Double Parton Interactions in *pp* and *pA* collisions

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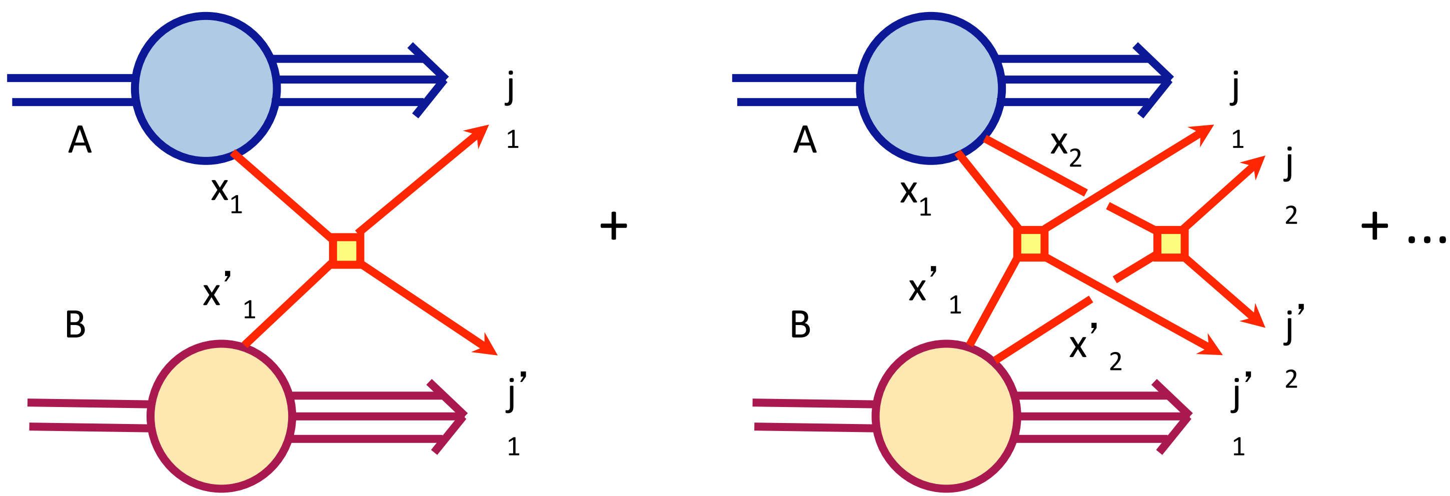
A simplest model for Double Parton Interactions

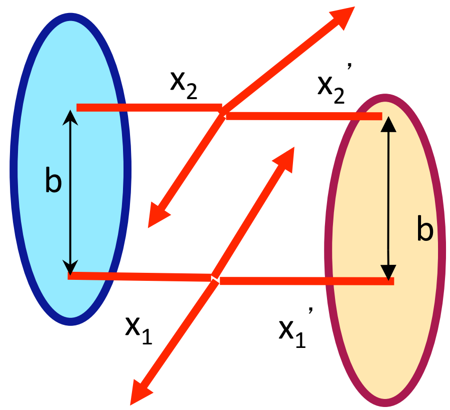
Multiple Parton Interactions have been introduced to solve the unitarity problem generated by the fast raise of the inclusive hard cross sections at small x. At small x the hard cross section can in fact become larger than the total inelastic cross section.

Given the final state, **multiple parton interactions** are the processes which **maximize the incoming parton flux**



Unitarity is restored by MPI because the **inclusive cross section** counts the **multiplicity of interactions** and, in this way, when the **average multiplicity** of interactions is **large** the inclusive cross section is no more bounded by the value of the total inelastic cross section.

The simplest case is **Double Parton Scattering.** The incoming parton flux is maximal when **the hard component of the interaction is disconnected** and, in the case of the **DPS,** one thus obtains the geometrical picture here below, where the non-perturbative components are factorized into a functions which depends on two fractional momenta and on the relative transverse distance *b* between the two interaction points



When neglecting spin and color, the inclusive double parton-scattering cross-section, for two parton processes A and B in a *pp* collision, is given by

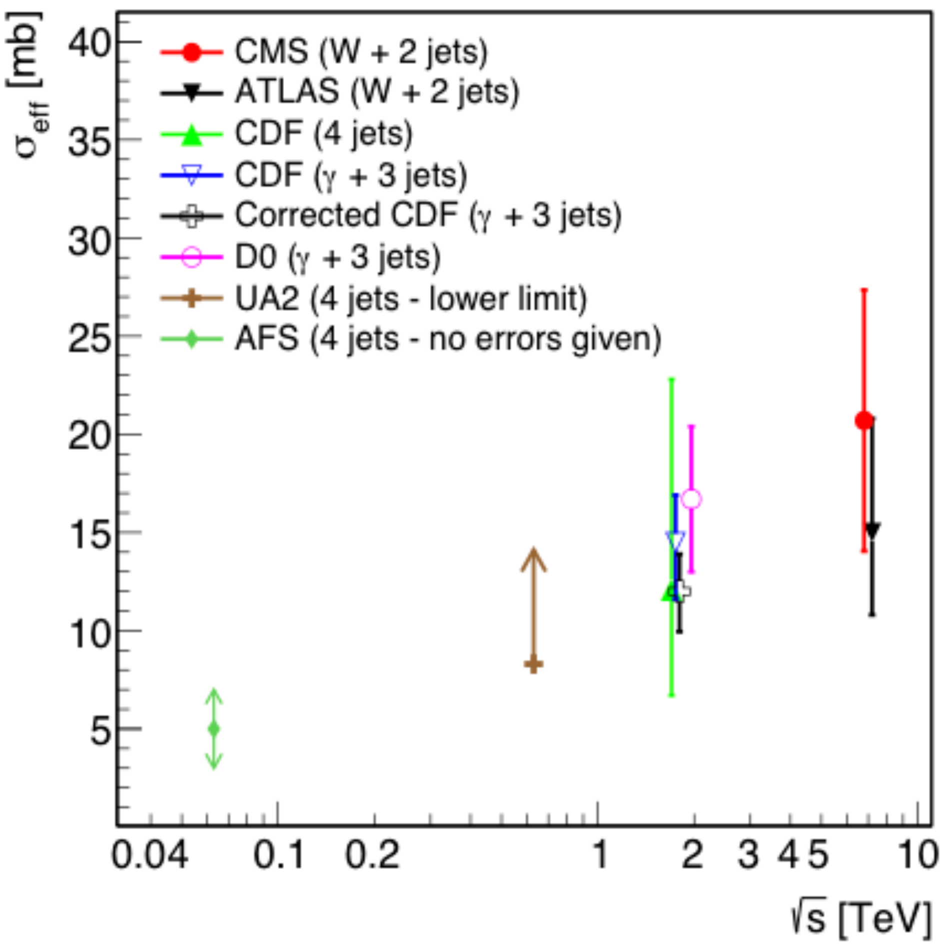
$$\boxed{\sigma^D_{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int D_{ij}(x_1, x_2; b) \hat{\sigma}^A_{ik}(x_1, x_1') \hat{\sigma}^B_{jl}(x_2, x_2') D_{kl}(x_1', x_2'; b) dx_1 dx_1' dx_2 dx_2' d^2 b}$$

Which, with leads to the "pocket formula" of the cross section utilized in the experimental analysis:

$$\boxed{\sigma^{pp\ (A,B)}_{double} = \frac{m}{2} \frac{\sigma_A \sigma_B}{\sigma_{eff}}}$$

Comparison with Experiment

In the "pocket formula" all unknowns are summarized in the value of a single quantity σ_{eff} wich, for the second interaction, plays "effectively" the role of the inelastic cross section.



Different results of the value of σ_{eff}, where the experimental DPI cross section is given by the "pocket formula":

$$\sigma_{double}^{(A,B)} = \frac{m}{2}\frac{\sigma_A \sigma_B}{\sigma_{eff}}$$

D0 Collaboration, Phys.Rev. D81 (2010) 052012

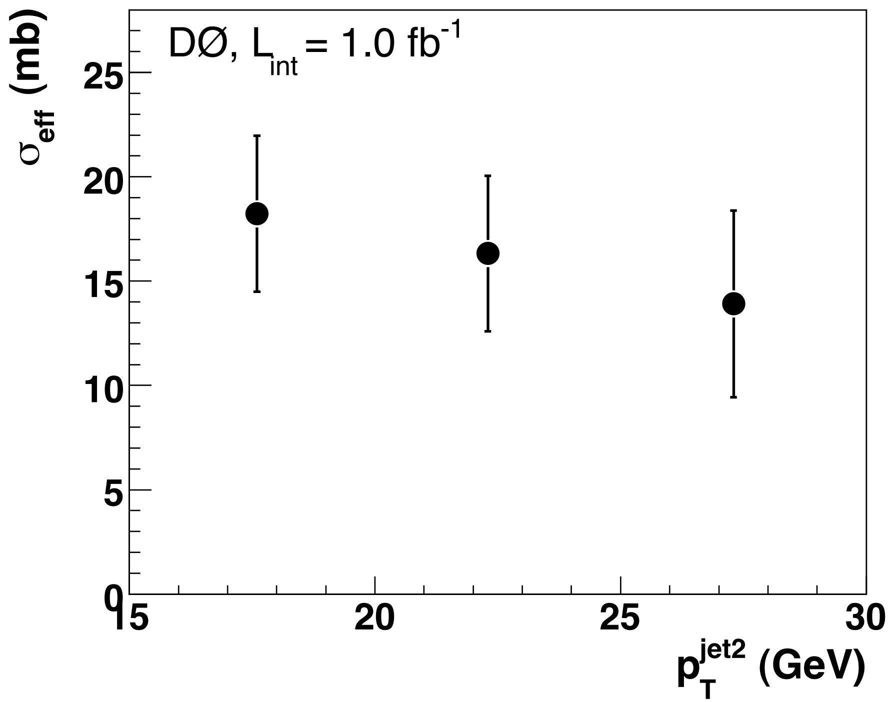
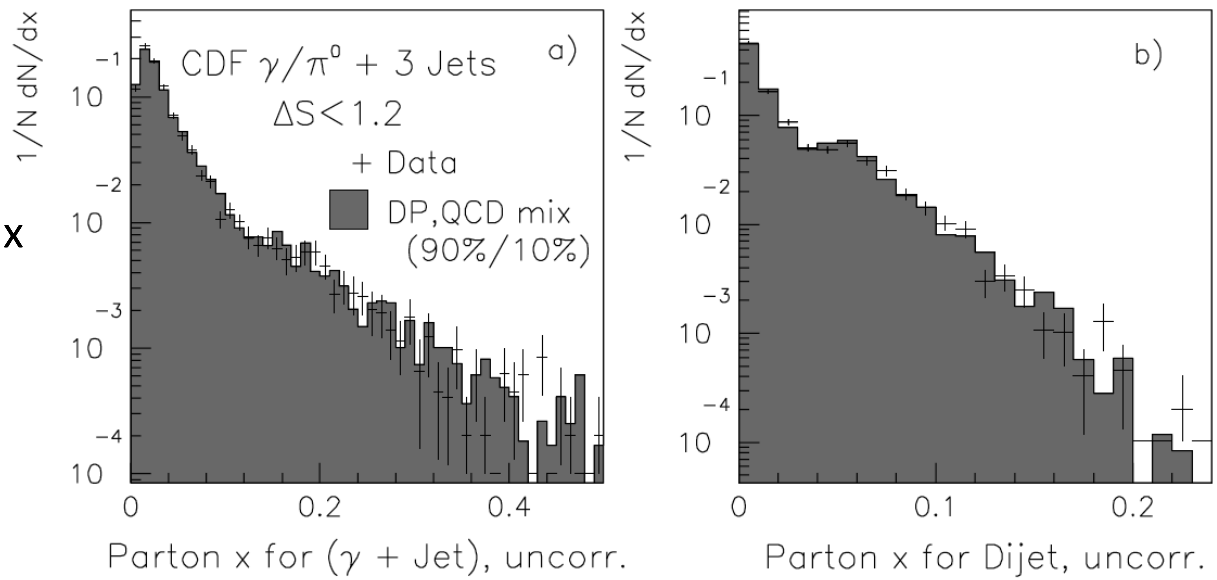


FIG. 11: Effective cross section $\sigma_{\rm eff}$ (mb) measured in the three $p_T^{\rm jet2}$ intervals.

F. Abe et al. [CDF Collaboration], Phys. Rev. D 56, 3811(1997).

Dependence of $\boldsymbol{\sigma}_{\mathbf{eff}}$ on x

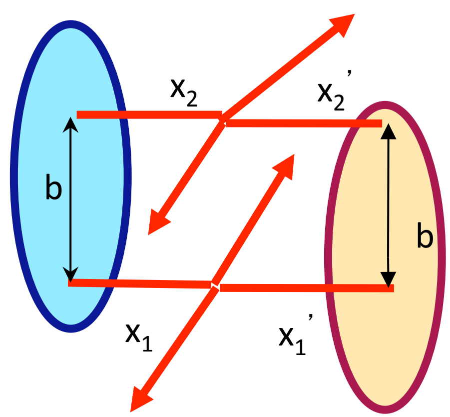


Distributions of x are plotted in Figs. 20(a) and 20(b), along with a prediction obtained by applying the DS < 1.2 selection to the admixture 90% MIXDP+10% PYTHIA.
No systematic deviation of the DP rate vs x, and thus **no x dependence to $\boldsymbol{\sigma}_{\mathbf{eff}}$**, is apparent over the x range accessible to this analysis (0.01 – 0.40 for the photon+jet scattering, 0.002–0.20 for the dijet scattering).

The the "pocket formula" of the inclusive cross-section has thus shown to be able to describe the experimental results of the direct search of double parton collisions in rather different kinematical regimes with a value of σ_{eff} compatible with a universal constant, while the study of CDF, of the dependence of σ_{eff} on the fractional momenta of the incoming partons, is again compatible with a value of σ_{eff} independent on x.

In the simples model, not inconsistent with present experimental evidence, DPS are therefore given by the disconnected contribution, which maximizes the incoming parton flux at small x, and leads to the "pocket formula" utilized in the experimental analyses, with a universal value of σ_{eff}

It is thus worth figuring out the expectations based on this simplest model for DPS



σ_{eff} and partonic correlations

One may write the double parton distribution functions as

$$\Gamma(x_1, x_2;\ b) = G(x_1, x_2) f_{x_1 x_2}(b), \qquad G(x_1, x_2) = K_{x_1 x_2} G(x_1) G(x_2)$$

where *f* is normalized to 1 and the transverse scales, characterizing *f*, may still depend on fractional momenta.

$$\int f_{x_1 x_2}(b) d^2 b = 1 \qquad G(x) = \langle n \rangle_x, \qquad G(x_1, x_2) = \langle n(n-1) \rangle_{x_1, x_2}$$

$$K_{x_1 x_2} = \frac{\langle n(n-1) \rangle_{x_1, x_2}}{\langle n \rangle_{x_1} \langle n \rangle_{x_2}}$$

In the simplest case one would have $K_{xx'} = 1$ which, after integrating on *b*, would be the case of a Poissonian multi-parton distribution in multiplicity.

In *pp* one thus has

correlations in multiplicity

$$\sigma^{pp\ (A,B)}_{double}(x_1,x_1',x_2,x_2') = \frac{m}{2}K_{x_1x_2}K_{x_1'x_2'}G(x_1)\hat\sigma_A(x_1,x_1')G(x_1')$$
$$\times G(x_2)\hat\sigma_B(x_2,x_2')G(x_2')\int f_{x_1x_2}(b)f_{x_1'x_2'}(b)db$$
$$= \frac{m}{2}\frac{K_{x_1x_2}K_{x_1'x_2'}}{\pi\Lambda^2(x_1,x_1',x_2,x_2')}\sigma_A(x_1,x_1')\sigma_B(x_2,x_2')$$

effective cross section

where $$\int f_{x_1x_2}(b)f_{x_1'x_2'}(b)db = \frac{1}{\pi\Lambda^2(x_1,x_1',x_2,x_2')}$$

typical transverse interaction area

$$\sigma^{pp\ (A,B)}_{double} = \frac{m}{2}\frac{\sigma_A\sigma_B}{\sigma_{eff}}$$

All new information on the hadron structure
is thus summarized in the effective cross section

Limiting cases

$$\Gamma(x_1, x_2;\ b) = G(x_1, x_2) f_{x_1 x_2}(b), \qquad G(x_1, x_2) = K_{x_1 x_2} G(x_1) G(x_2)$$

$$K_{x_1 x_2} = \frac{\langle n(n-1)\rangle_{x_1,x_2}}{\langle n\rangle_{x_1}\langle n\rangle_{x_2}}$$

If **partons** are **not correlated in multiplicity** one thus has $K_{x_1 x_2} = 1$

If **partons** are **not correlated in transverse coordinates** one may write:

$$\Gamma(x;b) = G(x) f_x(b), \qquad \int f_x(b) d^2b = 1, \qquad f_{x_1,x_2}(b) = \int f_{x_1}(b') f_{x_2}(b-b') d^2b'$$

In this way one however obtains **$\sigma_{eff} = \pi\Lambda^2 = 32$ mb**, which is about **a factor 2 too large** as compared with available experimental evidence

⇨ Either ***K*** **is NOT** equal to **1** or $\pi\Lambda^2$ **is NOT** equal to **32 mb** or both

$$\sigma_{eff}(x_1, x_1', x_2, x_2') = \frac{\pi\Lambda^2(x_1, x_1', x_2, x_2')}{K_{x_1x_2}K_{x_1'x_2'}}$$

Notice that the experimental indication is that the effective cross section depends only weakly on fractional momenta.

$\Rightarrow$ **weak dependence of Λ and *K* on fractional momenta**

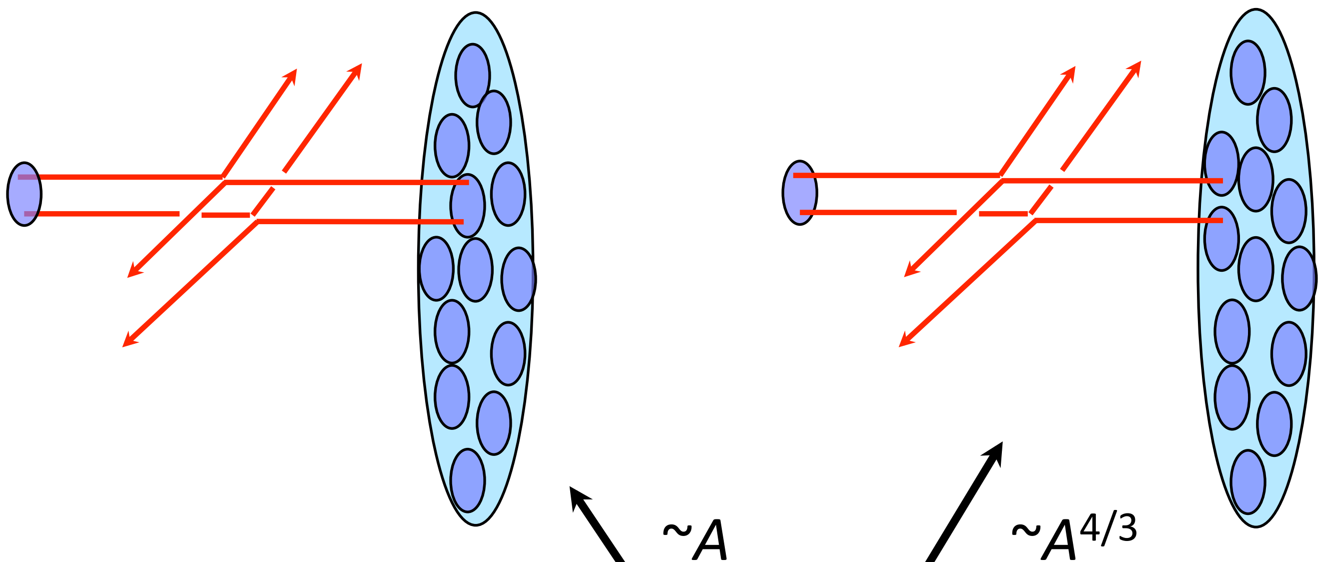
Since all new information on the hadron structure is summarized by a single quantity, **the effective cross section does not provide enough information to discriminate between Λ and *K***.

To obtain additional information on multi-parton correlations one may study MPI in *pA* collisions.

In the case of a double parton interaction, in a collision of a proton with a nucleus, the effects of longitudinal and transverse correlations are in fact different when a single nucleon or both target nucleons participate in the hard process.

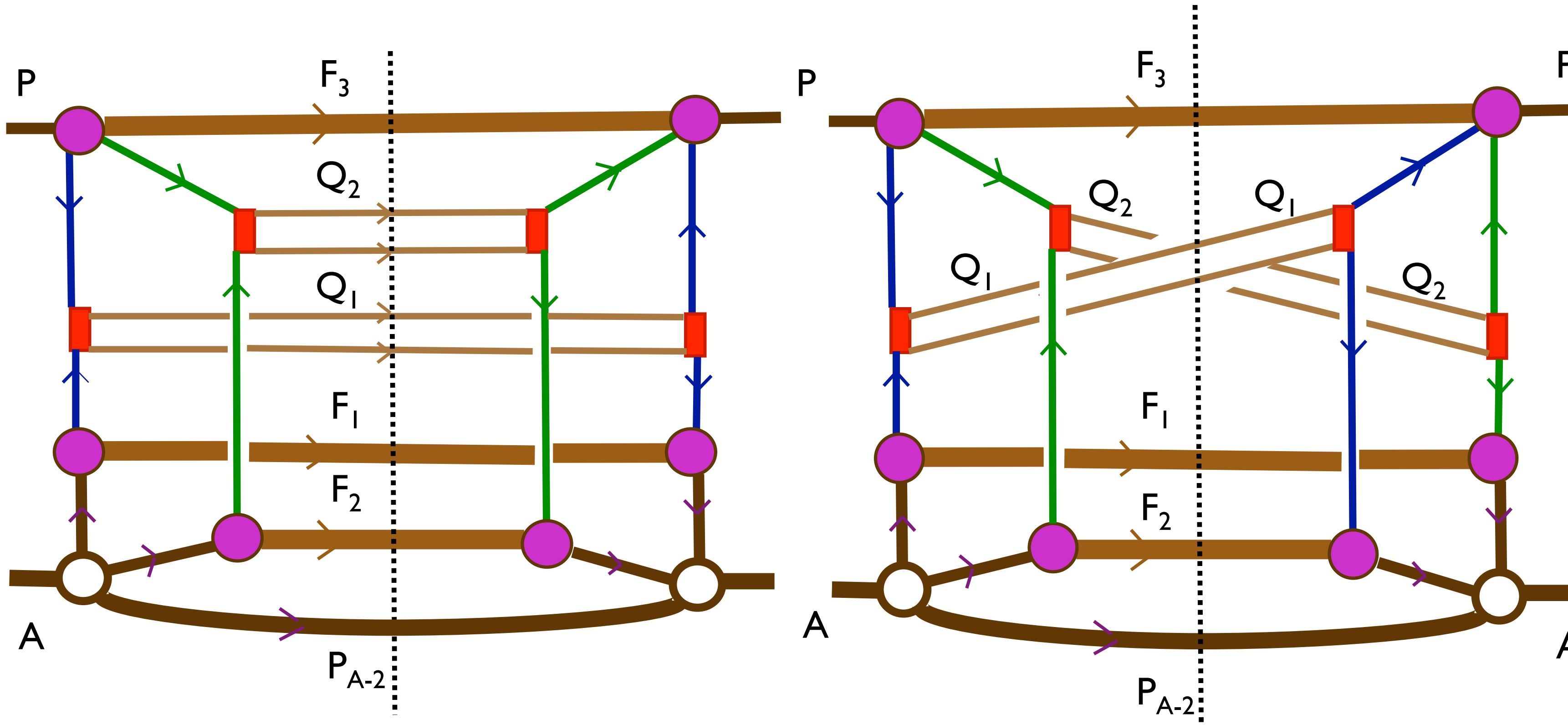
DPS in *p-A* collisions

Additional information can be obtained from DPS in *p-A* collisions:

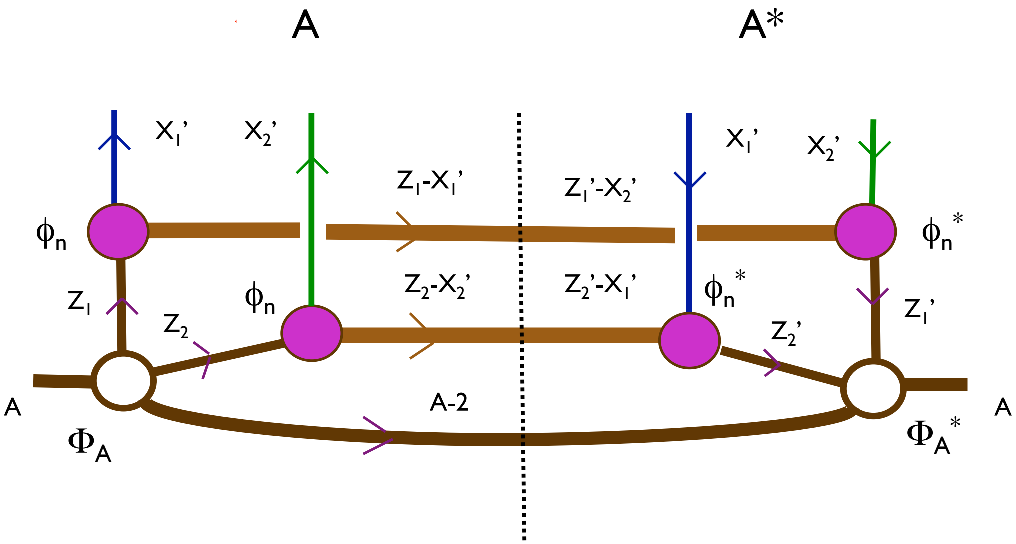


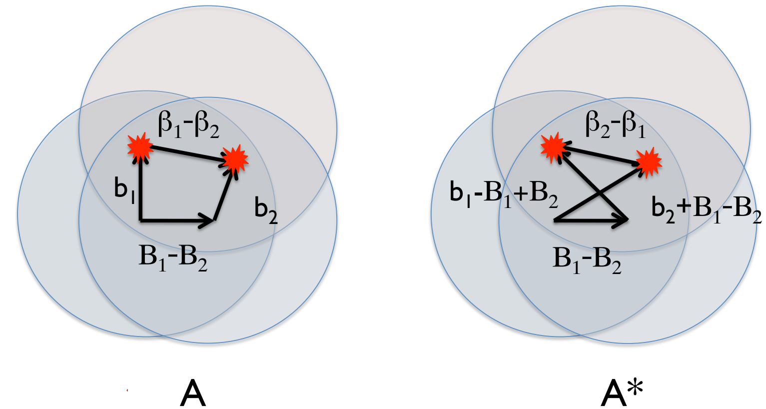
$$\sigma_D^A = \sigma_D^A\big|_1 + \sigma_D^A\big|_2$$

One has two different contributions to the forward scattering amplitude, in the case of two active target nucleons



In the case of two active target nucleons, when the two target partons are identical, in addition to the usually considered diagonal term one needs in fact to keep into account also the contribution of an interfrence term





In the interference term the nucleon's fractional momenta are different in the right and in the left hand side of the cut: $Z\text{-}Z'=x'_1\text{-}x'_2$ ⇨ the interference term is **proprotional to the nuclear form factor**, as a function of *Z-Z'*

x'_1 and x'_2 are measured in the final state. When $x'_1\text{-}x'_2$ is large, the contribution of the interference term therefore is small. In many cases of interest the contribution of the interference term may however be non negligible.

The two interactions are localized in two points in transverse space. Given two interaction points the parton with fractional momentum x'_1 may be provided by nucleon-1 and the parton with fractional momentum x'_2 by nucleon-2 (configuration **A**) or vice-versa (configuration **A***)

A simple case: *WJJ* production

The study of double parton correlations may be simpler when the off diagonal term is absent. An intersting case is *WJJ* production where the partons undergoing the double interaction are a quark and a gluon.

Here below the cross section, with ***K***, Λ and $\boldsymbol{\sigma}_{\mathbf{eff}}$ independent on x

$$\sigma^{pA}(WJJ) = \sigma_S^{pA}(WJJ) + \sigma_D^{pA}(WJJ), \text{ where } \quad \sigma_D^{pA}(WJJ) = \sigma_D^{pA}(WJJ)\big|_1 + \sigma_D^{pA}(WJJ)\big|_2$$

$$\sigma_D^{pA}(WJJ)\big|_1 = \frac{1}{\sigma_{eff}}\left[Z\sigma_S^{p[p]}(W)\sigma_S^{p[p]}(JJ) + (A-Z)\sigma_S^{p[n]}(W)\sigma_S^{p[n]}(JJ)\right]$$

$$\sigma_D^{pA}(WJJ)\big|_2 = K\left[\frac{Z}{A}\sigma_S^{pp}(W) + \frac{A-Z}{A}\sigma_S^{pn}(W)\right]\sigma_S^{pp}(JJ) \times\left[\int T(B)^2 d^2B - 2\int \rho(B,z)^2 d^2B dz \times r_c \mathcal{C}_K\right]$$

short range nuclear correlation

anti-shadowing contribution

nuclear thickness function
=> growth as $A^{4/3}$

nuclear density
=> growth as A

DPS in *p-Pb* collisions: Expectations according with the simplest model

When assuming K, Λ (and thus $\sigma_{\text{eff}} = \pi\Lambda^2/K^2$) independent on x one may consider two extreme cases:

a) $K^2 = 1$ and $\pi\Lambda^2 = \sigma_{\text{eff}}$ **:** No correlation in multiplicity, σ_{eff} gives the typical value of the transverse area where the DPS takes place

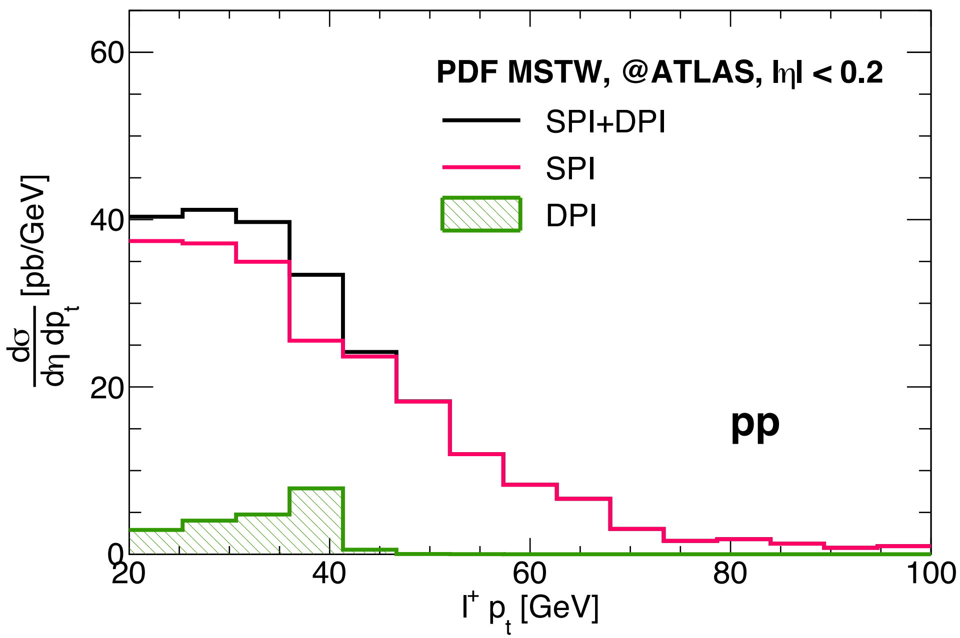
$$\frac{\sigma_D^{pA}|_2}{\sigma_D^{pA}|_1} \approx 2$$

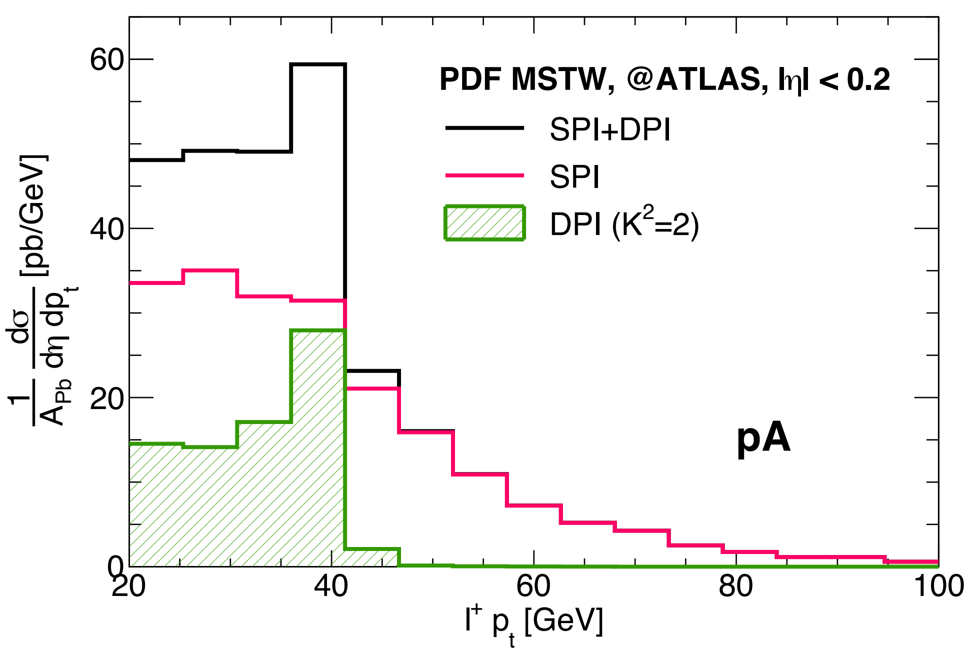
[200% anti-shadowing corrections]

b) $K^2 = 2$ and $\pi\Lambda^2 = K^2\sigma_{\text{eff}}$ **:** The observed value of σ_{eff} is completely due to the correlation in multiplicity

$$\frac{\sigma_D^{pA}|_2}{\sigma_D^{pA}|_1} \approx 3$$

[300% anti-shadowing corrections]





p_t spectrum of the charged lepton from the W^+ decay, in W^+JJ production in *pp* and in *pA* collisions at the LHC.

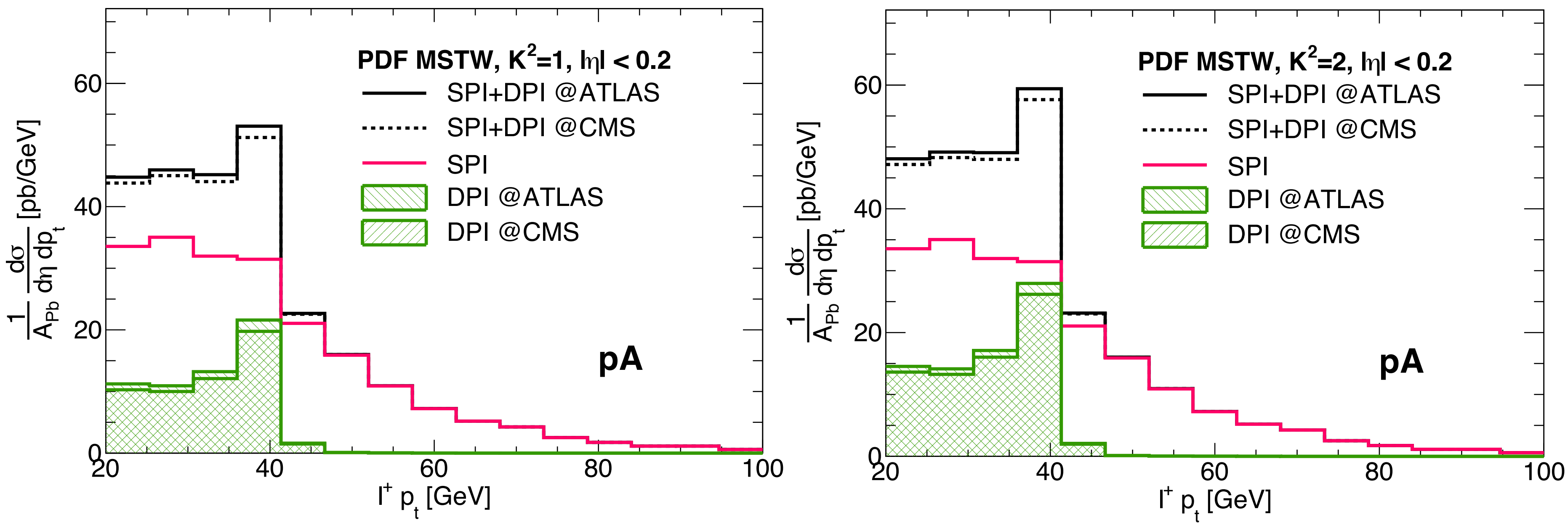
While the contribution of DPI is small in *pp* it becomes remarkably large in *pA*.

Kinematics is the same as in the case of the measurement of DPI in W^+JJ production in *pp* collisions by ATLAS.

The nucleus is *Pb* and, in the case of *pA*, one assumed that partons are not correlated in the relative transverse distance.

Notice that, by measuring the height of the spectrum at 40 GeV one obtains indications both on correlations in transverse distance and on correlations in multiplicity

p_t spectrum of the *W* decay-lepton in *p-Pb*: dependence on K and on the measured value of σ_{eff} in *p-p* collisions



The spectra in *p-Pb* collisions do not change much, when σ_{eff} increases from 15 to 20 mb in *p-p* collisions. The effect of increasing K^2 from 1 to 2 is on the contrary sizable and, at $p_t \sim 40$ GeV, in *p-Pb* collisions one expects an increase of the DPS contribution from 60 to 90 % of the SPS contribution to the cross section

Multiple production of b-bar pairs

At the LHC the inclusive production of two $b\bar{b}$ pairs is dominated by DPS.

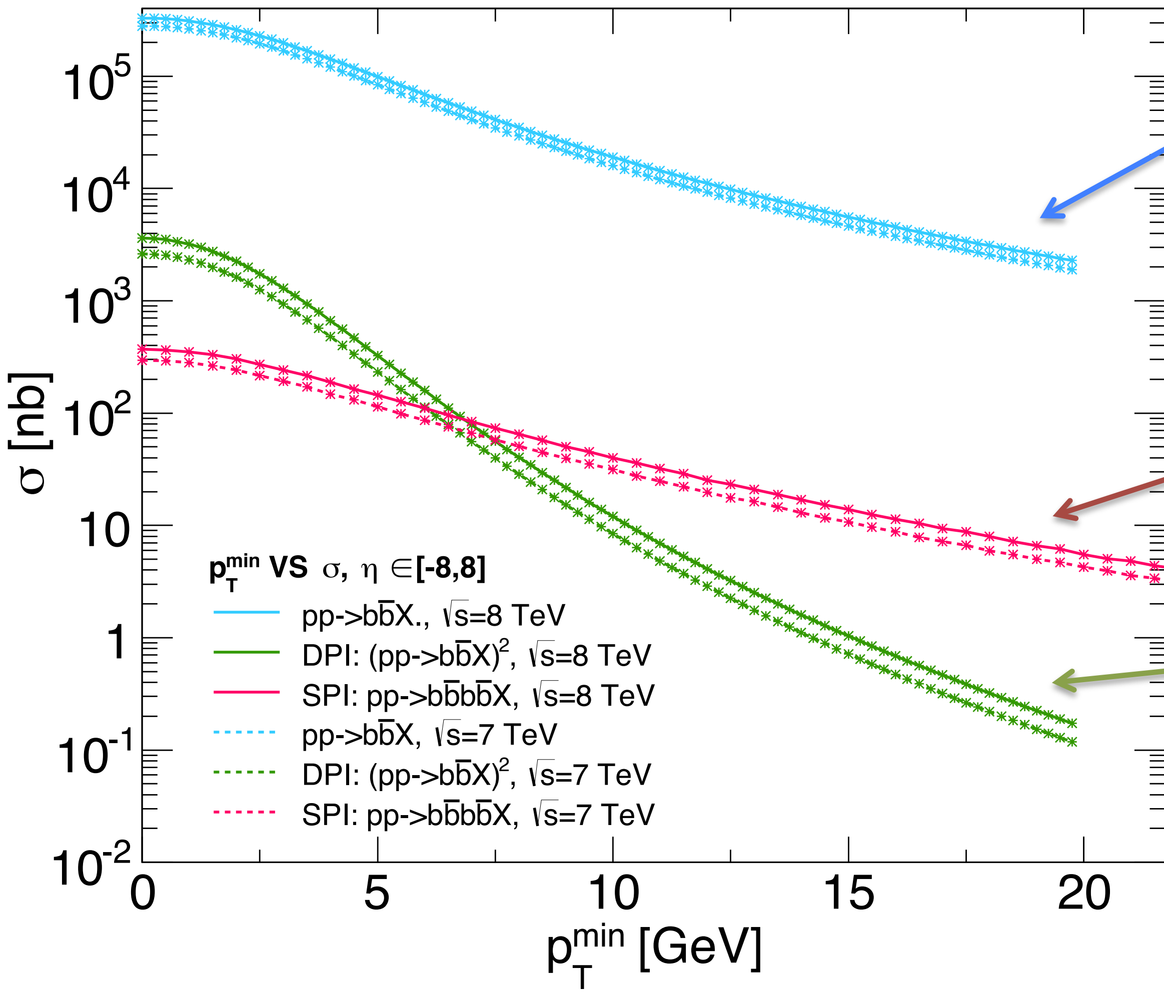
In *pp* collisions at 8 TeV c.m. energy, **the DPS contribution** to the integrated inclusive cross section is in fact expected to be **one order of magnitude larger** as compared to **the SPS contribution**.

The amount of $b\bar{b}b\bar{b}$ pairs produced in a *pp* collisions will thus exceed by a factor 10 the rate expected according with the leading QCD production mechanism.

Notice that b quarks are produced strongly and decay weakly. As a consequence the integrated amount of heavy quarks, produced in a hadronic collision, does not depend on final state interactions.

The integrated inclusive cross section to produce two $b\bar{b}$ pairs can thus provide a direct measurement of the DPS contribution.

$b\bar{b}$ and $b\bar{b}b\bar{b}$ production in pp collisions at the LHC: Integrated cross sections

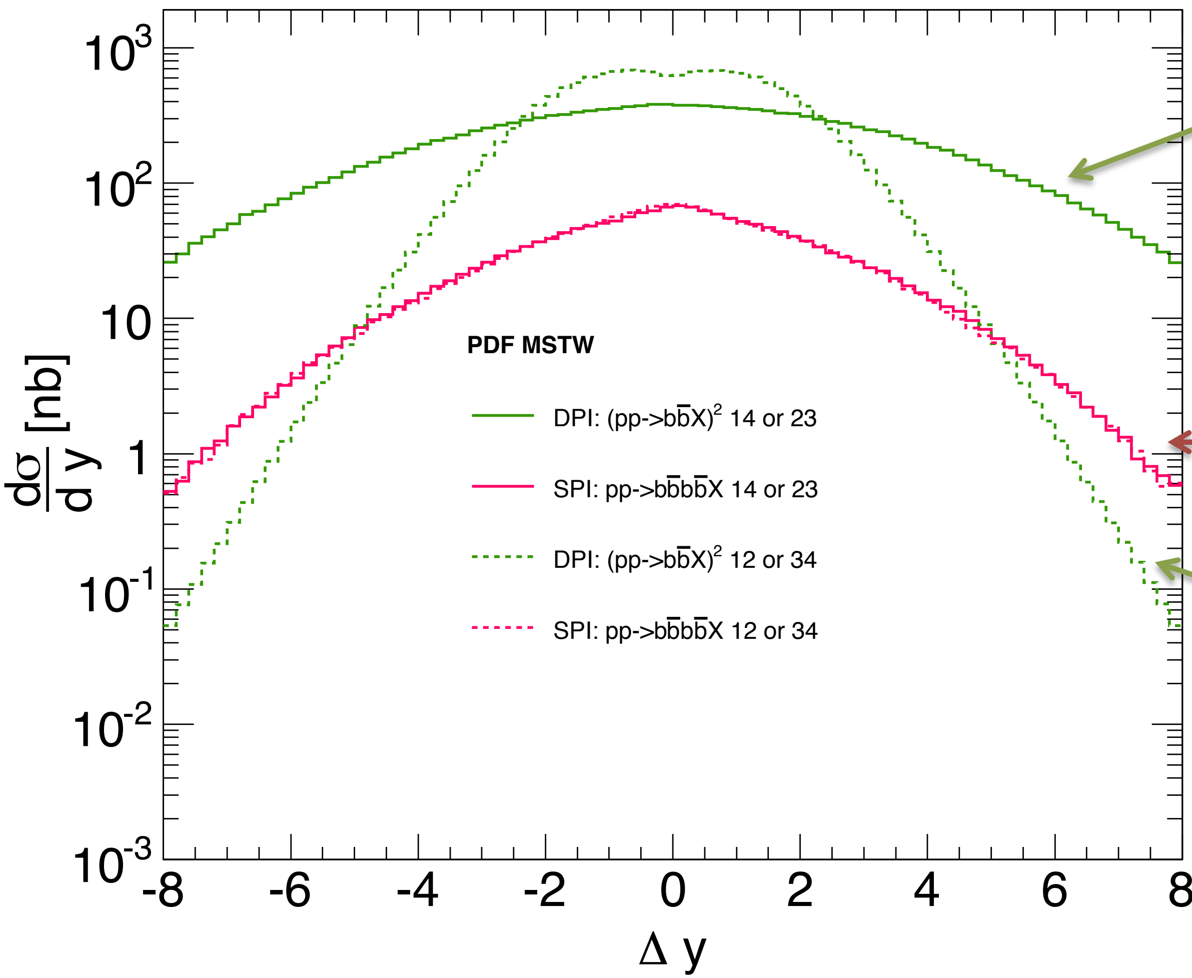


$b\bar{b}$ production: Integrated cross section, with $p_T > p_T^{min}$

$b\bar{b}b\bar{b}$ production by <u>single parton scattering</u>: Integrated cross section, with $p_T > p_T^{min}$

$b\bar{b}b\bar{b}$ production by <u>double parton scattering</u>: Integrated cross section, with $p_T > p_T^{min}$

$b\bar{b}b\bar{b}$ production in pp collisions at the LHC: Correlations in rapidity



Double parton scattering: correlation between two b-quarks originated in two different elementary interactions

Single parton scattering

Double parton scattering: correlation between two b-quarks originated in the same elementary interaction

In ***p-Pb*** **collisions** the dominand contribution to $b\bar{b}b\bar{b}$ production is due to the "anti-shadowing contribution" to DPS, where two target nucleons play and active role in the production process.

The value of **the DPS cross section** for $b\bar{b}b\bar{b}$ production, in *p-Pb* collisions at 8 TeV proton-nucleon c.m. energy, **may range between 1 and 2 mb**.

By comparison, the expected $b\bar{b}b\bar{b}$ production cross section due to **the leading QCD mechanism,** in *p-Pb* collisions at 8 TeV proton-nucleon c.m. energy, **may be** about 80 μb, namely **20 – 30 times smaller**.

In this case, **the interference term may give a sizable contribution** and needs to be the taken into account in the anti-shadowing contribution to DPS.
Although the interference term will be depressed by the factors of color and spin which, for gluon initated processes, will reduce its contribution by a factor 16.

Concluding Summary

In the simpest model for DPS, not inconsistent with present experimental evidence, only disconnected hard interactions are taken into account and $\sigma_{\mathbf{eff}}$ does not dependent on fractional momenta.

In the model $\sigma_{\mathbf{eff}}$ is given by the ratio of the typical transverse interaction area ($\pi\Lambda^2$) and the multiplicity of parton pairs ($\boldsymbol{K}^2$). DPS in *p-p* collisions can thus provide information only on the ratio between Λ and $\boldsymbol{K}$.

DPS is different in *p-p* and *p-A* collisions. In *p-A* collisions two different target nucleons can in fact play an active role in the hard collision process.
The picture of the interaction is simpler for processes, which do not involve identical target partons, as in the case of *WJJ* production.

The contribution, where two different target nucleons play an active role in DPS, gives a positive (anti-shadowing) contribution to the cross sectio which, in *p-Pb*, may be 2-300% larger then the term involging only a single target nucleon in the DPS process.

The anti-shadowing correction term is **a**) *proportional to the factor **K***, which gives the multiplicity of parton pairs in the projectile proton and **b**) it *depends weakly on the partonic correlations in the transverse coordinates*.

By measuring the amount of anti-shadowing, in *WJJ* production in *p-Pb* collisions, one thus obtains a **direct indication on the multiplicity of parton pairs in the projectile, which represents an unprecedented piece of information on the correlated partonic structure of the hadron**.

When an estimate of ***K*** is available, one can obtain Λ from $\sigma_{\mathbf{eff}}$ and, in this way, valuable information on the 3D structure of the proton.

In *pp* collisions at the LHC, **DPS** is the dominant production mechanism of multiple pairs of b-quarks, which is expected to be **one order of magnitude larger** as compared to **the SPS contribution**.

in *p-Pb* $b\bar{b}b\bar{b}$ production by **DPS** is expected to exceed the SPS contribution by **a factor 20 – 30**, the **dominant contribution** being the **anti-shadowing term**.

The **interference contribution** to the anti-shadowing term **can provide valuable information on off-diagonal parton distributions**.

Thank you