# The 3D Nucleon Structure

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Università di Pavia & INFN

How can we built up
a multidimensional picture
of the nucleon?

# Charges

$$\boxed{\frac{1}{2P^+}\langle p^+,\vec{0}_\perp,\Lambda'|\bar{\psi}(0)\Gamma\psi(0)|p^+,\vec{0}_\perp,\Lambda\rangle}$$

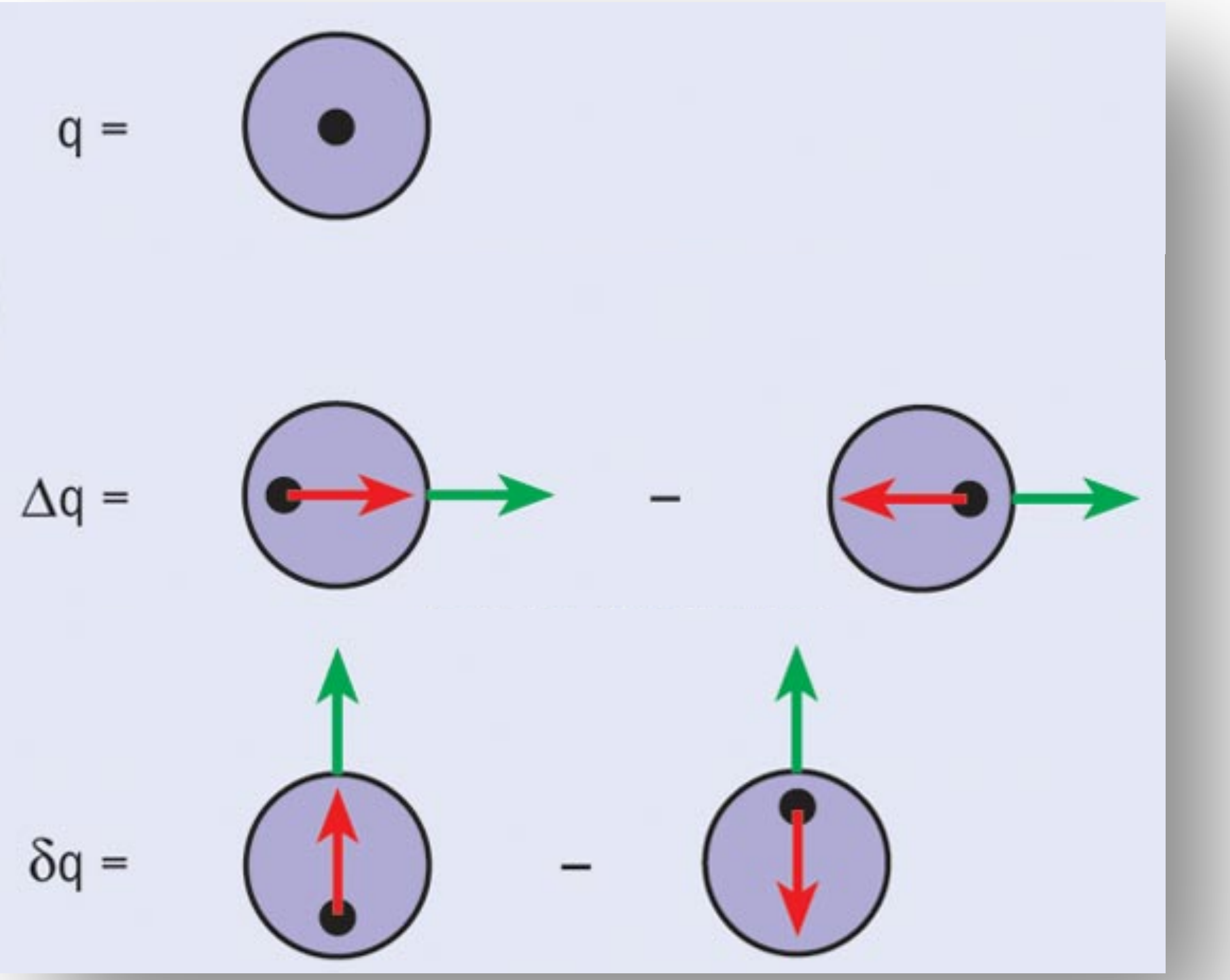
Depend on

$\Lambda,\ \Lambda',\ \Gamma$ : nucleon and quark polarizations

**Vector:** $\Gamma = \gamma^+$
Parton number

**Axial:** $\Gamma = \gamma^+\gamma_5$
Parton helicity

**Tensor:** $\Gamma = i\sigma^{+i}\gamma_5$
Parton transversity



Charges

# Form Factors (FFs)

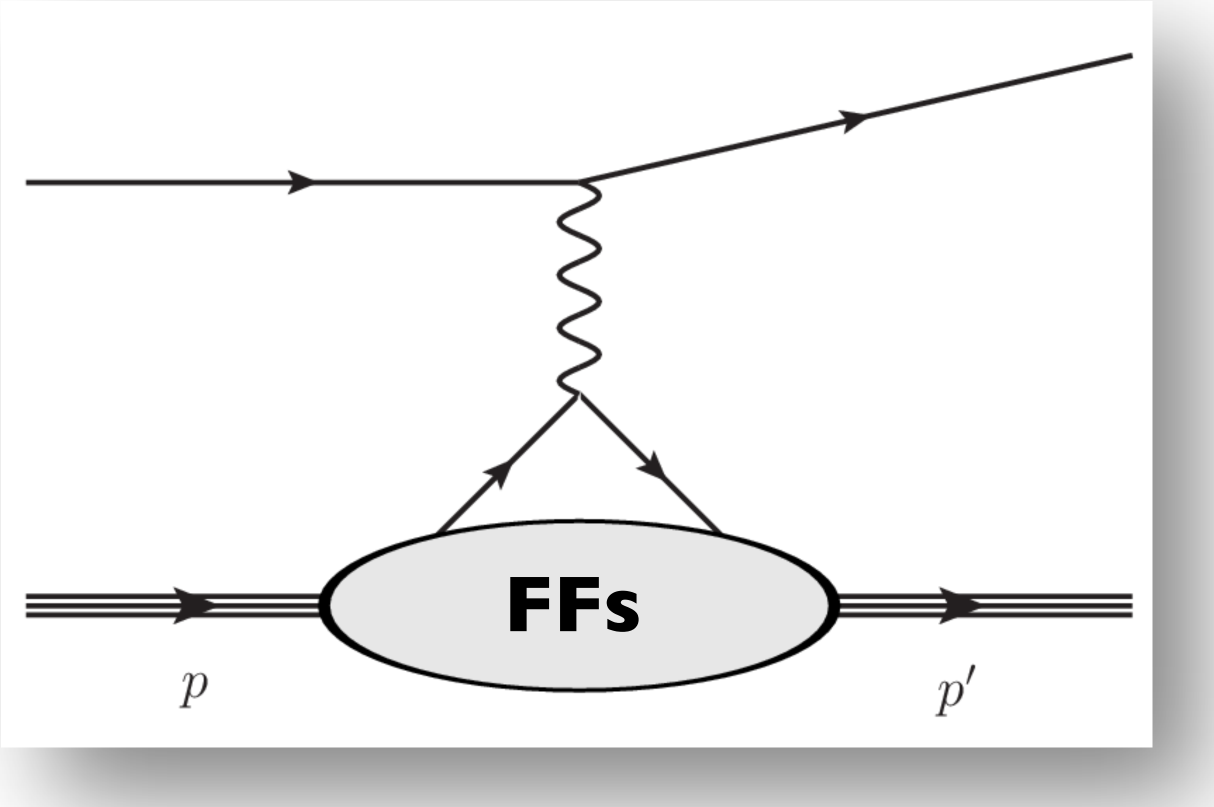
$$\boxed{\frac{1}{2P^+}\langle p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda' | \bar{\psi}(0)\,\Gamma\,\psi(0) | p^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda\rangle}$$

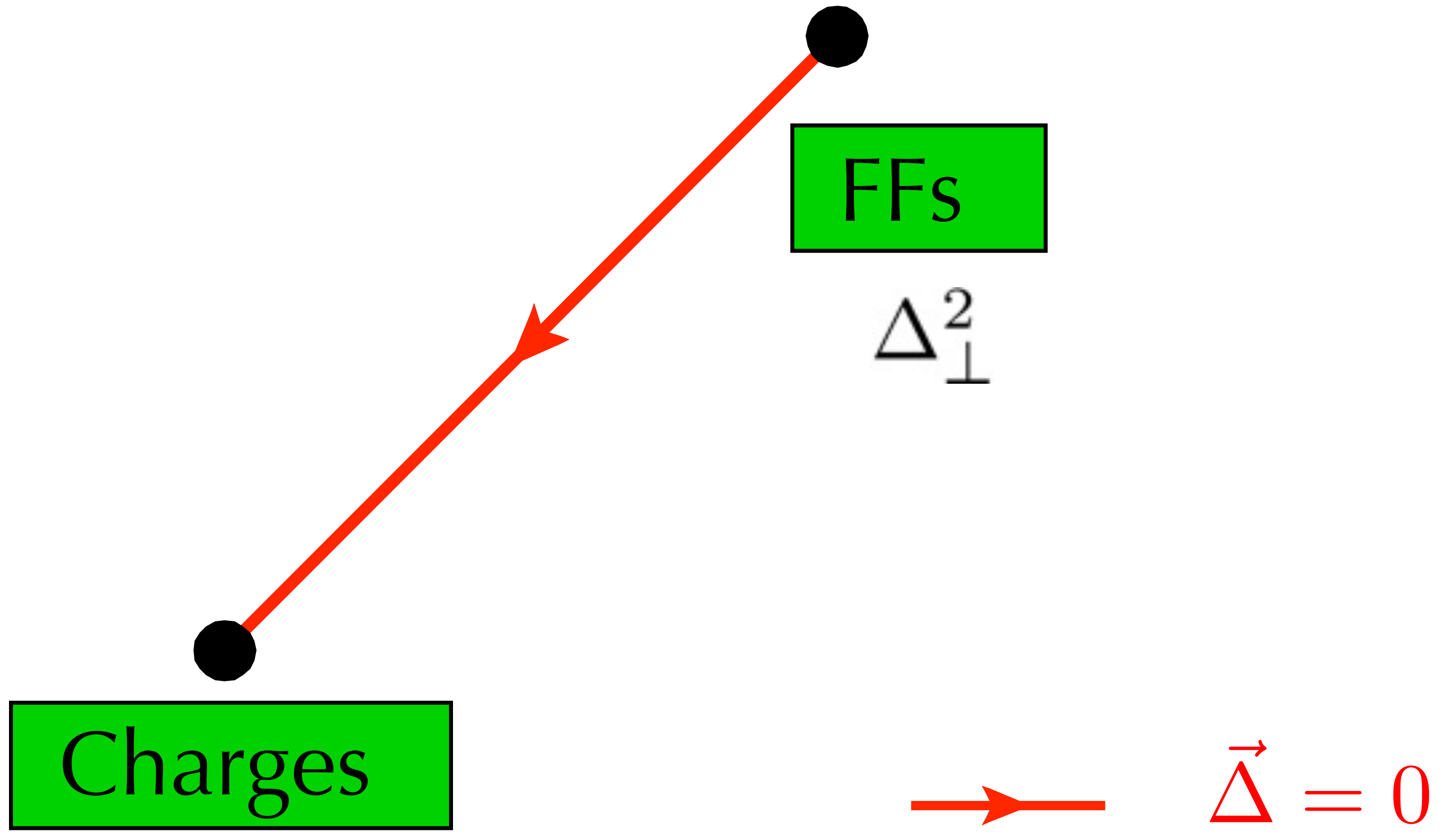
Depend on

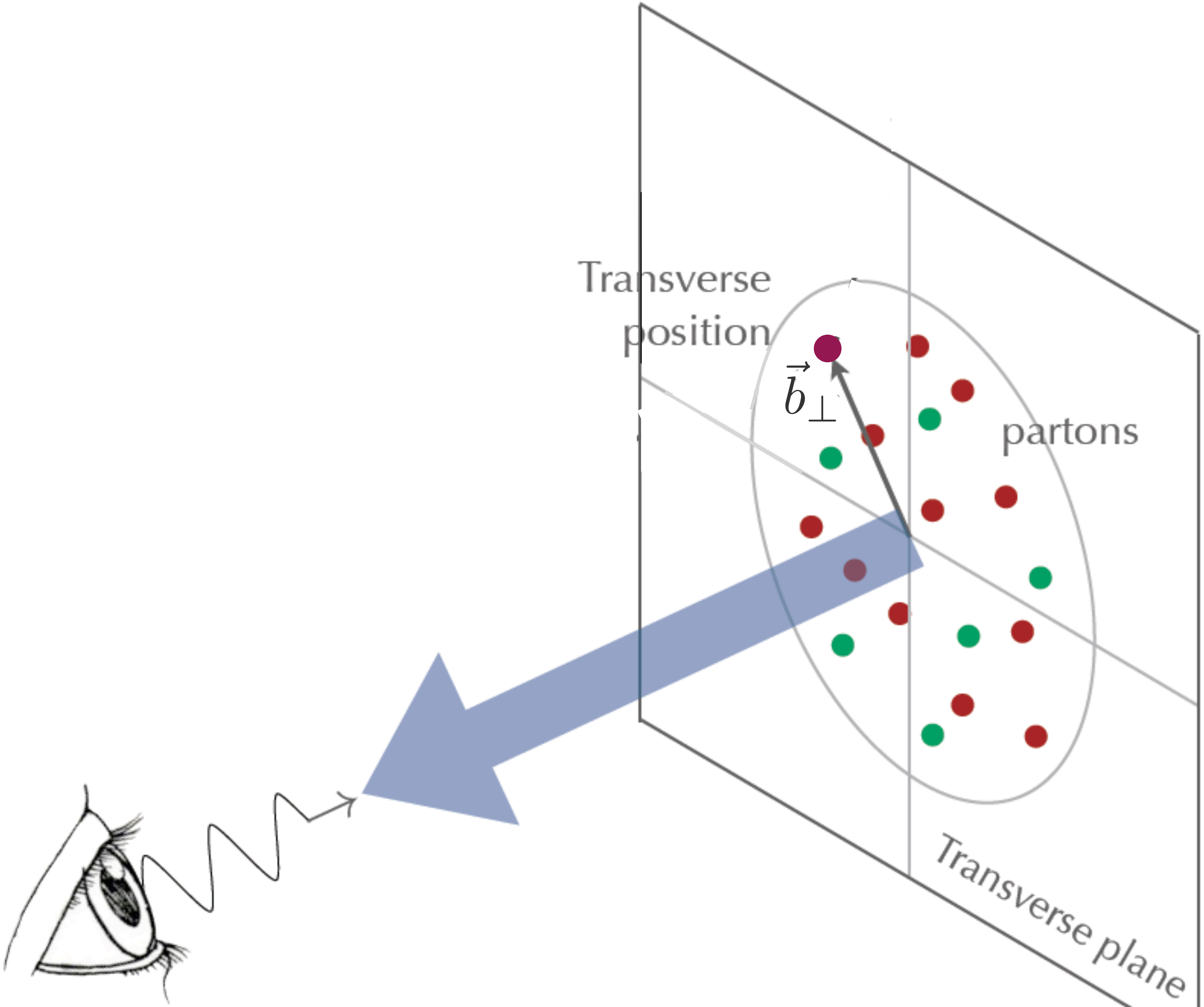
$\Lambda, \Lambda', \Gamma$ : nucleon and quark polarizations

$\Delta$ : momentum transfer $\vec{\Delta}_\perp \xleftrightarrow{\text{FT}} \vec{b}_\perp$ : impact parameter

**Elastic Scattering**






Transverse position
$\vec{b}_\perp$
partons
Transverse plane

# Parton Distribution Functions (PDFs)

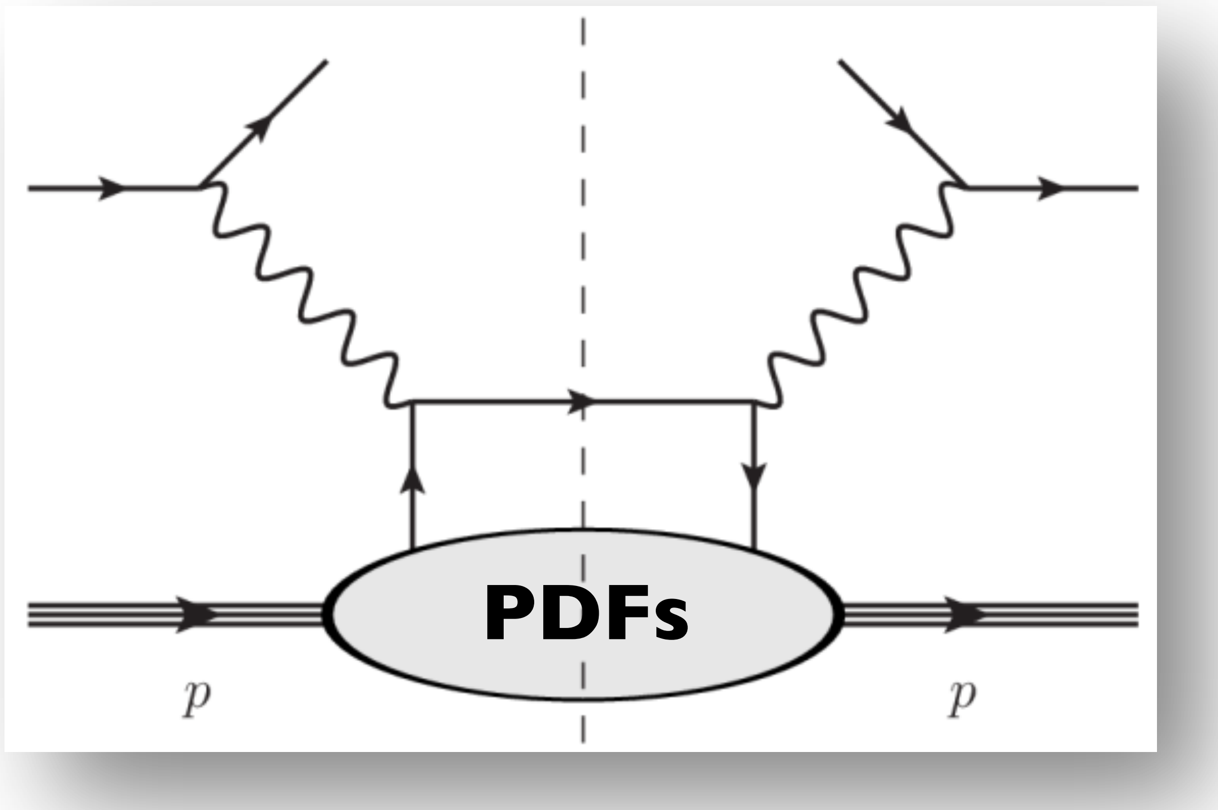
$$\boxed{\frac{1}{2}\int\frac{\mathrm{d}z^-}{2\pi}e^{ik^+z^-}\langle p^+,\,\vec{0}_\perp,\,\Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+,\,\vec{0}_\perp,\,\Lambda\rangle_{z^+=0,z_\perp=0}}$$

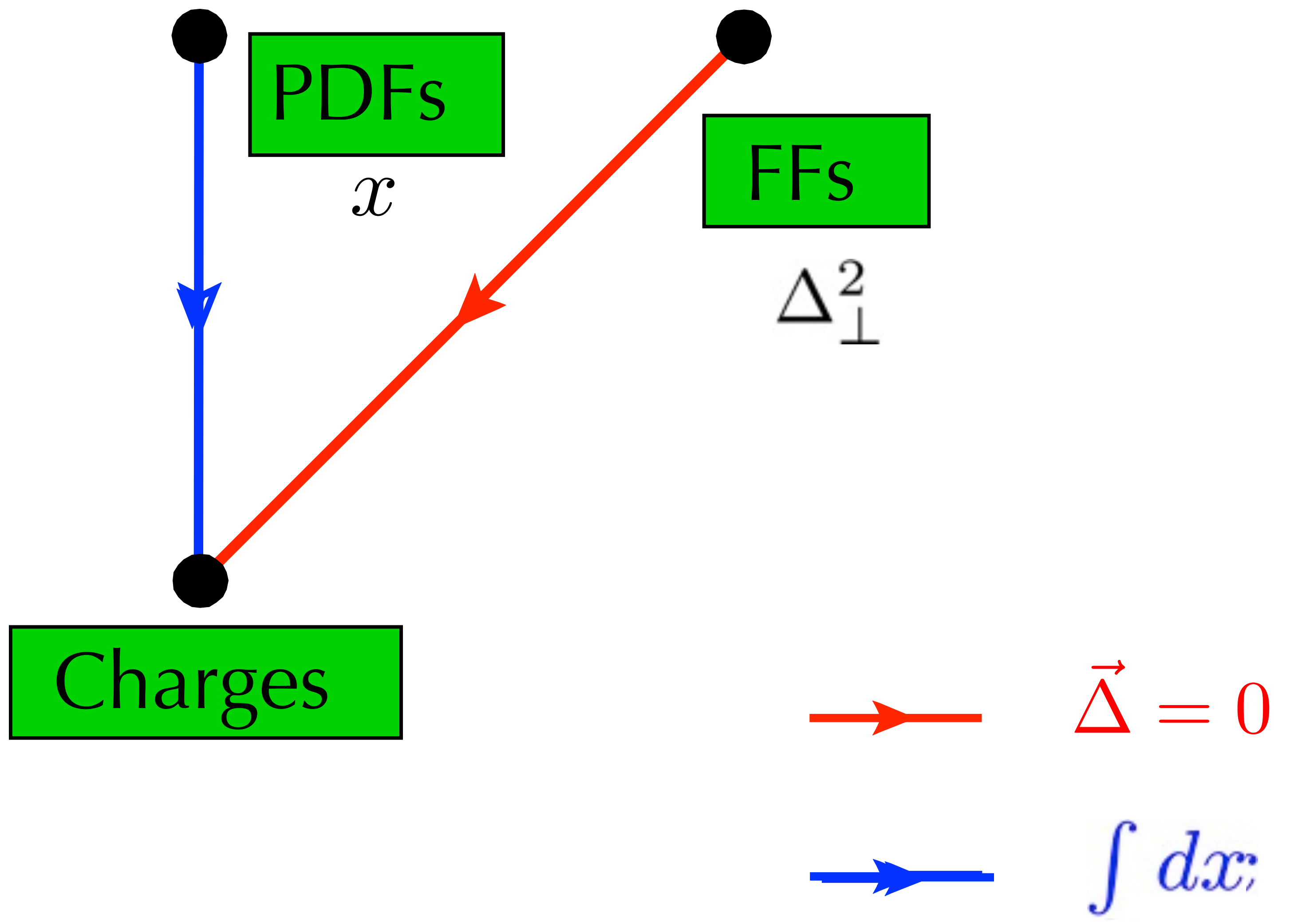
Depend on

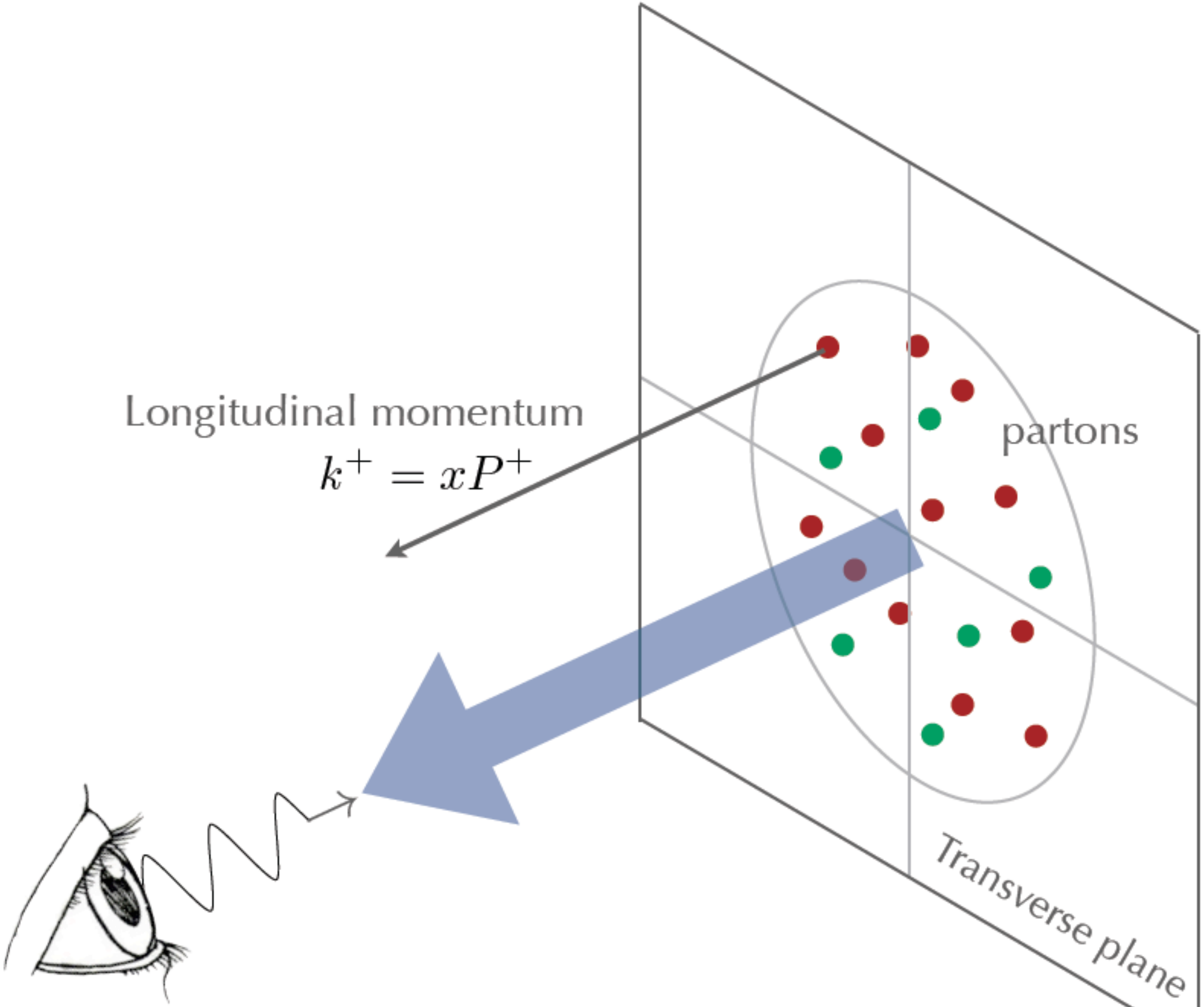
$\Lambda,\ \Lambda',\ \Gamma$ : nucleon and quark polarizations

$x = \dfrac{k^+}{p^+}$ : longitudinal momentum fraction

**Deep Inelastic Scattering**






Longitudinal momentum
$k^+ = xP^+$
partons
Transverse plane

# Generalized Parton Distributions (GPDs)

$$\boxed{\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ik^+z^-}\langle p'^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda\rangle_{z^+=0,z_\perp=0}}$$

Depend on

$\Lambda,\ \Lambda',\ \Gamma$ : nucleon and quark polarizations

$x = \frac{k^+}{p^+}$ : longitudinal momentum fraction

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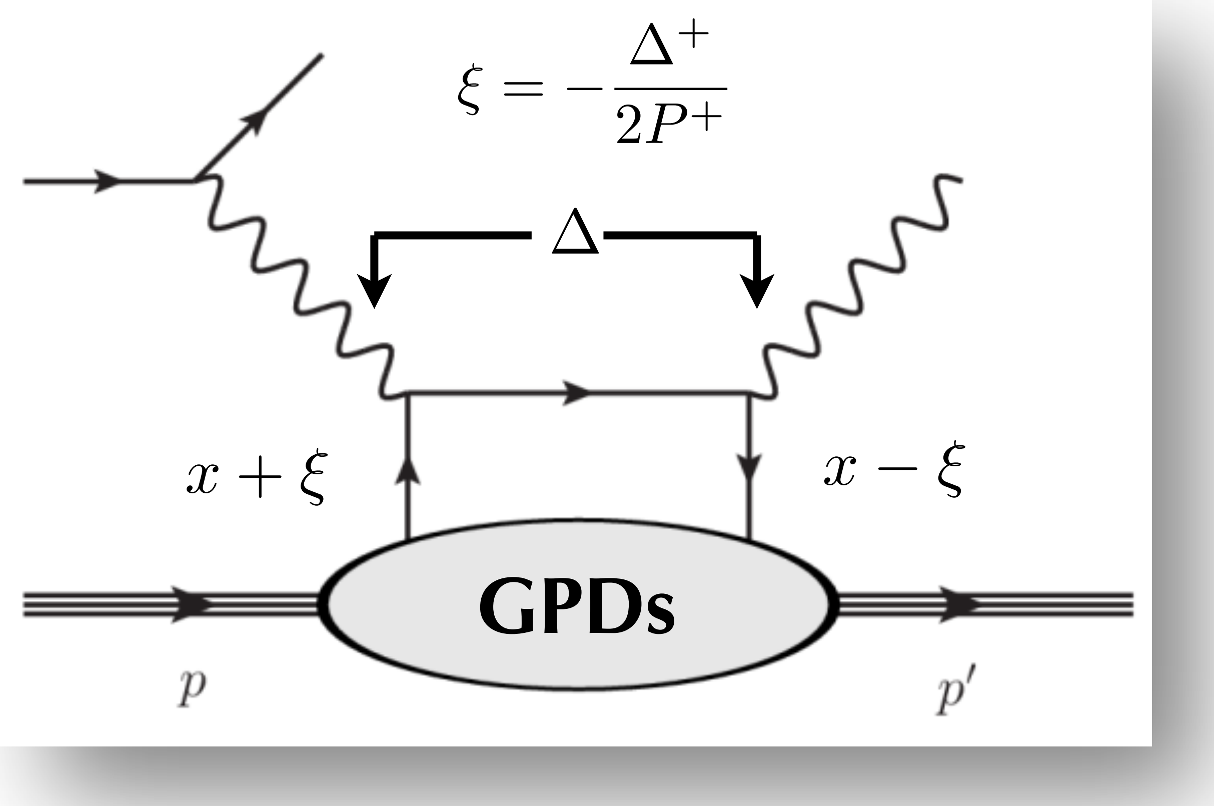
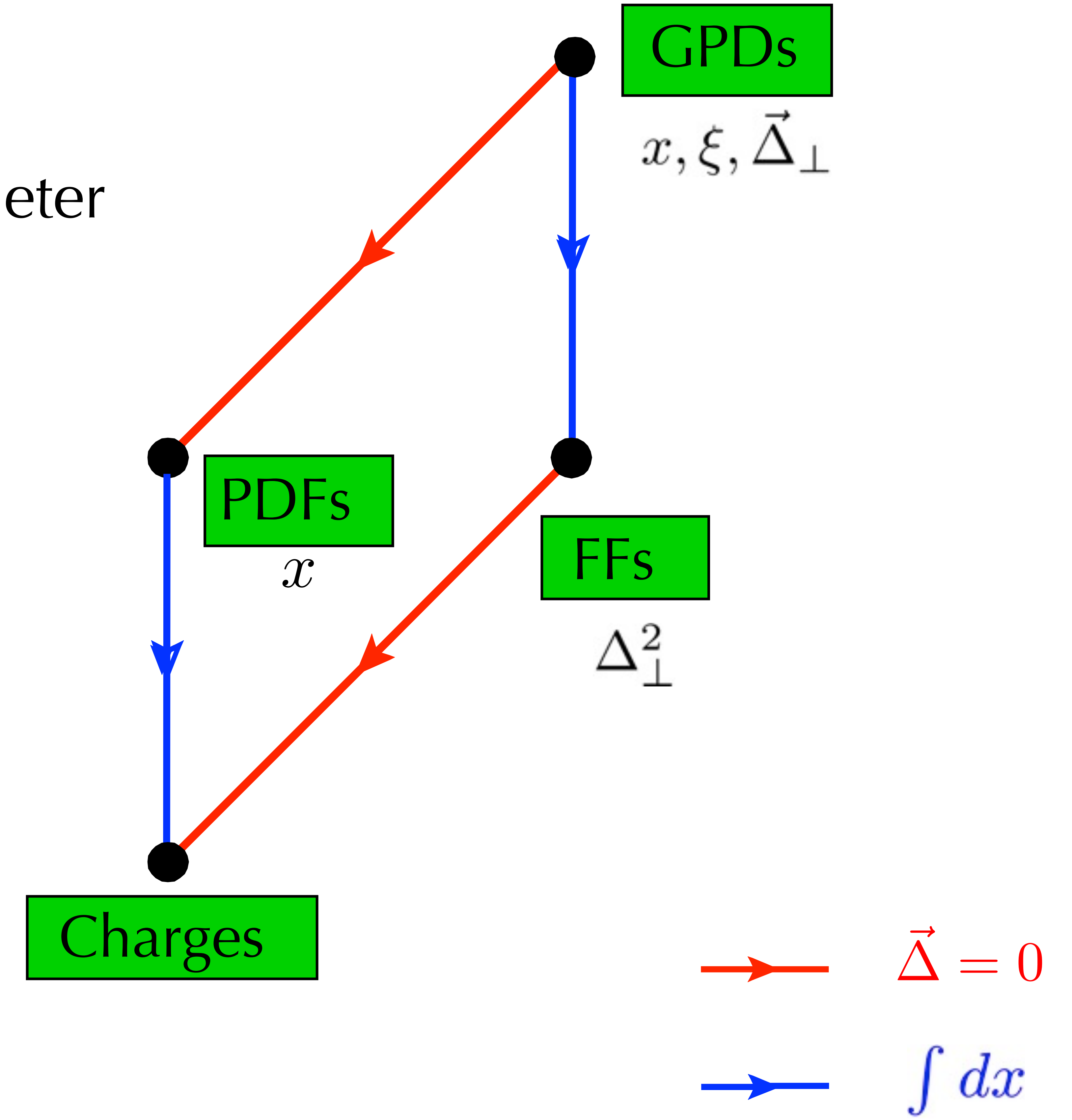
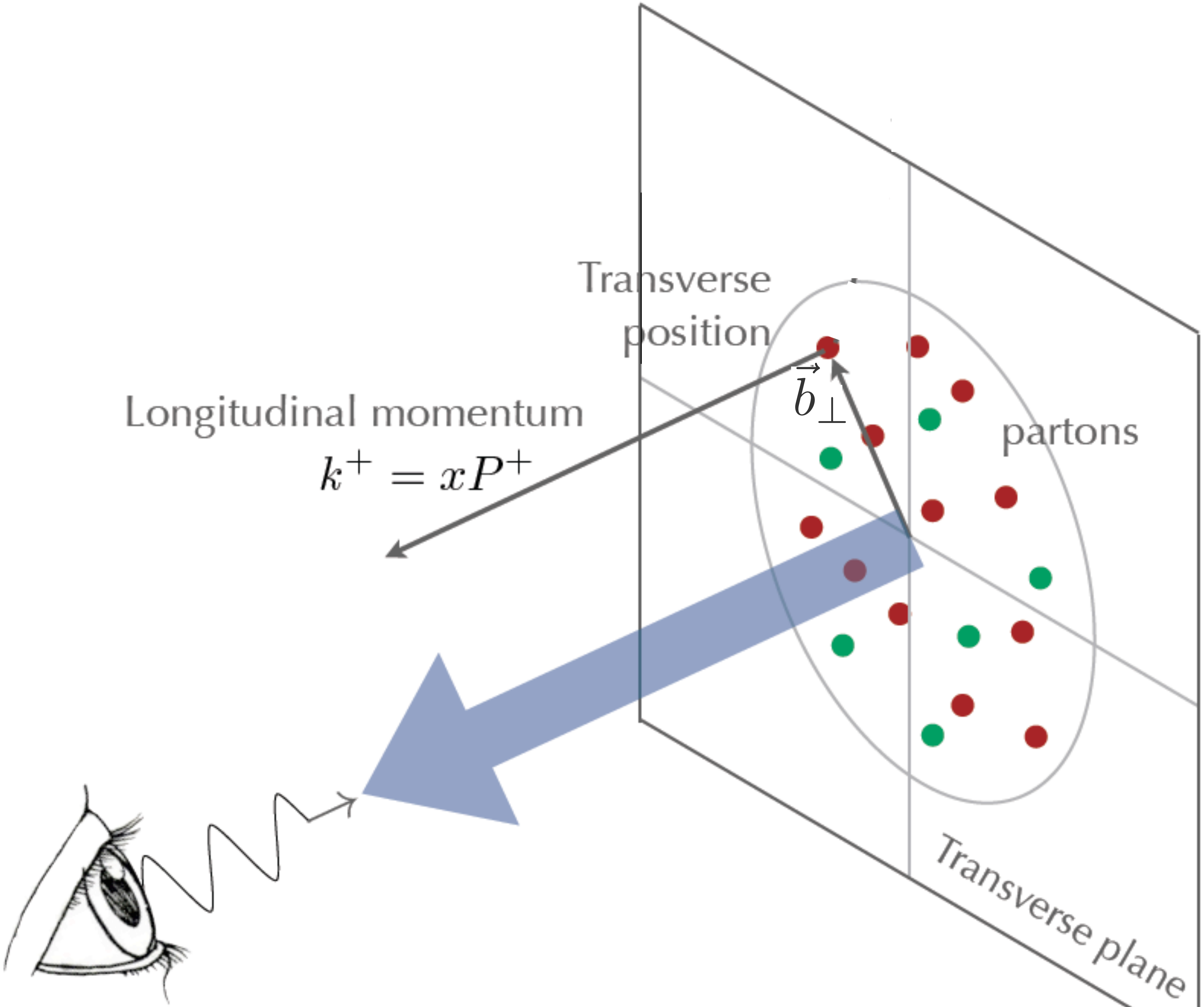
**Deeply Virtual Compton Scattering**






Transverse position
Longitudinal momentum
$k^+ = xP^+$
$\vec{b}_\perp$
partons
Transverse plane

# Transverse Momentum PDFs (TMDs)

$$\boxed{\frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2 z_\perp}{(2\pi)^3}\, e^{ik\cdot z}\langle p^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda\rangle_{z^+=0}}$$

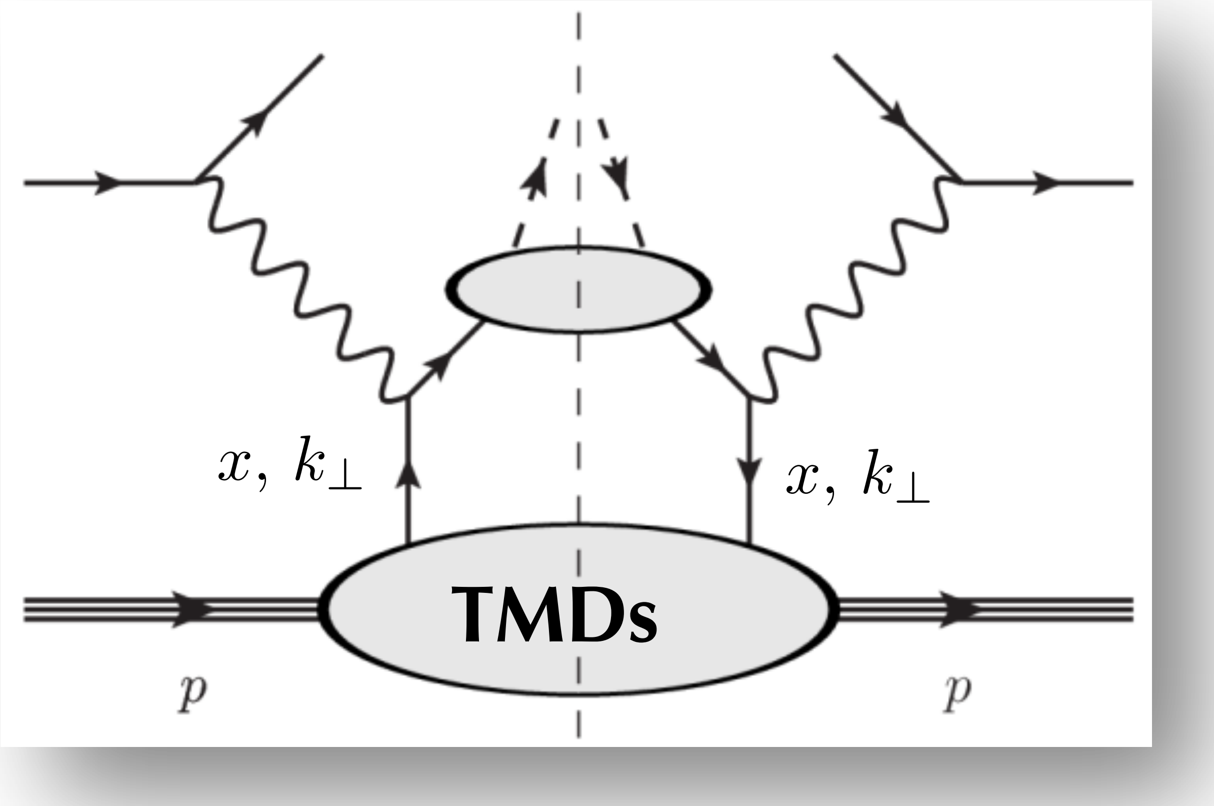
Depend on

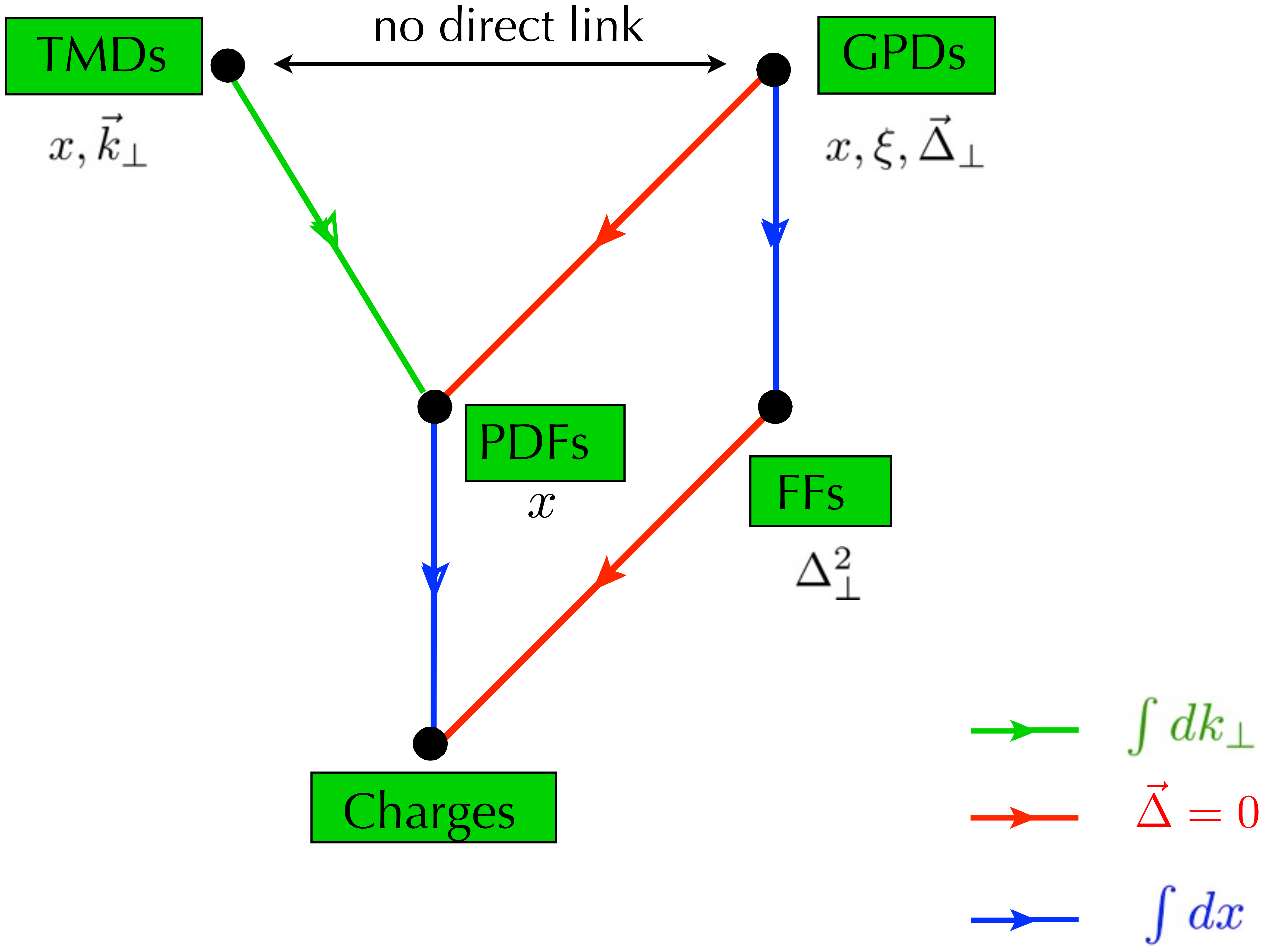
$\Lambda,\ \Lambda',\ \Gamma$ : nucleon and quark polarizations

$x = \dfrac{k^+}{p^+}$ : longitudinal momentum fraction

$k_\perp$ : parton transverse momentum

**Semi-Inclusive**
**Deep Inelastic Scattering**





TMDs $x, \vec{k}_\perp$

no direct link

GPDs $x, \xi, \vec{\Delta}_\perp$

PDFs $x$

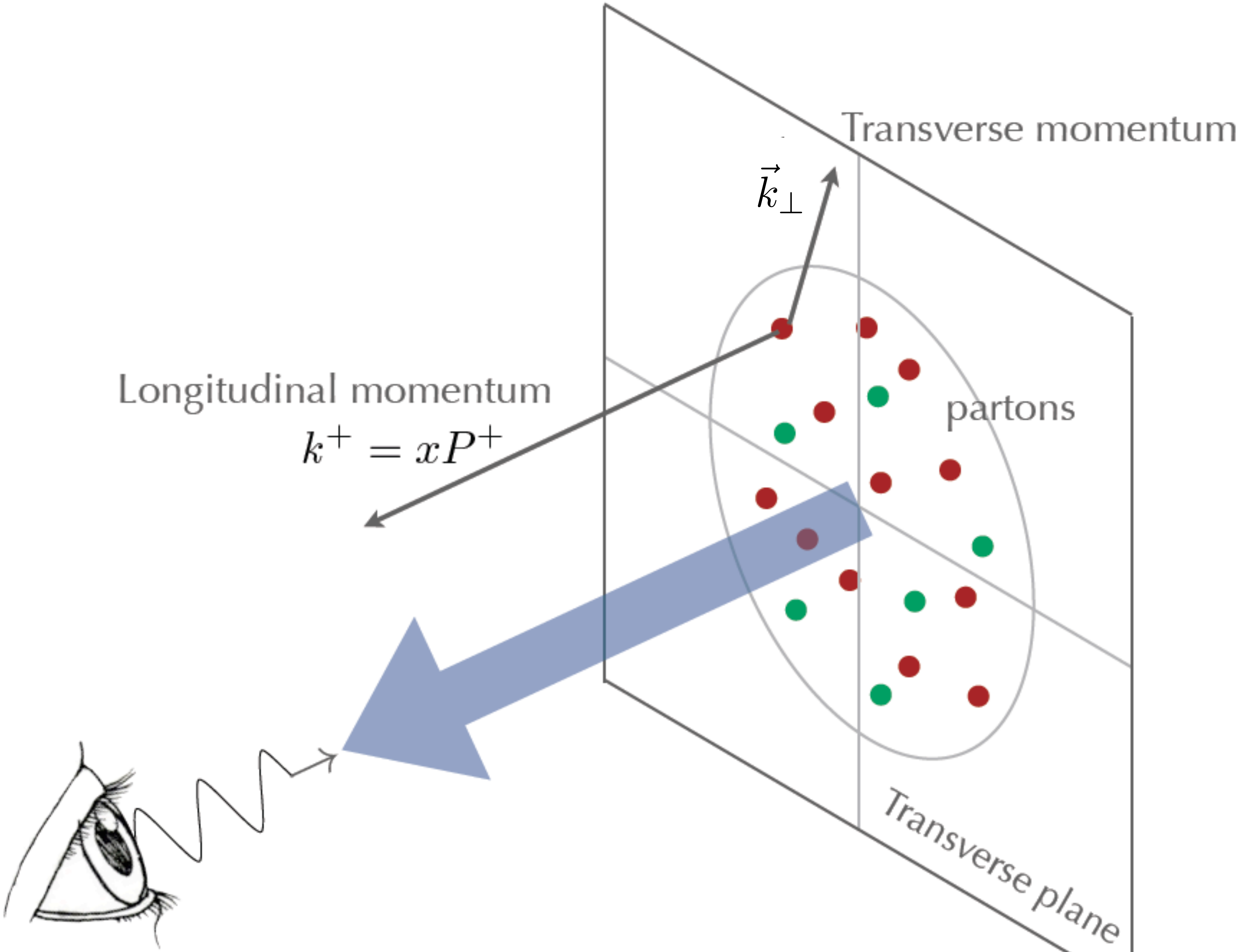
FFs $\Delta_\perp^2$

Charges

→ $\int dk_\perp$

→ $\vec{\Delta} = 0$

→ $\int dx$


Transverse momentum
$\vec{k}_\perp$
Longitudinal momentum
$k^+ = xP^+$
partons
Transverse plane

# Generalized TMDs (GTMDs)

$$\frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2 z_\perp}{(2\pi)^3} e^{ik\cdot z}\langle p^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\frac{z}{2})|p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda\rangle_{z^+=0}$$

Depend on

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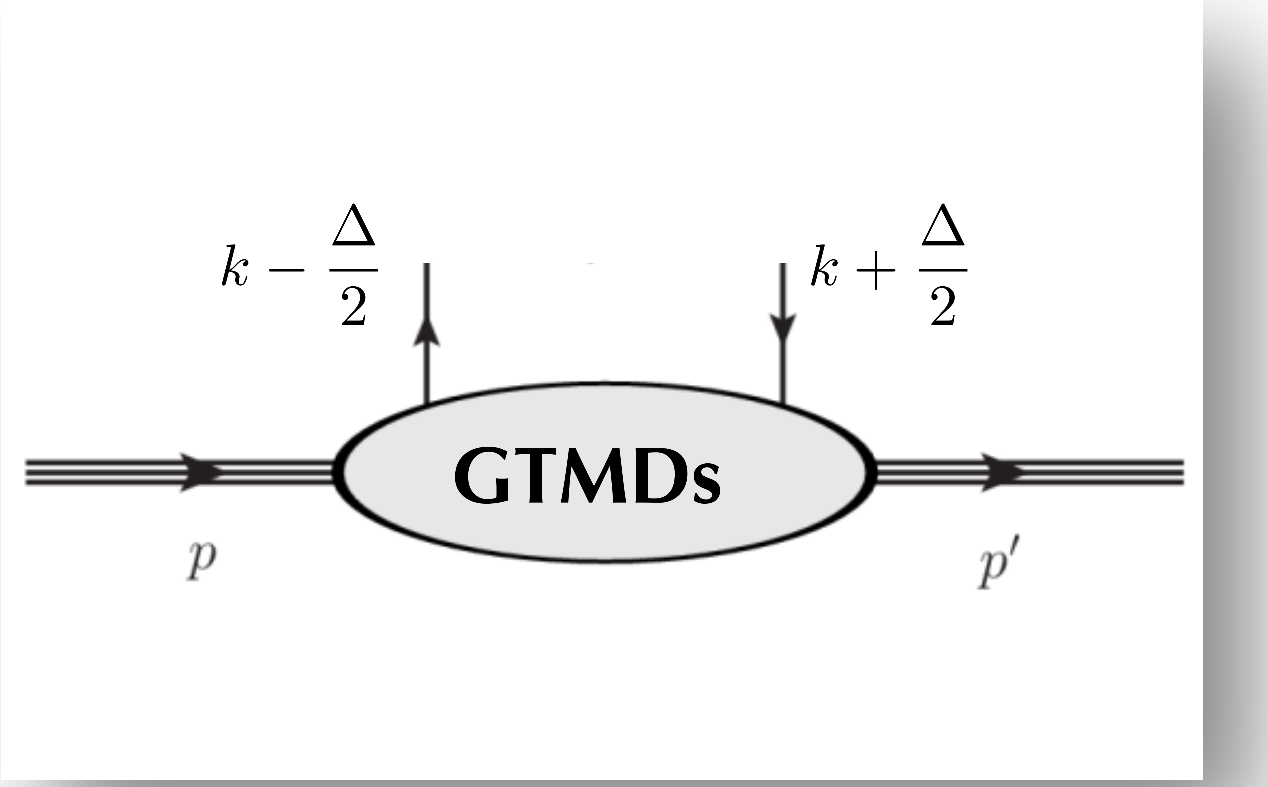
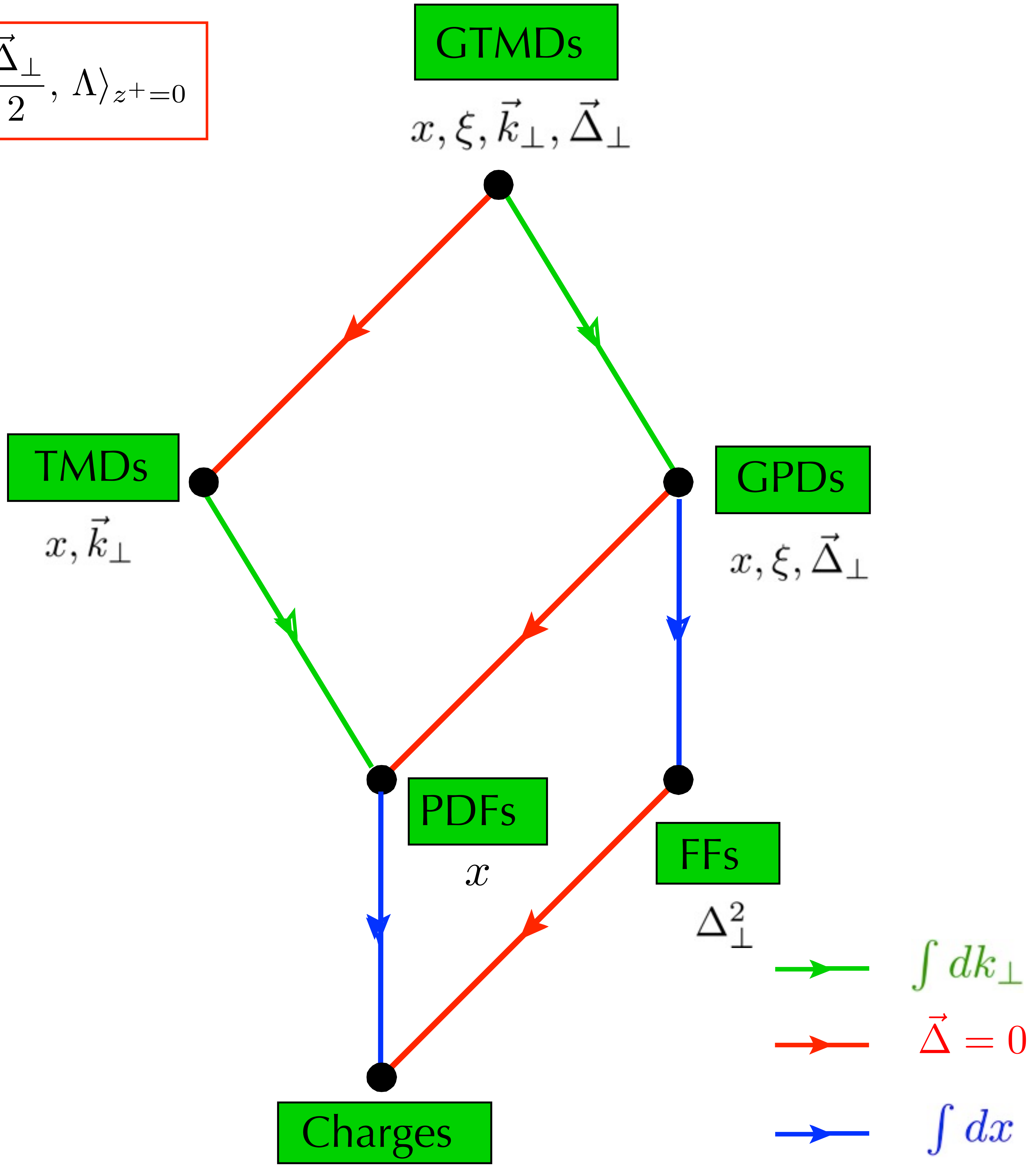
$x = \frac{k^+}{p^+}$ : longitudinal momentum fraction

$\Delta$ : momentum transfer

$k_\perp$ : parton transverse momentum

**???????**

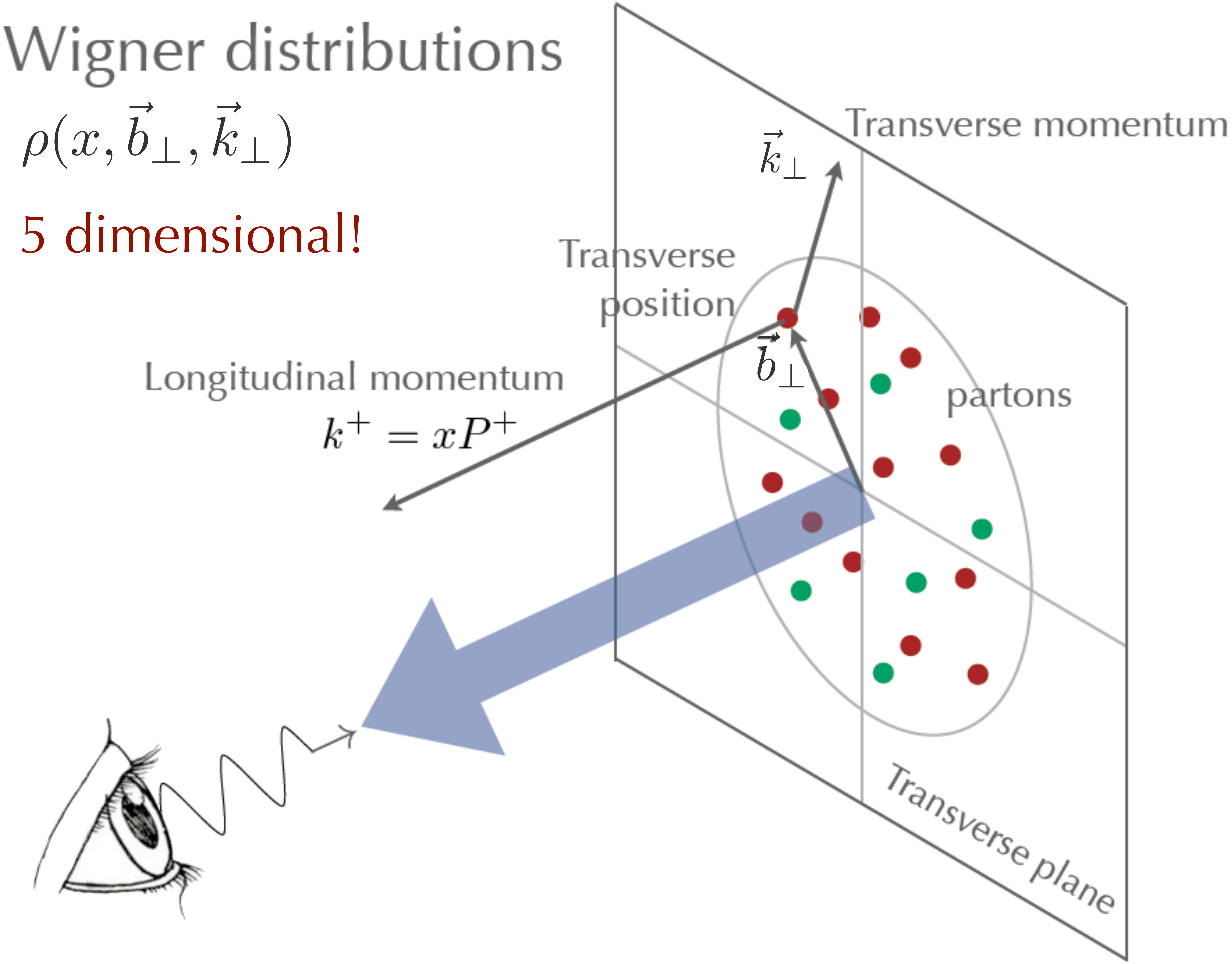


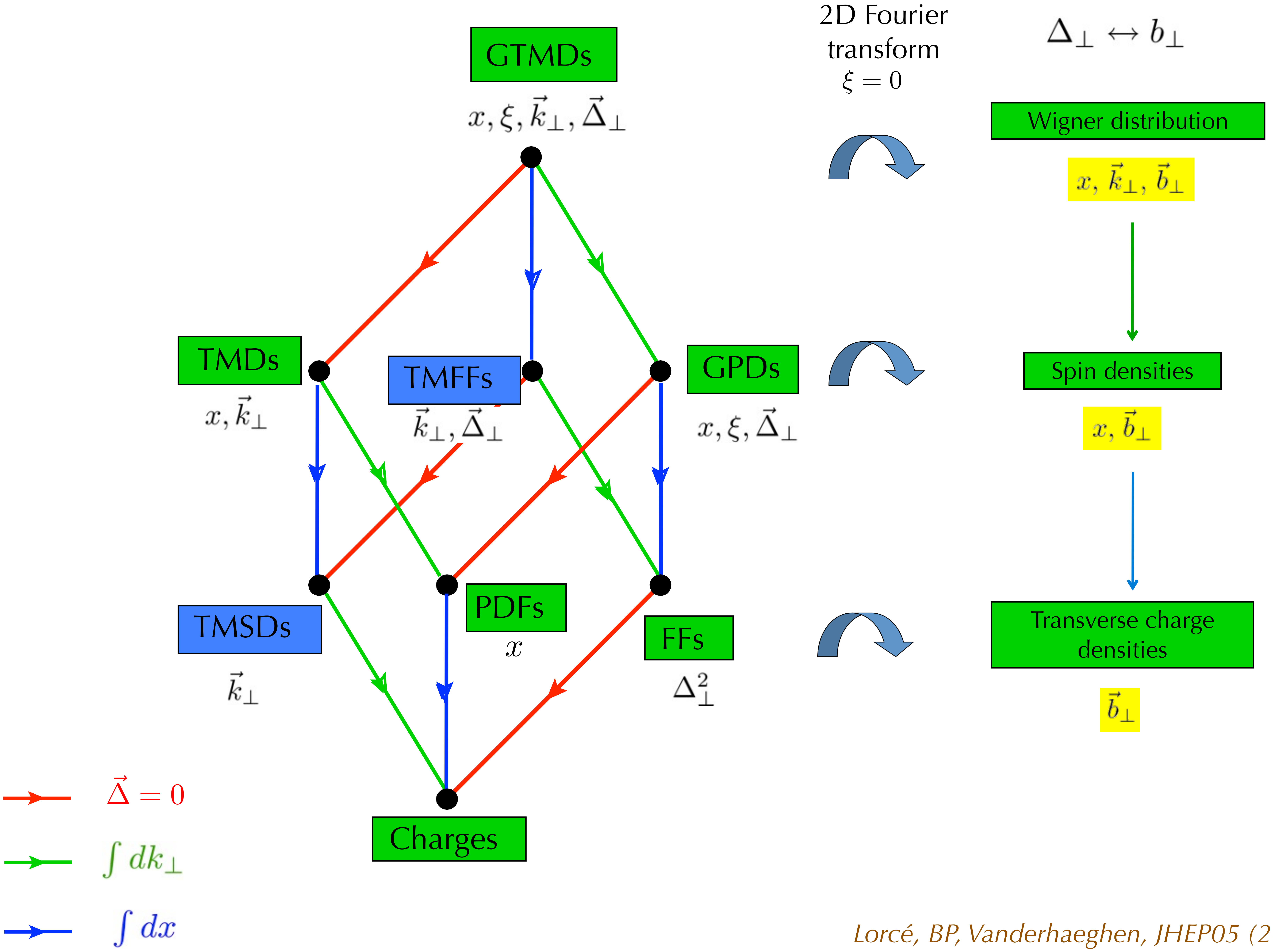


# Wigner distributions

$\rho(x, \vec{b}_\perp, \vec{k}_\perp)$

5 dimensional!





2D Fourier transform
$\xi = 0$

$\Delta_\perp \leftrightarrow b_\perp$

GTMDs
$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs
$x, \vec{k}_\perp$

TMFFs
$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs
$x, \xi, \vec{\Delta}_\perp$

TMSDs
$\vec{k}_\perp$

PDFs
$x$

FFs
$\Delta_\perp^2$

Charges

Wigner distribution
$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities
$x, \vec{b}_\perp$

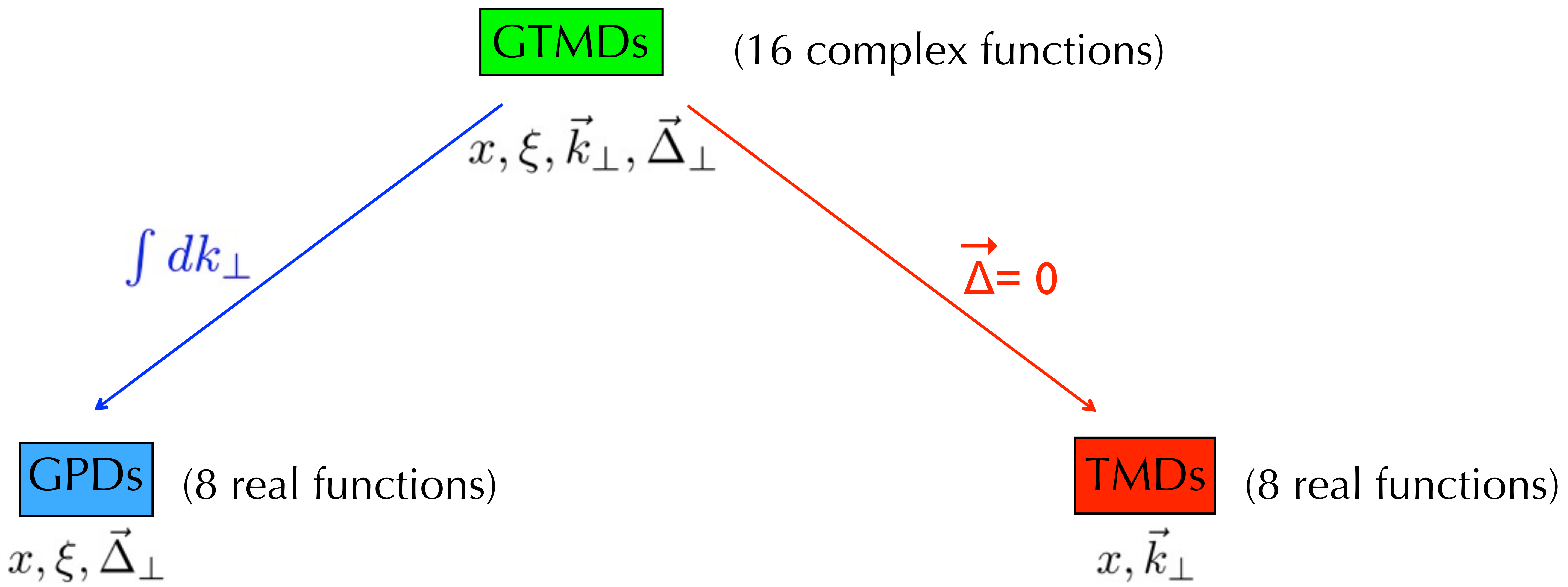
Transverse charge densities
$\vec{b}_\perp$

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

*Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041*



GTMDs (16 complex functions)

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\int dk_\perp$

$\vec{\Delta} = 0$

GPDs (8 real functions)

$x, \xi, \vec{\Delta}_\perp$

TMDs (8 real functions)

$x, \vec{k}_\perp$

Quark polarization

Nucleon polarization

| | $U$ | $T$ | $L$ |
|---|---|---|---|
| $U$ | $H$ | $\mathcal{E}_T$ | |
| $T$ | $E$ | $H_T, \tilde{H}_T$ | $\tilde{E}$ |
| $L$ | | $\tilde{E}_T$ | $\tilde{H}$ |

Quark polarization

Nucleon polarization

| | $U$ | $T$ | $L$ |
|---|---|---|---|
| $U$ | $f_1$ | $h_1^\perp$ | |
| $T$ | $f_{1T}^\perp$ | $h_1,\ h_{1T}^\perp$ | $g_{1T}$ |
| $L$ | | $h_{1L}^\perp$ | $g_{1L}$ |

each distribution contains unique information

the distributions in red vanish if there is no quark orbital angular momentum

the distributions in black survive in the collinear limit

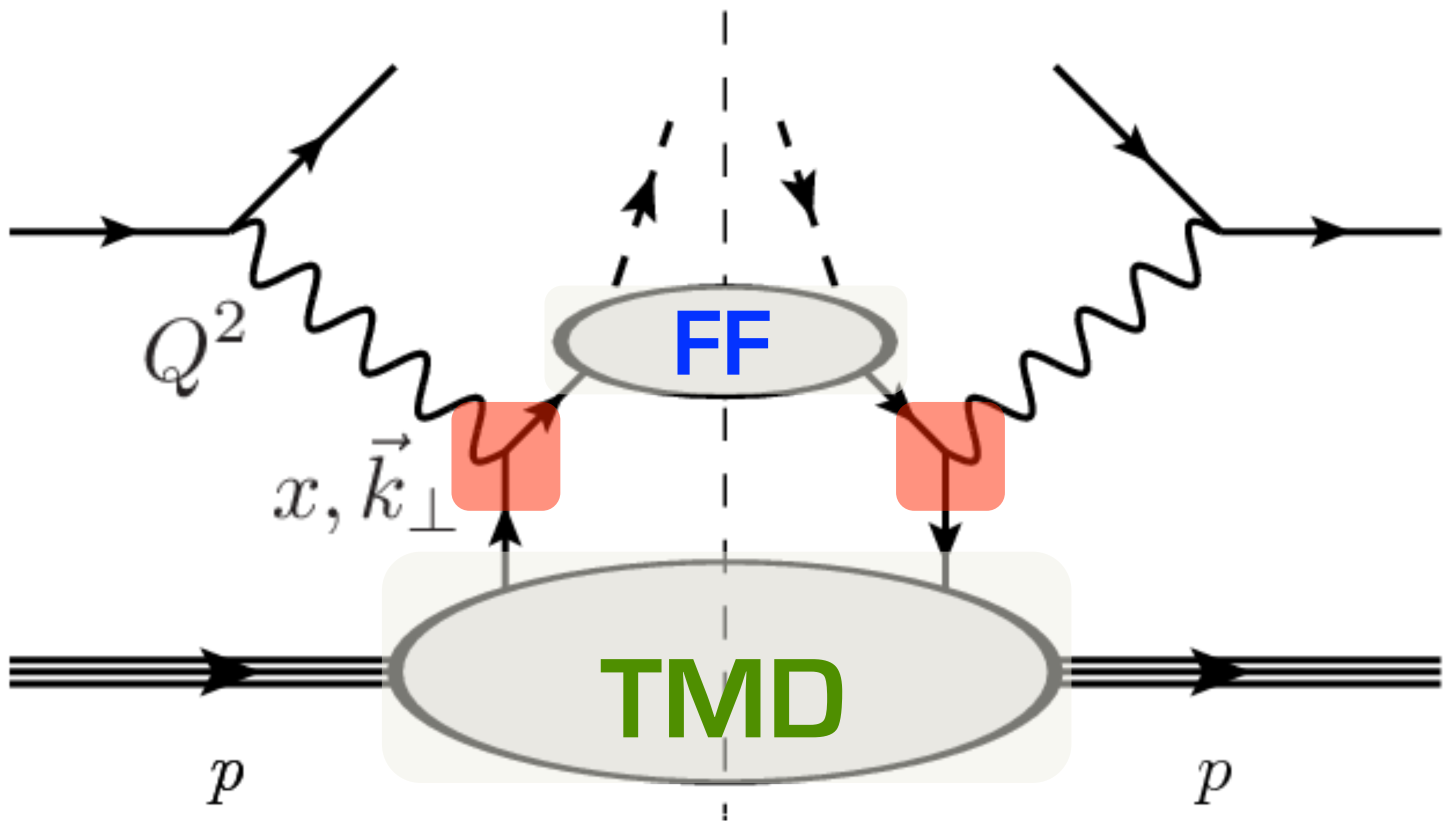
# Key information from TMDs

- Spin-Spin and Spin-Orbit Correlations of partons
- Transverse momentum size
- Test what we can calculate with QCD (perturbative and lattice)
- Non-perturbative structure we cannot calculate with QCD

A. Bressan: TMDs in experiments M. Boglione: TMD phenomenology M. Contalbrigo: 3D future

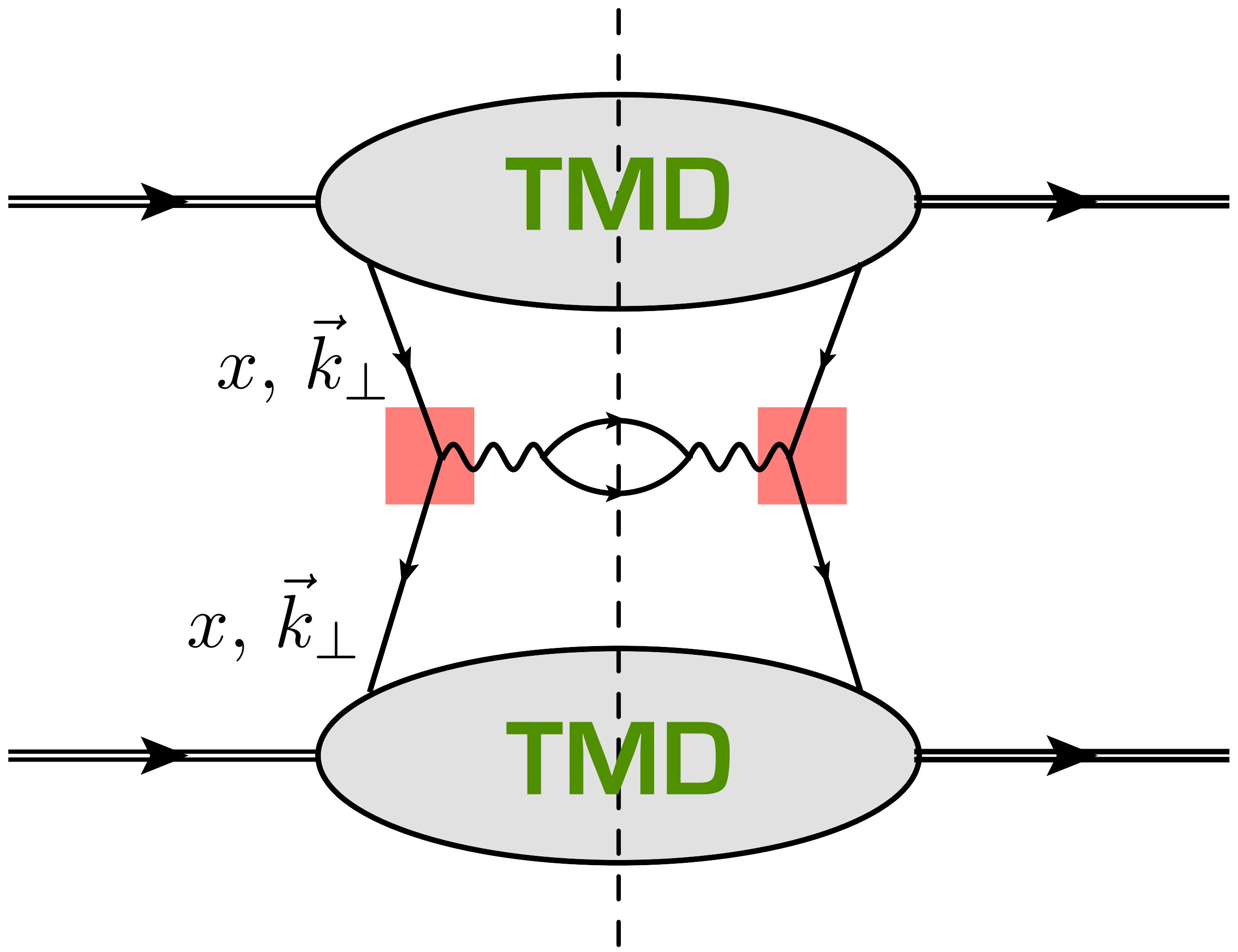
# How to measure the TMDs

$\ell(l) + N(P) \to \ell(l') + h(P_h) + X$



$$d\sigma \sim \sum \text{TMD}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard} \otimes \text{FF}(z, \vec{p}_\perp) + \mathcal{O}(\frac{P_T}{Q})$$

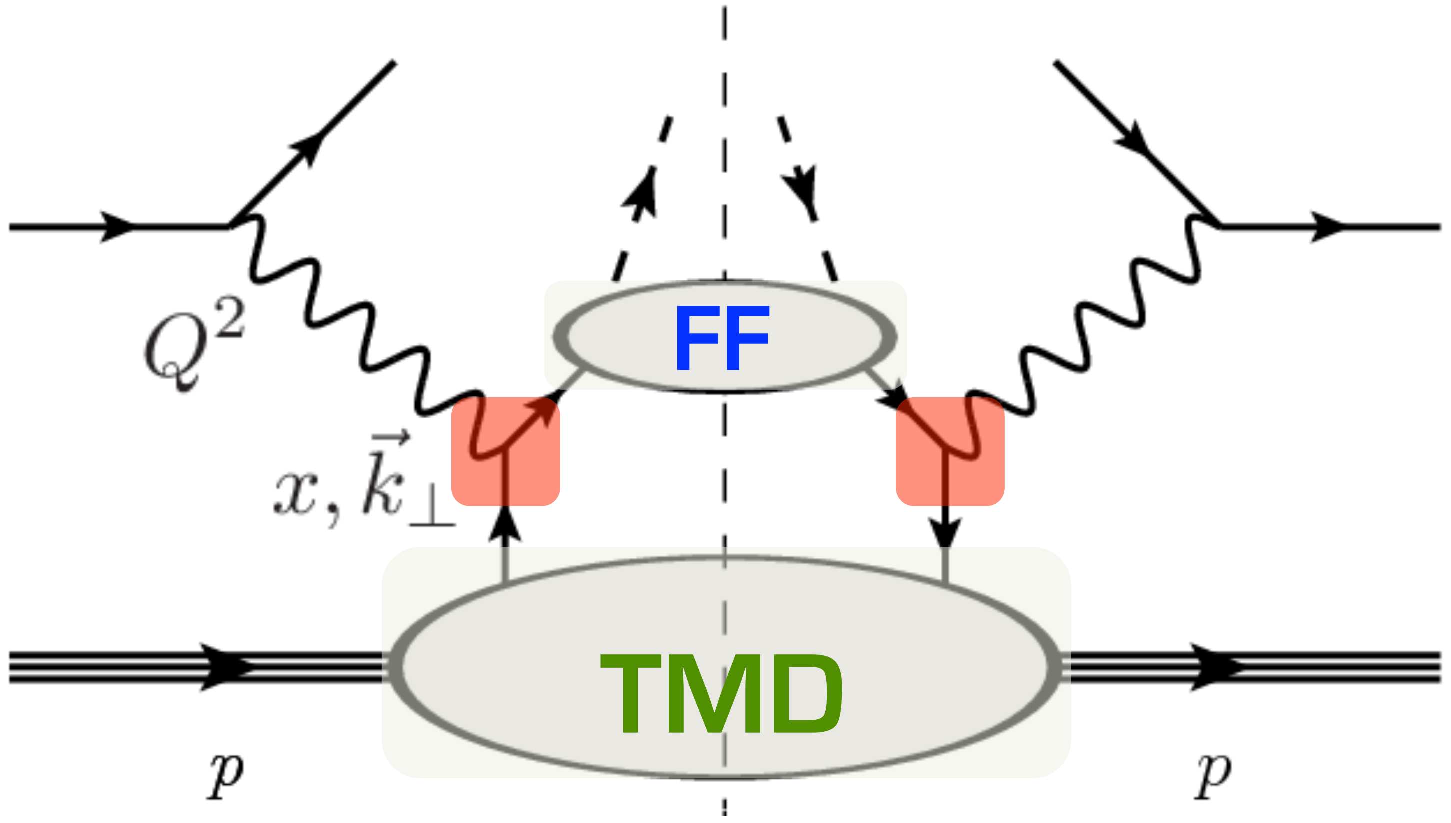
$h(P_1) + h(P_2) \to \ell^+(l) + \ell^-(l')$



$$d\sigma \sim \sum \text{TMD}(x, \vec{k}_\perp) \otimes \overline{\text{TMD}}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard}$$

# How to measure the TMDs

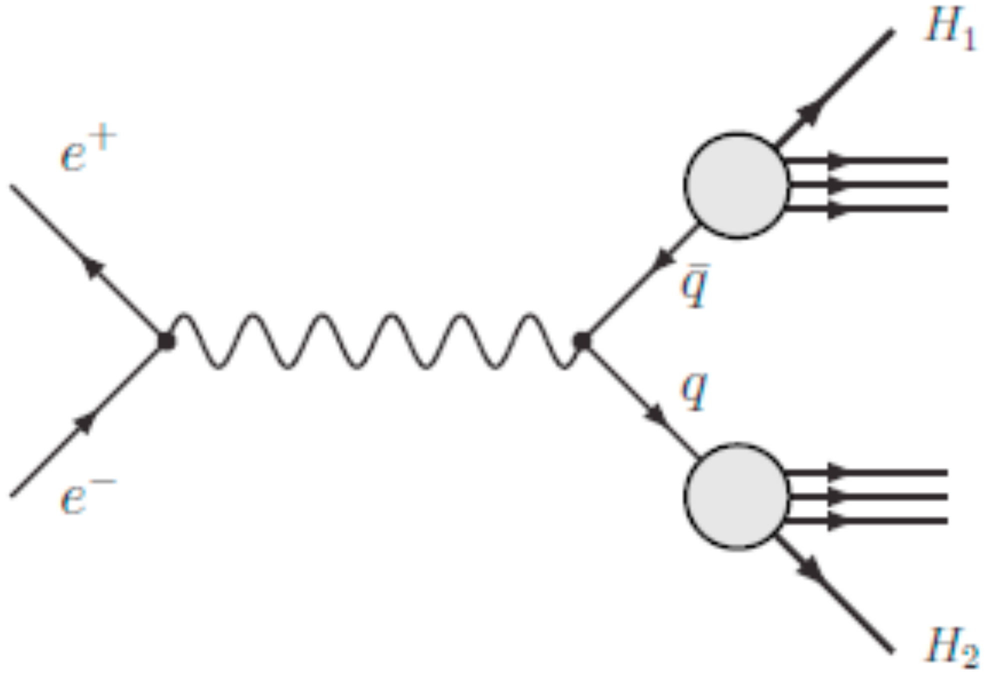
$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X$$



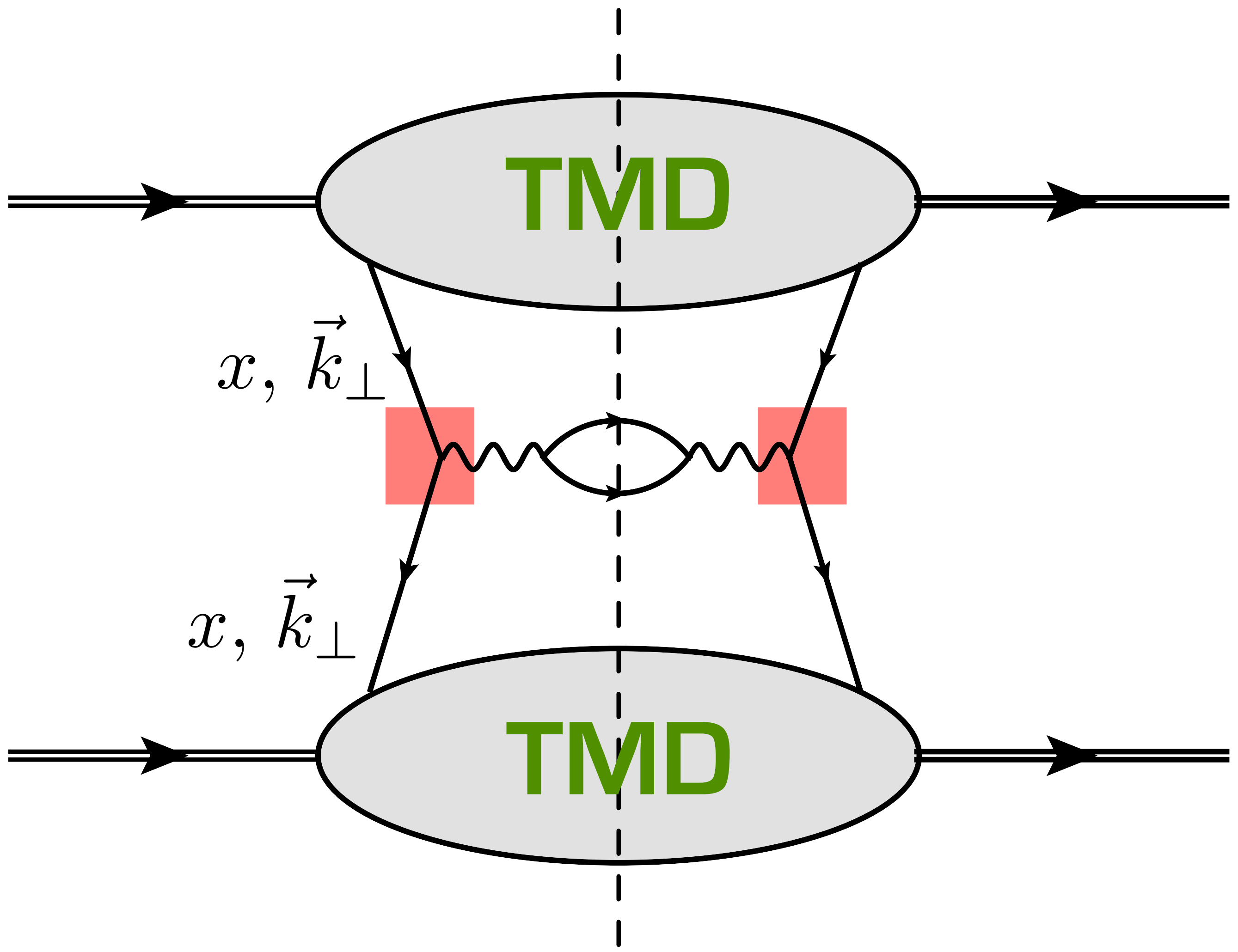
$$d\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard} \otimes \mathrm{FF}(z, \vec{p}_\perp) + \mathcal{O}(\frac{P_T}{Q})$$

Fragmentation Functions

$e^+e^- \to hh'X$



$$h(P_1) + h(P_2) \to \ell^+(l) + \ell^-(l')$$

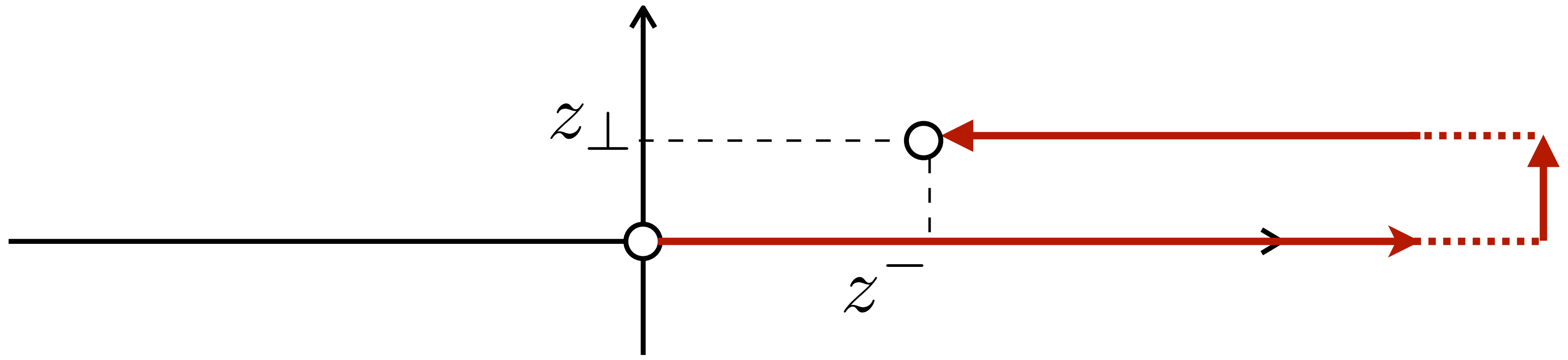


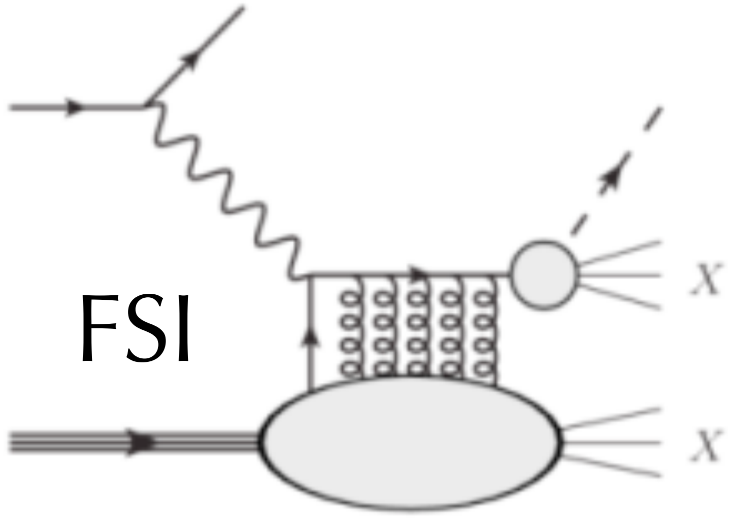
$$d\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes \overline{\mathrm{TMD}}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard}$$

# Gauge link dependence of TMDs

$$\frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2 z_\perp}{(2\pi)^3} e^{i(k^+ z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}\langle p^+,\, 0_\perp,\, \Lambda'|\bar{\psi}(0)\,\gamma^+ \boxed{\text{GaugeLink}}\,\psi(0,z^-,z_\perp)|p^+,\, 0_\perp,\, \Lambda\rangle$$

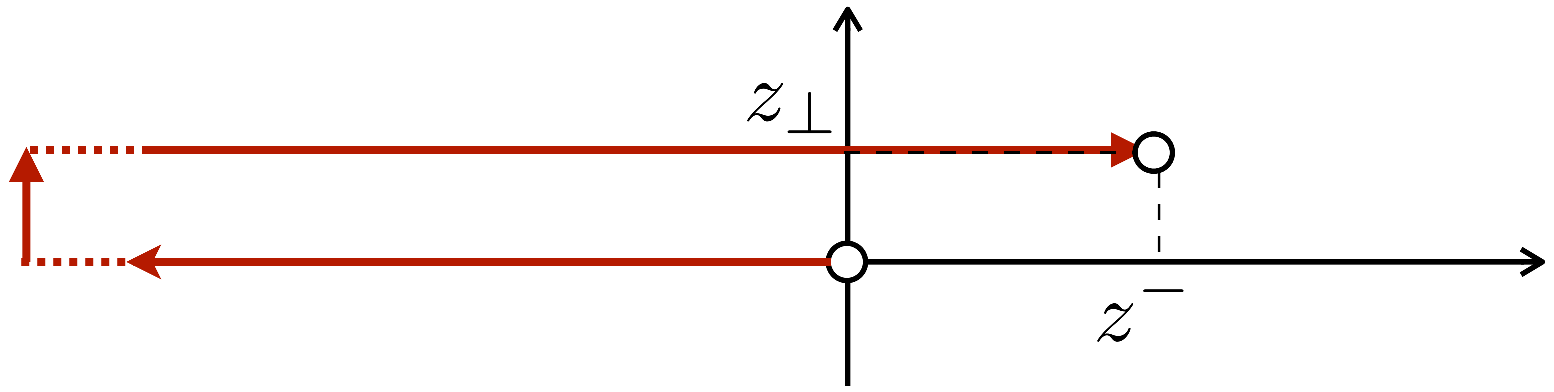
SIDIS

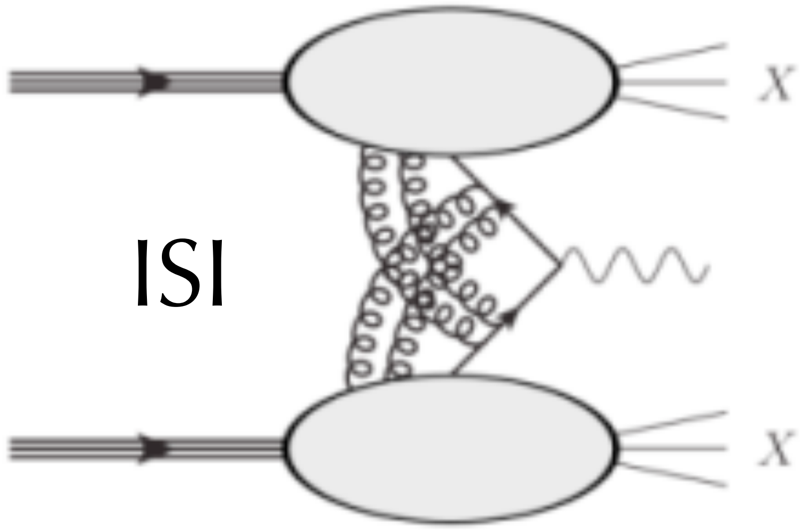




FSI

Drell-Yan



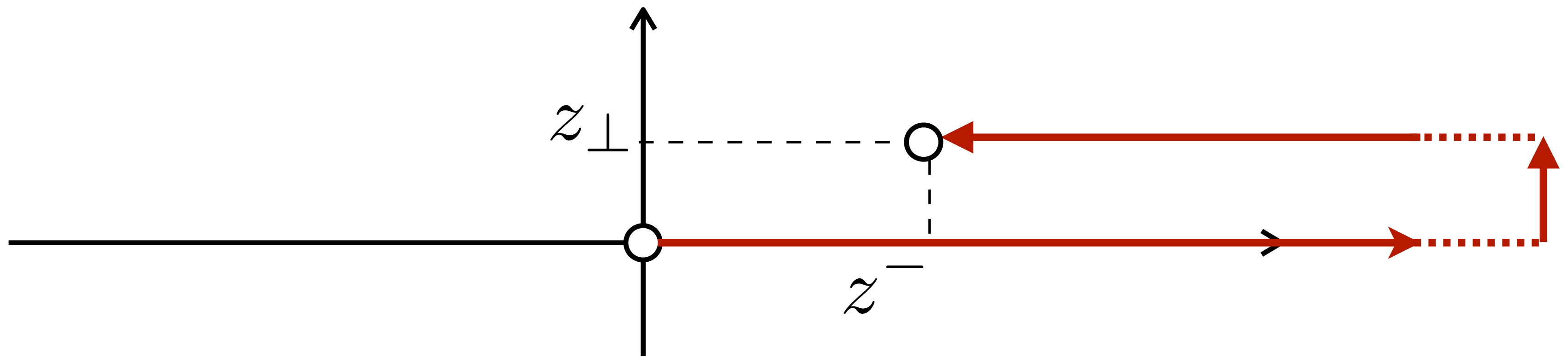


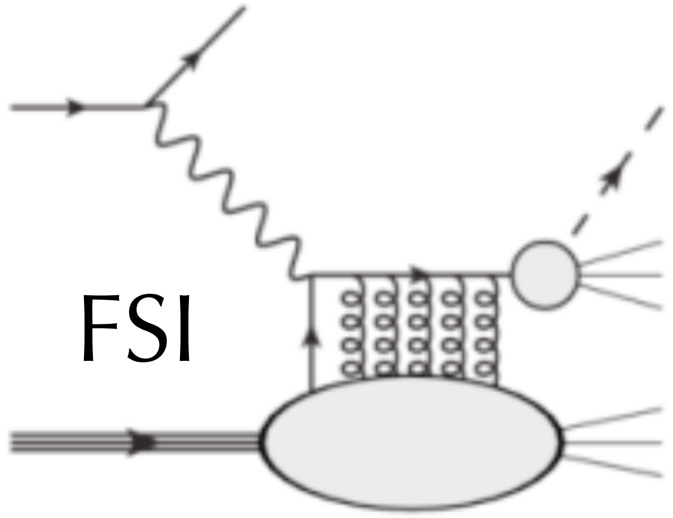
ISI

# Gauge link dependence of TMDs

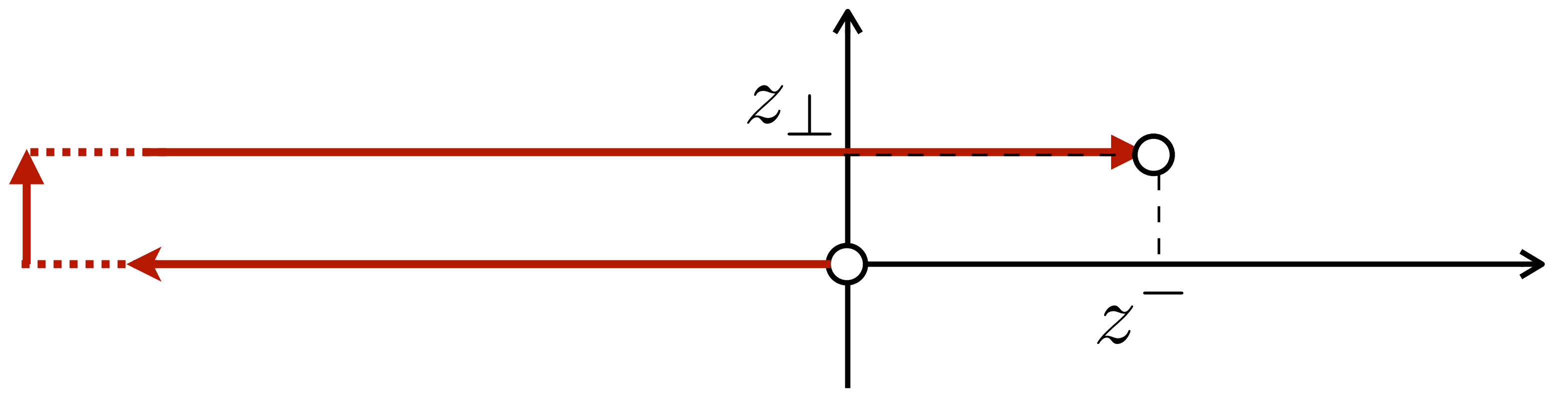
$$\frac{1}{2}\int\frac{\mathrm{d}z^-\,\mathrm{d}^2 z_\perp}{(2\pi)^3}\,e^{i(k^+ z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}\langle p^+,\,0_\perp,\,\Lambda'|\bar{\psi}(0)\,\gamma^+\boxed{\text{GaugeLink}}\psi(0,z^-,z_\perp)|p^+,\,0_\perp,\,\Lambda\rangle$$

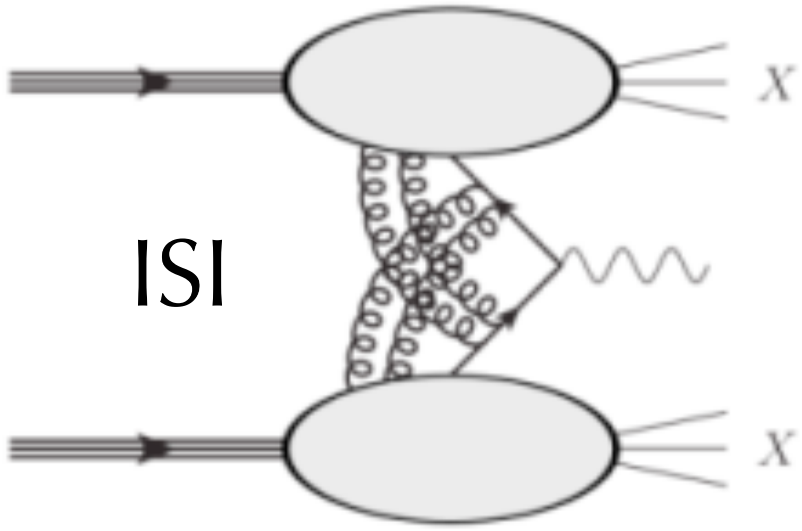
SIDIS





Drell-Yan





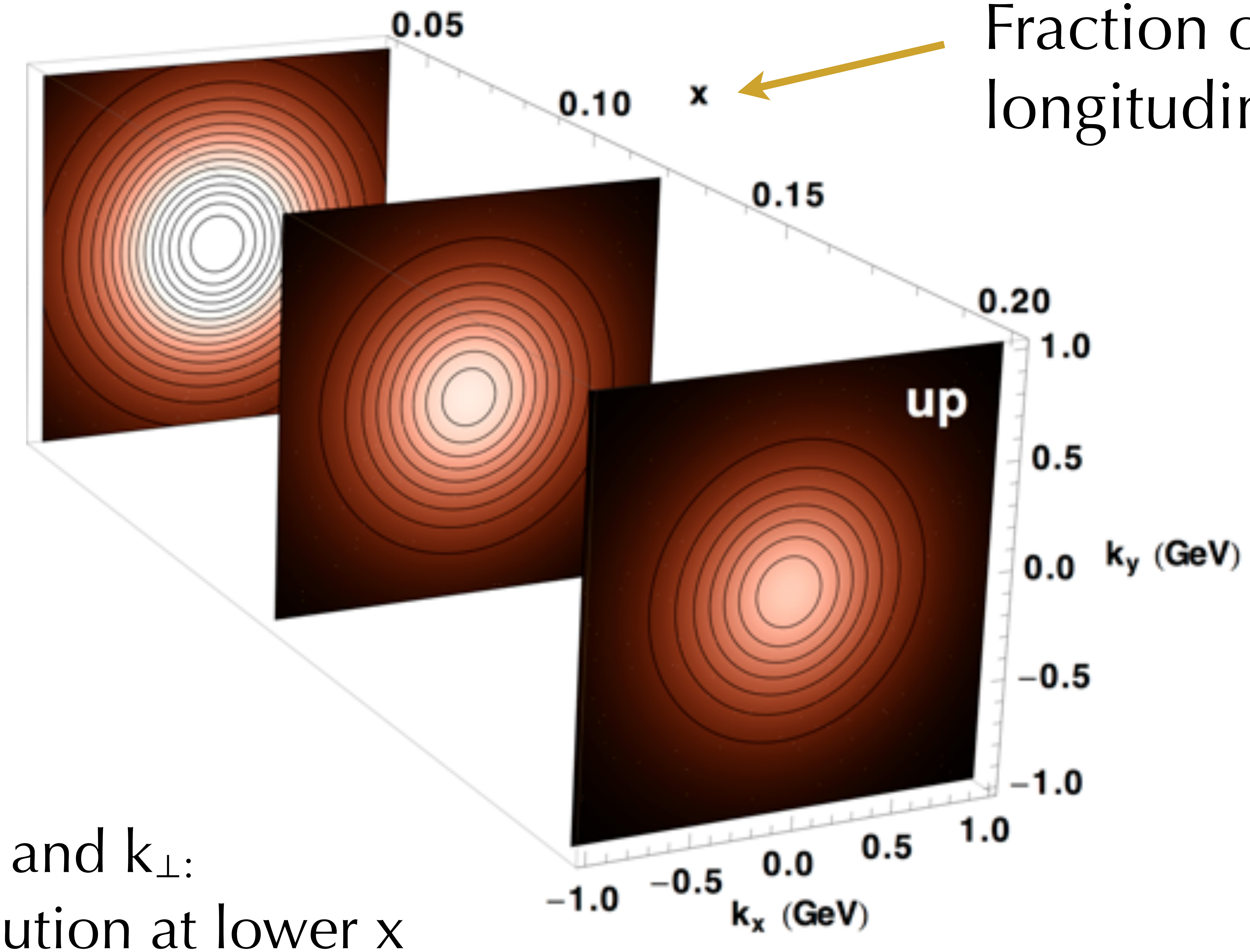
Sivers function$_{\text{SIDIS}}$ = – Sivers function$_{\text{Drell-Yan}}$

Boer-Mulders function$_{\text{SIDIS}}$ = – Boer-Mulders function$_{\text{Drell-Yan}}$

Strong QCD prediction. Needs to be tested.

*Collins, PLB 536 (2002) 43*

# The unpolarized TMD $f_1$

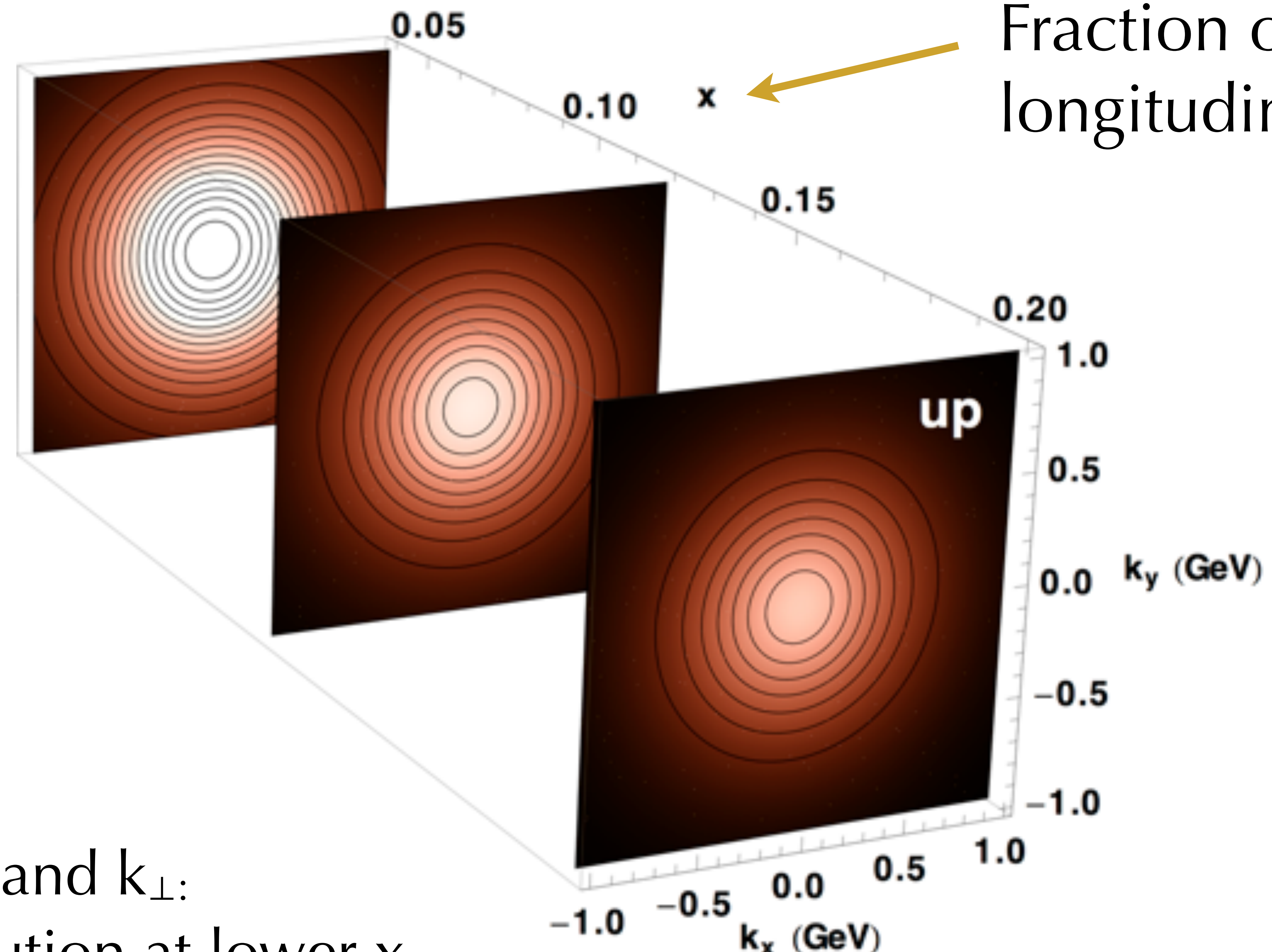


Fraction of
longitudinal momentum

Transverse momentum

Correlation between x and $k_\perp$:
widening of the distribution at lower x

# The unpolarized TMD $f_1$



Fraction of longitudinal momentum

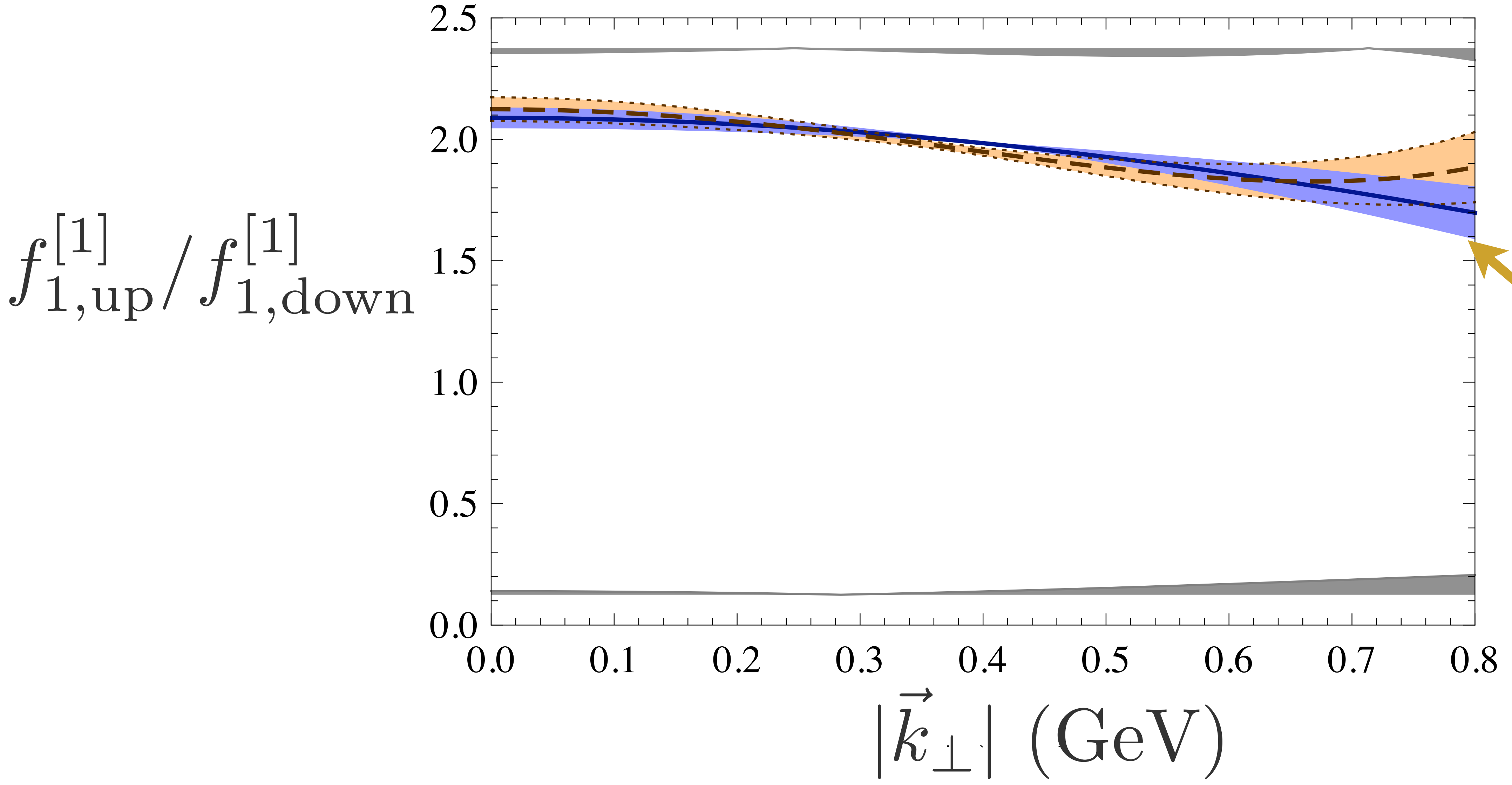
Transverse momentum

Correlation between x and $k_\perp$:
widening of the distribution at lower x

We know the integrated PDF very well.
We know the TMD still poorly.

# Flavor structure of TMDs: indications from lattice

$$f_{1,\,q}^{[1]}(\vec{k}_\perp^2) = \int_0^1 \mathrm{d}x\, f_{1,\,q}(x,\vec{k}_\perp^2) \longrightarrow$$ number of quarks as function of transverse momentum

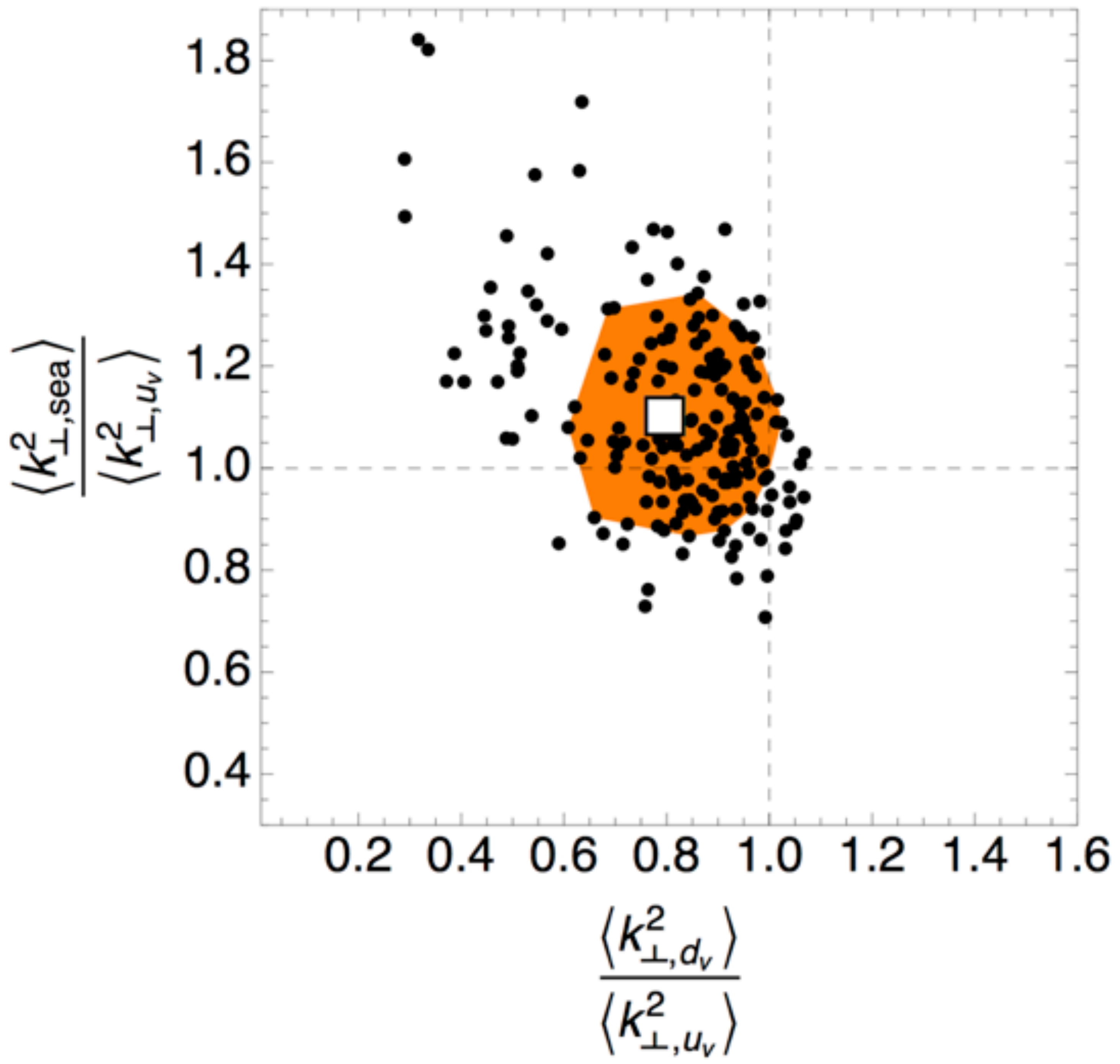


“less” up quarks

Pioneering lattice-QCD studies hint at a down distribution being wider than up

*Musch, Hagler, Negele, Schaefer, PRD**83** (2011) 094507*

# Flavor structure of TMDs: indications from data



Ratio of width of sea /
width of up valence

Ratio width of down valence/
width of up valence

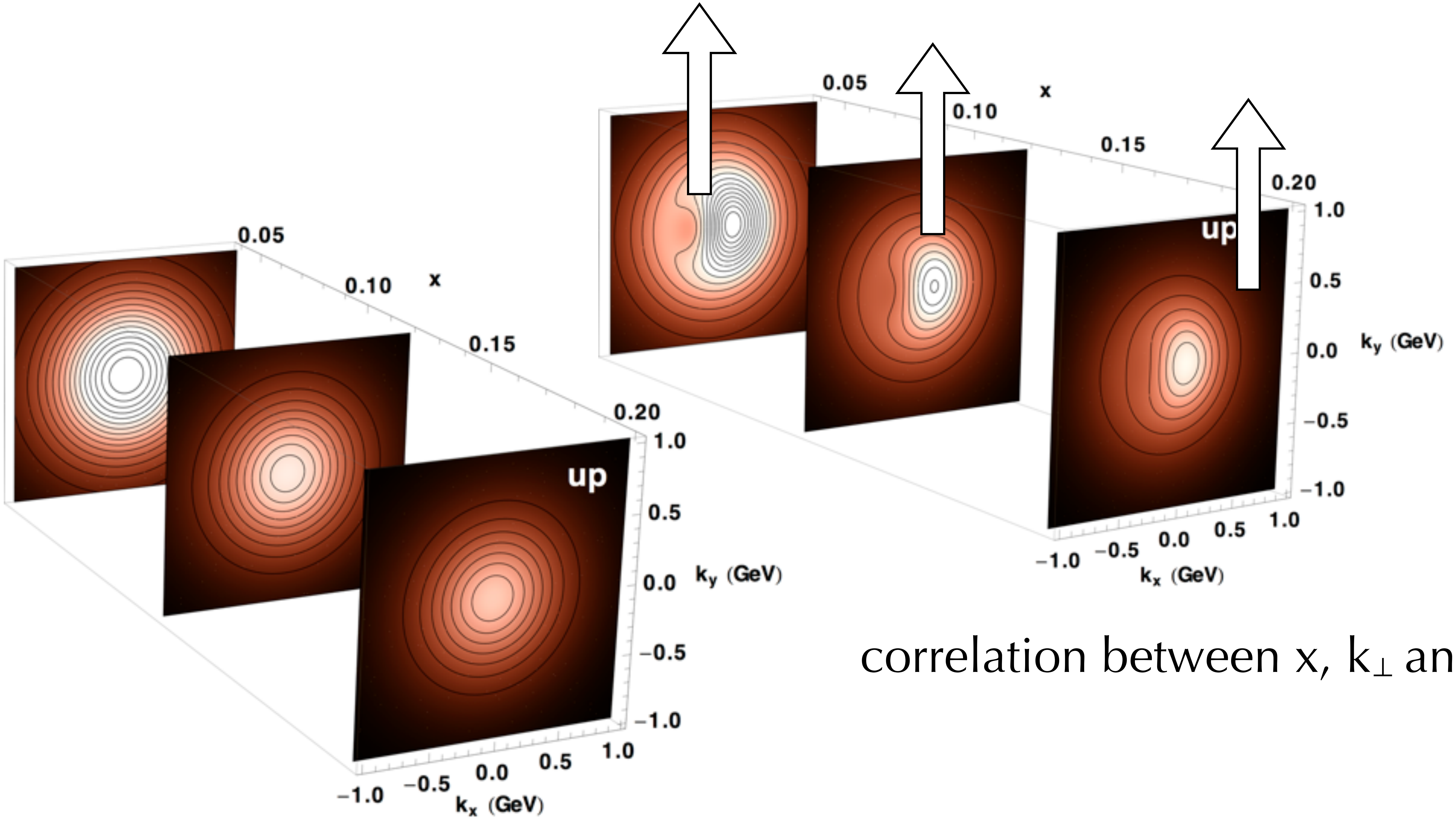
fit to SIDIS multiplicities from HERMES:

$$\langle k^2_{\perp,d_v} \rangle < \langle k^2_{\perp,u_v} \rangle < \langle k^2_{\perp,sea} \rangle$$

*Signori, Bacchetta, Radici, Schnell, JHEP 1311 (13)*

*talk of M. Boglione*

# Adding the spin

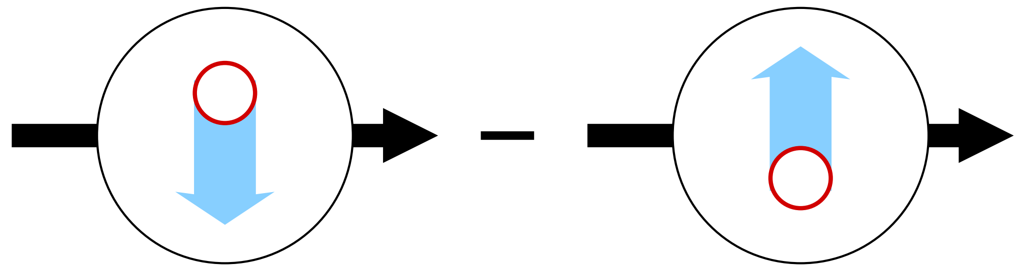


correlation between x, $k_\perp$ and spin

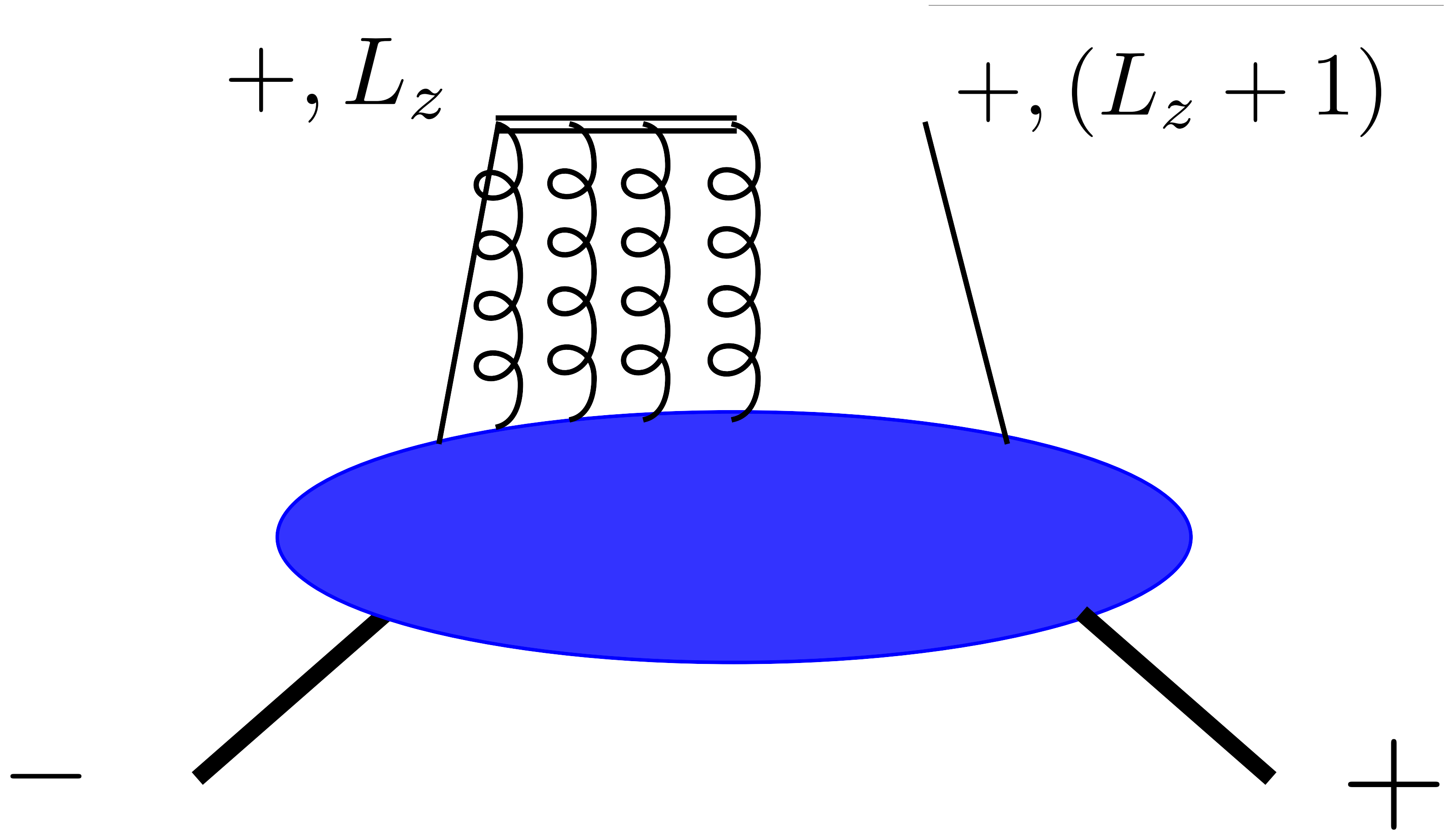
correlation between x and $k_\perp$

# Sivers function

$f_{1T}^{\perp} =$



unpolarized quarks in ⊥ pol. nucleon

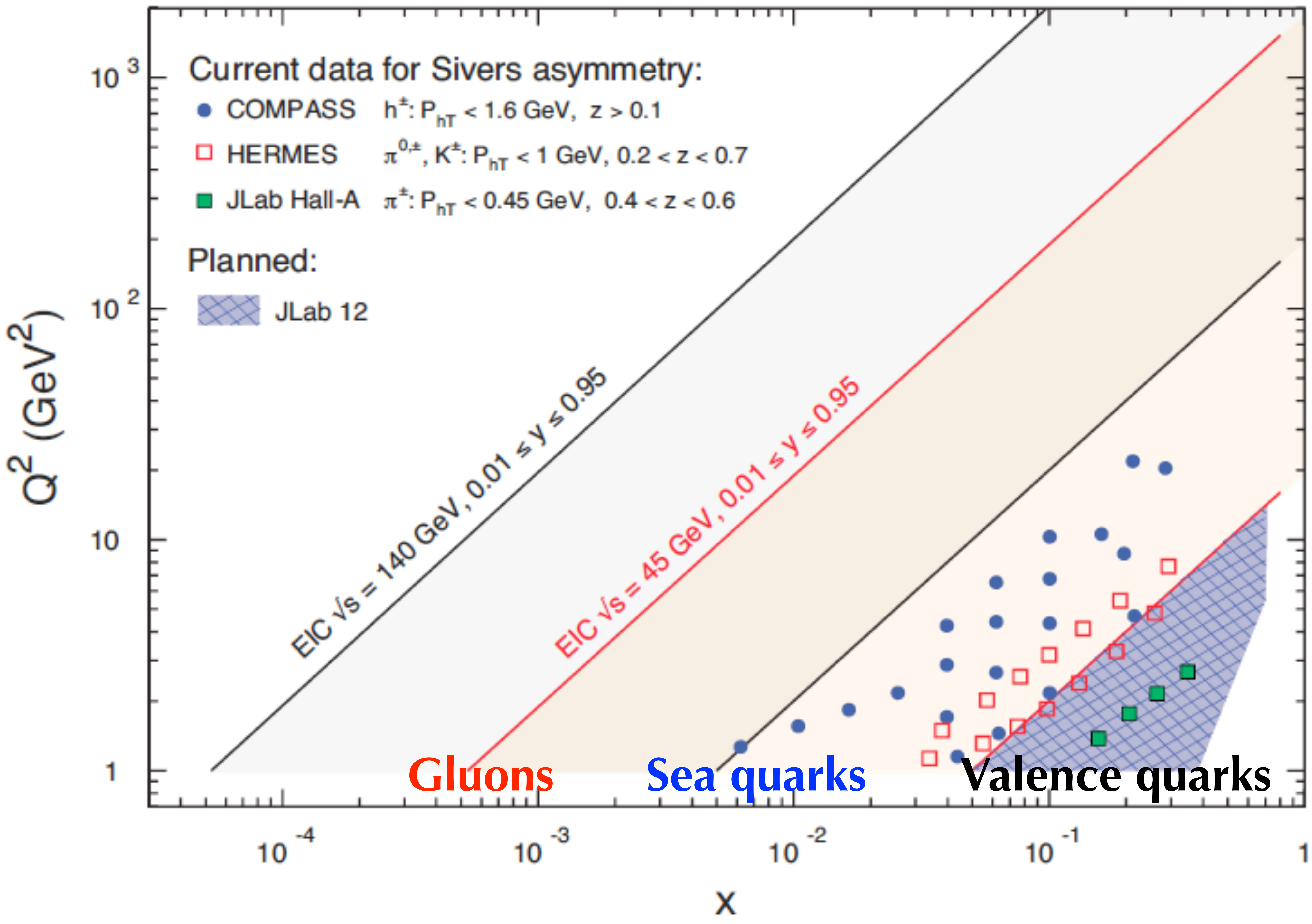


$$f_{1T}^{\perp}|_{\text{SIDIS}} = -f_{1T}^{\perp}|_{\text{DY}}$$

non-zero ONLY with final-state interaction

the helicity mismatch requires orbital angular momentum

# Paste, present and future TMD measurements



hermes

Jefferson Lab

COMPASS

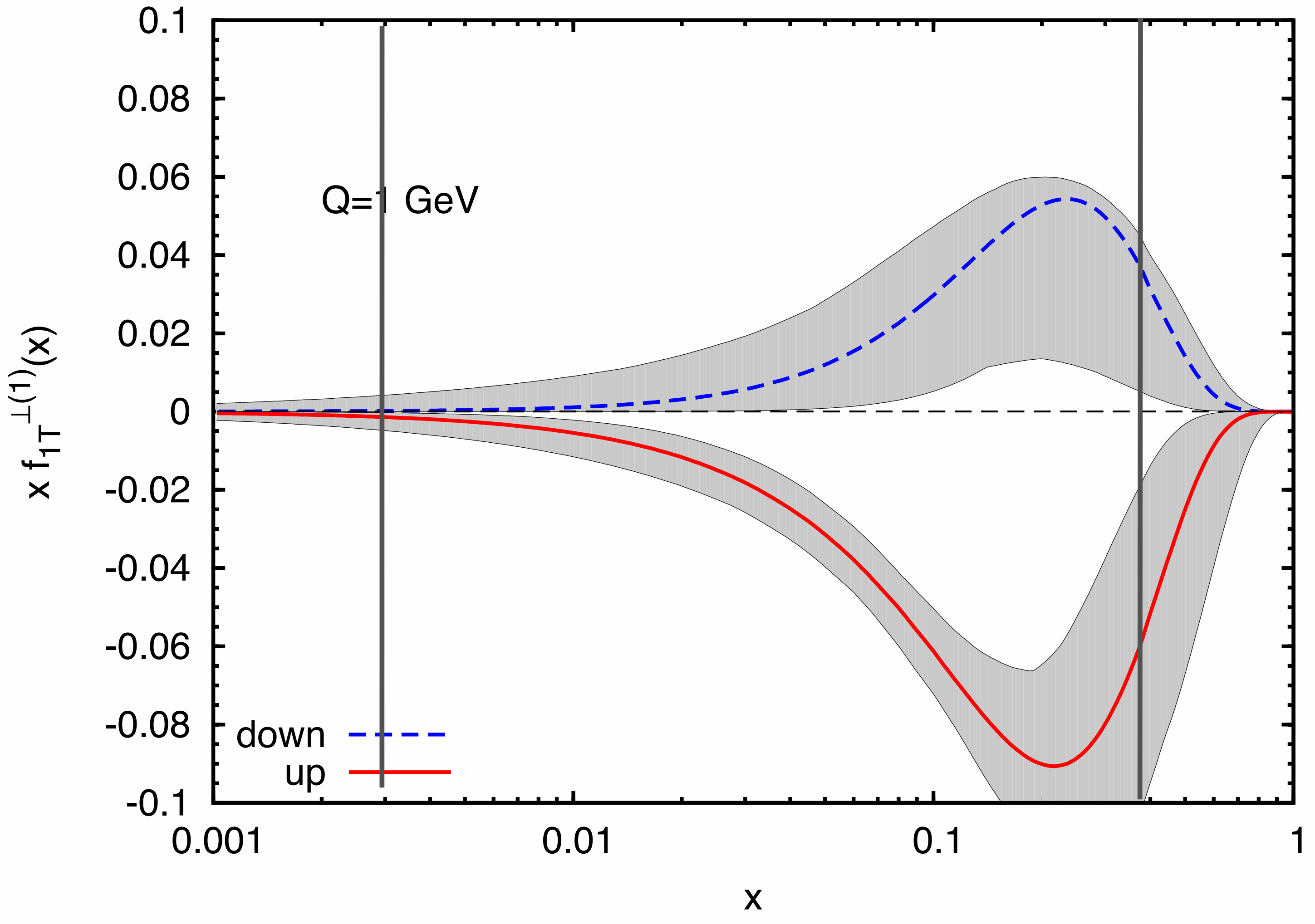
EIC

*Accardi et al., The Electron Ion Collider: the next QCD Frontier*
*arXiv:1212.1701*

*talks of A. Bressan, M. Boglione and M. Contalbrigo*

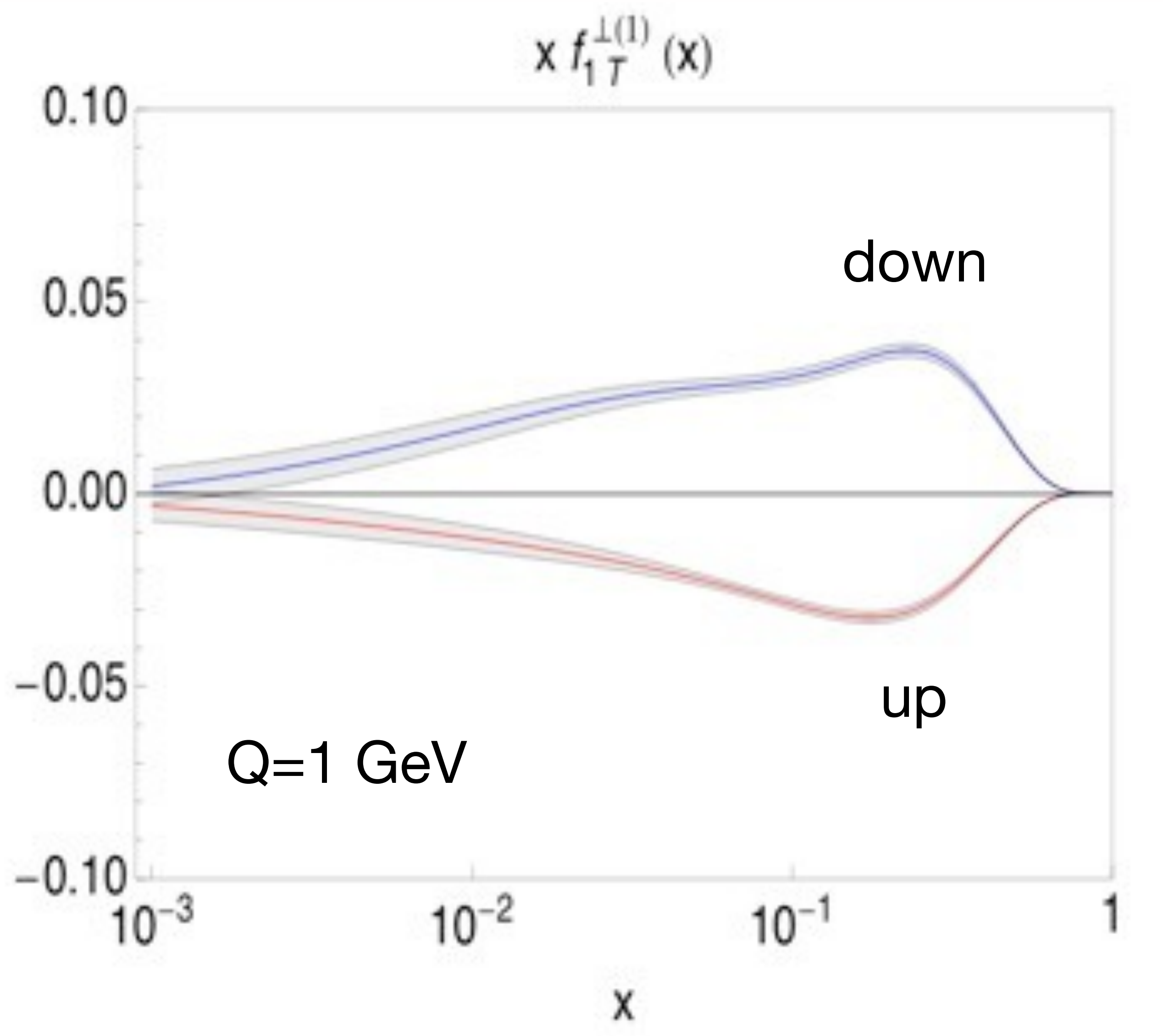
# Sivers function has been extracted

## Torino 2012 update



*adapted by Stefano Melis from*
*Anselmino et al., PRD**86** (2012) 014028*

## Pavia 2011



*Bacchetta, Radici, PRL**107** (2011) 012001*

*talk of M. Boglione*

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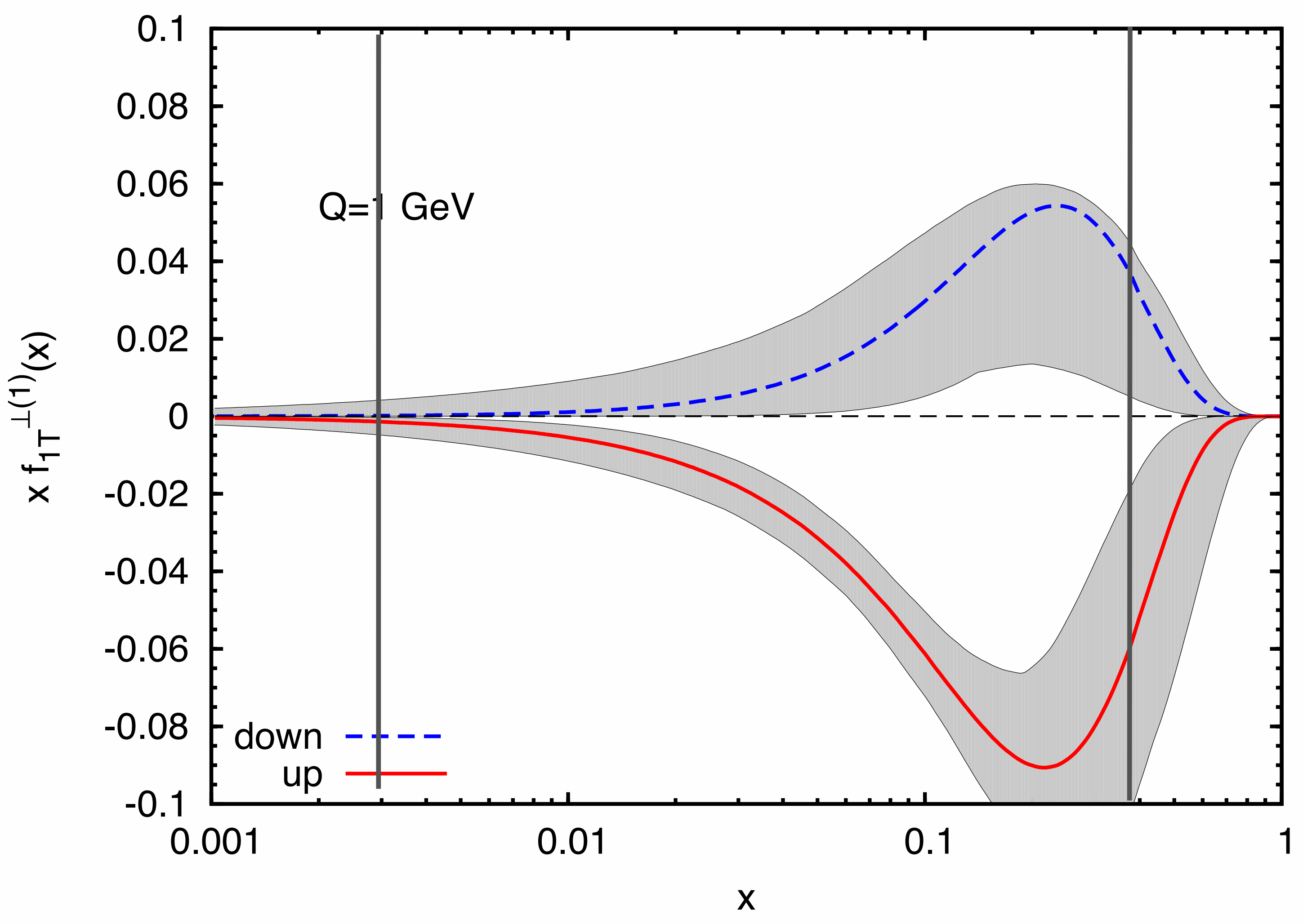

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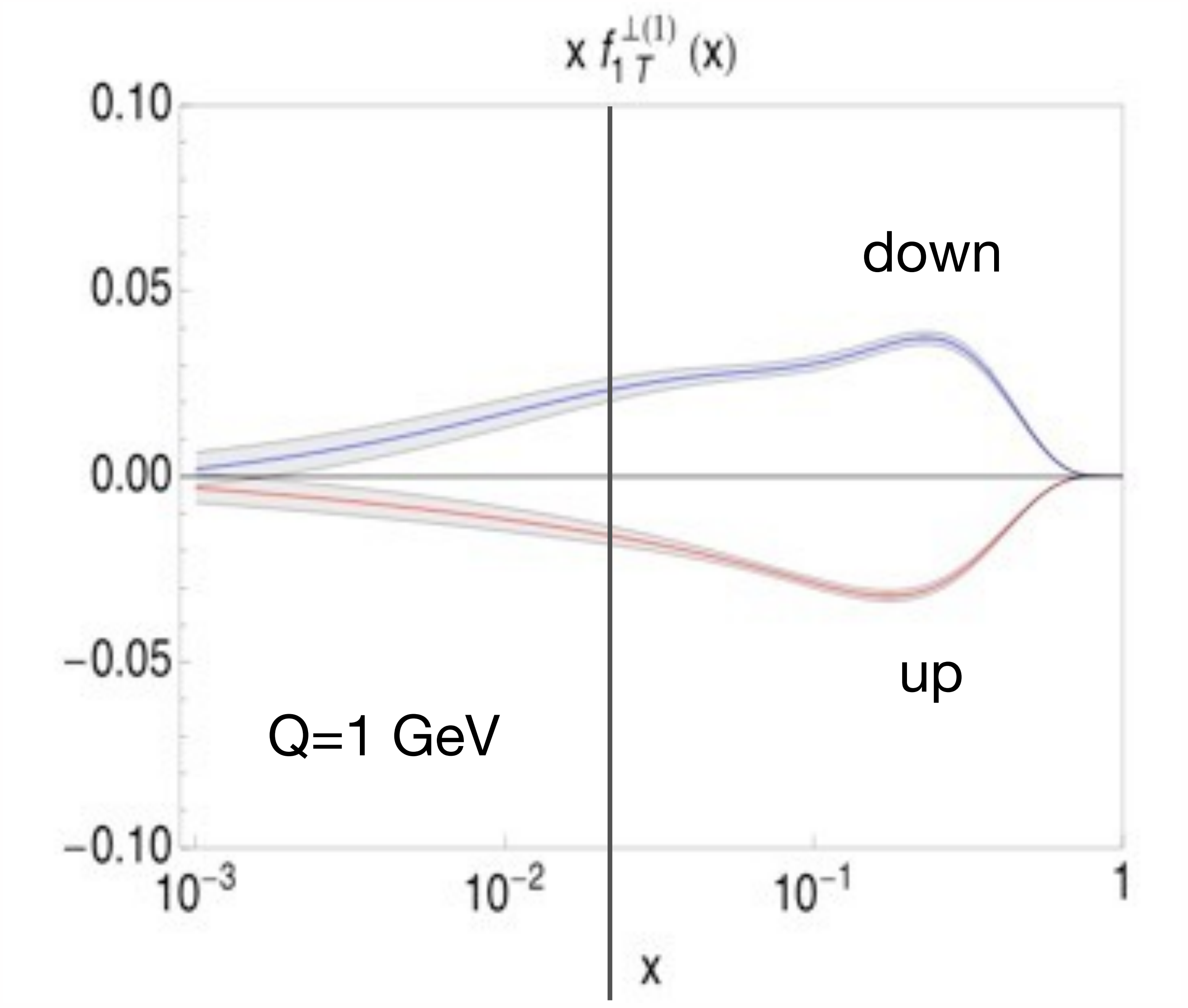
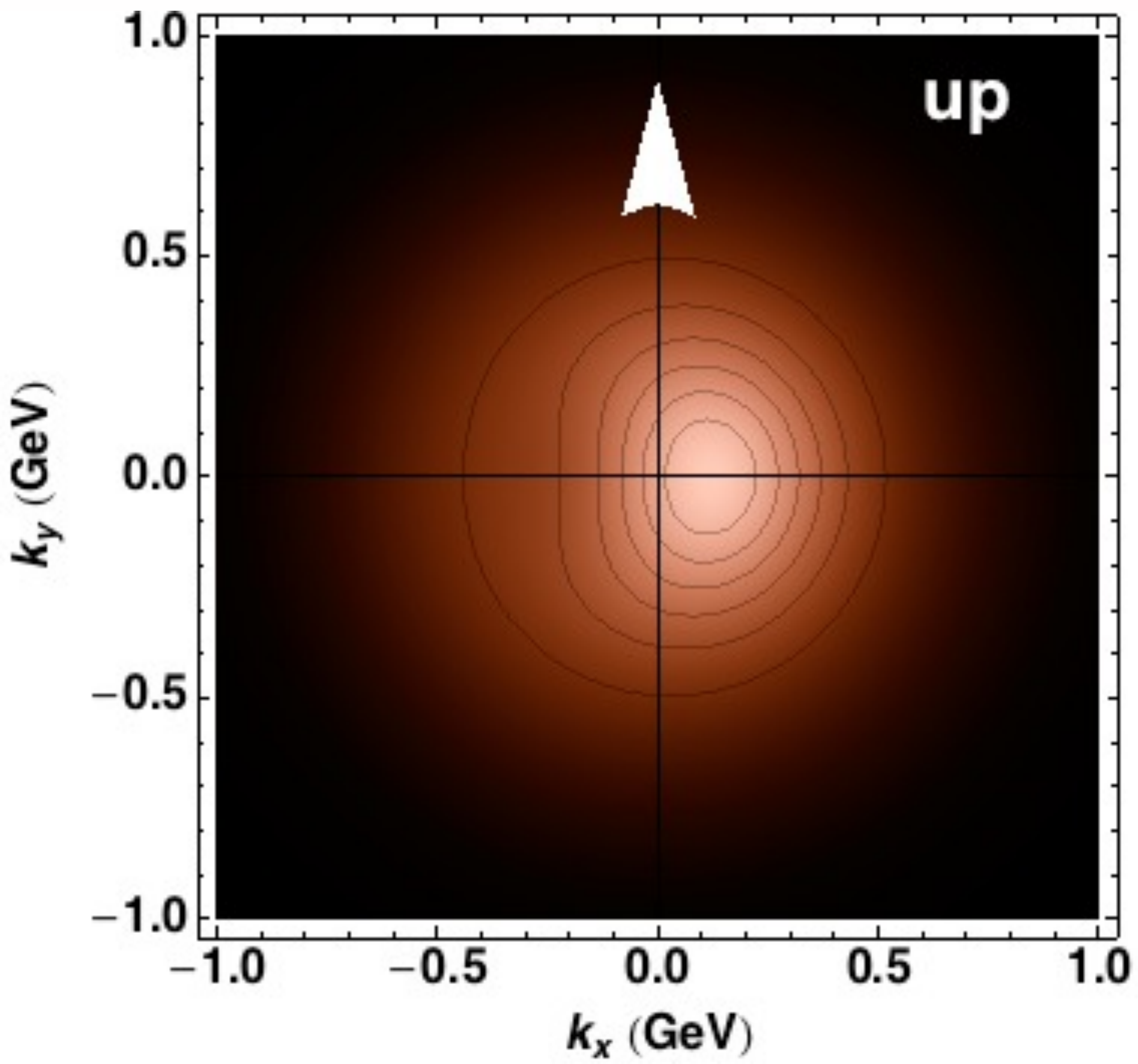


*Bacchetta, Radici, PRL**107** (2011) 012001*

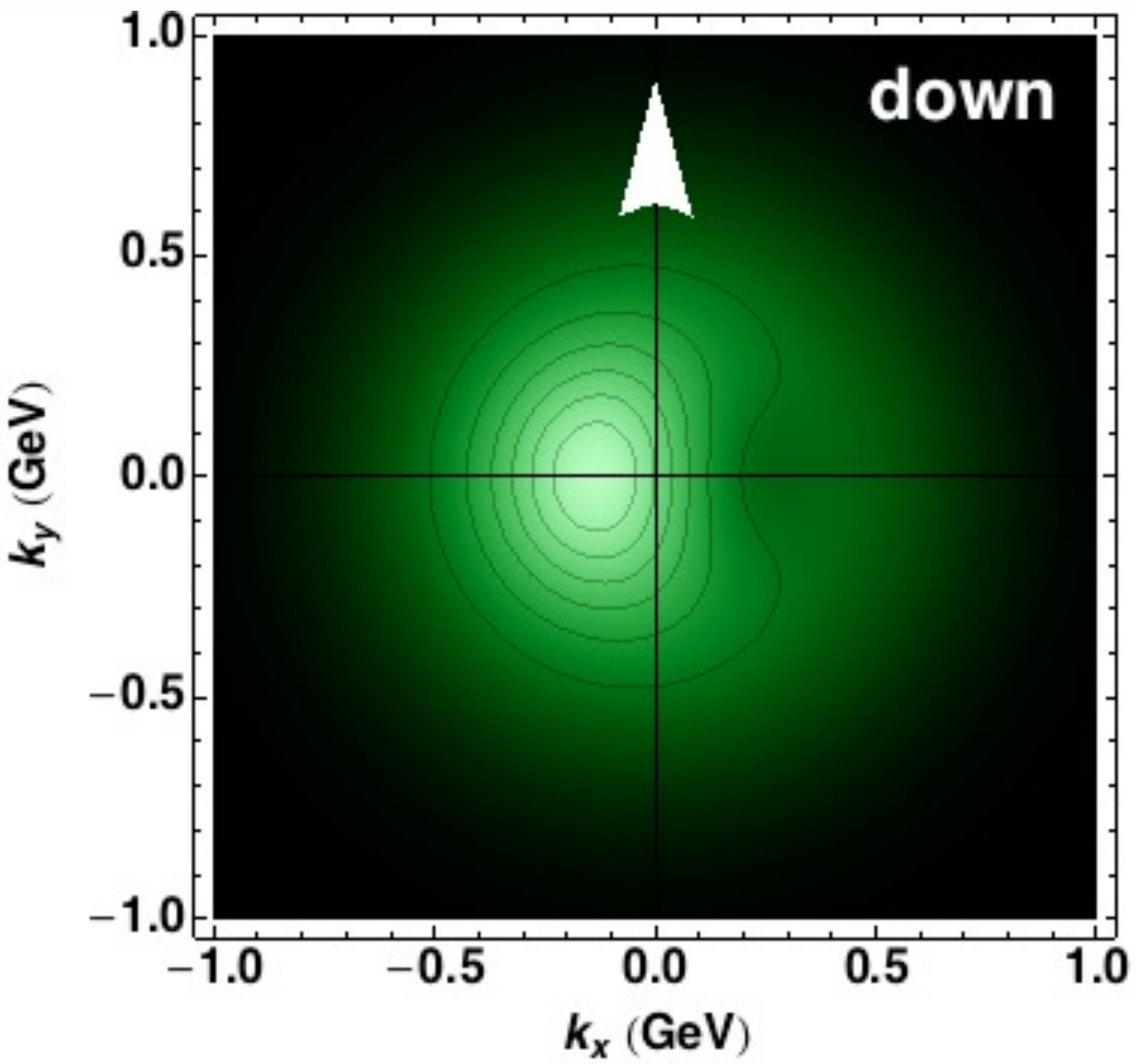
*talk of M. Boglione*

# distribution of unpolarized q in ⊥ polarized $\mathrm{p}^\uparrow$

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\,\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot\mathbf{S}}{M}$$



$S_y$
polarization



deformation induced by Sivers function

*Bacchetta & Contalbrigo, The proton in 3D*
*Il Nuovo Saggiatore* **28** (12) n.1,2

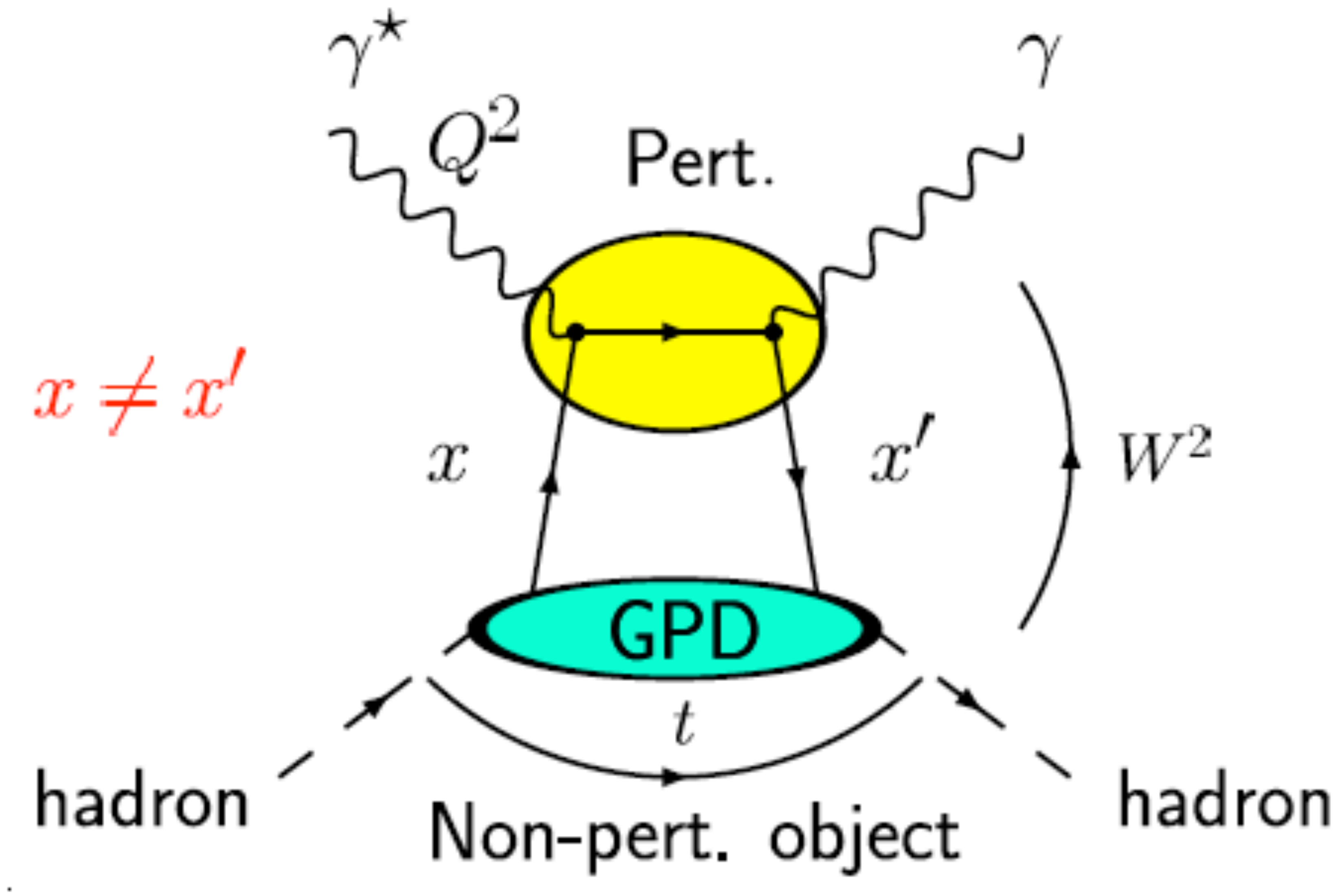
# Key information from GPDs

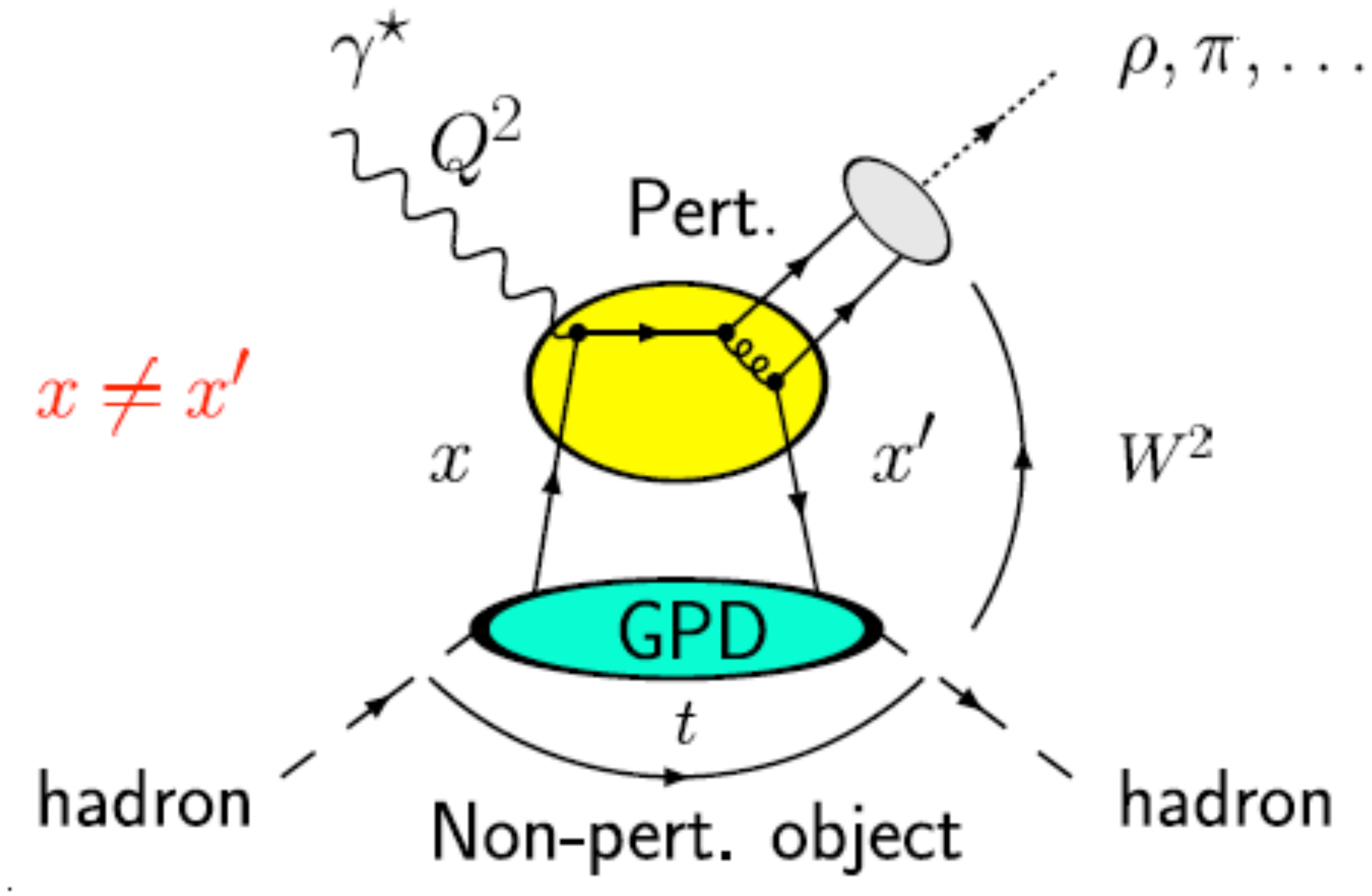
- Transverse position size
- Decomposition of Form Factors w.r.t. x
- Sum rule for Angular Momentum
- Access to Form Factors of Energy Momentum Tensor

$\longrightarrow$ “mechanical” properties of the nucleon

S. Pisano: GPDs in experiments M. Guidal: GPD phenomenology M. Contalbrigo: 3D future

# How to measure the GPDs





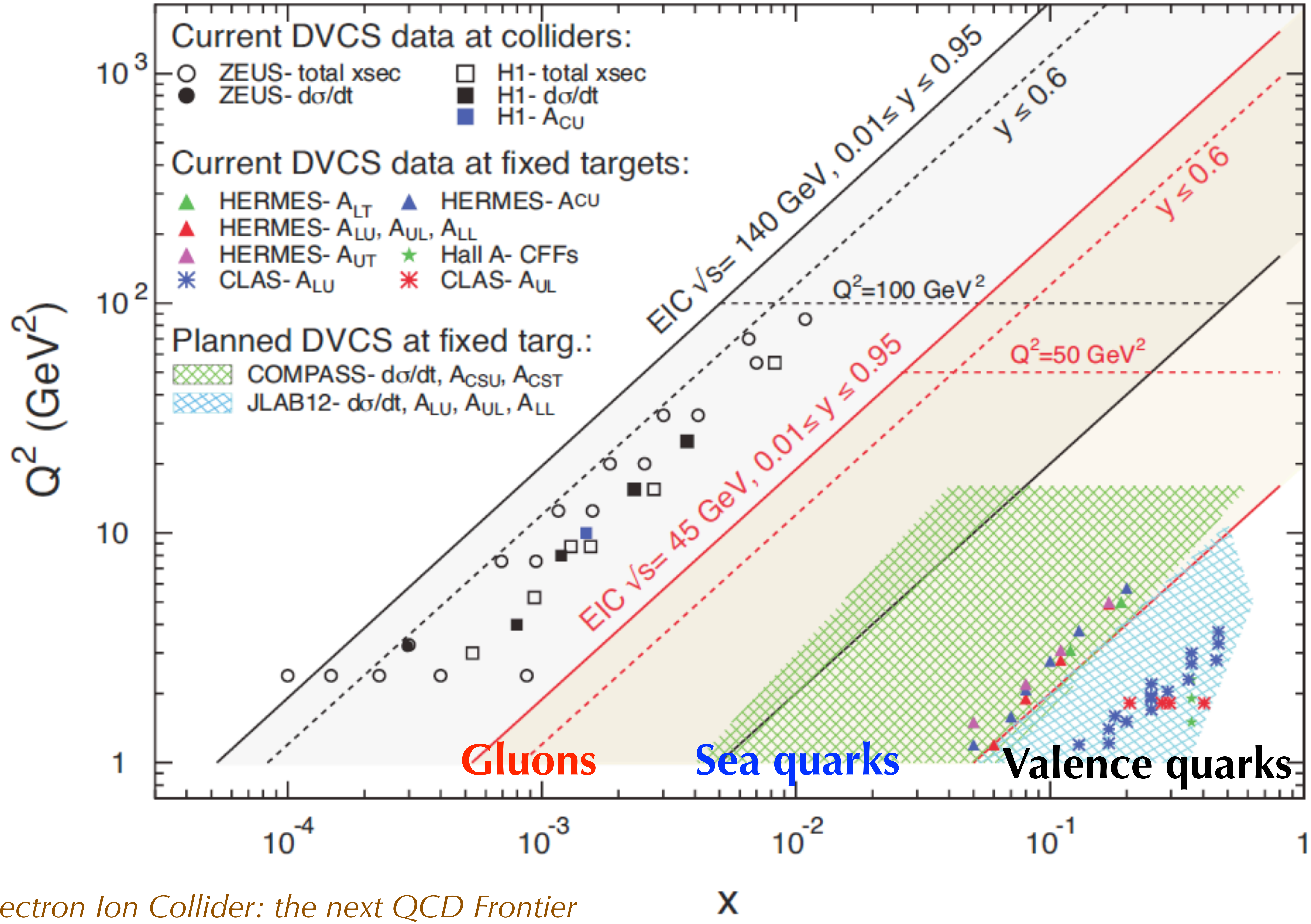
- accessible in exclusive reactions
- factorization for large $Q^2$, $|t| << Q^2$ , $W^2$
- depend on 3 variables: $x, \xi, t$

Compton Form Factors

$$\mathrm{Im}\, \mathcal{H}(\xi, t) \overset{\mathrm{LO}}{=} H(\xi, \xi, t)$$

$$\mathrm{Re}\, \mathcal{H}(\xi, t) \overset{\mathrm{LO}}{=} \mathcal{P} \int_{-1}^{1} \mathrm{d}x\ \mathrm{H}(x, \xi, t) \frac{1}{x-\xi}$$

# Paste, present and future DVCS experiments



DESY

hermes

Jefferson Lab

COMPASS

EIC

*Accardi et al., The Electron Ion Collider: the next QCD Frontier*
*arXiv:1212.1701*

*See talks of S. Pisano, M. Guidal, M. Contalbrigo*

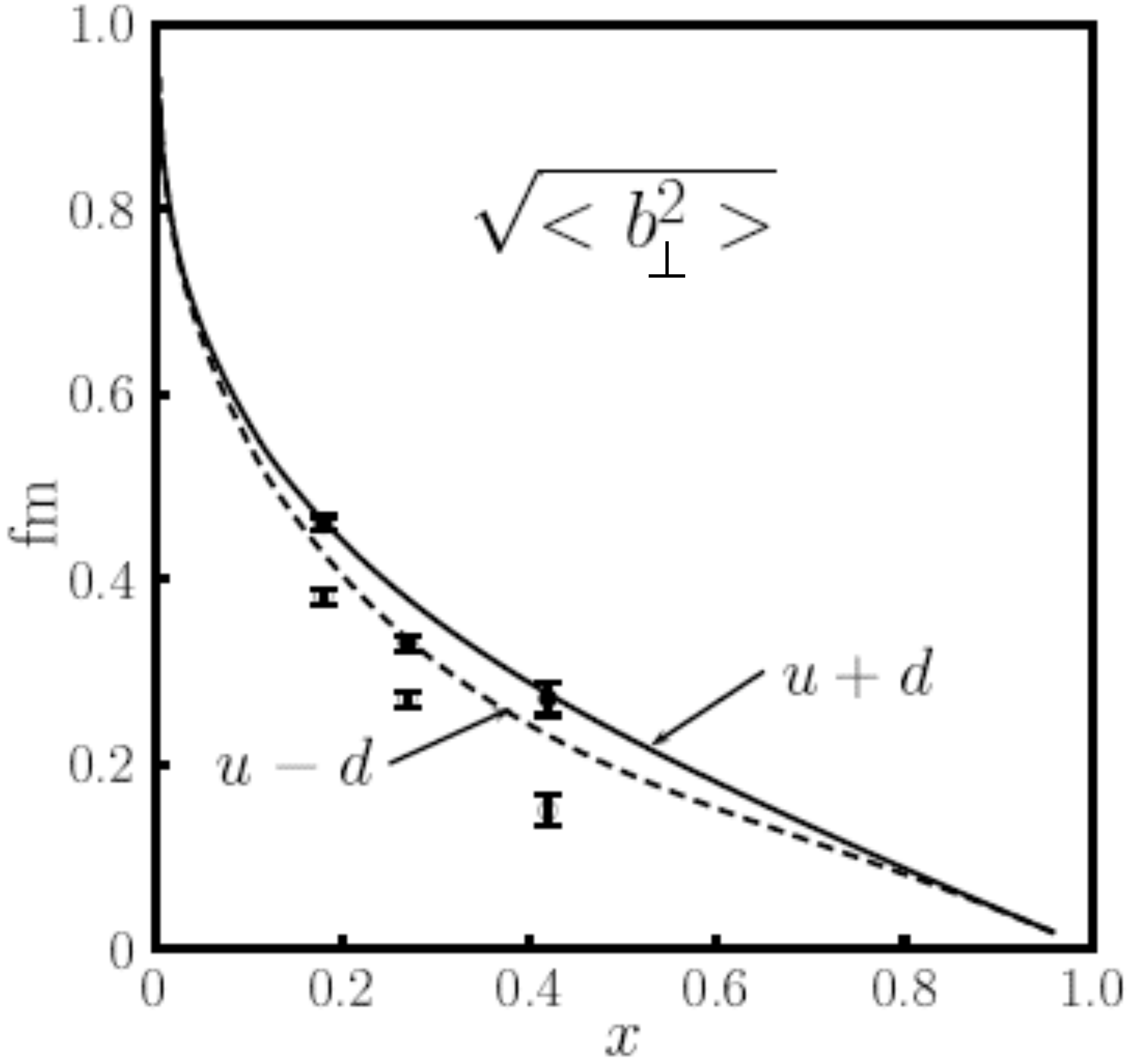
# The unpolarized GPD H

$$F_1(t) = \int \mathrm{d}x \, H(x,0,t)$$

$$H(x,0,\vec{b}_\perp) = \int \mathrm{d}^2\Delta_\perp \, H(x,0,t) \, e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp}$$

$$t = -\vec{\Delta}_\perp^2$$

As x ⟶ 1, the active parton carries all the momentum
and represents the centre of momentum



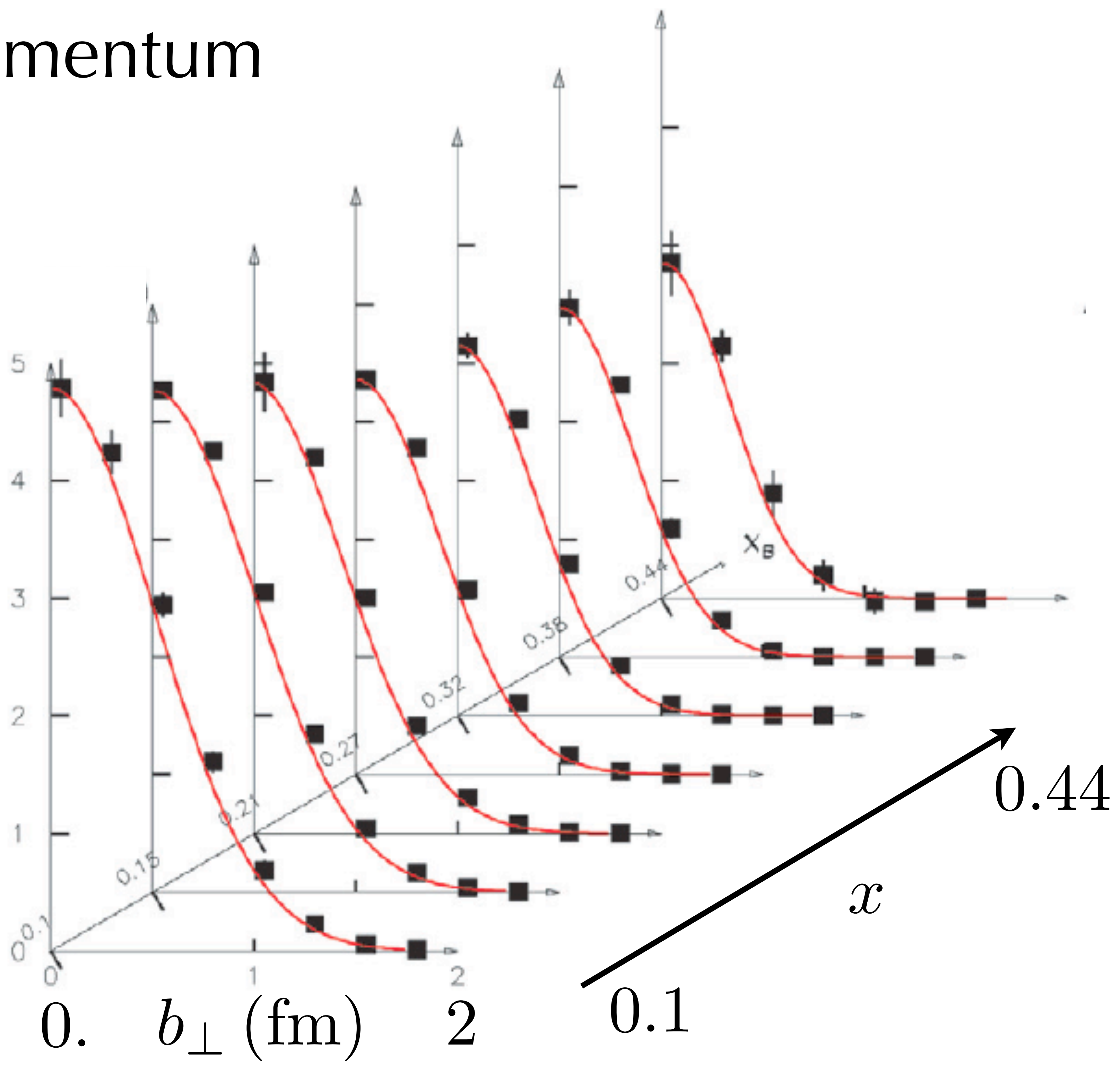
Lattice results

*Negele et al., NPB Proc. Suppl.* ***128*** *(2004) 170*

$H^q(x,0,b_\perp)$

$\downarrow$

extrapolation from data



From experimental data

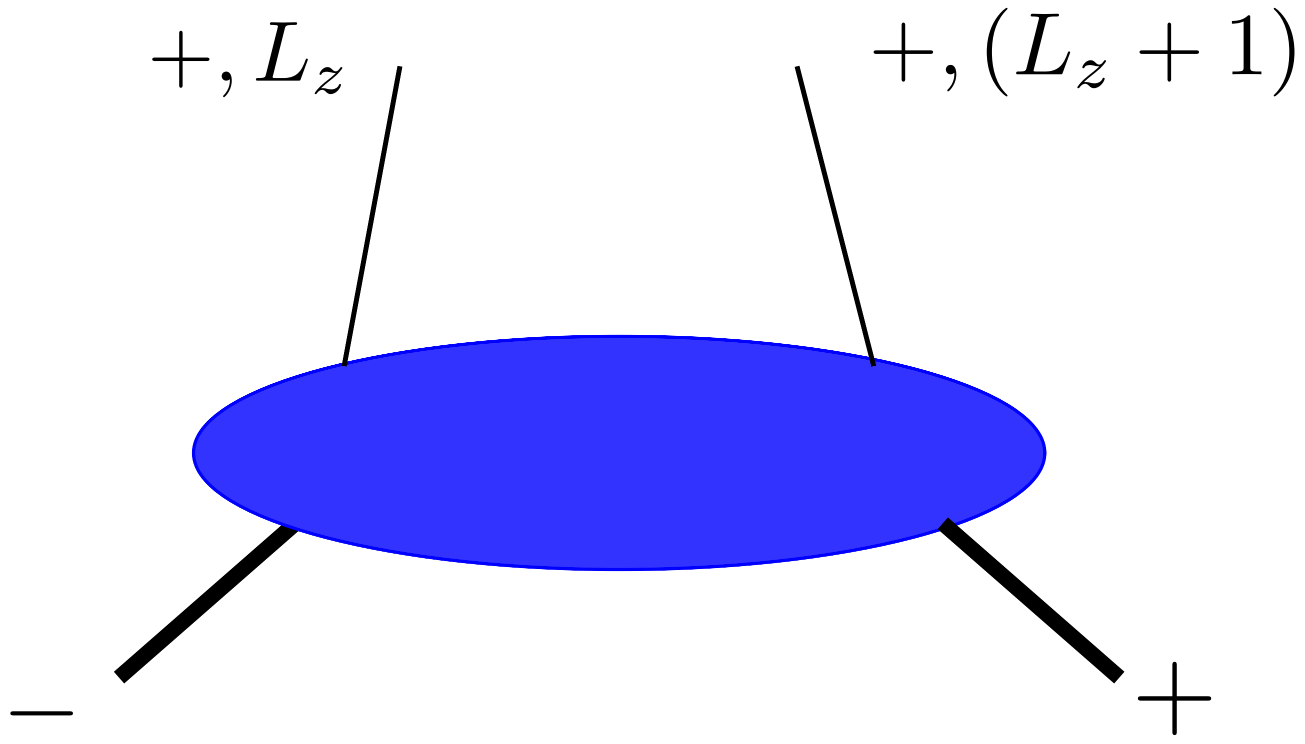
*Guidal et al., Rep. Prog. Phys.* ***76*** *(2013) 066202*

# Unpolarized quarks in transversely pol. nucleon

"Helicity mismatch" requires orbital angular momentum

- $F_2(t) = \int \mathrm{d}x \, E(x, \xi, t)$
- no-forward limit to PDF

GPD E



unpolarized quarks
in ⊥ pol. nucleon
↓
"partner" of Sivers function

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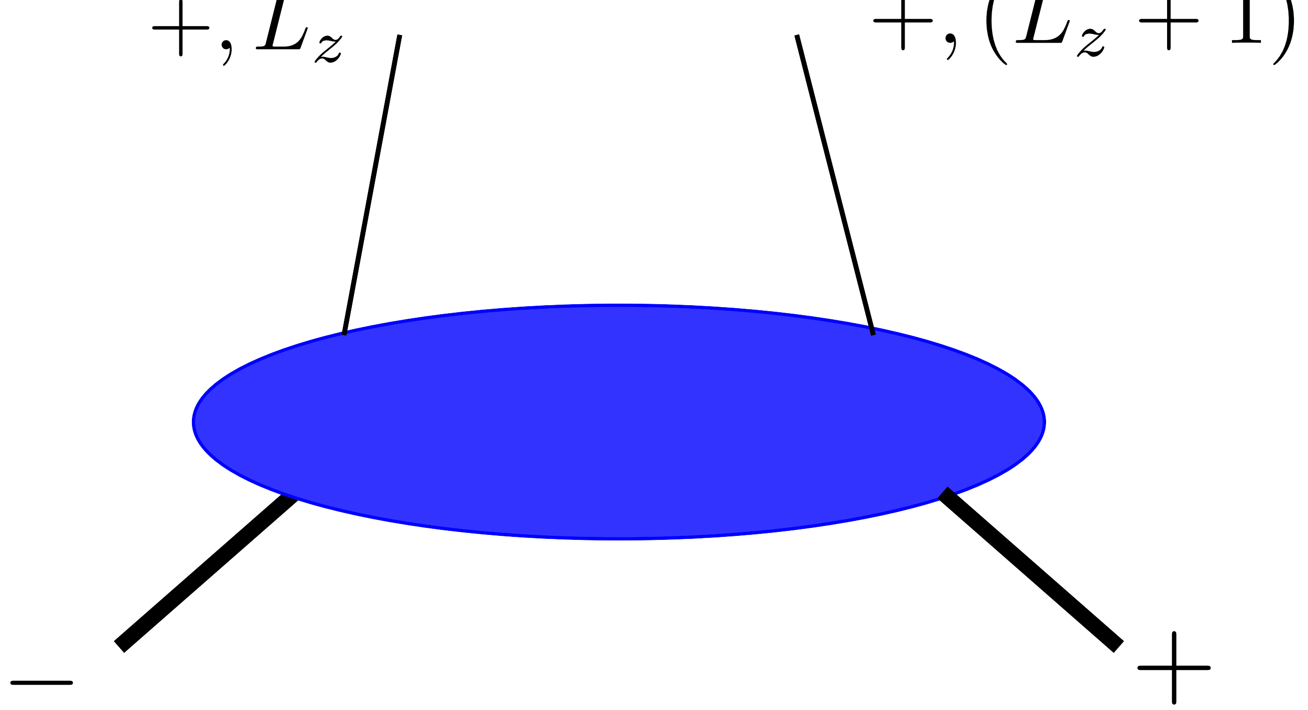


unpolarized quarks
in ⊥ pol. nucleon
↓
"partner" of Sivers function

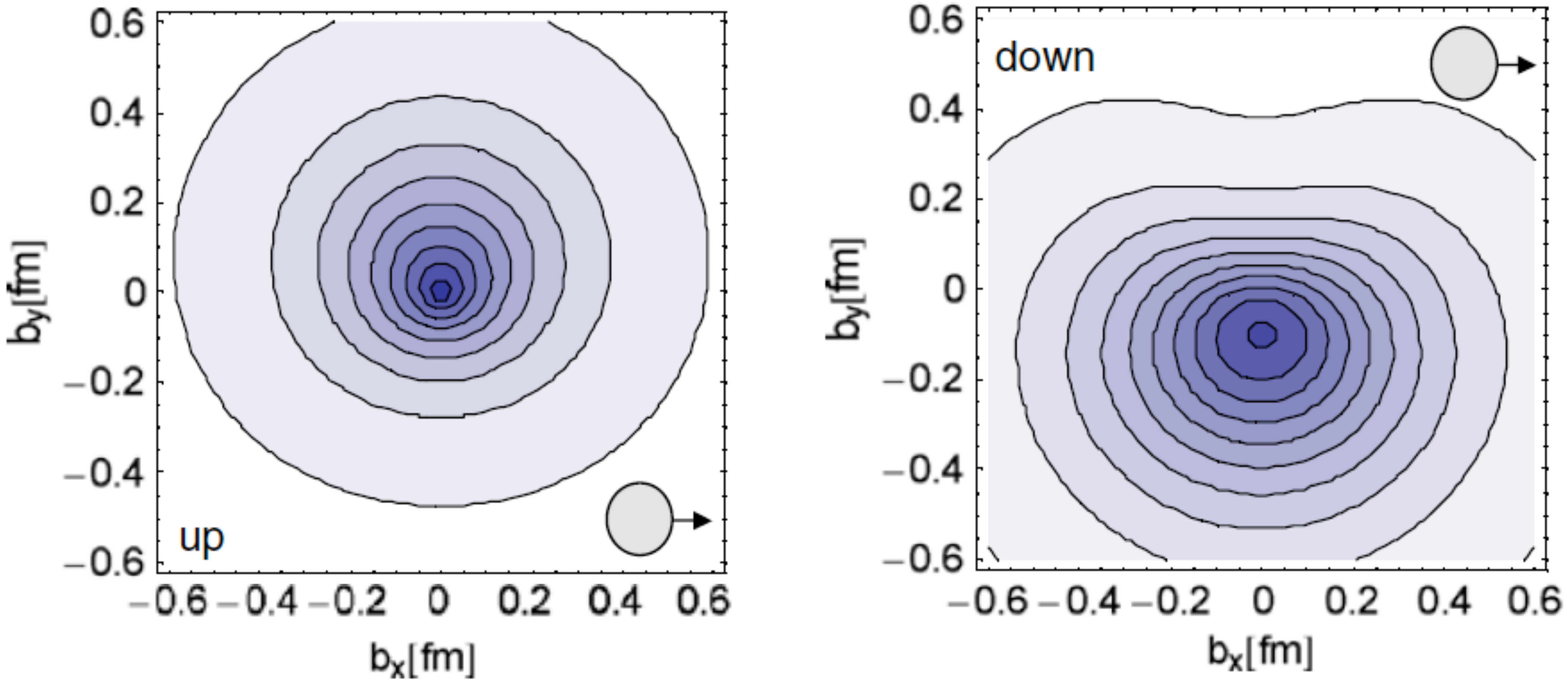
Transverse dipole moment:

$$d_y^q = \frac{\kappa^q}{2M}$$

$$\kappa^u = 1.86 \quad \kappa^d = -1.57$$

quark contribution to proton anomalous magnetic moment

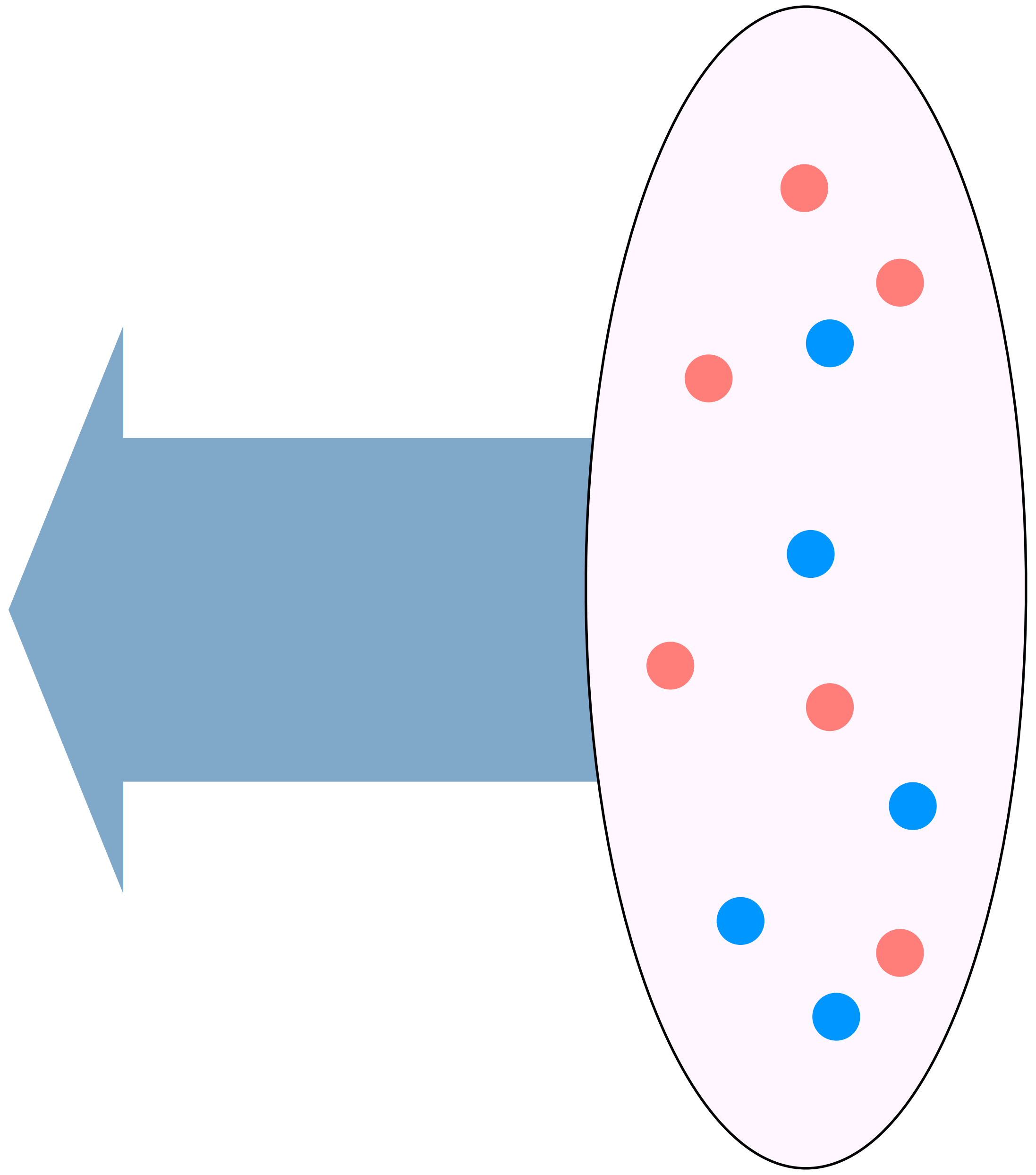
Lattice calculation



*Goeckeler et al., Phys. Rev. Lett.* ***98*** *(2007) 222001*

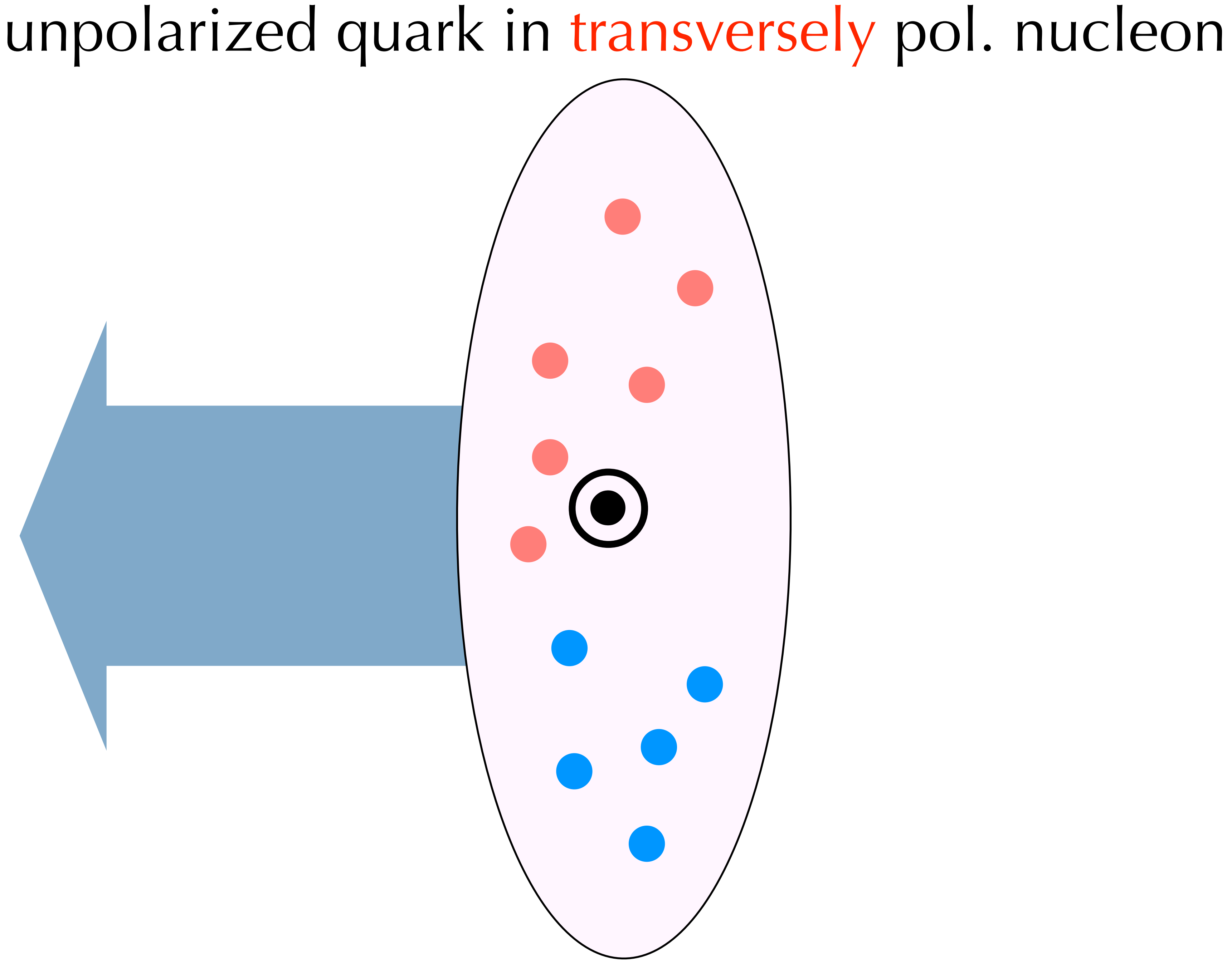
# Model relation TMD ⟷ GPD

unpolarized quark in unpolarized nucleon



*Burkardt, PRD **66** (2002) 114005*

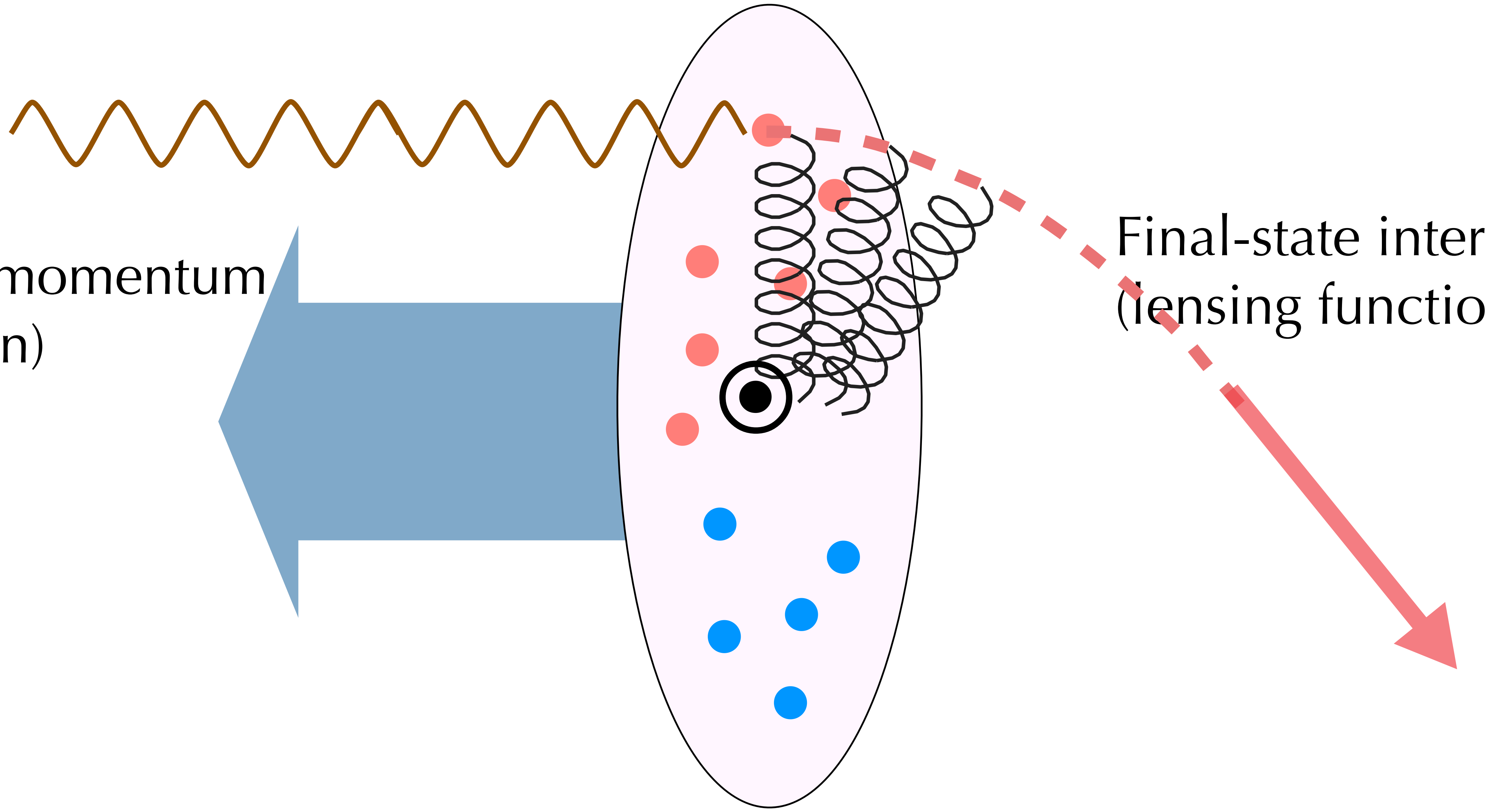
# Model relation TMD $\longleftrightarrow$ GPD



Distortion in impact parameter
(related to GPD E)

*Burkardt, PRD* ***66*** *(2002) 114005*

# Model relation TMD ⟷ GPD

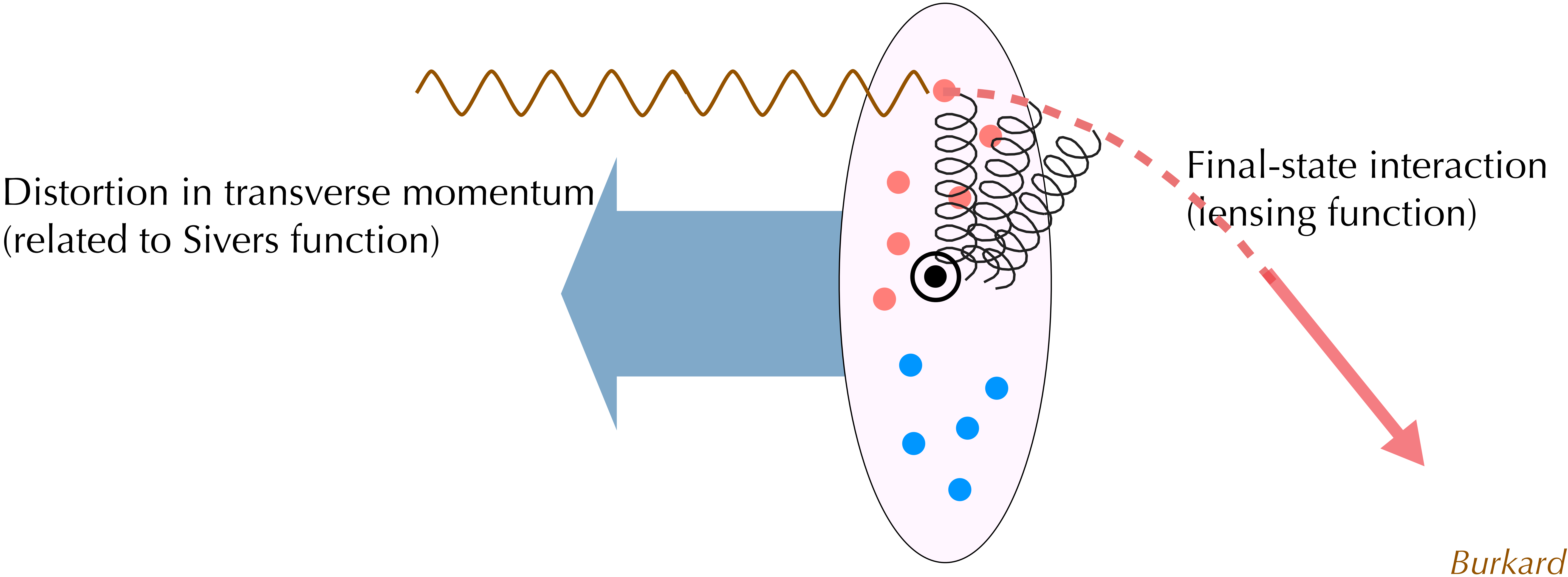


Distortion in transverse momentum
(related to Sivers function)

Final-state interaction
(lensing function)

*Burkardt, PRD **66** (2002) 114005*

# Model relation TMD ⟷ GPD



Distortion in transverse momentum
(related to Sivers function)

Final-state interaction
(lensing function)

*Burkardt, PRD* ***66*** *(2002) 114005*

$$-\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^2) \simeq \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x,\vec{b}_T^2)\right)'$$

Sivers function

Lensing function

F.T. of E(x,0,t)

Successful phenomenological applications:

*Bacchetta, Radici, PRL* ***107*** *(2011) 212001*

*Gamberg, Schlegel, PLB* ***685*** *(2010) 95*

# Angular Momentum Relation ("Ji's Sum Rule")

*X. Ji, PRL **78** (1997) 610*

quark and gluon contribution to the nucleon spin

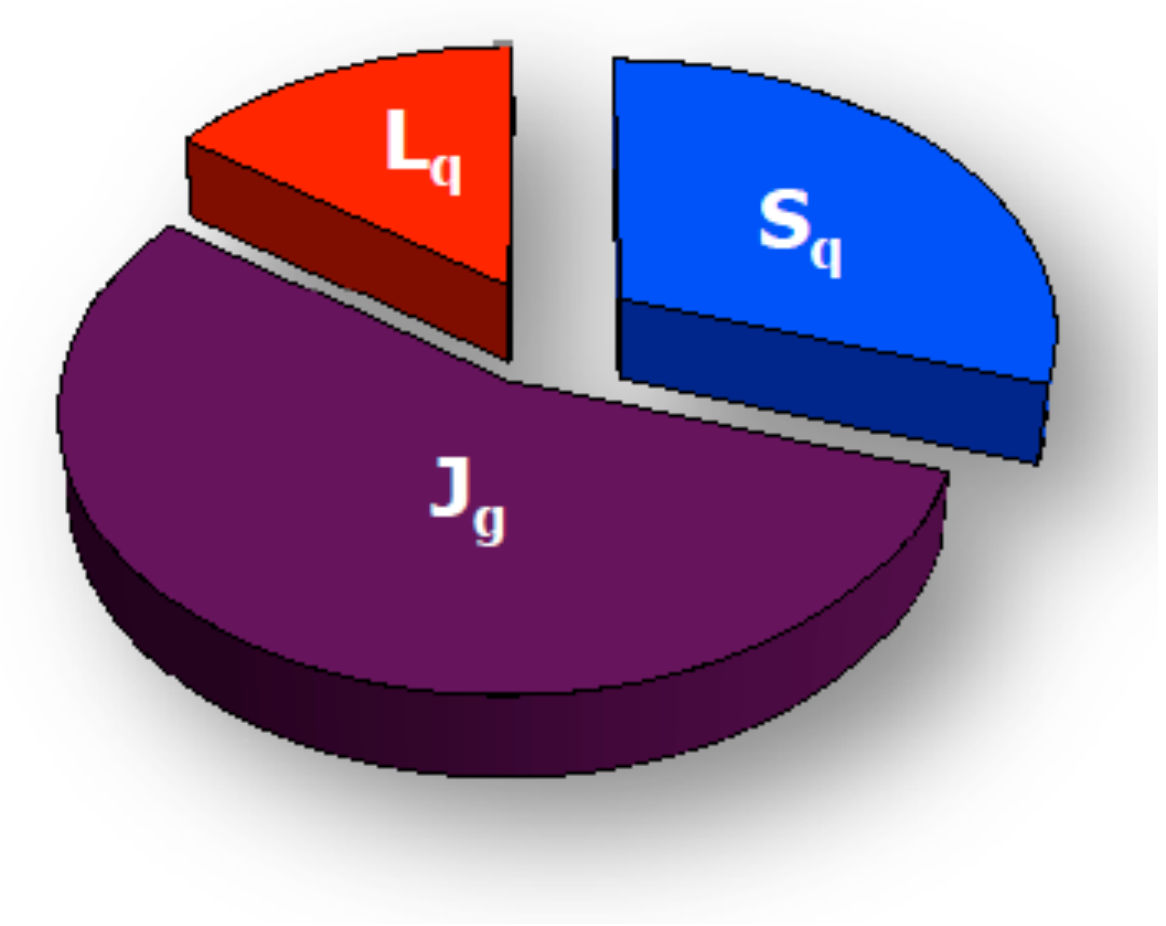
$$J^{q,g} = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x \left( \underset{\Big\downarrow}{H^{q,g}}(x,0,0) + E^{q,g}(\underset{\Big\downarrow}{x},0,0) \right)$$

not directly accessible

## Proton spin decomposition

$$J^q = L^q + \overset{\frac{1}{2}\Delta\Sigma \text{ from DIS}}{\overset{\uparrow}{S^q}}$$

gauge invariant decomposition



$J^g$

no further gauge-invariant decomposition

# Angular Momentum Relation (“Ji’s Sum Rule”)

*X. Ji, PRL **78** (1997) 610*

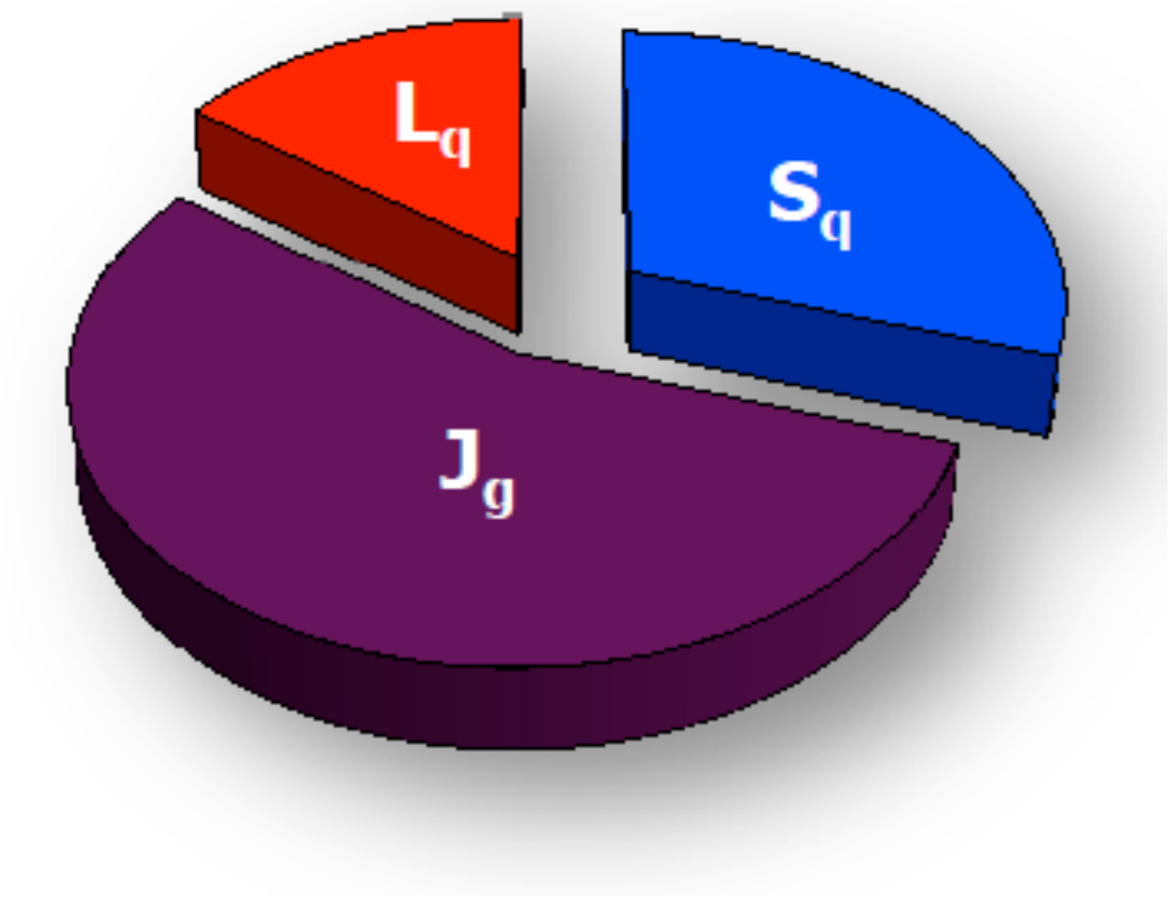
quark and gluon contribution to the nucleon spin

$$J^{q,g} = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x \left( \underset{\substack{\downarrow \\ \text{unpolarized PDF}}}{H^{q,g}(x,0,0)} + \underset{\substack{\downarrow \\ \text{not directly accessible}}}{E^{q,g}(x,0,0)} \right)$$

Proton spin decomposition

$$J^q = L^q + \overset{\substack{\frac{1}{2}\Delta\Sigma \text{ from DIS} \\ \uparrow}}{S^q}$$

gauge invariant decomposition



$$J^g$$

no further gauge-invariant decomposition

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Proton spin decomposition

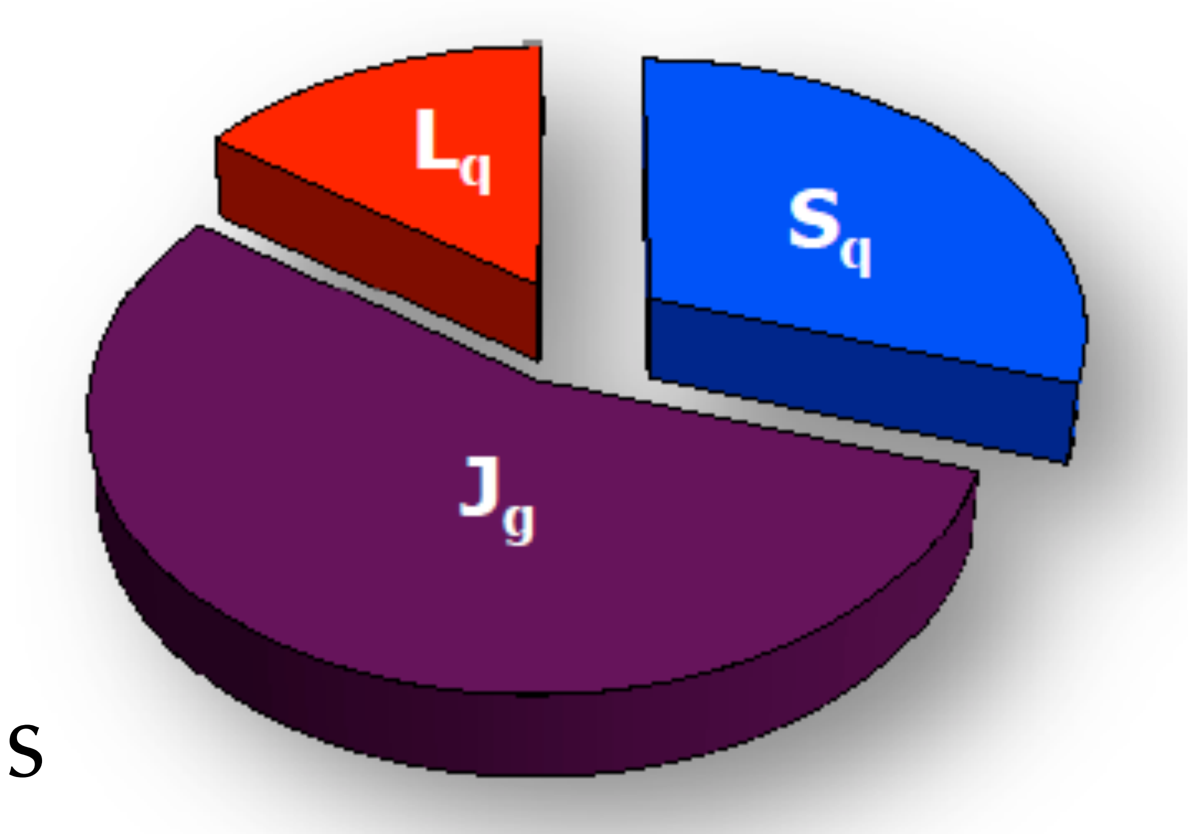
$$\overset{\overset{\frac{1}{2}\Delta\Sigma \text{ from DIS}}{\uparrow}}{} $$

$$J^q = L^q + \overset{\overset{\frac{1}{2}\Delta\Sigma \text{ from DIS}}{\uparrow}}{S^q}$$

gauge invariant decomposition

sum rule for $L^q$ from twist-3 GPDs

→ *talk of S. Pisano*



$J^g$

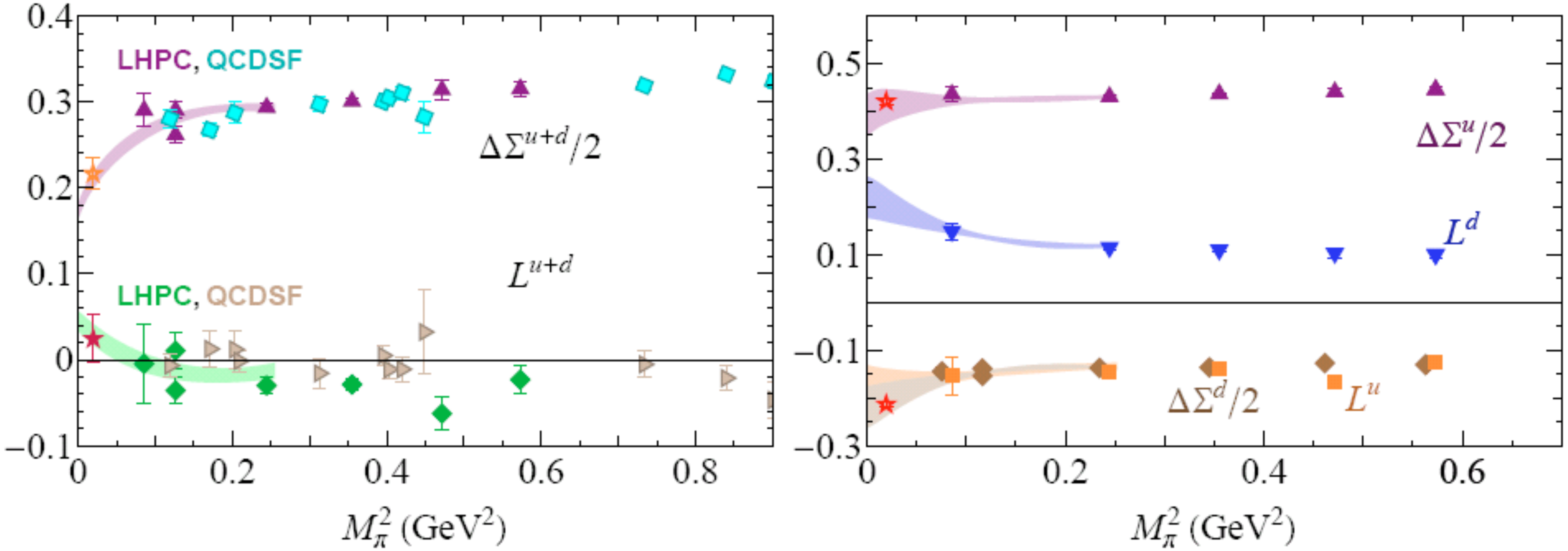
no further gauge-invariant decomposition

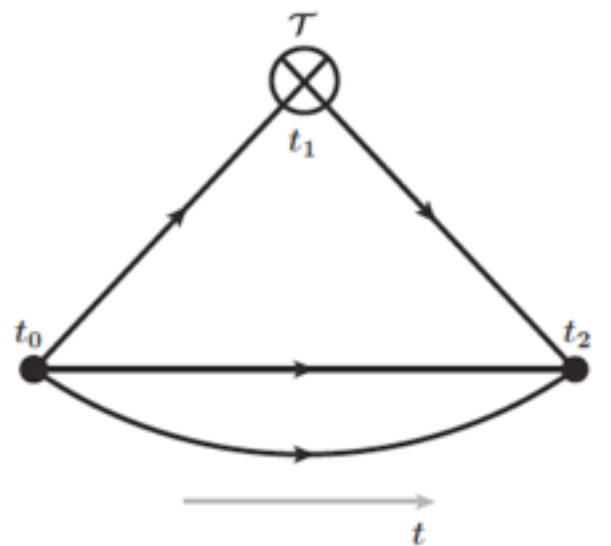
# Lattice Calculations of Angular Momentum

without disconnected insertion

$L^u + L^d \approx 0$

$J^u > 0, \quad J^d \approx 0$

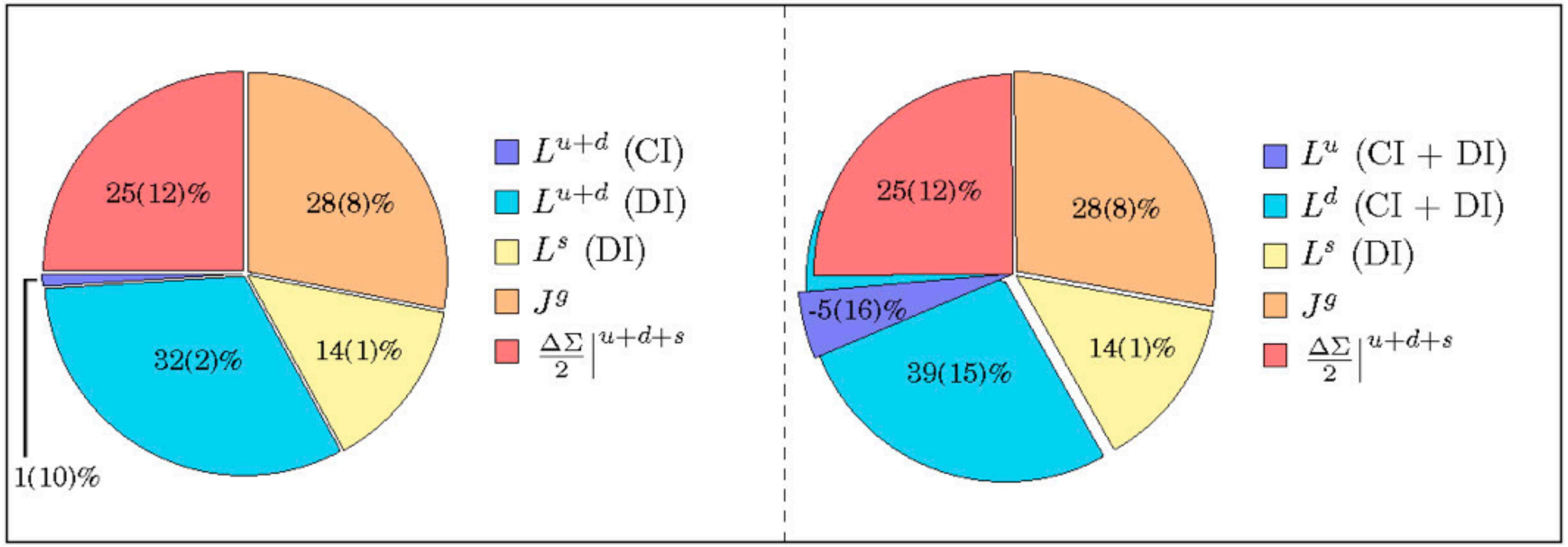


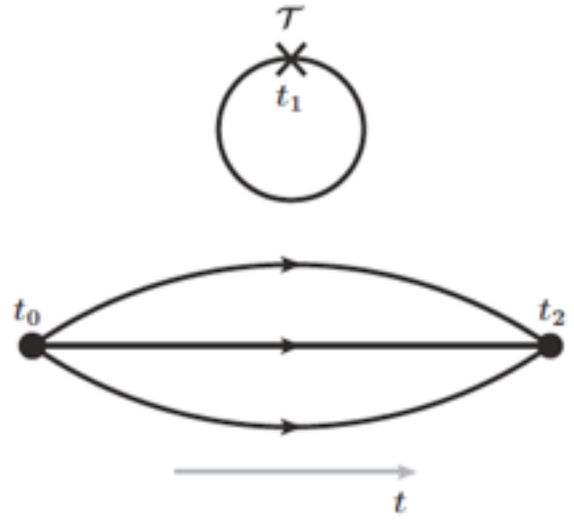


connected insertions (CI)

with disconnected insertion

$L^u + L^d \approx 33\%$



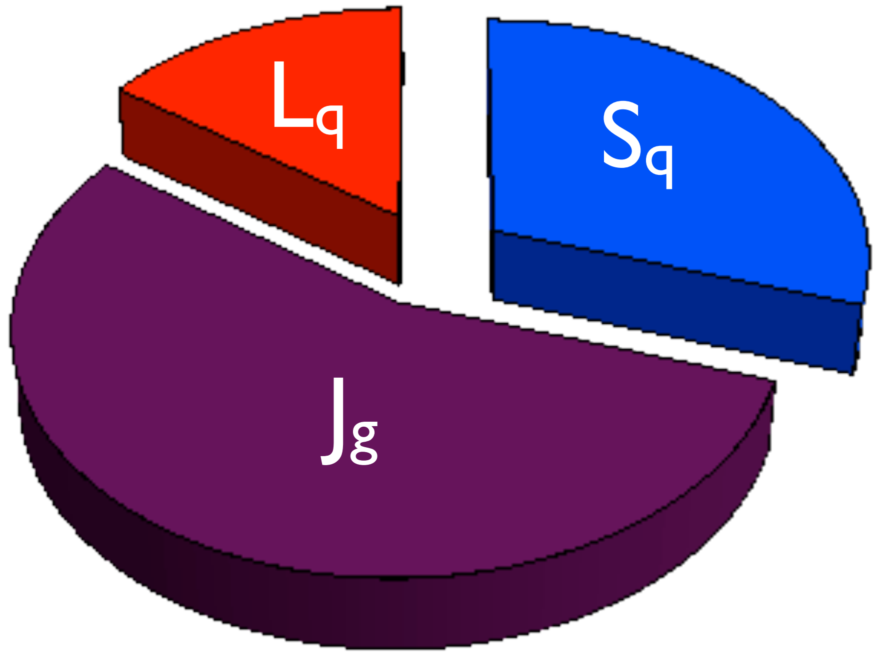


disconnected insertions (DI)

*Deka et al., PRD* ***91*** *(2015) 014505*

# Different definitions of OAM

## Ji's sum rule



Pros:

- Each term is gauge invariant
- Accessible in DIS and DVCS
- Can be calculated in Lattice QCD

Cons:

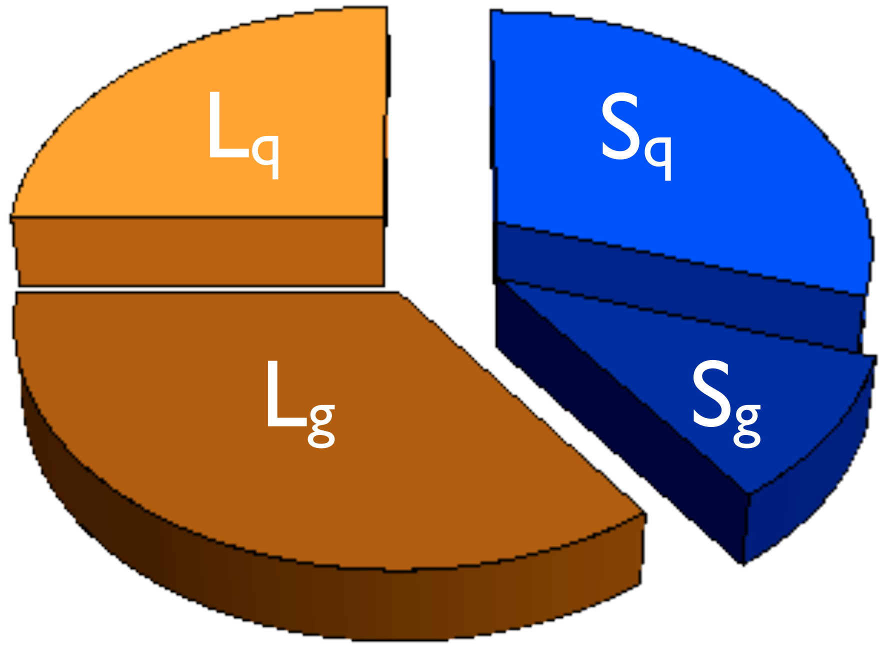
- Does not satisfy canonical commutation relations
- No decomposition of $J_g$ in spin and orbital part

Improvements:

- Complete decomposition

$$J^g = L^g + \Delta g$$

## Jaffe-Manohar



Pros:

- Satisfies canonical relations
- Complete decomposition

Cons:

- Gauge-variant decomposition
- Missing observables for the OAM

($\Delta g$ and $\Delta\Sigma$ measured by COMPASS, HERMES , RHIC)

Improvements:

- OAM accessible via Wigner distributions and it can be calculated on the lattice

*Lorcé, Leader, Phys. Rep.* ***541*** *(2014) 163*

# Quark Orbital Angular Momentum

$$\ell_z^q = \int \mathrm{d}x \, \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

⬇

Wigner distribution for
Unpolarized quark in a Longitudinally pol. nucleon

*Lorcé, BP, PRD* ***84*** *(2011) 014015*

# Quark Orbital Angular Momentum

$$\ell_z^q = \int \mathrm{d}x\,\mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

$$= \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}^q_\perp \rangle \longrightarrow \langle \vec{k}^q_\perp \rangle = \int \mathrm{d}x\,\mathrm{d}\vec{k}_\perp\, \vec{k}_\perp \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

*Lorcé, BP, PRD* ***84*** *(2011) 014015*

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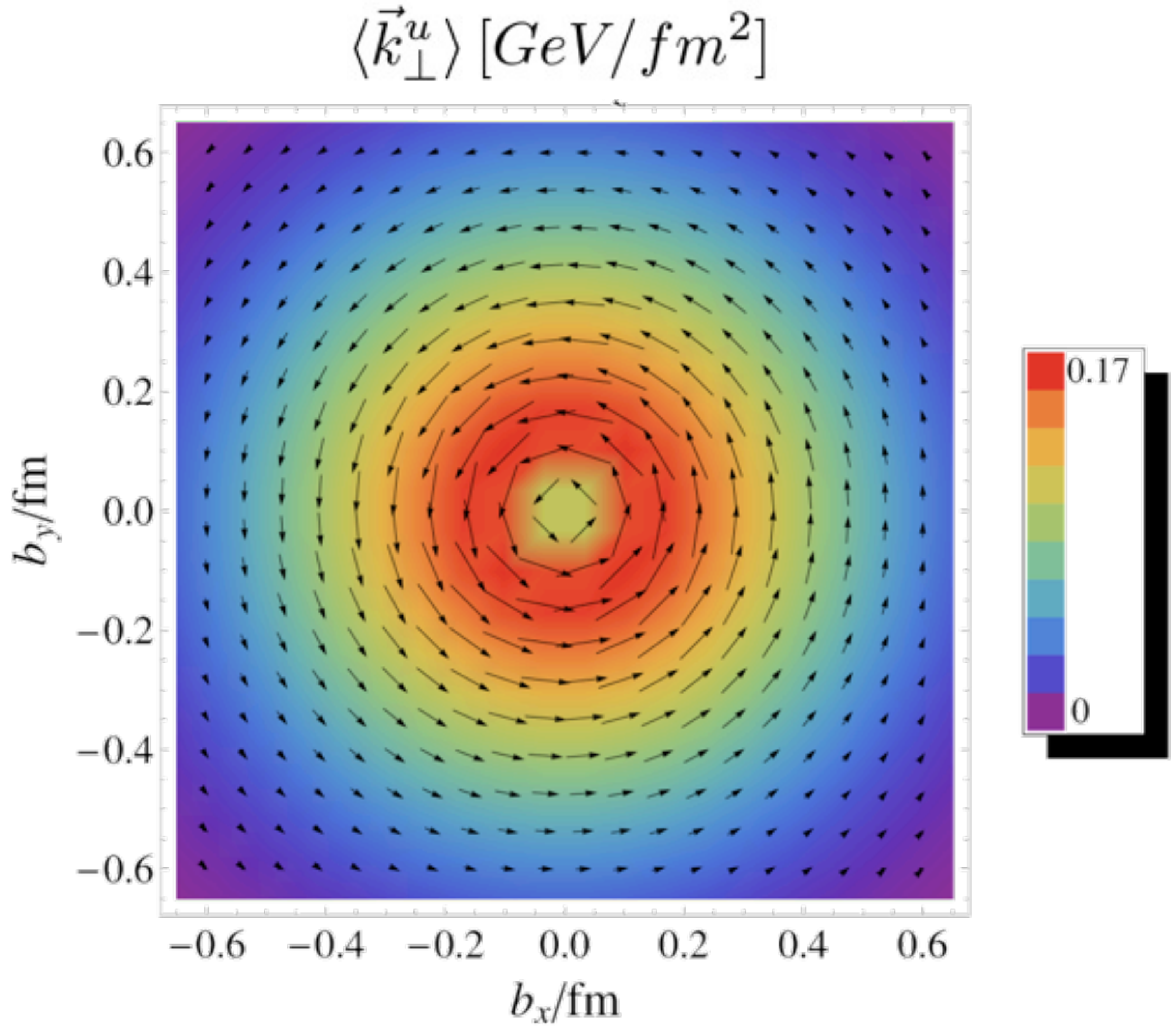

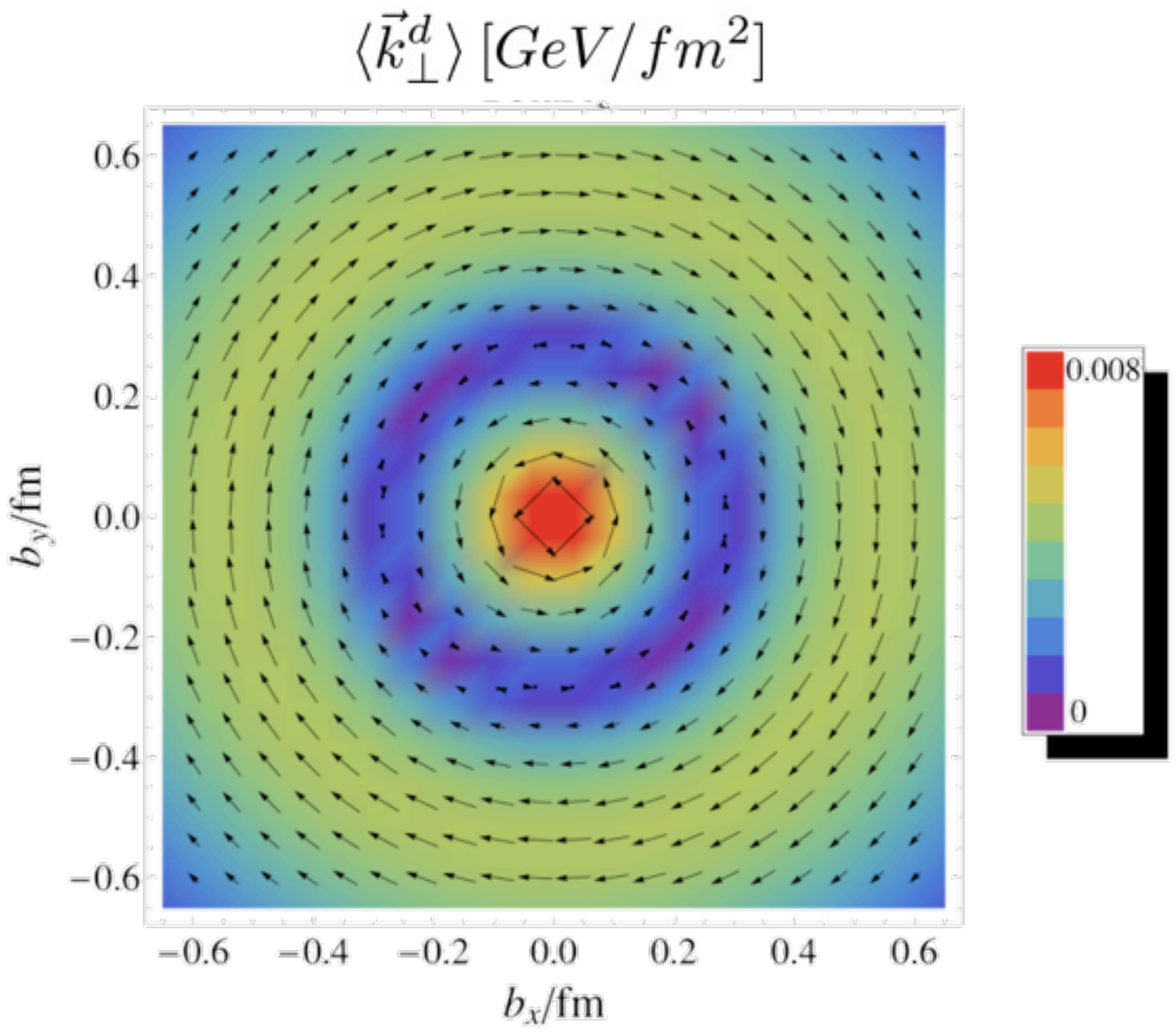

Results in a light-front constituent quark model:

*Lorcé, BP, PRD **84** (2011) 014015*
*Lorcé, BP, Xiong, Yuan, PRD **85** (2012) 114006*

⟶ Proton spin
⟶ u-quark OAM
⟵ d-quark OAM

# Quark Orbital Angular Momentum

$$\ell_z^q = \int \mathrm{d}x\,\mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

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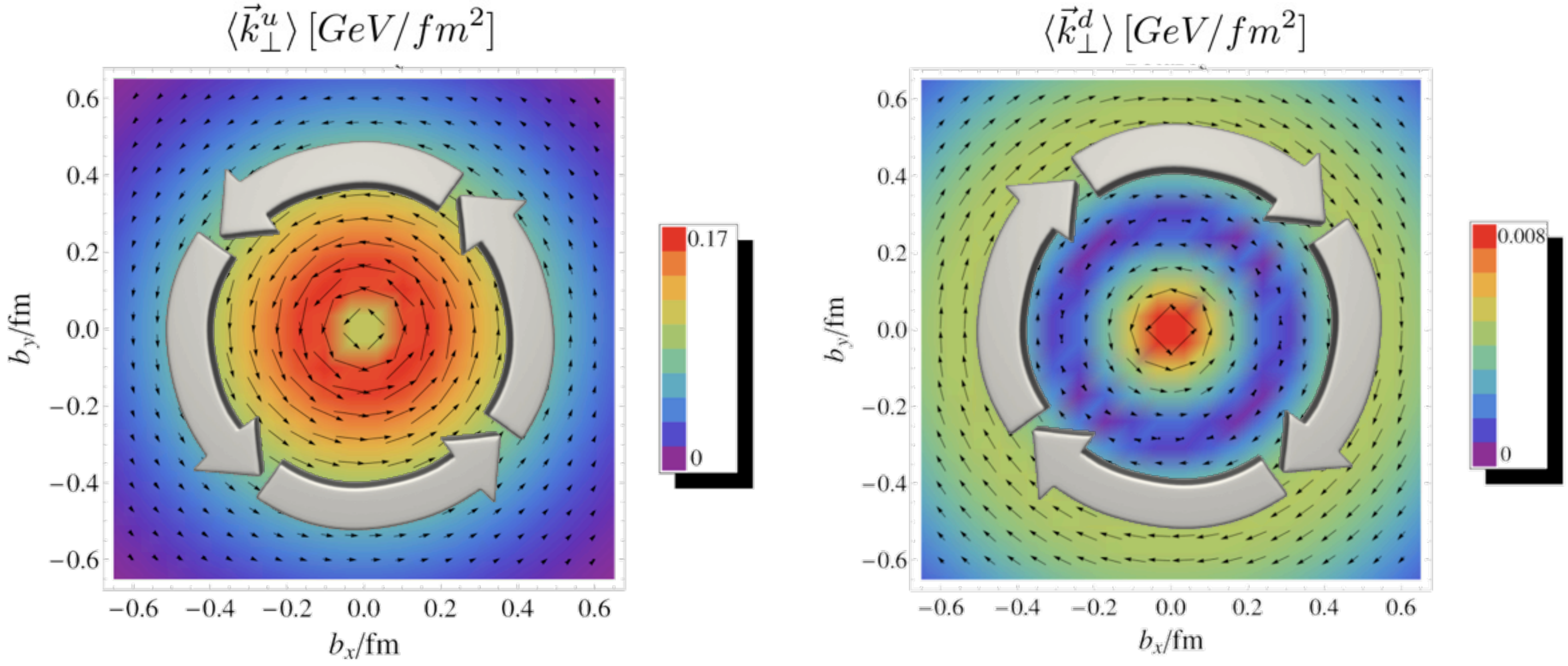

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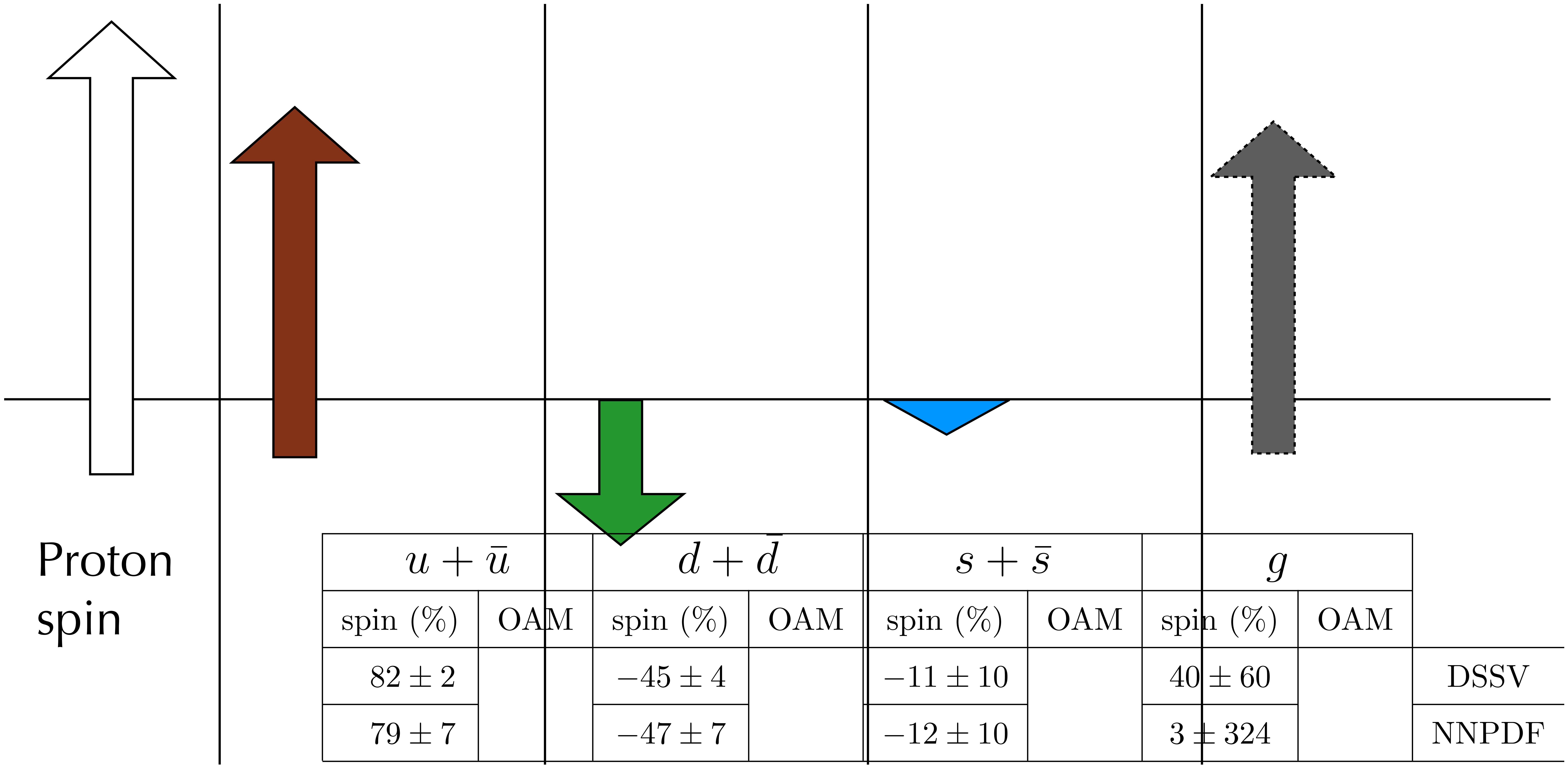
→ Proton spin
→ u-quark OAM
← d-quark OAM

# Status of spin sum rule

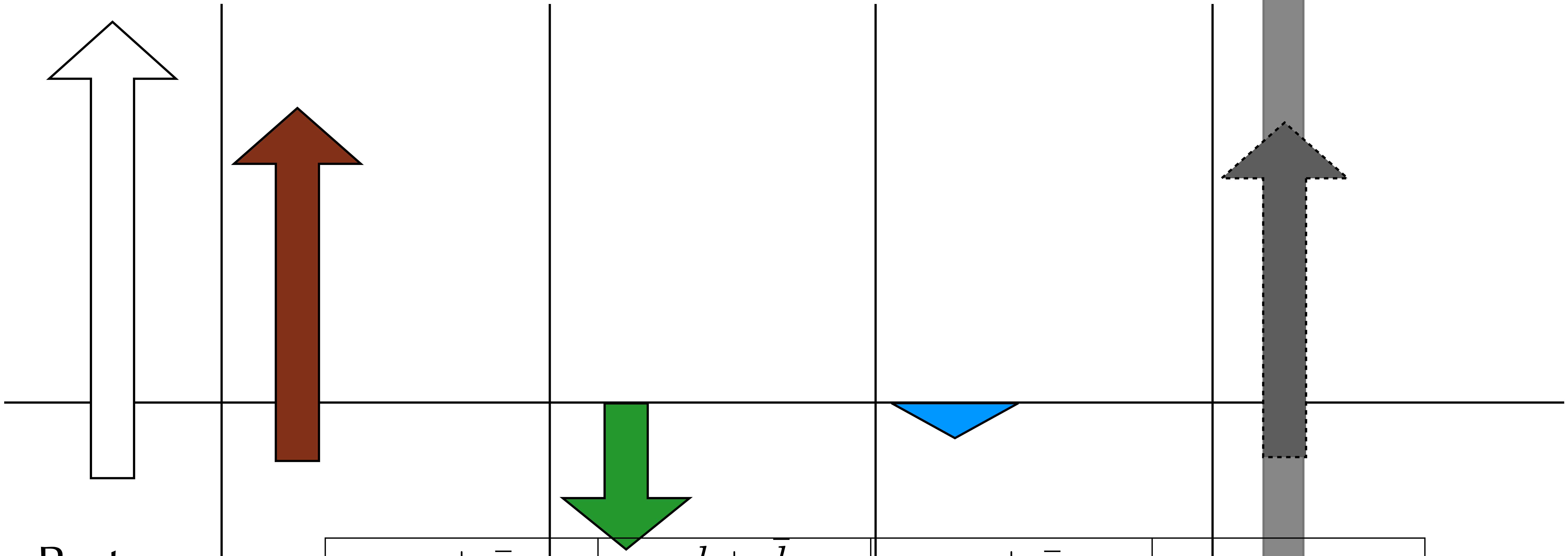


| $u+\bar{u}$ | | $d+\bar{d}$ | | $s+\bar{s}$ | | $g$ | | |
|---|---|---|---|---|---|---|---|---|
| spin (%) | OAM | spin (%) | OAM | spin (%) | OAM | spin (%) | OAM | |
| $82 \pm 2$ | | $-45 \pm 4$ | | $-11 \pm 10$ | | $40 \pm 60$ | | DSSV |
| $79 \pm 7$ | | $-47 \pm 7$ | | $-12 \pm 10$ | | $3 \pm 324$ | | NNPDF |

*de Florian, Sassot, Stratmann, Vogelsang, PRL 113 (14)*

*NNPDF, Ball... Nocera... NPB 887 (14), Tab. 12, 13*

# Status of spin sum rule



Proton spin

| $u+\bar{u}$ | | $d+\bar{d}$ | | $s+\bar{s}$ | | $g$ | | |
|---|---|---|---|---|---|---|---|---|
| spin (%) | OAM | spin (%) | OAM | spin (%) | OAM | spin (%) | OAM | |
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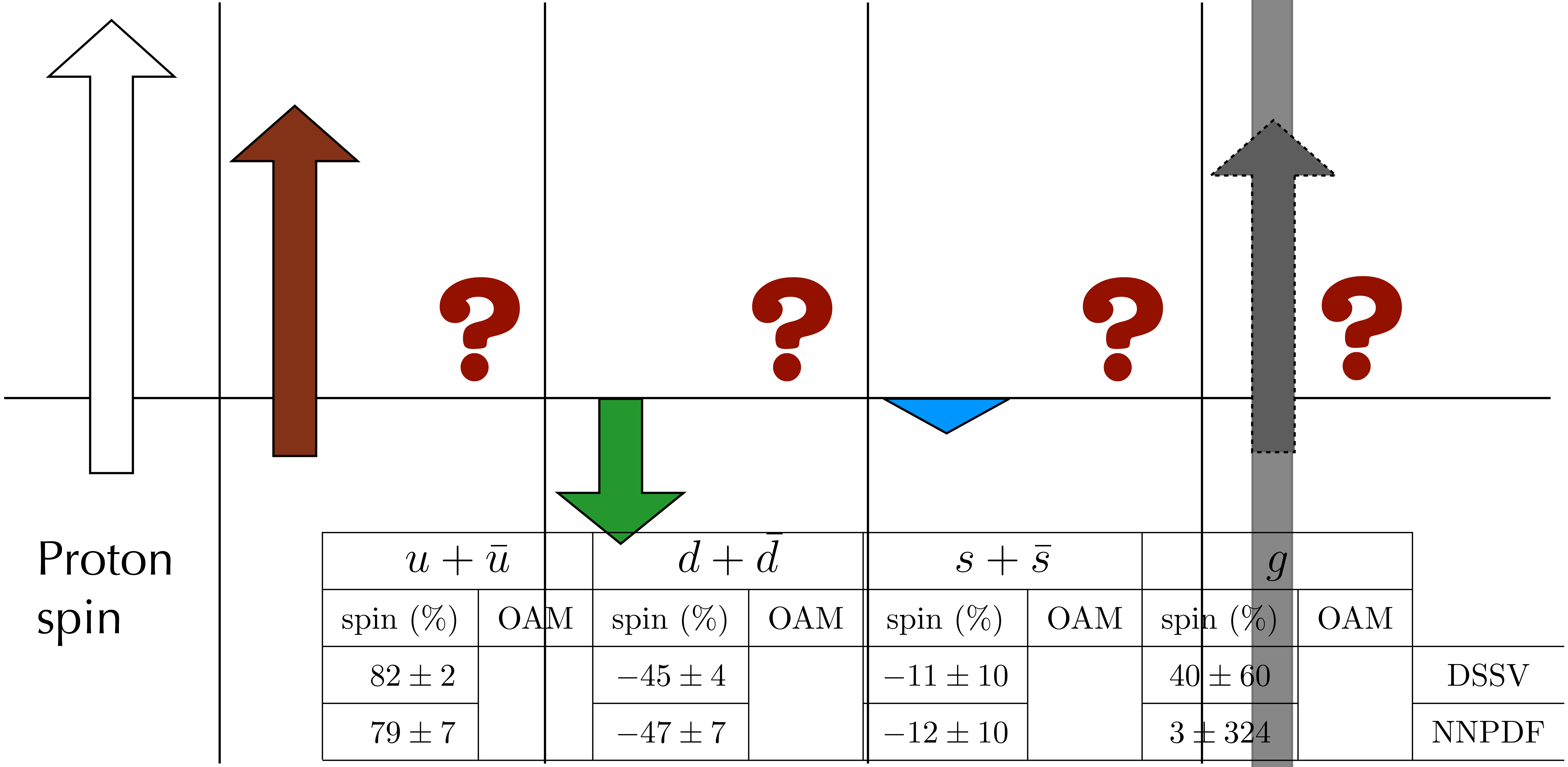

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|---|---|---|---|---|---|---|---|---|
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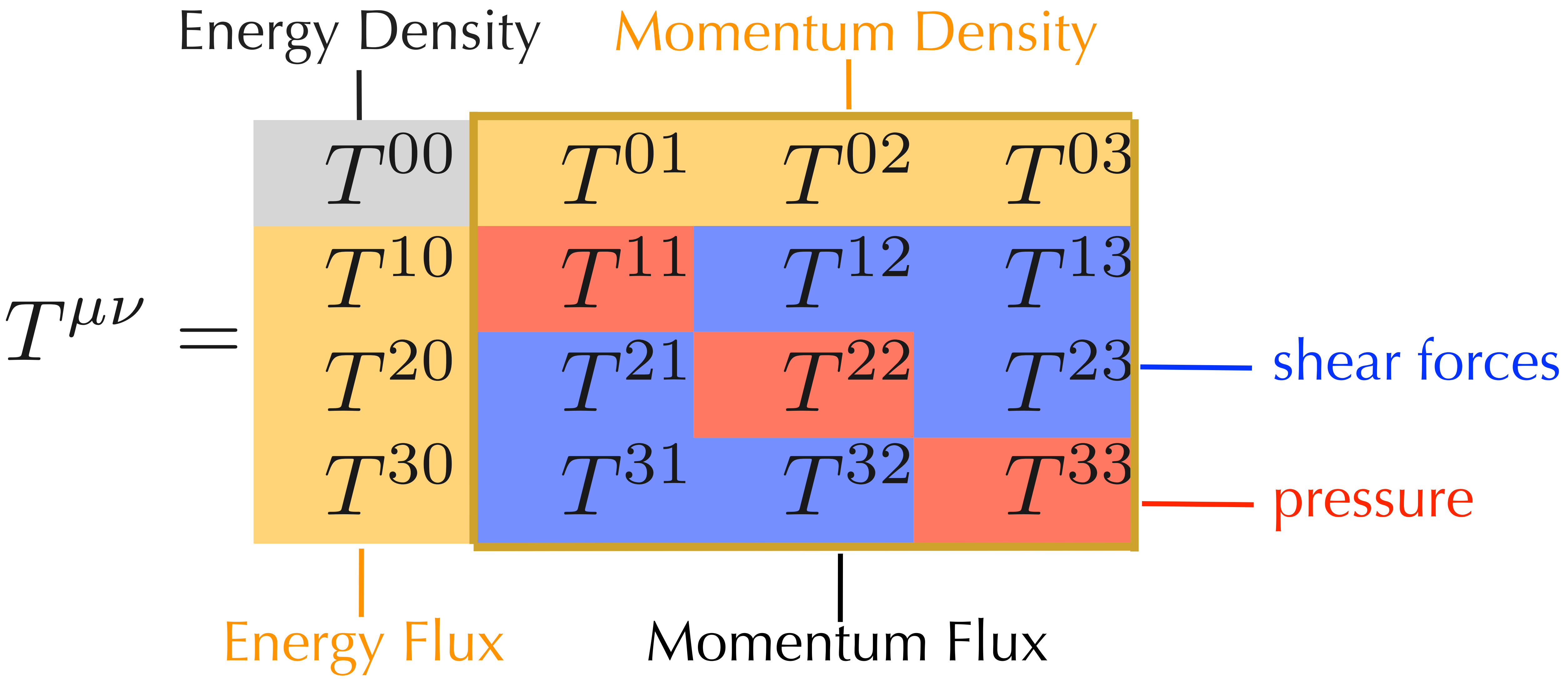
*de Florian, Sassot, Stratmann, Vogelsang, PRL 113 (14)*

*NNPDF, Ball... Nocera... NPB 887 (14), Tab. 12, 13*

# Form factors of Energy Momentum tensor



Energy Density

Momentum Density

$$T^{\mu\nu} = \begin{matrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{matrix}$$

shear forces

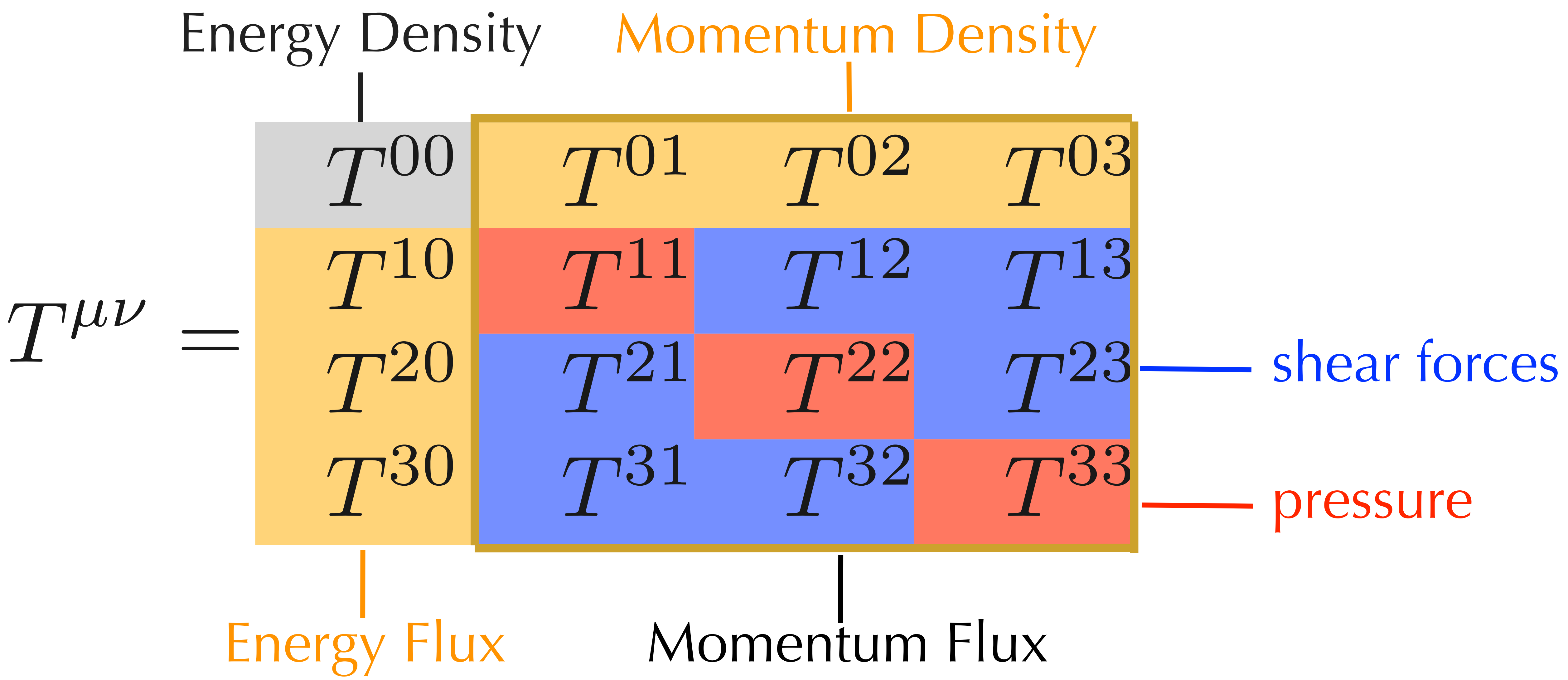
pressure

Energy Flux

Momentum Flux

$$\langle P'|T^{Q,G}_{\mu\nu}|P\rangle = \bar{u}(P')[M_2^{Q,G}(t)\,\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}]u(P)$$

# Form factors of Energy Momentum tensor



Energy Density

Momentum Density

$$T^{\mu\nu} = \begin{array}{cccc} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{array}$$

shear forces

pressure

Energy Flux

Momentum Flux

$$\langle P'|T^{Q,G}_{\mu\nu}|P\rangle = \bar{u}(P')[M_2^{Q,G}(t)\,\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}]u(P)$$

Relation with second-moments of GPDs:

$$\sum_q \int \mathrm{d}x\, x\, H^q(x,\xi,t) = M_2^Q(t) + \frac{4}{5}\, d_1^Q(t)\xi^2$$

$$\sum_q \int \mathrm{d}x\, x\, E^q(x,\xi,t) = 2J^Q(t) - M_2^Q(t) - \frac{4}{5}\, d_1^Q(t)\xi^2$$

“Charges” of the EM Tensor Form Factors at t=0

| | |
|---|---|
| $M_2(0)$ | nucleon momentum carried by parton |
| $J(0)$ | angular momentum of partons |
| $d_1(0)$ | D-term related to “stability” of the nucleon |

# Fourier transform in coordinate space

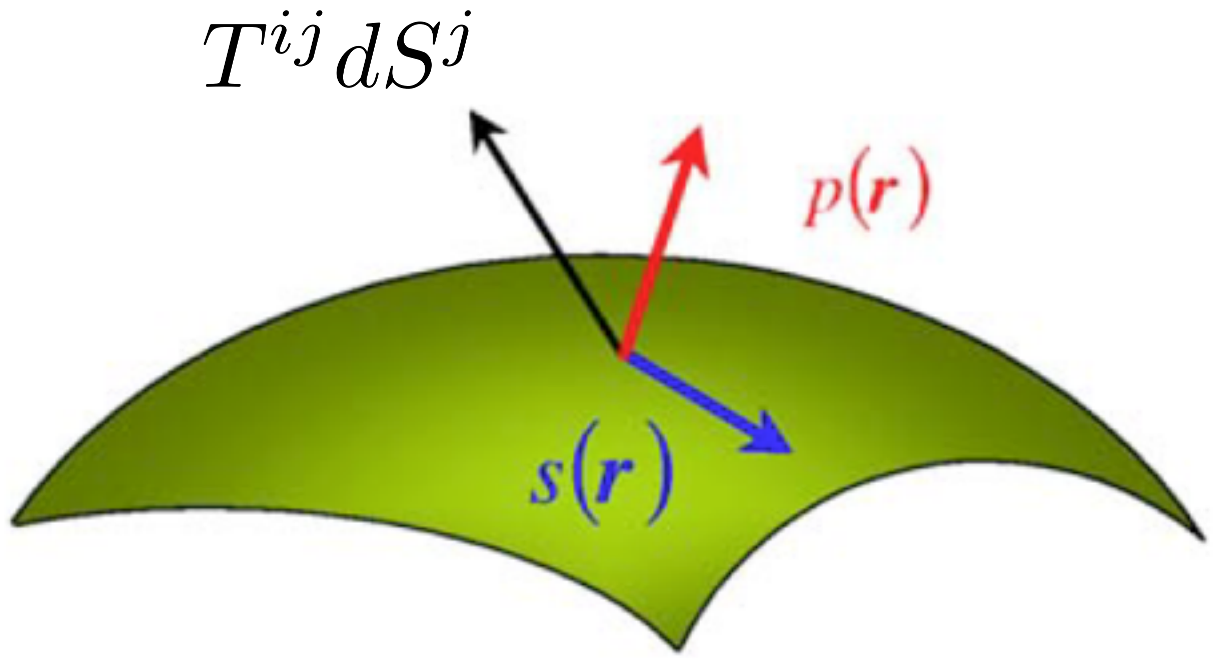
$$T_{ij}^{Q}(\vec{r}) = s(\vec{r})\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(\vec{r})\,\delta_{ij}$$

shear forces

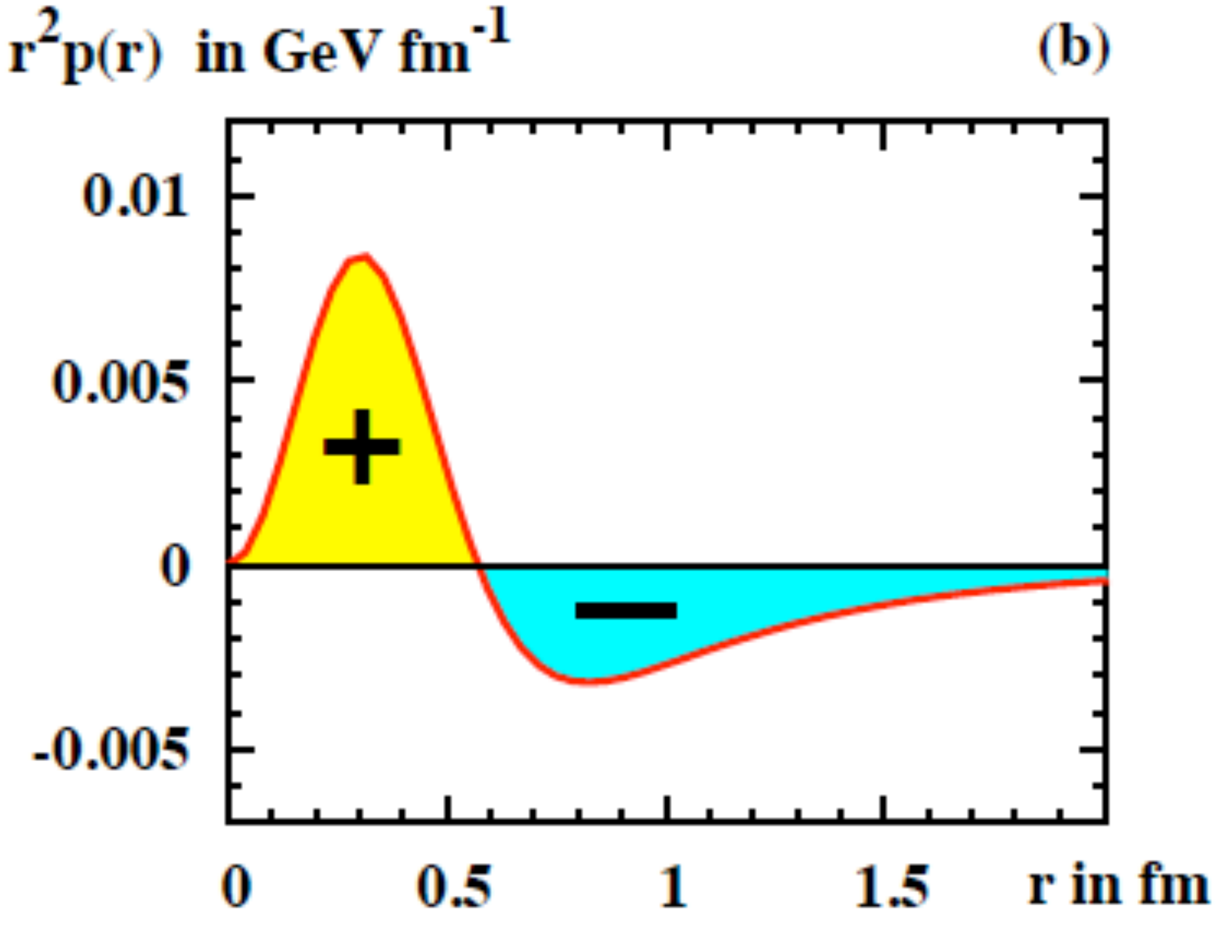
pressure

$$\boxed{d_1^Q(0) = 5\pi M_N \int_0^\infty \mathrm{d}r\, r^4\, p(r)}$$

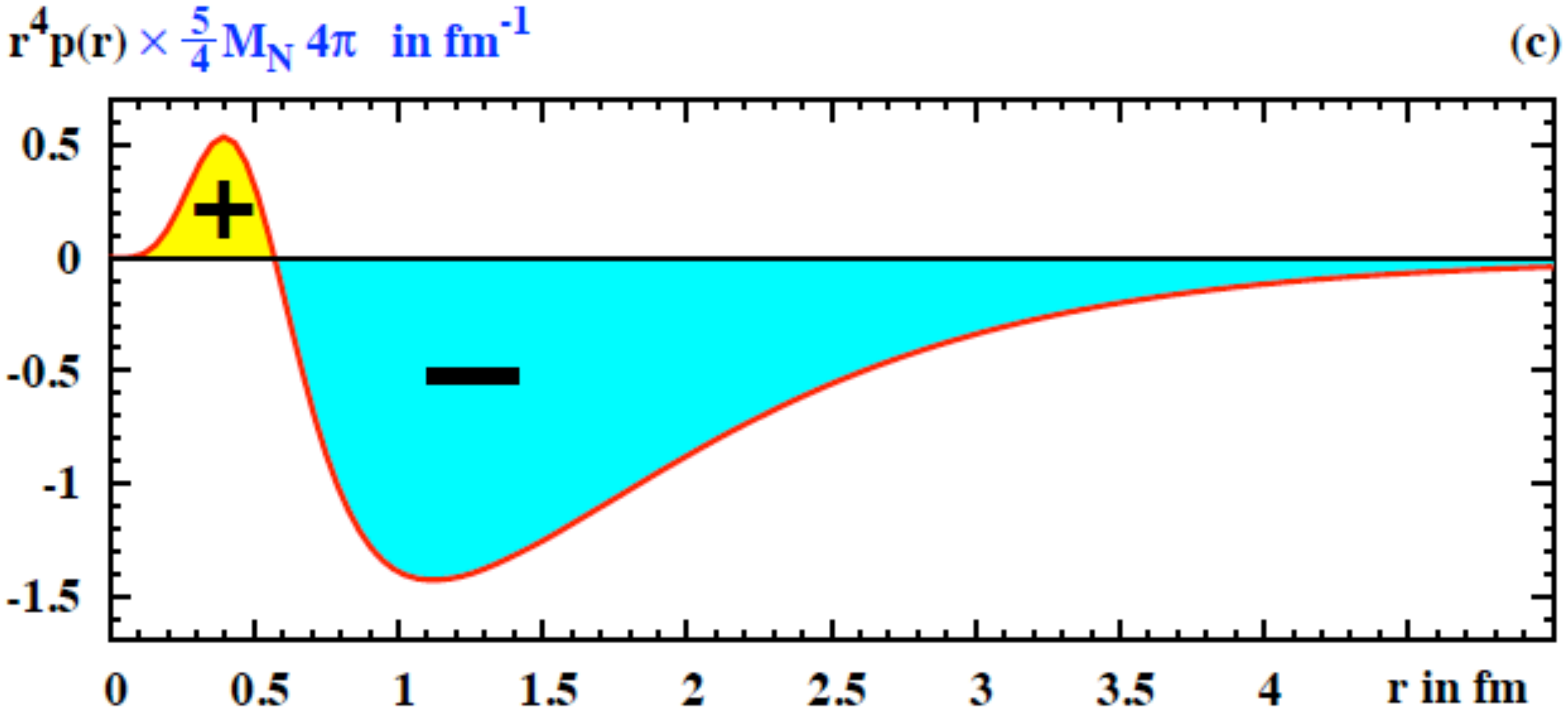
## "mechanical properties" of nucleon



*M. Polyakov, PLB **555** (2003) 57*



$$\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0$$

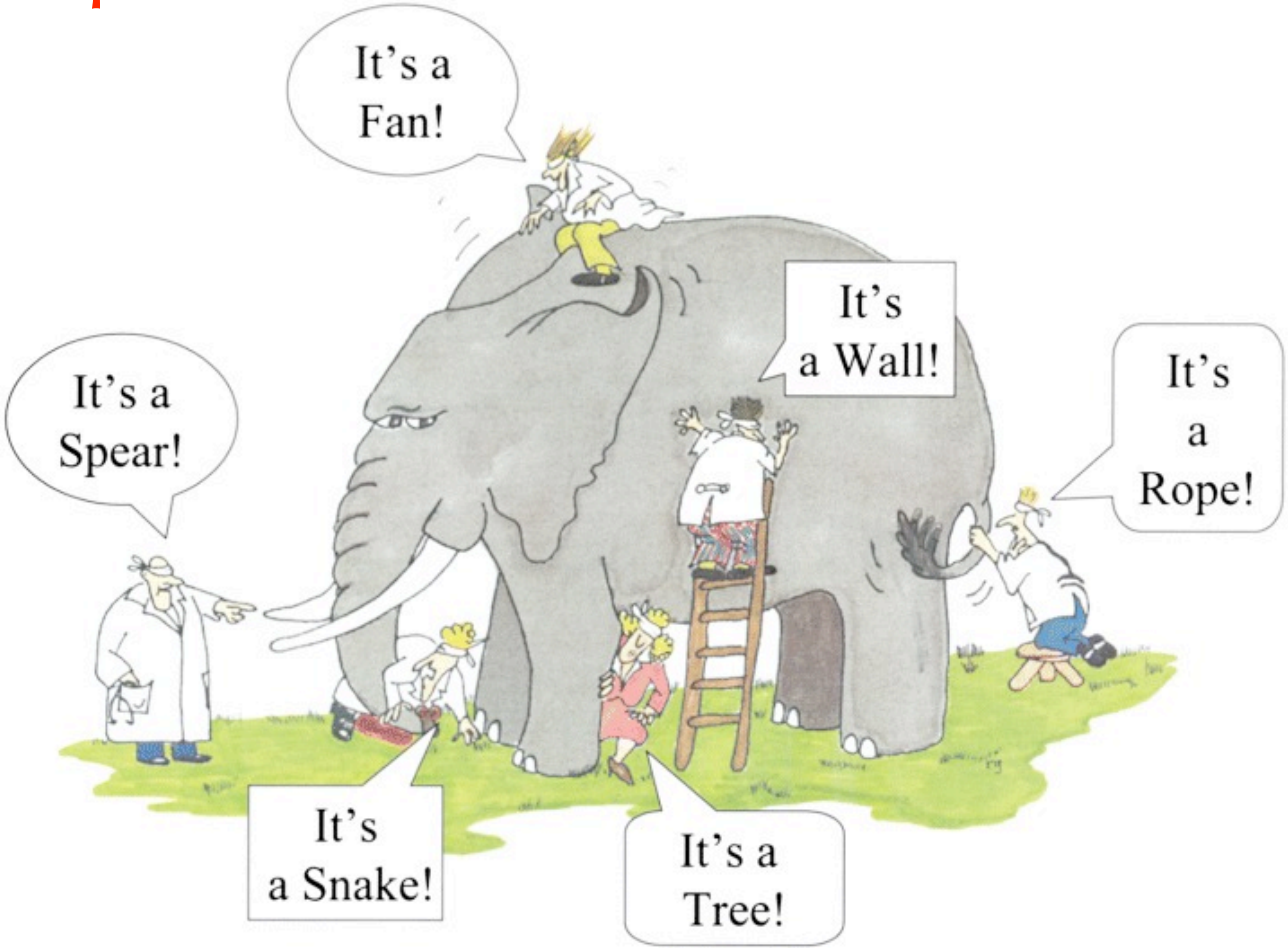


$$\int_0^\infty \mathrm{d}r\, r^4\, p(r) < 0$$

*Schweitzer et al., PRD **75** (2007) 094021*

# The blind men and the elephant

from H. Avakian



TMDs and GPDs provide different and complementary information
and need to talk to each other
to reconstruct the full 3D picture of the nucleon

# Recent achievement

ERC press release 12.03.2015

European Research Council

Established by the European Commission

**3D$^{\text{SPIN}}$**

Alessandro Bacchetta
ERC Consolidator grant
University of Pavia + INFN

3 PhD students
3 Post-docs