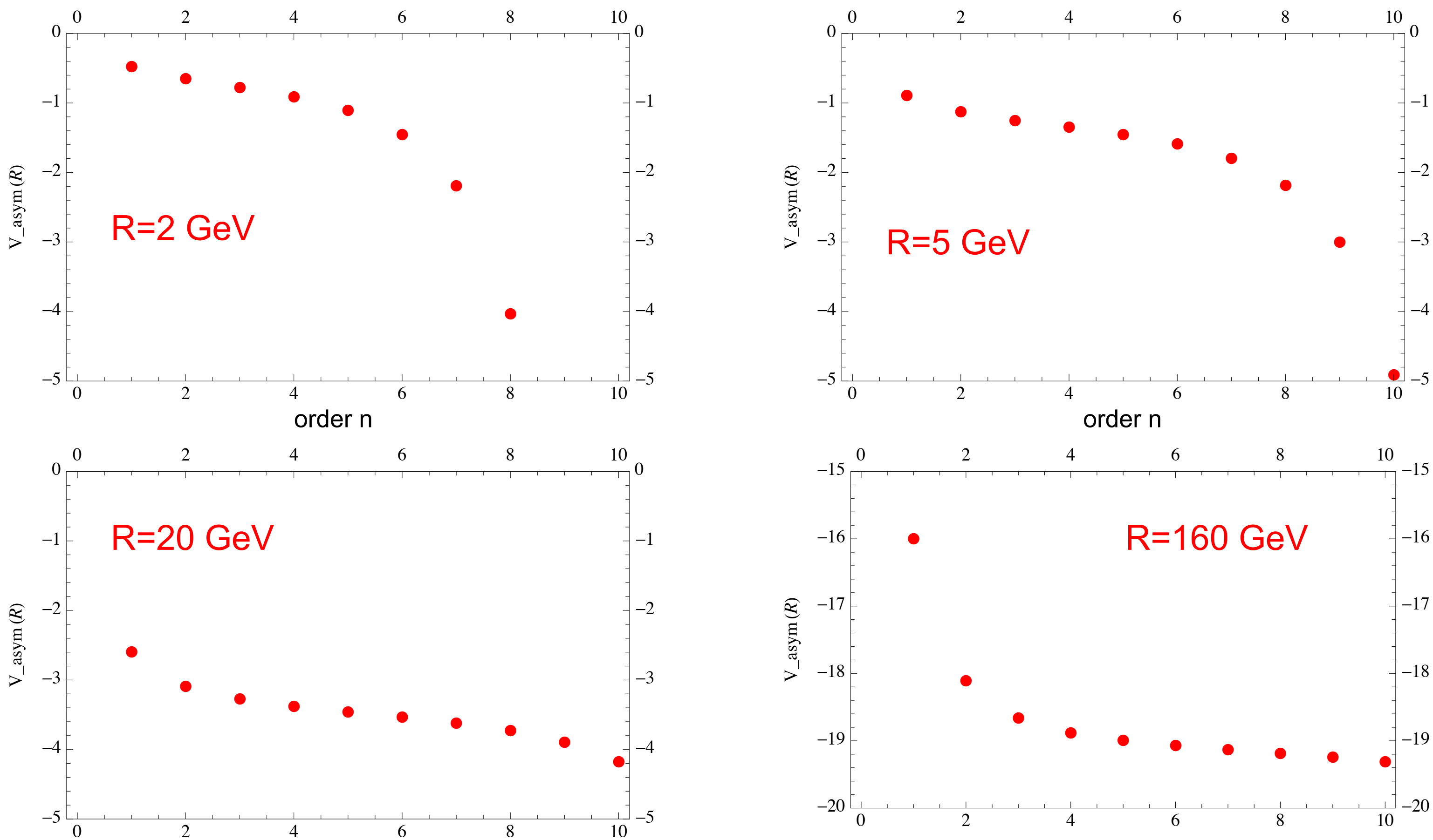


Series with a Renormalon

→ Behavior depends on the typical scale R of the observable ?

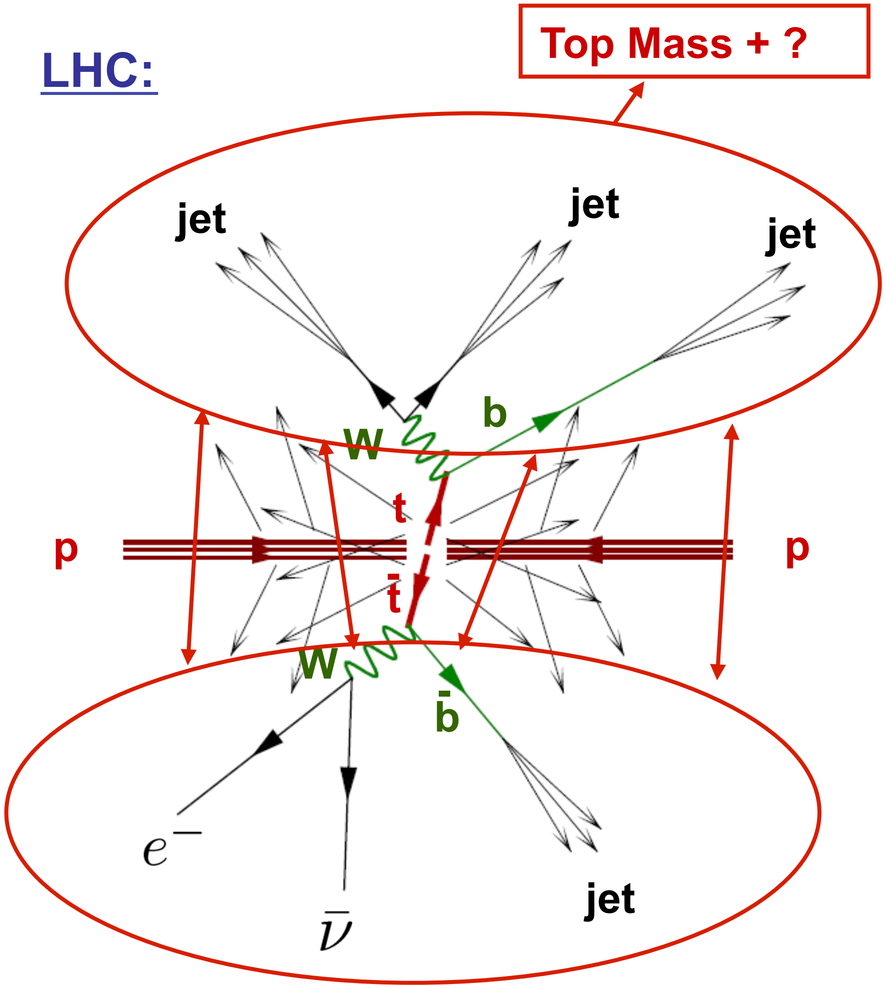
→ Series for large R converge longer, but size of corrections at lower orders are large

→ Formal ambiguity always the same: $\Lambda_{\mathrm{QCD}} \approx 0.5 \ \mathrm{GeV}$



Description of Jets

LHC:



Principle of mass measurements:

Identification of the top decay products

$$“\ m_{\rm top}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2\ ”$$

Problem is non-trivial !

- Measured object does not exist a priori, but only through the experimental prescription for the measurement. **Quantum effects !!**

The idea of a - by itself - well defined object having a well defined mass is incorrect !!

Details and uncertainties of the parton shower and the hadronization models in den MC's influence the measured top quark mass.

NLL Numerical Analysis

Double differential invariant mass distribution:

$$Q = 5 \times 172 \text{ GeV}$$

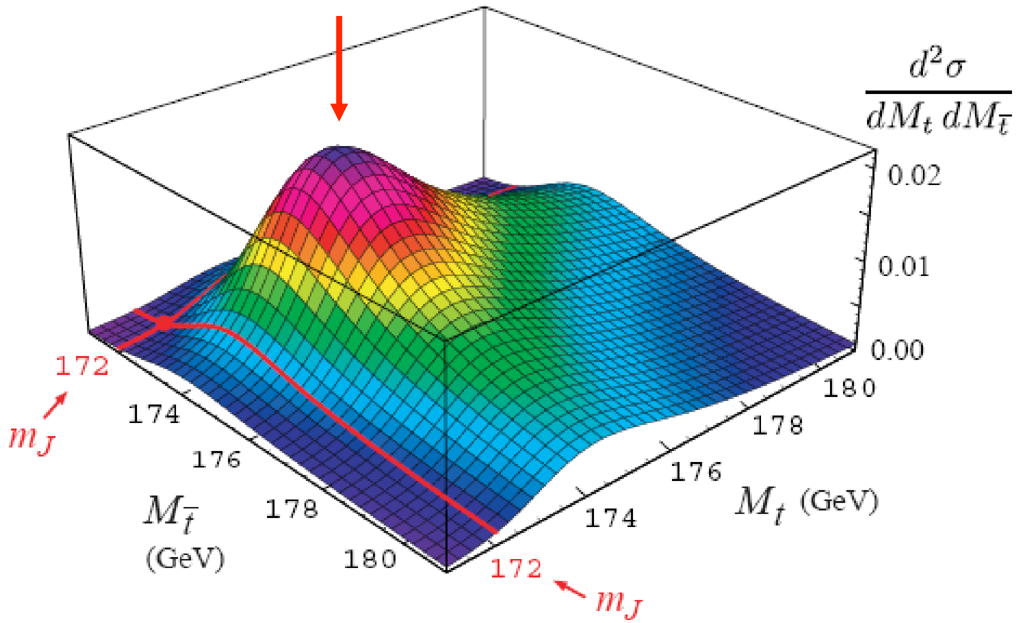
$$\Gamma = 1.43 \text{ GeV}$$

$$m_J(2\,\text{GeV}) = 172 \text{ GeV}$$

$$\mu_\Gamma = 5 \text{ GeV}$$

$$\mu_\Lambda = 1 \text{ GeV}$$

$$a = 2.5\,, \quad b = -0.4$$
$$\Lambda = 0.55 \text{ GeV}$$



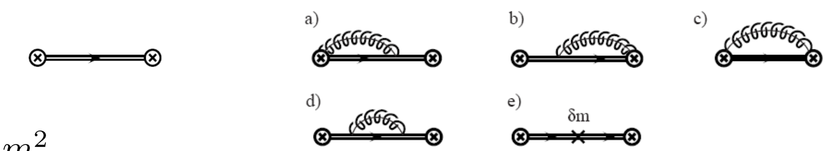
Non-perturbative effects shift the peak by +2.4 GeV
and broaden the distribution.

Reconstructed Top Jets (ILC)

bHQET jet function:

$$B_+(2v_+\cdot k) \;=\; \frac{-1}{8\pi N_c m}\,\mathrm{Disc}\int d^4x\, e^{ik\cdot x}\,\langle 0|\mathrm{T}\{\bar{h}_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)\}|0\rangle$$

- perturbative, <u>any mass scheme</u>
- depends on m_t, Γ_t
- Breit-Wigner at tree level



$$B_\pm(\hat{s},\Gamma_t) = \frac{1}{\pi m_t}\frac{\Gamma_t}{\hat{s}^2+\Gamma_t^2} \qquad \hat{s} = \frac{M^2-m_t^2}{m_t}$$

- Describes soft cross talk of the top (and its decay b quark) with the anti-top (and its decay anti-b quark) in the <u>top rest frame</u>
- Soft function describes soft radiation in the <u>lab frame</u>

Issues sorted out for the first time.

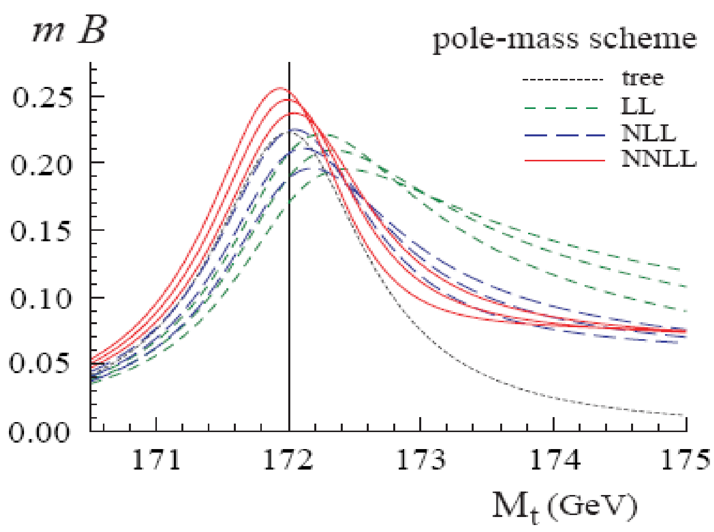
Results still true for LHC (but additional issues to resolved there)

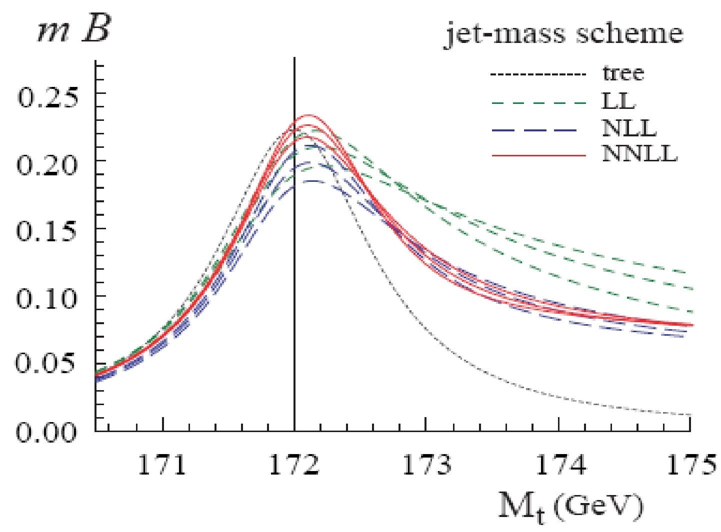
Reconstructed Top Jets (ILC)

→ Jet function has an $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon in the pole mass scheme

$$\mathcal{B}_\pm(\hat{s},0,\mu,\delta m) = -\frac{1}{\pi m}\,\frac{1}{\hat{s}+i0}\left\{1+\frac{\alpha_s C_F}{4\pi}\left[4\ln^2\left(\frac{\mu}{-\hat{s}-i0}\right)+4\ln\left(\frac{\mu}{-\hat{s}-i0}\right)+4+\frac{5\pi^2}{6}\right]\right\} - \frac{1}{\pi m}\,\frac{2\delta m}{(\hat{s}+i0)^2}$$

$$\delta m = m_t^{\rm scheme} - m_t^{\rm pole}$$





Jain, Scimemi, Stewart
PRD77, 094008(2008)

$$m_{\rm pole} = m_J(\mu) + e^{\gamma_E} R\,\frac{\alpha_s(\mu)\,C_F}{\pi}\left[\ln\frac{\mu}{R}+\frac{1}{2}\right] + \mathcal{O}(\alpha_s^2)$$

$$\boxed{R \sim \Gamma_t}$$

Reconstructed Top Jets (ILC)

Why is the pole mass not visible?

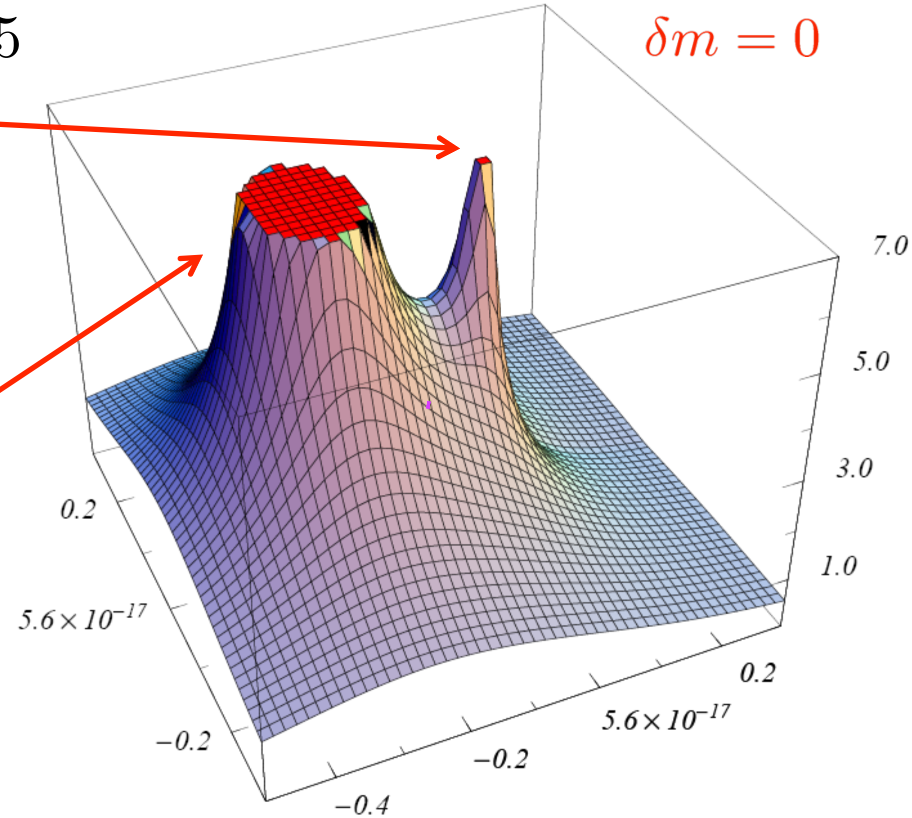
$$\hat{s} = \frac{M_t^2 - m_t^2}{m_t}$$

$$\Gamma = \Gamma_t/5$$

$$|\mathcal{B}_\pm(\hat{s}, \Gamma_t, \mu)|^2$$

$$\delta m = 0$$

pole mass peak



observable peak

→ jet mass is observable

- Located at the visible peak
- Short-distance mass

QCD Factorization

Fleming, Mantry, Stewart, AHH

Phys.Rev.D77:074010,2008
Phys.Rev.D77:114003,2008
Phys.Lett.B660:483-493,2008

$$Q \gg m_t \gg \Gamma_t > \Lambda_{\rm QCD}$$

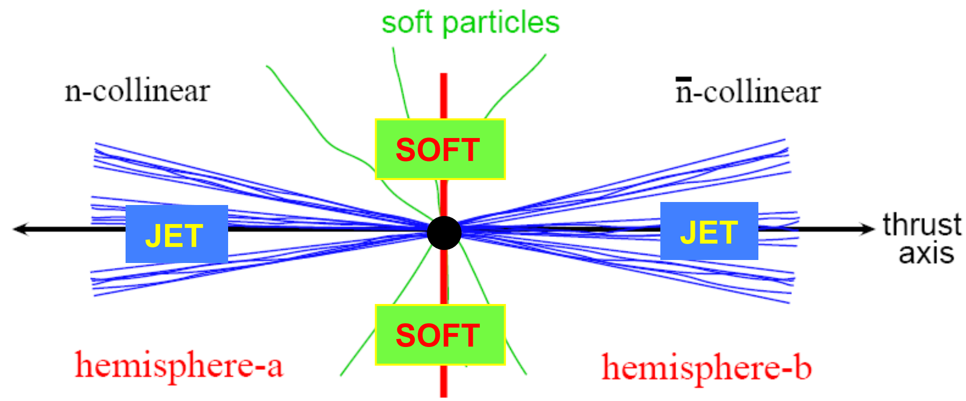
QCD

↓

Soft-Collinear-Effective-Theory

↓

Heavy-Quark-Effective-Theory + **Unstable-Particle-Effective-Theory**



Faktorization Formula

$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \qquad \hat{s} = \frac{M_t^2 - m_J^2}{m_J}$$

$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

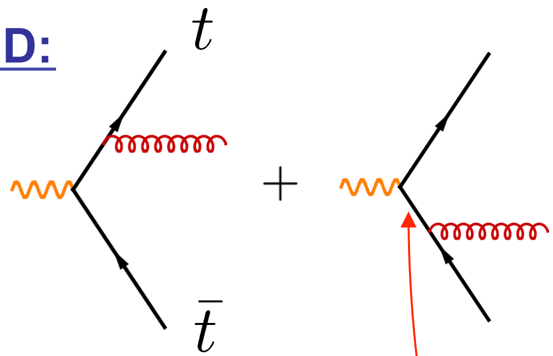
JET **JET** **SOFT**

QCD Factorization

$k_+ = k_0 - k_3$

$k_- = k_0 + k_3$

full QCD:

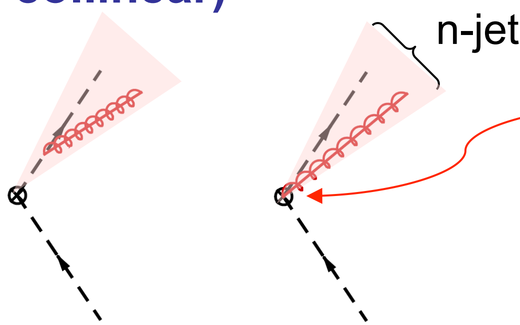


3 phase space regions: $\lambda \sim m_t/Q$

- n-collinear: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, 1, \lambda)$
- $\bar{\text{n}}$-collinear: $(k_+, k_-, k_\perp) \sim Q(1, \lambda^2, \lambda)$
- soft: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2)$

$$\frac{1}{(p_{\bar{t}}+k)^2 - m_t^2} \qquad (p_{\bar{t}}^2 \approx m_t^2\,,\ \ \bar{n}^2 = 0)$$

Gluon collinear to the top:
(n-collinear)



$$\frac{1}{Q\ \ \bar{n}.k}$$

$$W_n^\dagger(\infty, x) = \mathrm{P}\ \exp\left(ig\int_0^\infty ds\bar{n}\cdot A_+(ns+x)\right)$$

$h_{v_+}(x)$ → **gauge dependent**

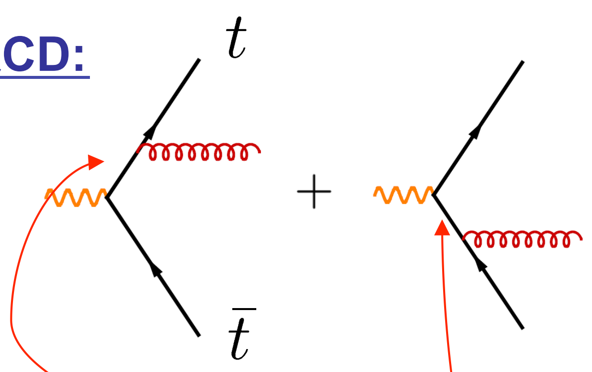
$W_n^\dagger(\infty, x)\, h_{v_+}(x)$ → **gauge independent**

$$B_+(\hat{s}, \Gamma_t, \mu) \ = \ \mathrm{Im}\left[\frac{-i}{12\pi m_J}\int d^4x\, e^{ir.x}\, \langle 0|\, T\, \{\bar{h}_{v_+}(0) W_n(0)\, W_n^\dagger(x) h_{v_+}(x)\}\, |0\rangle\right]$$

$k_+ = k_0 - k_3$
$k_- = k_0 + k_3$

QCD Factorization

full QCD:



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- n̄-collinear: $(k_+, k_-, k_\perp) \sim Q(1, \lambda^2, \lambda)$
- soft: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2)$

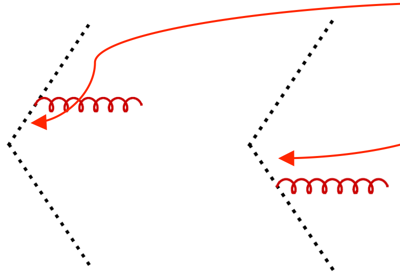
$$\frac{1}{(p_{t,\bar{t}}+k)^2 - m_t^2}$$

$$(p_{t,\bar{t}}^2 \approx m_t^2\,,\;\; n^2 = 0\,,\;\; \bar{n}^2 = 0)$$

Gluon soft:

$$\frac{1}{Q\;\; n.k}$$

$$\frac{1}{Q\;\; \bar{n}.k}$$



$$Y_n(x) = \overline{\mathrm{P}}\,\exp\Big(-ig\int_0^\infty ds\, n\cdot A_s(ns+x)\Big)$$

$$\overline{Y_{\bar{n}}}(x) = \overline{\mathrm{P}}\,\exp\Big(-ig\int_0^\infty ds\, \bar{n}\cdot \overline{A}_s(\bar{n}s+x)\Big)$$

$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\langle 0|\,(\overline{Y}_{\bar n})^{cd}\,(Y_n)^{ce}(0)\,\delta(\ell^- - (\hat{P}_a^+)^\dagger)\delta(\ell^- - \hat{P}_b^-)\,(Y_n^\dagger)^{ef}(\overline{Y}^\dagger_{\bar n})^{df}(0)\,|0\rangle$$

QCD Factorization

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Jet functions:

$$B_+(\hat s, \Gamma_t, \mu) = \operatorname{Im}\left[\frac{-i}{12\pi m_J} \int d^4x\, e^{ir.x} \langle 0| T\{\bar h_{v_+}(0) W_n(0)\, W_n^\dagger(x) h_{v_+}(x)\}|0\rangle\right]$$

- perturbative
- dependent on <u>mass, width, color charge</u>

$$B_\pm^{\text{Born}}(\hat s, \Gamma_t) = \frac{1}{\pi m_t} \frac{\Gamma_t}{\hat s^2 + \Gamma_t^2} \qquad \hat s = \frac{M^2 - m_t^2}{m_t}$$

Soft function:

$$S_{\text{hemi}}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k_s^{+a}) \delta(\ell^- - k_s^{-b}) \langle 0| \overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle \langle X_s| Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger(0)|0\rangle$$

- non-perturbative
- analogous to the pdf's
- dependent on <u>color charge, kinematics</u>

Independent of the mass !

MC Mass

- Concept of mass in the MC depends on the **structure and reliability of the perturbative part** and the **interplay of perturbative and nonperturbative part** in the MC.

Parton shower (kinematic expansion)

Perturbative corrections

Hadronization Models

top mass definition decided here !

scheme-dependent separation (shower cut)

affect all top mass dependent observables

- Assume that the MC is a good QCD box (LO of s.th. more precise): How can one pin down the relation between m_t^{Pythia} and the Lagrangian mass ?
- Is the MC really a good QCD box ? Is the MC more a model or more QCD ?

Answer for m_t^{Pythia} might be process- and observable-dependent if the MC is not a good QCD box !