Topical Workshop

"Top mass: challenges in definition and determination"

$$\text{\textbf{\textit{What is }} } m_t \text{ \textbf{?}}$$

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introduction
- the obvious
- motivation
- sketch of a clean top mass determination
- linear collider threshold scan

mass definitions
- pole mass
- infrared ambiguity
- short distance mass and threshold mass

mass determinations
- m_t from cross-section like observables
- m_t from invariant mass of decay products

toy analysis
- $t\bar{t}$ at NLO
- scheme dependence

summary/conclusion

- the top is unstable, no top has ever be seen, only its decay products

- it is usually said that m_t is a fundamental parameter of the SM, but there are infinitely many (different) m_t

- the parameter in the Lagrangian is the bare mass $\mathcal{L} \ni m_0$

- stating a numerical value for m_t without giving a precise definition of what is meant by it is meaningless

- could have $m_t \simeq 165$ GeV ($\overline{\mathrm{MS}}$ -scheme) or $m_t = 173$ GeV (pole scheme) or anything (in between)

- at LO $m_t^{\text{any scheme}} = m_0$, beyond LO need to fix a renormalization scheme (usually also includes choice of one or more scales)

- same for other fundamental parameters: by e.g. $\alpha_s = 0.118$ we mean α_s in the $\overline{\mathrm{MS}}$ -scheme at the scale $\mu = M_Z$ is 0.118 (or whatever)

- there is no 'best' renormalization scheme, hence no 'best' definition of m_t

- in principle !?!, there is a perturbatively computable relation between m_t in two different schemes

- if you don't think a precise determination of the top mass is important, there is still time to leave the room
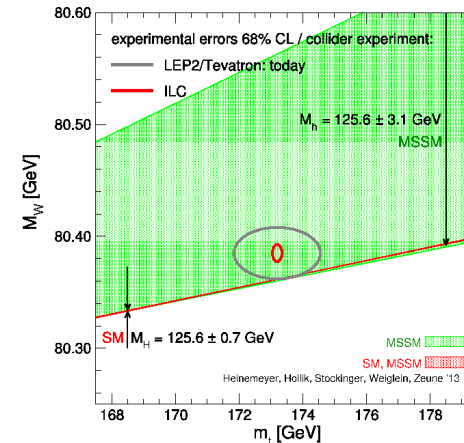
- 2014 'world average' (Atlas, CDF, CMS, D0) [1403.4427]

$$m_t = 173.34 \pm 0.27 \, (\text{stat}) \pm 0.71 \, (\text{sys}) \, \text{GeV}$$

- 'all' exp results going into the result above have been obtained 'in the same way' (from invariant mass of decay products)

- other determinations (e.g. from m_t dependence of cross section) lead to considerably larger error

- there are 1001 issues that need to be discussed and understood for the above result (colour reconnection, hadronization, parton showers . . .)

- I have nothing to say or add to 999 of them

- this leaves me with the infrared ambiguity of pole mass and scheme dependence of top mass extractions

- this value for m_t is/will be taken as input for many other observables
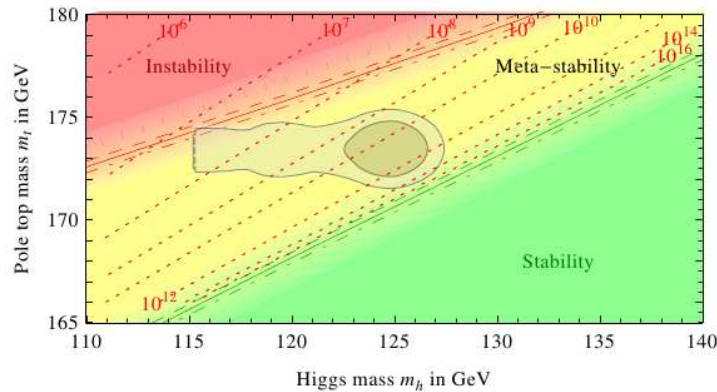
Degrassi et al. [1205.6497]   [Heinemeyer et al. 2013]

- we have to understand how this quantity m_t is related to a well defined (renormalized) mass parameter

- the issue is not (and never was!!) whether this is the pole mass or the $\overline{\text{MS}}$ mass

- many (all) issues discussed here are irrelevant if $\delta m_t \sim 2\,\text{GeV}$ but are definitely relevant if $\delta m_t \sim 0.5\,\text{GeV}$

- many (all) issues discussed here have been discussed before in the context of determinations of m_b (even m_c) and m_t from a linear collider threshold scan

a clean way to determine m_t (or any other fundamental parameter)

- choose a 'good' observable (total cross section near threshold)
  - very sensitive to m_t
  - easy to measure
  - can get reliable and precise theoretical prediction
- choose a 'good' renormalization scheme RS_1
  - mass parameter is well defined (to the required accuracy)
  - perturbative expansion is under control
- compare theory vs experiment $\rightarrow$ extract m_{RS_1} and δm_{RS_1} and do not stop there
- repeat this procedure for another (good) renormalization scheme RS_2
  $\rightarrow$ extract m_{RS_2} and δm_{RS_2}
- check consistency: relate (perturbatively) m_{RS_2} to m_{RS_1}
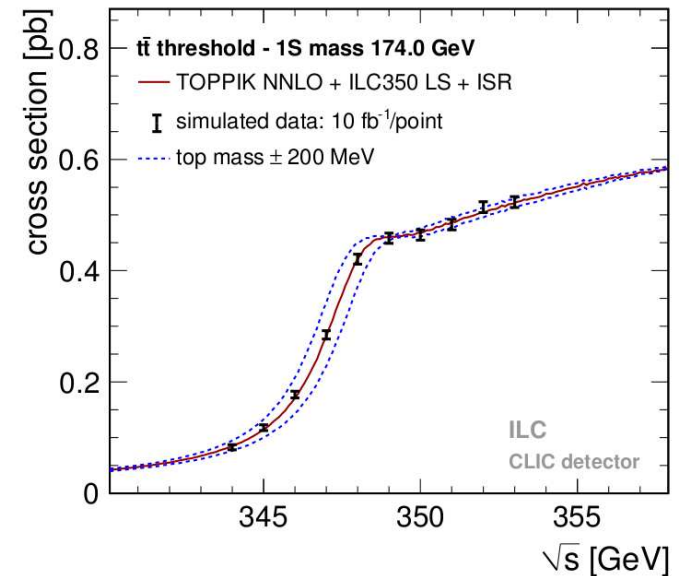
$$m_{\mathrm{RS}_1} = m_{\mathrm{RS}_2}^{(0)} + \sum_{i=1}^{n} \alpha^i \, c^{(i)}(m_{\mathrm{RS}_2}) + \text{higher order}$$

- add scheme dependence as 'systematic theory error'

a 'theory' plot
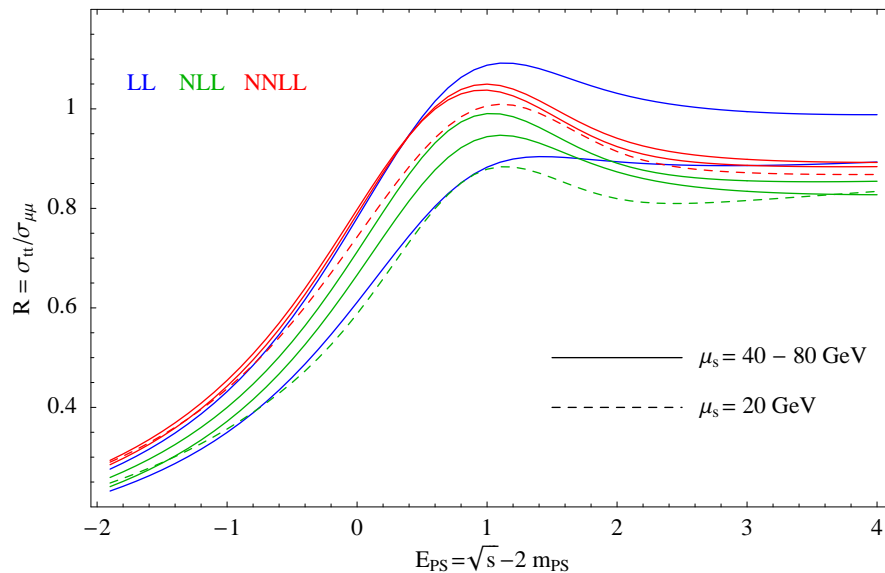
a 'real' plot ISR and beamstrahlung

- calculation done at NNLO and NNLL (and very soon NNNLO)
  [Beneke et al; Pineda et al; Hoang et al; . . .]

- peak (in theory plot) is remnant of 'want-to-be' (1S) bound states

- different mass schemes in use: PS mass, RS mass, 1S mass but not the pole mass

- cannot compute directly in $\overline{MS}$ scheme, but can convert to $\overline{MS}$ mass after extraction

- consider top quark propagator $\dfrac{1}{\not{p} - m_0} \xrightarrow{\text{h.o.}} \dfrac{1}{\not{p} - m_0 - \Sigma}$

- full self energy involves all scales, also $k \lesssim \Lambda_{\text{QCD}}$

$$\Sigma = \Sigma_{\text{div}} + \Sigma_{\text{fin}} = \int_0^\infty d^D k \ldots$$
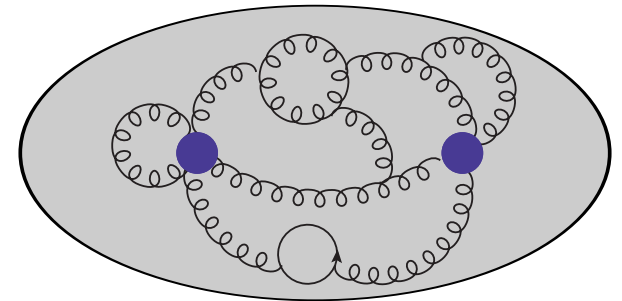
- pole mass defined as (real part) of position of pole of propagator

$$(m_0 + \Sigma_{\text{div}} + \Sigma_{\text{fin}}) \Rightarrow m_{\text{pole}}$$

- many nice properties (e.g. no infrared singularity) and 'physical' mass for leptons

- the pole mass has an intrinsic uncertainty of order Λ_{QCD} (since Σ_{fin}(all scales))

consider (fictitious) meson:

$$\underbrace{M}_{\text{well def. pole mass}} = \underbrace{2\, m_{\text{pole}}}_{\text{pert. ambiguity}} + \underbrace{V_{\text{Coul}}(q^2)}_{\text{pert. ambiguity}}$$

- pole mass is a threshold mass but NOT a short-distance mass

- masses of bottom and charm mass are never given in the pole scheme $\delta m < \Lambda_{\text{QCD}}$

- $\overline{\text{MS}}$ mass does not have the problem of infrared sensitivity (only pure UV is absorbed into mass definition)

$$(m_0 + \Sigma_{\text{div}}) \Rightarrow m_{\overline{\text{MS}}} \quad \rightarrow \quad \text{short distance mass}$$

- but the pole of the propagator is far away from $m_{\overline{\text{MS}}}$ $\rightarrow$ NOT a threshold mass

- this is not a problem as such, but

- for physical process at threshold cannot use $\overline{\text{MS}}$ mass, but have to use threshold mass (differs by at most $\alpha_s^2 m_t$ from m_{pole}) ($m_{\overline{\text{MS}}} \sim 165 \text{ GeV}$, $m_{\text{pole}} \sim 173 \text{ GeV}$)

- then relate threshold mass to $m_{\overline{\text{MS}}}$; 4-loop exact [Marquard et al.]
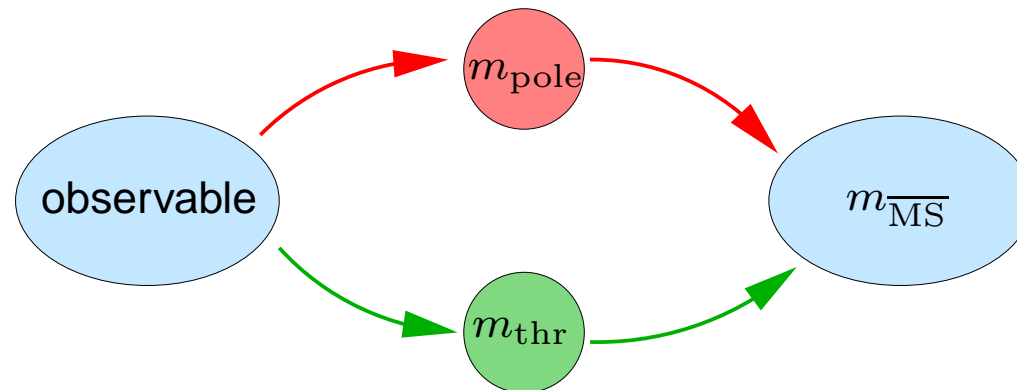
we can have the cake and eat it

- many options for short distance threshold mass definitions:

    - potential subtracted mass (PS mass) [Beneke]

    $$m_{\mathrm{PS}}(\mu_{\mathrm{PS}}) \equiv m_{\mathrm{pole}} + \frac{1}{2} \int\limits_{|\vec{q}|<\mu_{\mathrm{PS}}} \frac{d^3\vec{q}}{(2\pi)^3} V_{\mathrm{Coul}}(q) \quad \text{with} \quad \mu_{\mathrm{PS}} \sim m\,\alpha_s$$

    $$m_{\mathrm{pole}} = m_{\mathrm{PS}}(\mu_{\mathrm{PS}}) + \mu_{\mathrm{PS}} \left[\frac{\alpha_s}{2\pi}\delta_1 + \frac{\alpha_s^2}{(2\pi)^2}\delta_2 + \dots \right]$$

    - renormalon subtracted mass (RS mass) [Pineda]

      identify source and subtract

    - 1S mass [Hoang] $m_{1\mathrm{S}} = M_{1S}/2$

- its not clear whether we have to use short distance masses in the case of top (recall $\delta m_t \sim 0.7\ \mathrm{GeV}$) but we certainly are allowed to do so!

- whether we have to use a threshold mass or not depends on the observable:
  e.g. $t\bar{t}$ cross section near threshold (linear collider) or invariant mass of (reconstructed) top, but NOT e.g. total cross section $t\bar{t}$ at hadron collider
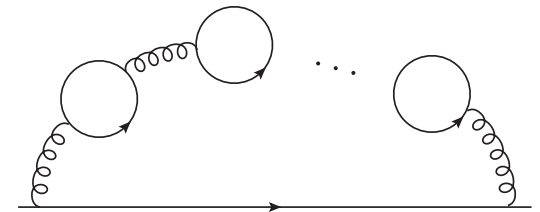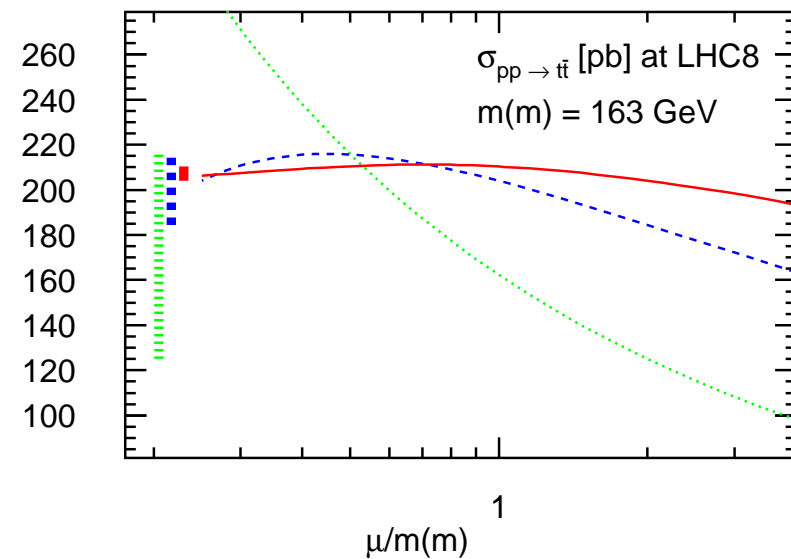
find observable with large m_t sensitivity and compute beyond LO

example 1:

determination of m_{pole} and/or $m_{\overline{\mathrm{MS}}}$ through total cross section [Dowling, Moch, 1305.6422]
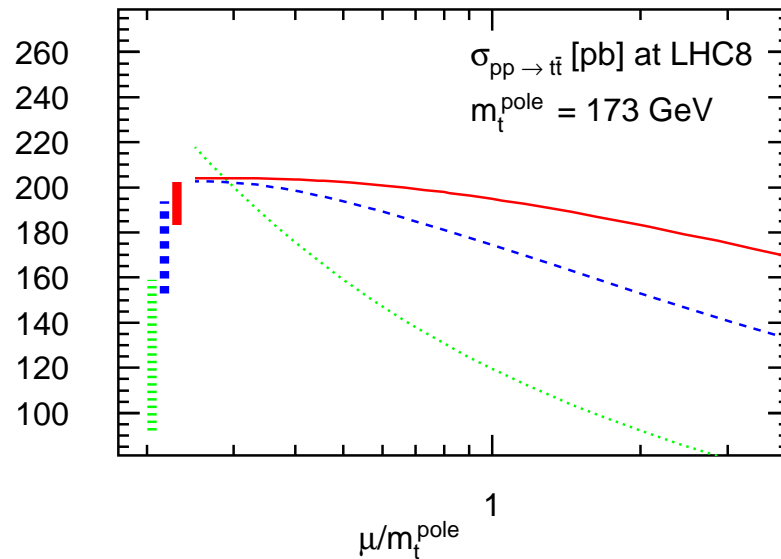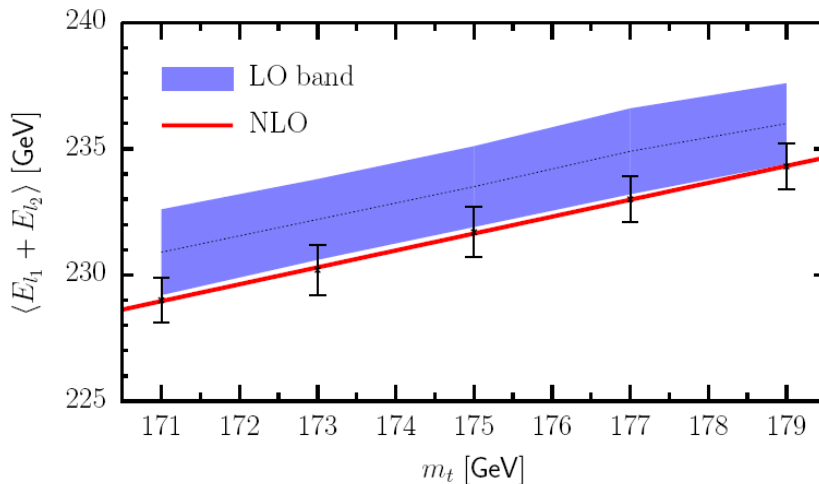

pole scheme

$\overline{\mathrm{MS}}$ scheme (better !?)

claim: $\delta m_t \sim 2.5\ \mathrm{GeV}$
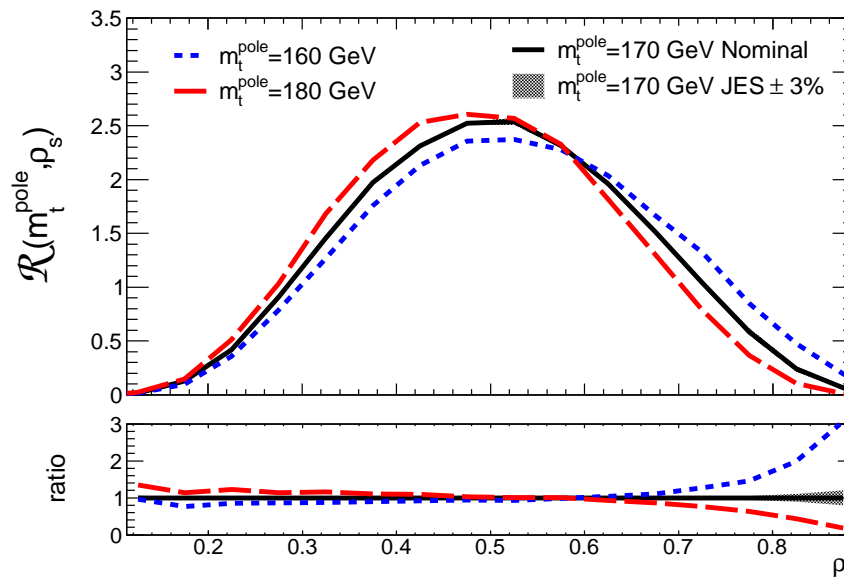
example 2: [Biswas, Melnikov, Schulze, 1006.0910]

m_{pole} through $\langle E_\ell + E_{\ell'} \rangle$

compare $\delta_{\mathrm{th}} m$ (PDF, higher order) with m_t sensitivity

claimed $\delta_{\mathrm{th}} m$: 1.7 (LO) $\rightarrow$ 1 GeV (NLO)
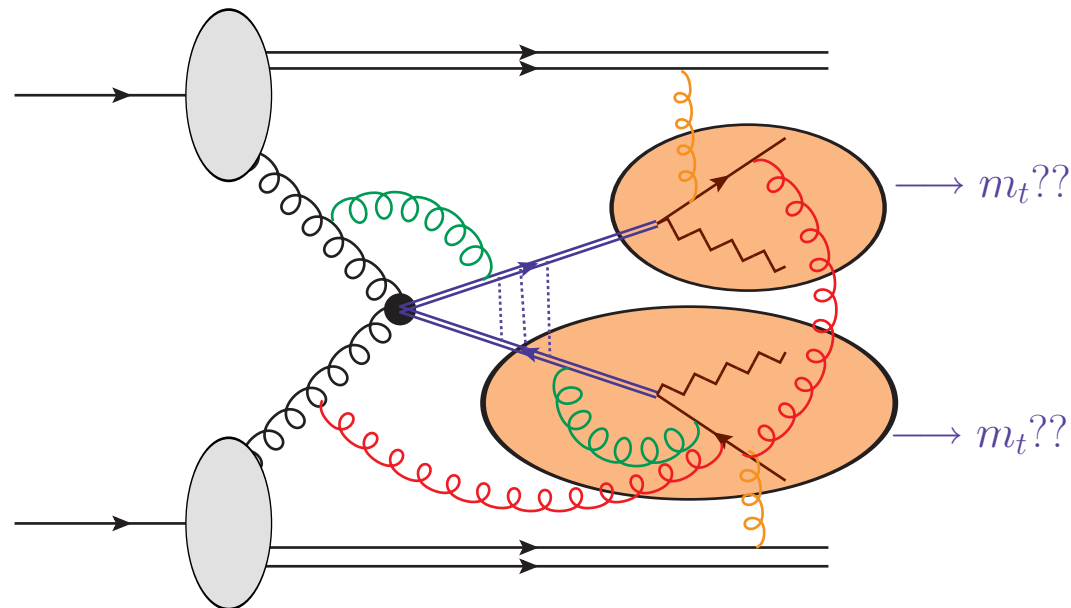
example 3: [Alioli et al. 1303.6415]

$$\mathcal{R}(m_{\mathrm{pole}}, \rho) = \frac{1}{\sigma_{t\bar{t}j}} \frac{d\sigma_{t\bar{t}j}}{d\rho}(m_{\mathrm{pole}}, \rho)$$

could (should?) use other mass schemes as well

claimed $\delta m_{\mathrm{pole}} \sim 1\,\mathrm{GeV}$

m_t from invariant mass of decay products

- many effects!! some non-perturbative

- here only 2 out of 1001...: consider partonic calculation of invariant mass of reconstructed top i.e. $M_t \equiv M(J_b, W) \equiv \sqrt{(p_{J_b} + p_W)^2}$

- better do this beyond LO !! ($\rightarrow$ need to consider off-shell top quarks [Bevilacqua et al; Denner et al; Heinrich et al.]) and in more than one mass scheme !! [Falgari et al.]

via effective theory approach [Falgari et al. 1303.5299]

integrate out hard modes $\rightarrow$ effective Lagrangian

$$\mathcal{L} = \phi^\dagger B\phi + c_p\,\phi(\Pi\psi_i) + c_d\phi(\Pi\chi_j) + c_b\,(\Pi\psi_i\chi_j) + \bar\psi D_s\psi + \dots$$
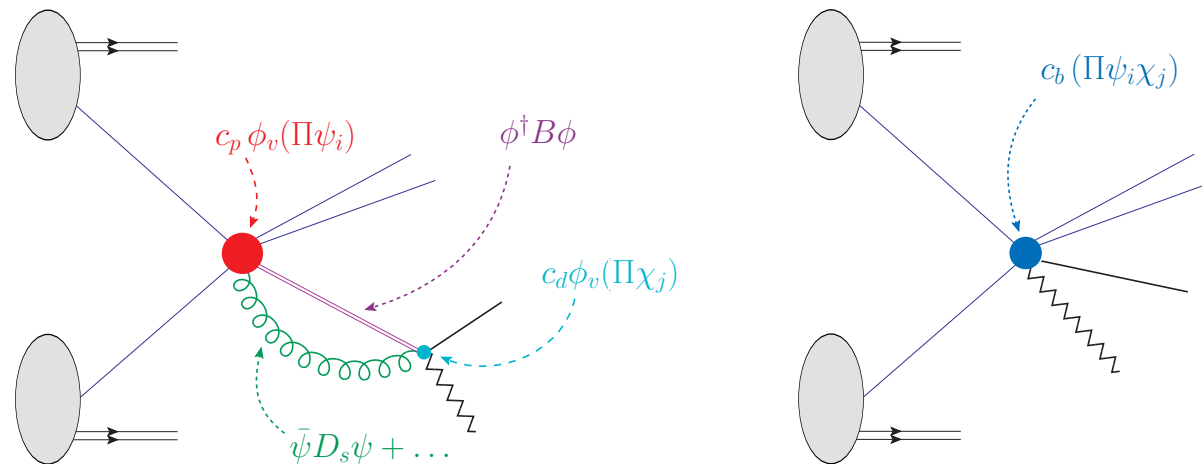

- matching coefficients c_i contain effects of hard modes

- matching done on shell, $p_X^2 = \bar{s} = m_X^2 + \mathcal{O}(\delta)$, with $\bar{s}$ the complex position of pole

- soft (and collinear ...) d.o.f. still dynamical

- can be combined with further resummations (e.g. non-relativistic $\rightarrow$ ET has more complicated structure)
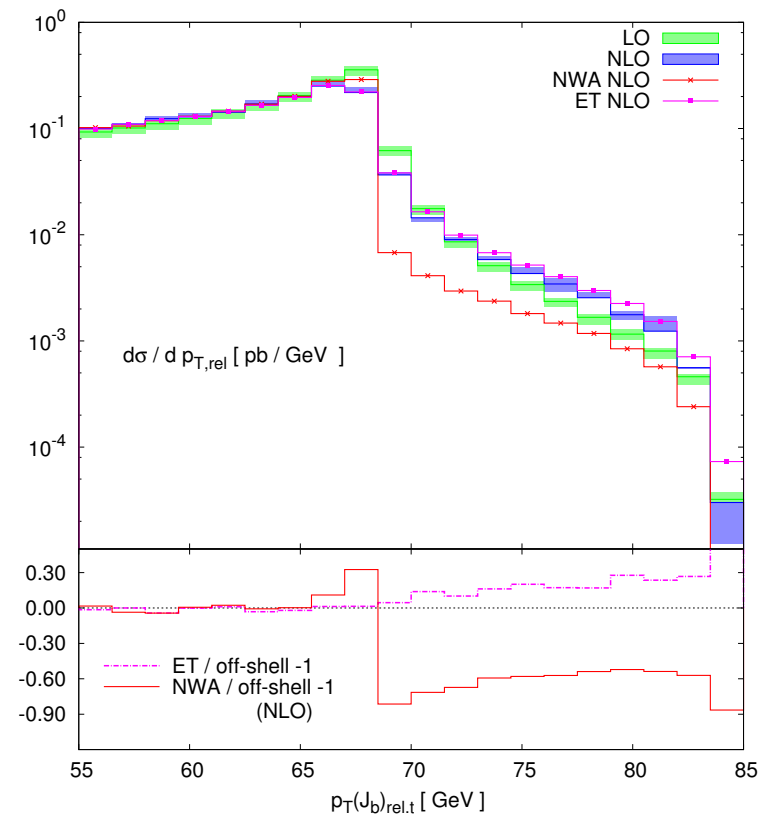
comparison EFT approach vs complex mass scheme

calculation for single top $\Rightarrow$ good agreement [Papanastasiou et al. 1305.7088]

invariant mass
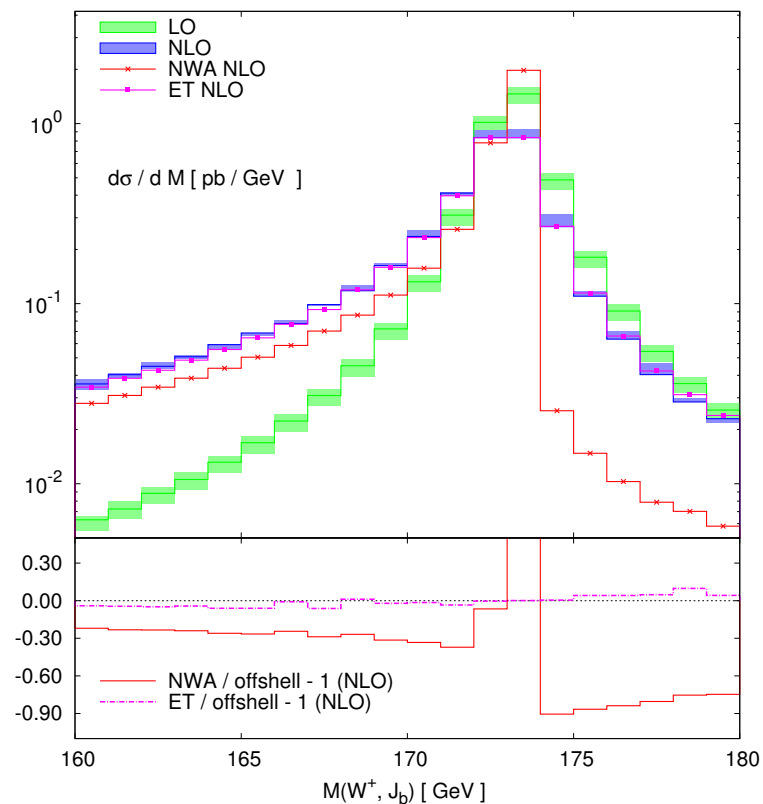
relative transverse b-jet momentum

$t\bar{t}$ at NLO with short-distance (threshold) mass [Falgari et al. 1303.5299]

- toy analysis with some jet definition ($k_\perp$, $R = 0.7$) and some cuts on final state particles/jets (decay of W in NWA)

- consider mass scheme different from pole mass m_{pole}
    - check scheme dependence
    - avoid infrared sensitivity of pole mass

- example used here: potential subtracted mass m_{PS} [Beneke]

$$m_{\text{PS}}(\mu_{\text{PS}}) = m_{\text{pole}} + \frac{1}{2} \int\limits_{q < \mu_{\text{PS}}} \frac{d^3\vec{q}}{(2\pi)^3} V_{\text{coul}}(q) \quad \text{with} \quad \mu_{\text{PS}} \sim m\,\alpha_s \sim \delta^{1/2}$$

- note $m_{\text{PS}}(\mu_{\text{PS}} = 0) = m_{\text{pole}}$ and $\mu_{\text{PS}} \lesssim 20\ \text{GeV}$ to have threshold mass
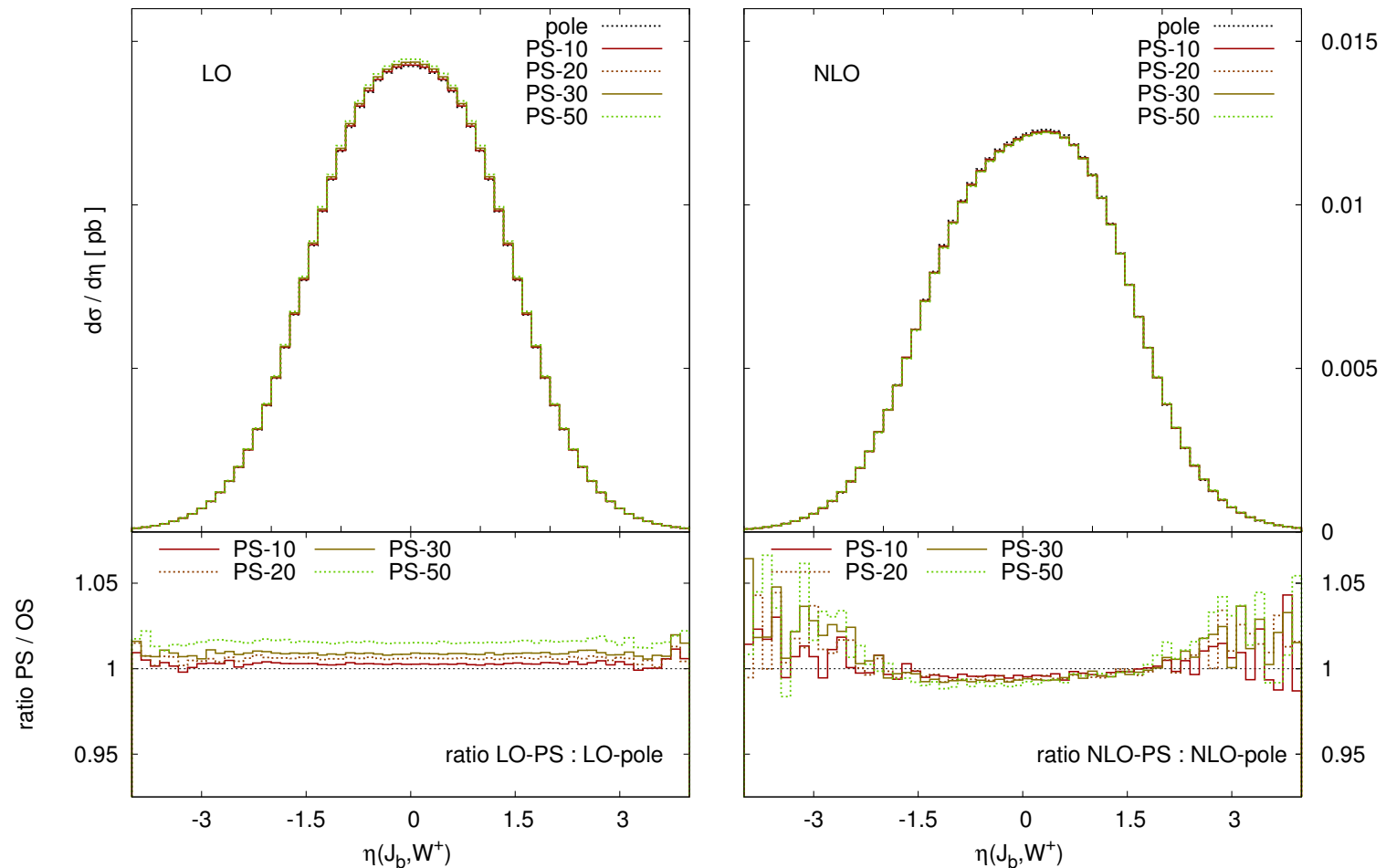
- express everything in terms of m_{PS}

$$m_{\text{pole}} = m_{\text{PS}}(\mu_{\text{PS}}) + \mu_{\text{PS}} \left[\frac{\alpha_s}{2\pi} \delta_1 + \frac{\alpha_s^2}{(2\pi)^2} \delta_2 + \ldots \right]$$

- (inverse of) propagator (counting $\delta \sim \alpha_{\text{ew}} \sim \alpha_s^2$):

$$\underbrace{p^2 - m_{\text{PS}}^2 + i m_{\text{PS}}\Gamma}_{\sim \delta} - \underbrace{\frac{\alpha_s}{\pi}\, \delta_1 \mu_{\text{PS}}\, m_{\text{PS}}}_{\sim \delta} - \underbrace{\frac{\alpha_s^2}{2\pi^2}\, \delta_2 \mu_{\text{PS}}\, m_{\text{PS}}}_{\sim \delta^{3/2}} + \ldots$$
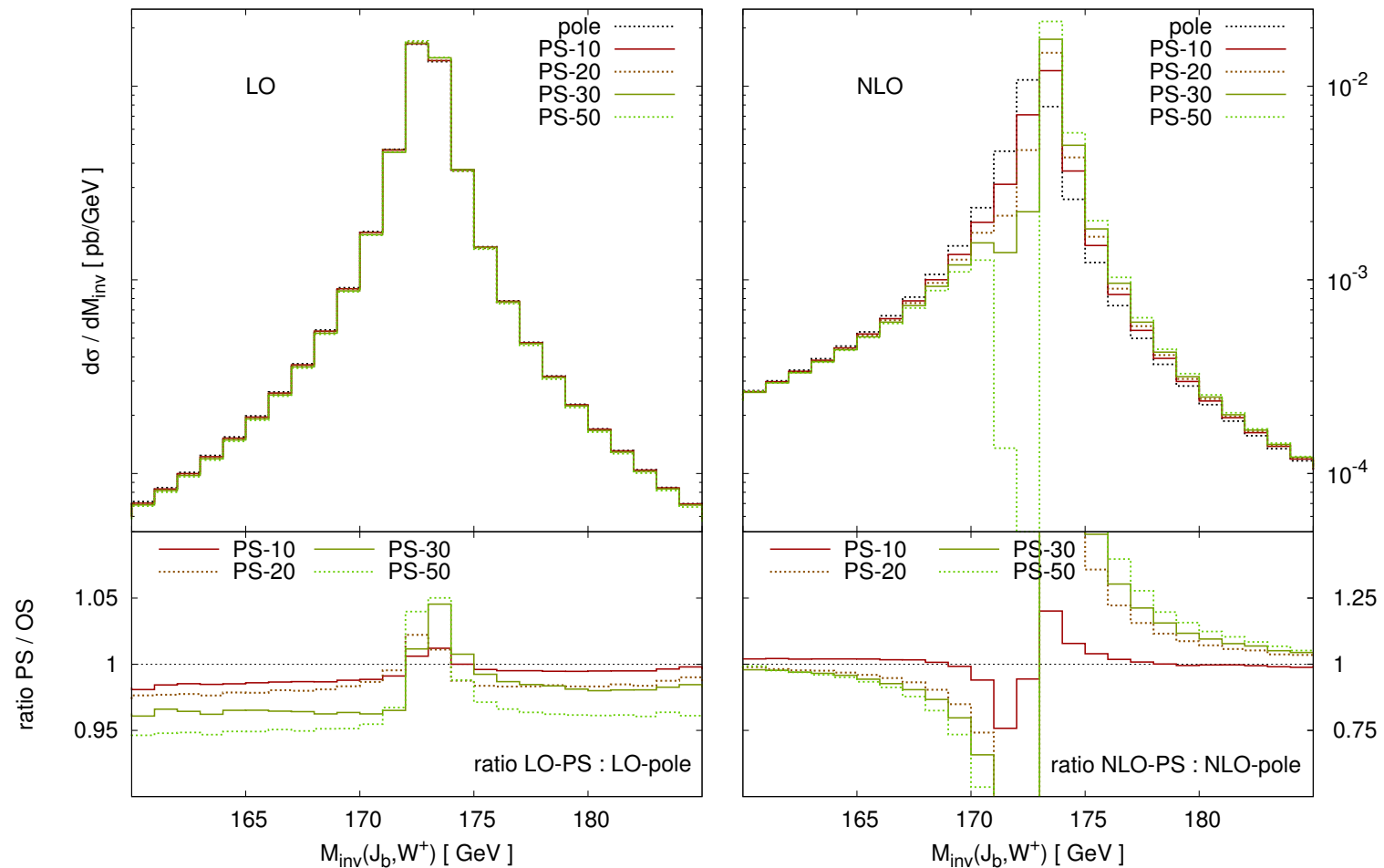
results in PS scheme $\mu_{\mathrm{PS}} \in \{0, 10, 20 \; !?, 30 \; ??, 50 \; ???\} \; \mathrm{GeV}$

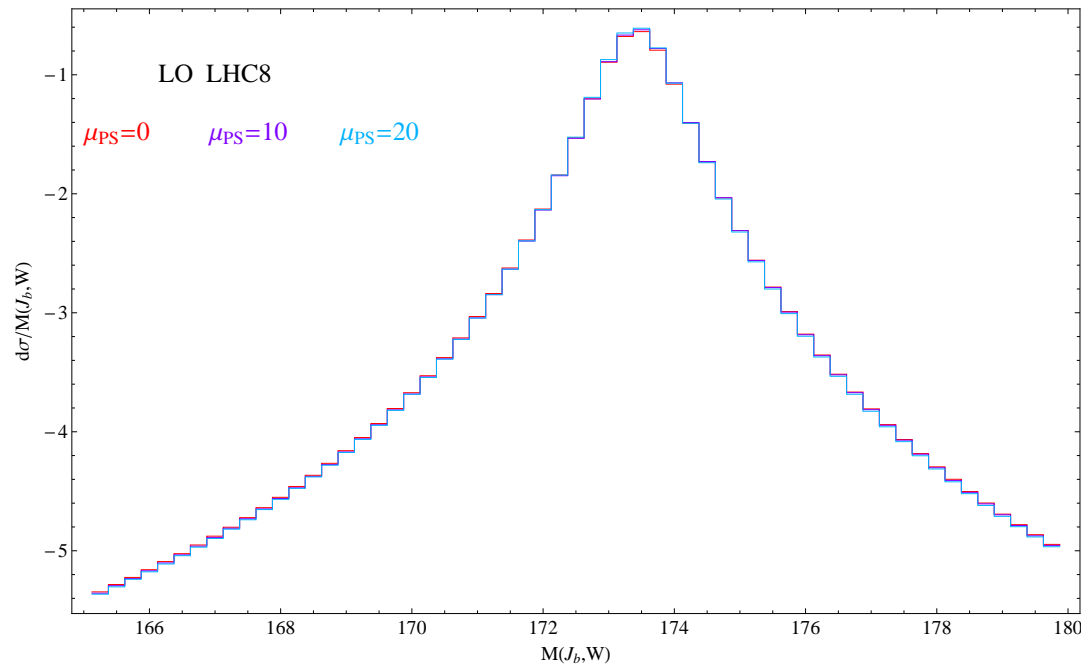example of non-sensitive observable (pseudo-rapidity of 'top') (here Tevatron, $q\bar{q}$ only)

results in PS scheme $\mu_{\mathrm{PS}} \in \{0, 10, 20 \ !?, 30 \ ??, 50 \ ???\} \ \mathrm{GeV}$

example of sensitive observable (invariant mass of 'top') $\Rightarrow \mu_{\mathrm{PS}} \lesssim 20 \ \mathrm{GeV}$

extract m_t at LO

- assume distribution for $m_{\mathrm{pole}} = m_{\mathrm{PS}}(0) = 173.3\ \mathrm{GeV}$ is 'true' distribution

- adjust $m_{\mathrm{PS}}(10)$ and $m_{\mathrm{PS}}(20)$ to fit this 'true' distribution
- result at LO: $m_{\mathrm{PS}}(10) = 172.8\ \mathrm{GeV}$ and $m_{\mathrm{PS}}(20) = 172.4\ \mathrm{GeV}$

extract m_t at NLO assume again 'true' distribution is the one with $m_{\text{pole}} = 173.3 \text{ GeV}$

- extract mass at NLO:
  $m_{\text{PS}}(10) = 172.6 \text{ GeV}$ and
  $m_{\text{PS}}(20) = 172.1 \text{ GeV}$

- perturbative behaviour very good for $\mu_{\text{PS}} = 10 \text{ GeV}$ and resonable for $\mu_{\text{PS}} = 20 \text{ GeV}$

- $\mu_{\text{PS}} \gtrsim 30 \text{ GeV} \rightarrow$ 'bad' scheme
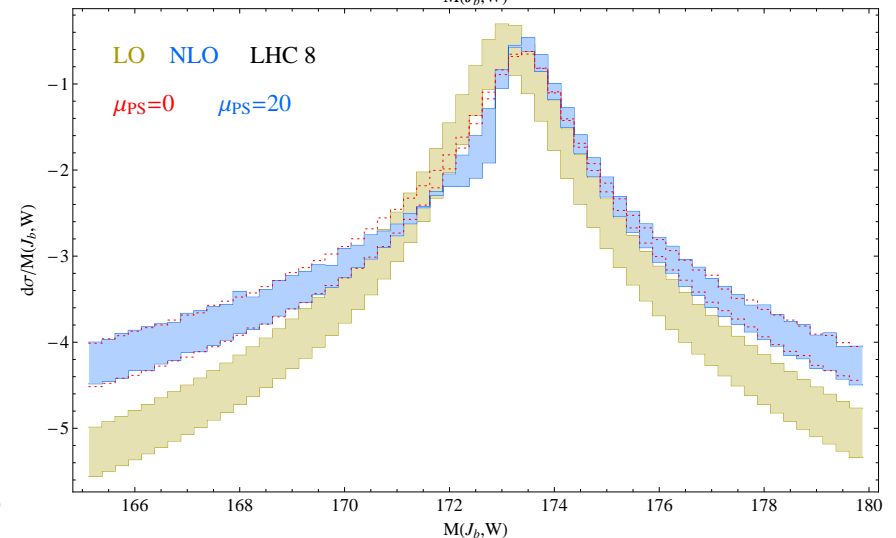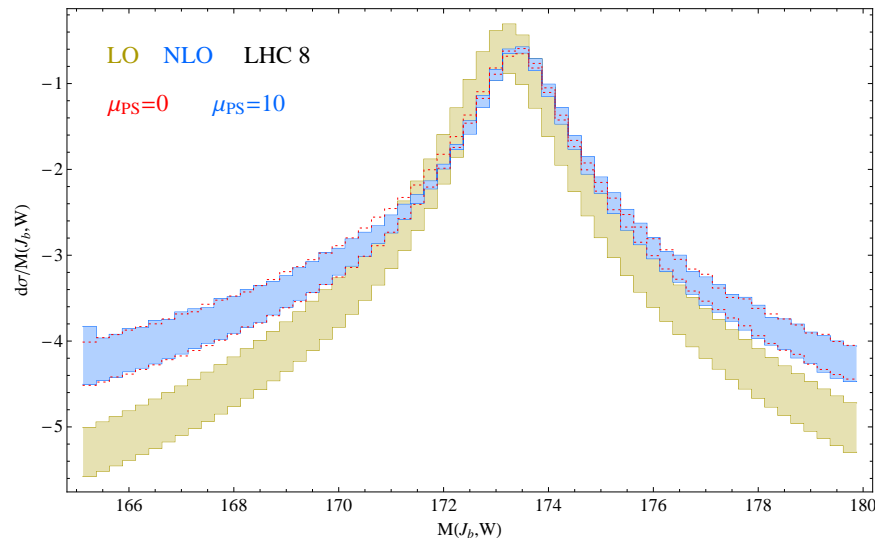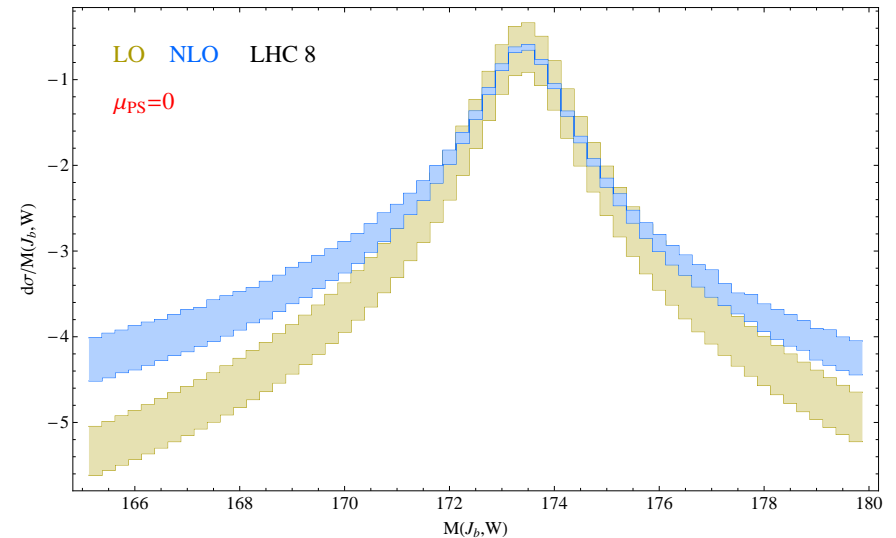
consider scheme dependence of mass extraction or what is the best value for $m_{\overline{\mathrm{MS}}}$

μ_{PS}	LO			NLO		
	m_{exp}	$m_{\overline{\mathrm{MS}}}$	m_{pole}	m_{exp}	$m_{\overline{\mathrm{MS}}}$	m_{pole}
0	173.3	162.6	173.3	173.3	162.6	173.3
10	172.8	163.1	173.9	172.6	162.9	173.7
20	172.4	163.3	174.2	172.1	163.0	173.9
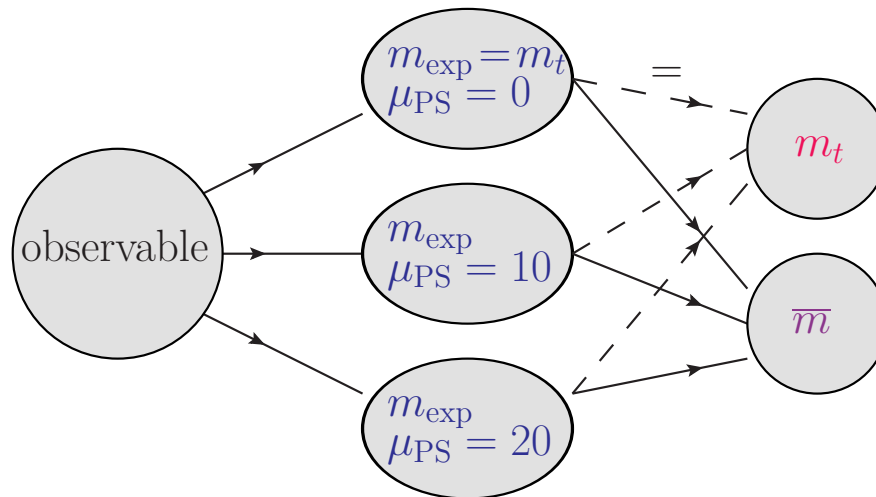
- conversion at NNNLO (+ Pade approximation)

- scheme ambiguity $\sim 500 - 900 \ \mathrm{MeV}$ at LO

- scheme ambiguity $\sim 300 - 600 \ \mathrm{MeV}$ at NLO

- $\overline{\mathrm{MS}}$ scheme somewhat more stable

- issue 1: infrared sensitivity of $m_{\rm pole}$ scale $\mathcal{O}(\Lambda_{\rm QCD})$

    - principal limitation on precision for $\delta m_{\rm pole}$
    - does not yet seem to be a show stopper for $\delta m_{\rm pole} \sim 0.7 \text{ GeV}$
    - will get ever more important for decreasing $\delta m_{\rm pole}$

- issue 2: scheme dependence of m_t scale $\mathcal{O}(\Gamma_t)$

    - needs theory input at least at NLO
    - can use 'cross section' like observables (NLO standard, soon NNLO)
    - for m_t from invariant mass of decay products, need NLO in this quantity !!
    - e.g. PS scheme seems to be perfectly acceptable for $\mu_{\rm PS} \lesssim 20 \text{ GeV}$
    - there is a sizeable scheme dependence $\delta m_t = (0.5 \ldots 1) \text{ GeV}$ of extracted top mass in parton-level toy analysis!!
    - not clear (at least to me) to what extent such effects are modelled / included / washed out in parton showers
    - but setting $m_{\rm MC} = m_{\rm pole}$ is just plain wrong, $m_{\rm MC} \simeq m_{\rm pole}$ is fine but at some point (aleady?) not sufficient any longer