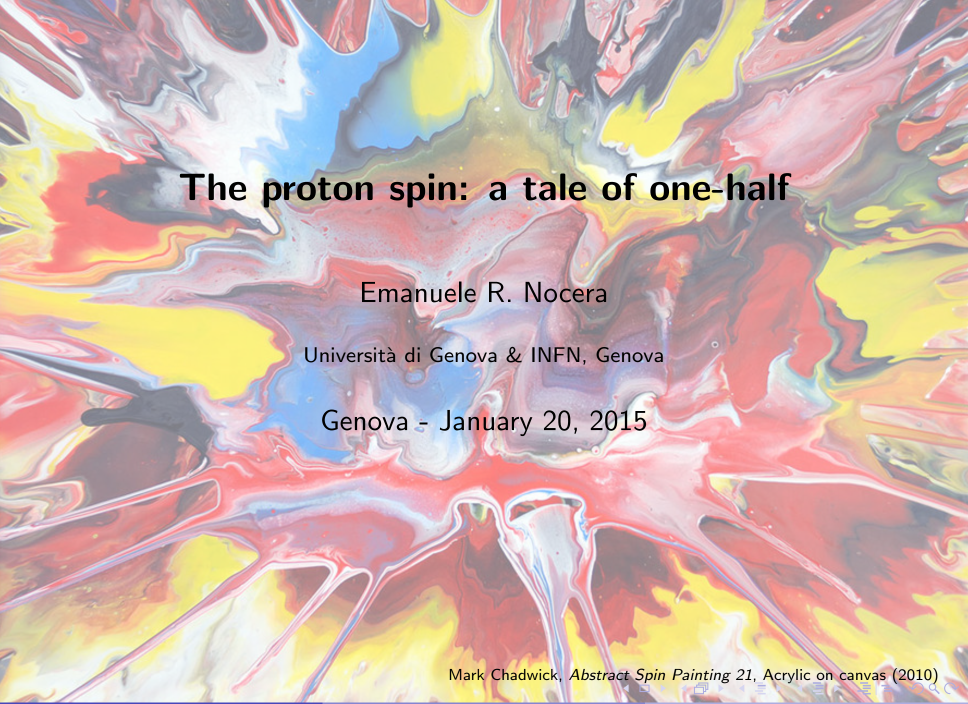

The proton spin: a tale of one-half

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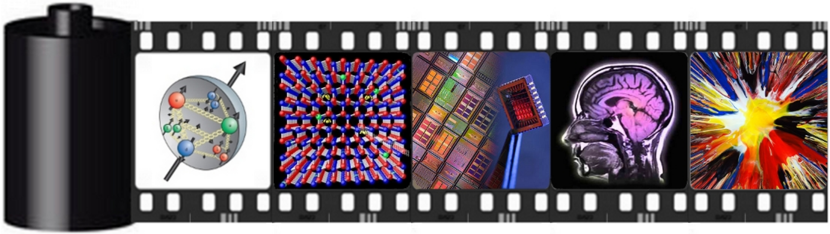
Genova - January 20, 2015

Mark Chadwick, *Abstract Spin Painting 21*, Acrylic on canvas (2010)

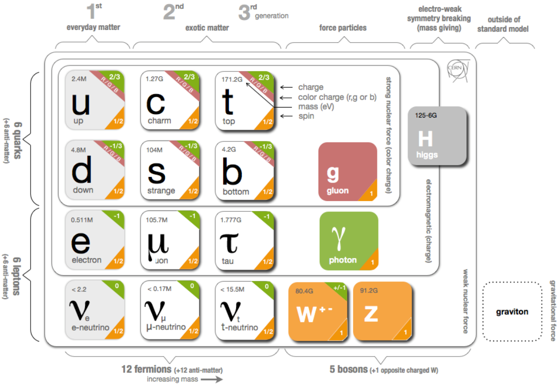
Spin, a fundamental concept in physics

1. SPIN is deeply rooted in the symmetries and structure of space-time
   - *an intrinsic form of angular momentum*
   - seminal role in Quantum Mechanics
2. SPIN determines whether a particle follows Fermi or Bose statistics
   - implications in the structure of matter and the stability of many-body systems
   - lays the foundations for chemistry and biology
3. SPIN has evident effects even at large scales, almost in everyday life
   - spintronics-based memory chips
   - nuclear magnetic resonance imaging



Spin in particle physics

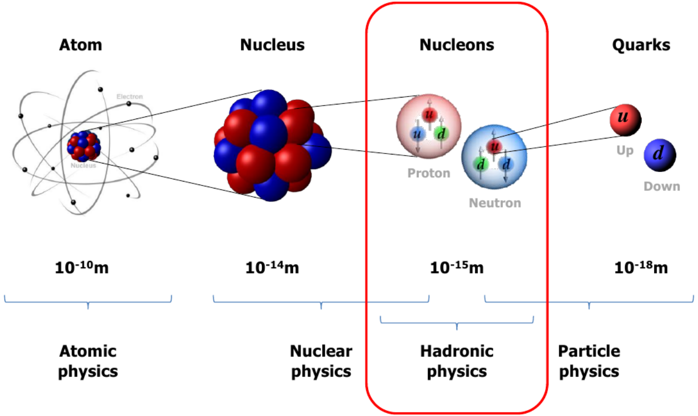
All elementary particles in the SM carry spin, except the recently observed Higgs boson
Among them, the particles that are subject to the strong interactions: quarks and gluons



SPIN is intimately entwined with Quantum Chromodynamics

Spin in hadron physics

Nucleons, protons and neutrons, are bound states with spin one-half
They make up all nuclei and hence most of the visibile mass in the Universe



They have internal structure, and such a structure is intimately entwined with SPIN

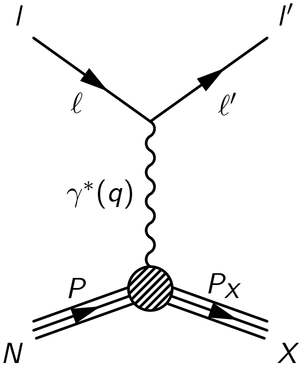
Outline

1. Partons in QCD
   - Distributions, polarization, and all that
2. A global determination of polarized PDFs
   - Data, methodology, (most recent) results
3. The path forward
   - Future opportunities at JLAB, RHIC and EIC
4. Drawing conclusions

1. Partons in QCD

Blake Brasher, *Deep Inelastic Scattering*, Acrylic, ink, and holographic glitter on canvas (2014)

A modern realization of Rutherford's experiment



$$l(\ell) + N(P) \longrightarrow l'(\ell') + X(P_X)$$

$$\frac{d^3\sigma^{s_\ell, S}}{dxdyd\varphi} = \frac{\alpha_{em}^2 y}{2Q^4}\left[L_{\mu\nu}^{(S)} W^{\mu\nu(S)} - L_{\mu\nu}^{(A)} W^{\mu\nu(A)}\right]$$

$$L^{\mu\nu} = \sum_{s_{\ell'}}\left[\bar{u}(\ell, s_\ell)\gamma^\nu u(\ell', s_{\ell'})\bar{u}(\ell', s_{\ell'})\gamma^\mu u(\ell, s_\ell)\right]$$

$$W_{\mu\nu} = \frac{1}{2\pi}\sum_X \int d\Pi_X \left[\langle P, S|J_\nu^\dagger(q)|P_X\rangle\langle P_X|J_\mu(q)|P, S\rangle\right]$$

$$W_{\mu\nu}^{(A)} = \frac{2M\epsilon_{\mu\nu\rho\sigma}q^\rho}{P\cdot q}\left\{S^\sigma g_1(x) + \left[S^\sigma - \frac{S\cdot q}{P\cdot q}P^\sigma\right] g_2(x)\right\}$$

$$\frac{d^3\sigma^{\rightarrow;\Rightarrow}}{dxdy} - \frac{d^3\sigma^{\rightarrow;\Leftarrow}}{dxdy} = \frac{8\pi\alpha_{em}^2}{Q^2}\left[\left(2 - y - \frac{\gamma^2 y^2}{2}\right) g_1(x) - \gamma^2 y g_2(x)\right], \qquad \gamma = x\frac{M}{Q}$$

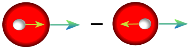
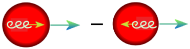
Helicity-dependent (polarized) parton distribution functions

1. Asymmetry as an incoherent sum of lepton-parton point-like interactions

$$\frac{d^2\sigma^{\rightarrow;\Rightarrow}}{dxdy} - \frac{d^2\sigma^{\rightarrow;\Leftarrow}}{dxdy} = \sum_q e_q^2 \Delta f(x) \left[\frac{d\hat{\sigma}^{\rightarrow;\rightarrow}}{dy} - \frac{d\hat{\sigma}^{\rightarrow;\leftarrow}}{dy} \right]$$

2. The momentum densities of partons with spin $(^{\uparrow})$ or $(^{\downarrow})$ *w.r.t* the nucleon

$$\Delta f(x) \equiv f^{\uparrow}(x) - f^{\downarrow}(x)\,, \qquad f = u, \bar{u}, d, \bar{d}, s, \bar{s}, g$$

$\Delta q(x) =$  $\Delta g(x) =$ 

3. Allow for a proper field-theoretic definition as matrix elements of bilocal operators

$$\Delta q(x) = \frac{1}{4\pi} \int dy^- e^{-iy^- xP^+} \langle P, S | \bar{\psi}(0, y^-, \mathbf{0}_\perp) \gamma^+ \gamma^5 \psi(0) | P, S \rangle$$

$$\Delta g(x) = \frac{1}{4\pi x P^+} \int dy^- e^{-iy^- xP^+} \langle P, S | G^{+\alpha}(0, y^-, \mathbf{0}_\perp) \tilde{G}_\alpha^+(0) | P, S \rangle$$

with light-cone coordinates and QCD field-strength tensor G ($A^+ = 0$ gauge)

$$y = (y^+, y^-, \mathbf{y}_\perp)\,, \quad y^+ = (y^0 + y^z)/\sqrt{2}\,, \quad y^- = (y^0 - y^z)/\sqrt{2}\,, \quad \mathbf{y}_\perp = (v^x, v^y)$$

$$G_{\mu\nu}^{\alpha} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c$$

Naive parton model expectations

1. In the naive parton model, one would expect

$$\frac{d^2\sigma^{+;\Rightarrow}}{dxdy} - \frac{d^2\sigma^{+;\Leftarrow}}{dxdy} = \frac{4\pi\alpha_{em}^2}{Q^2}\left[\sum_q e_q^2 \Delta q(x)(2-y)\right]$$

2. In terms of structure functions (assuming $n_f = 3$)

$$g_1^p(x) = \frac{1}{2}\sum_q e_q^2 \Delta q(x) = \frac{1}{9}\Delta\Sigma(x) + \frac{1}{12}\Delta T_3(x) + \frac{1}{36}\Delta T_8(x) \qquad g_2^p(x) = 0$$

$$\Delta\Sigma = \Delta u^+ + \Delta d^+ + \Delta s^+ \qquad \Delta T_3 = \Delta u^+ - \Delta d^+ \qquad \Delta T_8 = \Delta u^+ + \Delta d^+ - 2\Delta s^+$$

3. Relate the first moment of $\Delta\Sigma$ to the *m.e.* of the flavor singlet axial current

$$a_0 = \langle P, S | J_\Sigma^{\hat{z}} | P, S\rangle = \int_0^1 dx \Delta\Sigma(x) = 2\langle S^{q+\bar{q}}\rangle \approx 1$$

4. European Muon Collaboration result (1988)

$$a_0 = 0.098 \pm 0.076 \pm 0.113$$

The EMC experiment and the *proton spin crisis*

ELSEVIER

Physics Letters B

Volume 206, Issue 2, 19 May 1988, Pages 364–370

A measurement of the spin asymmetry and determination of the structure function g_1 in deep inelastic muon-proton scattering

European Muon Collaboration

Volume 206, number 2 PHYSICS LETTERS B 19 May 1988

The spin asymmetry in deep inelastic scattering of longitudinally polarised muons by longitudinally polarised protons has been measured over a large x range $(0.01 < x < 0.7)$. The spin-dependent structure function $g_1(x)$ for the proton has been determined and its integral over x found to be $0.114 \pm 0.012 \pm 0.026$, in disagreement with the Ellis–Jaffe sum rule. Assuming the validity of the Bjorken sum rule, this result implies a significant negative value for the integral of g_1 for the neutron. These values for the integrals of g_1 lead to the conclusion that the total quark spin constitutes a rather small fraction of the spin of the nucleon.

The EMC experiment and the *proton spin crisis*

Nuclear Physics B

Volume 328, Issue 1, 11 December 1989, Pages 1–35

An investigation of the spin structure of the proton in deep inelastic scattering of polarised muons on polarised protons

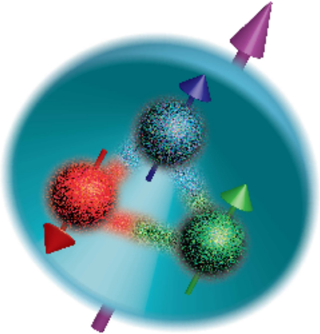
The European Muon Collaboration

J. Ashman et al. / Spin structure of proton

The spin asymmetry in deep inelastic scattering of longitudinally polarised muons by longitudinally polarised protons has been measured in the range $0.01 < x < 0.7$. The spin dependent structure function $g_1(x)$ for the proton has been determined and, combining the data with earlier SLAC measurements, its integral over x found to be $0.126 \pm 0.010(\text{stat.}) \pm 0.015(\text{syst.})$, in disagreement with the Ellis–Jaffe sum rule. Assuming the validity of the Bjorken sum rule, this result implies a significant negative value for the integral of g_1 for the neutron. These integrals lead to the conclusion, in the naïve quark parton model, that the total quark spin constitutes a rather small fraction of the spin of the nucleon. Results are also presented on the asymmetries in inclusive hadron production which are consistent with the above picture.

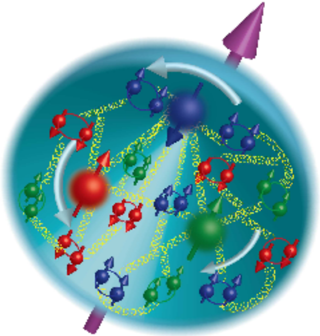
A realistic (QCD) picture of the nucleon

naive picture



three non-relativistic quarks

realistic picture



indefinite number of relativistic quarks and gluons

$$\xleftrightarrow[\textit{factorization},\textit{evolution}]{\textit{QCD}}$$

Factorization of physical observables

1. A variety of sufficiently inclusive processes allow for a factorized description

short-distance part		long-distance part
hard interaction of partons	$\xleftrightarrow[\text{scheme}]{\text{factorization}}$	nucleon spin structure
process-dependent kernels		universal parton distributions

2. Physical observables are written as a convolution of coefficient functions and PDFs

$$\mathcal{O}_I = \sum_{f=q,\bar{q},g} \Delta C_{If}(y, \alpha_s(\mu^2)) \otimes \Delta f(y, \mu^2) \qquad f \otimes g = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$

3. Coefficient functions allow for a perturbative expansion

$$\Delta C_{If}(y, \alpha_s) = \sum_{k=0} a_s^k \Delta C_{If}^{(k)}(y), \qquad a_s = \alpha_s/(4\pi)$$

4. For the structure function g_1 one has (at leading twist)

$$g_1(x, \mu^2) = \frac{\langle e^2 \rangle}{2} \left[\Delta C_{\rm NS} \otimes \Delta q_{\rm NS} + \Delta C_{\rm S} \otimes \Delta\Sigma + 2 n_f \Delta C_g \otimes \Delta g\right] \qquad \langle e^2 \rangle = n_f^{-1} \sum_{i=1}^{n_f} e_i^2$$

$$\Delta q_{\rm NS} \equiv \sum_{i=1}^{n_f} \left(\frac{e_i^2}{\langle e^2 \rangle} - 1\right) [\Delta q_i + \Delta \bar{q}_i] \qquad \Delta\Sigma \equiv \sum_{i=1}^{n_f} [\Delta q_i + \Delta \bar{q}_i]$$

$$\Delta C_{\rm NS}^{(0)} = \Delta C_S^{(0)} = \delta(1-x) \qquad \Delta C_g^{(0)} = 0$$

Scale-dependence of PDFs: DGLAP equations

1. A set of $(2n_f + 1)$ integro-differential equations, n_f is the number of active flavors

$$\frac{\partial}{\partial \ln \mu^2}\Delta f_i(x,\mu^2) = \sum_j^{n_f}\int_x^1 \frac{dz}{z}\Delta P_{ji}\left(z,\alpha_s(\mu^2)\right)\Delta f_j\left(\frac{x}{z},\mu^2\right)$$

2. Often written in a convenient basis of PDFs

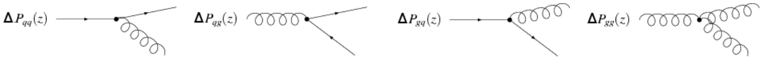
$$\Delta q_{\mathrm{NS};\pm} = (\Delta q_i \pm \Delta\bar{q}_i) - (\Delta q_j \pm \Delta\bar{q}_j) \quad \Delta q_{\mathrm{NS;v}} = \sum_i^{n_f}(\Delta q_i - \Delta\bar{q}_j) \quad \Delta\Sigma = \sum_i^{n_f}(\Delta q_i + \Delta\bar{q}_j)$$

$$\frac{\partial}{\partial \ln \mu^2}\Delta q_{\mathrm{NS};\pm,v}(x,\mu^2) = P^{\pm,\mathrm{v}}(x,\mu_F^2)\otimes\Delta q_{\mathrm{NS};\pm,v}(x,\mu^2)$$

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix}\Delta\Sigma(x,\mu^2)\\ \Delta g(x,\mu^2)\end{pmatrix} = \begin{pmatrix}\Delta P^{qq} & \Delta P^{gq}\\ \Delta P^{qg} & \Delta P^{gg}\end{pmatrix}\otimes\begin{pmatrix}\Delta\Sigma(x,\mu^2)\\ \Delta g(x,\mu^2)\end{pmatrix}$$

3. With perturbative computable splitting functions (computed up to NNLO)

$$\Delta P_{ji}(z,\alpha_s) = \sum_{k=0} a_s^{k+1}\Delta P_{ji}^{(k)}(z), \qquad a_s = \alpha_s/(4\pi)$$



The proton helicity sum rule

1. A decomposition of the nucleon spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(\mu^2) + \Delta G(\mu^2) + \mathcal{L}_q(\mu^2) + \mathcal{L}_g(\mu^2)$$

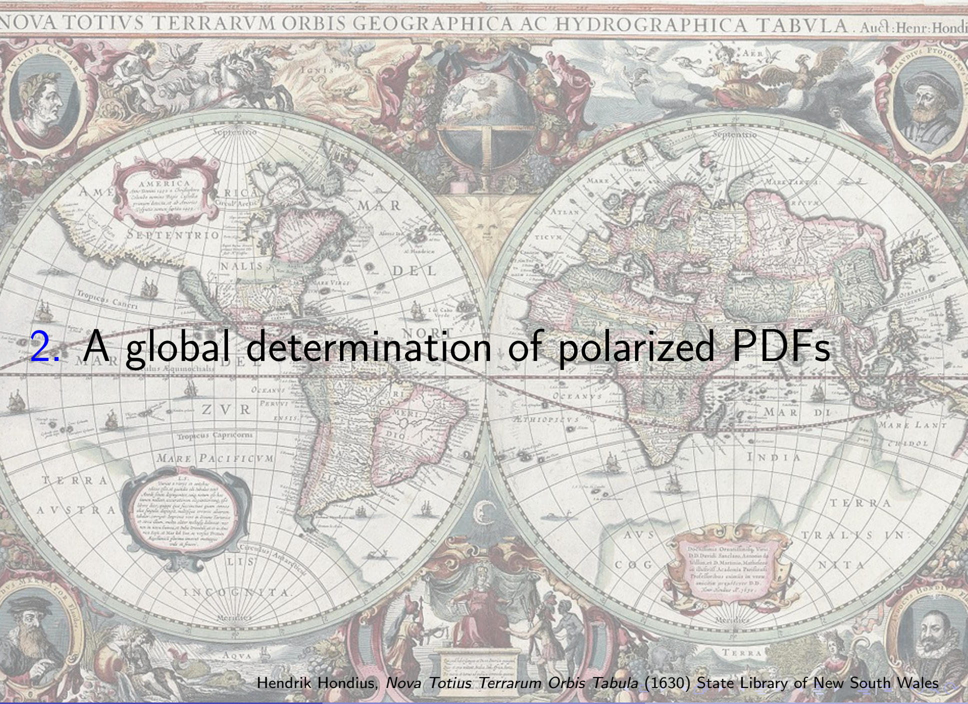
$$\Delta\Sigma(\mu^2) = \sum_{q=u,d,s}\int_0^1 [\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)] \qquad \Delta G(\mu^2) = \int_0^1 dx \Delta g(x,\mu^2)$$

2. All $\Delta\Sigma$, ΔG, $\mathcal{L}_q$ and $\mathcal{L}_g$ depend on the factorization scale μ (and scheme)
3. Beyond LO, the axial charge reads

$$a_0 = \Delta\Sigma(\mu^2) - n_f \frac{\alpha_s(\mu^2)}{2\pi}\Delta G(\mu^2)$$

4. According to DGLAP equations, ΔG evolves as $[\alpha_s(\mu^2)]^{-1}$
5. The gluon decouples from g_1, the naive parton model predictions are not recovered
6. Depending on ΔG there may be a cancellation with $\Delta\Sigma$ leading to $a_0 \approx 0$

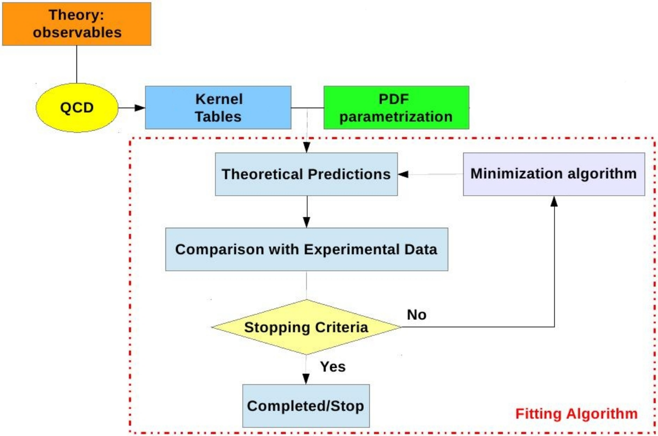
The rest of the talk will be on the determination of $\Delta\Sigma(\mu^2)$ and $\Delta G(\mu^2)$ in the $\overline{\mathrm{MS}}$ factorization scheme at NLO accuracy



2. A global determination of polarized PDFs

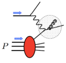
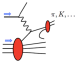
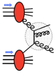
Hendrik Hondius, *Nova Totius Terrarum Orbis Tabula* (1630) State Library of New South Wales

Anatomy of a global QCD analysis of PDFs

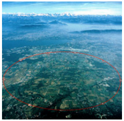


Obtain PDFs through global minimization of a figure of merit (*e.g.* χ^2)
Determine the set of best-fit parameters entering the assumed PDF parametrization
Need for a prescription to estimate and propagate uncertainties

Experimental data

Process	Reaction	Subprocess	PDFs probed	x	$Q^2/p_T^2/M^2$ [GeV^2]
 DIS	$\ell^\pm\{p,d,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$\Delta q + \Delta\bar{q}$ Δg	$0.003 \lesssim x \lesssim 0.8$	$1 \lesssim Q^2 \lesssim 70$
 SIDIS	$\ell^\pm\{p,d\} \to \ell^\pm hX$	$\gamma^* q \to q$	$\Delta u\ \Delta\bar{u}$ $\Delta d\ \Delta\bar{d}$ Δg	$0.005 \lesssim x \lesssim 0.5$	$1 \lesssim Q^2 \lesssim 60$
	$\ell^\pm\{p,d\} \to \ell^\pm DX$	$\gamma^* g \to c\bar{c}$	Δg	$0.06 \lesssim x \lesssim 0.2$	~ 10
 pp	$\vec{p}\vec{p} \to jet(s)X$	$gg \to qg$ $qg \to qg$	Δg	$0.05 \lesssim x \lesssim 0.2$	$30 \lesssim p_T^2 \lesssim 800$
	$\vec{p}p \to W^\pm X$	$u_L\bar{d}_R \to W^+$ $d_L\bar{u}_R \to W^-$	$\Delta u\ \Delta\bar{u}$ $\Delta d\ \Delta\bar{d}$	$0.05 \lesssim x \lesssim 0.4$	$\sim M_W^2$
	$\vec{p}\vec{p} \to \pi X$	$gg \to qg$ $qg \to qg$	Δg	$0.05 \lesssim x \lesssim 0.4$	$1 \lesssim p_T^2 \lesssim 200$

CERN



EMC, SMC, COMPASS

SLAC

E142, E143, E154, E155

DESY

HERMES

JLAB

HALL-A, CLAS

BNL

PHENIX, STAR

DIS&SIDIS

pp

A few issues

1. Limited (x, Q^2) kinematic coverage
   - difficult to get Δg from scaling violations in DIS and SIDIS
     $\rightarrow$ additional *direct* probes of Δg are needed (jets, open-charm)
   - need to use data down to $Q^2 = 1$ GeV^2
     $\rightarrow$ is perturbative QCD reliable?
     $\rightarrow$ how much higher twists affect the perturbative description of observables?
2. No neutrino DIS data
   - no quark-antiquark separation from inclusive DIS
     $\rightarrow$ limited set of $W^{\pm}$ production data in pp collisions
     $\rightarrow$ SIDIS data require knowledge of fragmentation functions (extra uncertainties)

---

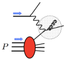
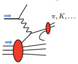
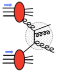
1. Relate the octet of axial-vector currents to β-decay of of spin-1/2 hyperons

$$a_3 = \int_0^1 dx \Delta T_3 = F + D = 1.2701 \pm 0.0025 \qquad a_8 = \int_0^1 dx \Delta T_8 = 3F - D = 0.585 \pm 0.025$$

2. Require the positivity of cross sections

$$|g_1(x,\mu^2)| \leq F_1(x,\mu^2) \overset{\text{LO}}{\Longleftrightarrow} |\Delta f(x,\mu^2)| \leq f(x,\mu^2)$$

Recent determinations of polarized PDFs ($\sim$ past 4 years)

collaboration	 DIS	 SIDIS	 pp	latest update	features
DSSV	☑	☑	☑ (jets, π^0)	[arXiv:1404.4293]	global fit
NNPDF	☑	☒	☑ (jets, $W^\pm$)	[arXiv:1406.5539]	minimally biased fit
JAM	☑	☒	☒	[arXiv:1310.3734]	large-x effects
LSS	☑	☑	☒	[arXiv:1010.0574]	higher-twist effects
BB	☑	☒	☒	[arXiv:1005.3113]	simultaneous fit of α_s

$+$ simultaneous determination of PDF uncertainties

N.B.: PDF uncertainties stem from three sources

1. the underlying data, affected by statistical and (correlated) systematic errors
2. the theory used to describe them, based on the truncation of a perturbative series
3. the procedure used to determine PDFs from data

Available PDF sets are all based on item 2, but may differ significantly for items 1 and 3

Methodological detour: the standard methodology

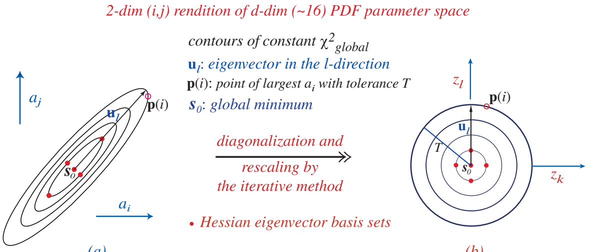
1. Simple analytical parametrization of PDFs

$$x\Delta f_i(x, \mu_0^2) = \eta_i x^{a_i}(1-x)^{b_i}\left(1 + \rho_i x^{\frac{1}{2}} + \gamma_i x\right)$$

$\Rightarrow$ **potential bias if the parametrization is too rigid**

2. Hessian propagation of errors
   - expand the χ^2 about its global minimum at first order
   - diagonalize the Hessian matrix and take the hypersphere of radius $\sqrt{\Delta\chi^2} = 1$

$\Rightarrow$ **is linear approximation adequate? do we need a *tolerance* T $= \sqrt{\Delta\chi^2} > 1$?**



(a)
Original parameter basis

(b)
Orthonormal eigenvector basis

Methodological detour: the NNPDF methodology

1. Neural network parametrization of PDFs
   - redundant and flexible parametrization, $\mathcal{O}(200)$ parameters
   - requires a proper minimization algorithm and stopping criterion

$\Rightarrow$ **reduce the theoretical bias due to the parametrization**

2. Monte Carlo propagation of errors
   - generate experimental data replicas assuming multi-Gaussian probability distribution
   - validate against experimental data to determine the sample size ($N_{\rm rep} \sim 100$)

$\Rightarrow$ **no need to rely on linear error propagation, no tolerance needed**

PDF replicas are equally probable members of a statistical ensemble which samples the probability density $\mathcal{P}[f_i]$ in the space of PDFs

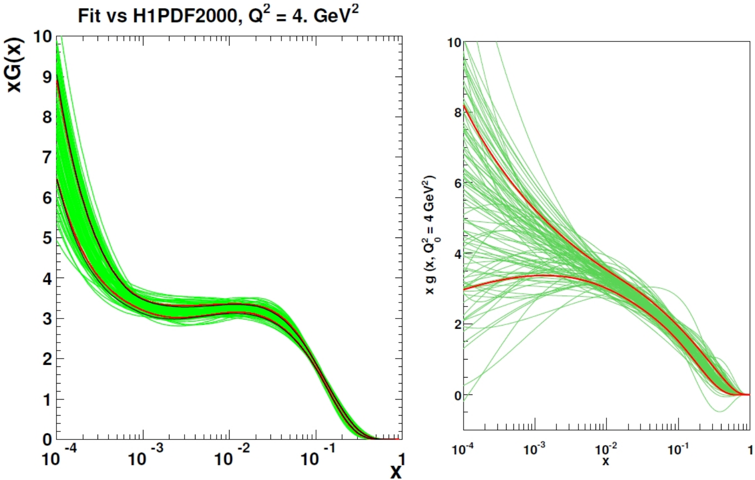
$$\langle \mathcal{O} \rangle = \int \mathcal{D}f_i \mathcal{P}[f_i]\mathcal{O}[f_i]$$

Expectation values for observables are Monte Carlo integrals

$$\langle \mathcal{O}[f_i(x,Q^2)] \rangle = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} \mathcal{O}[f_i^{(k)}(x,Q^2)]$$

Methodological detour: potential consequences

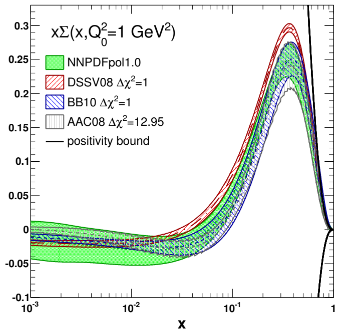
HERA-LHC 2009 PDF benchmark

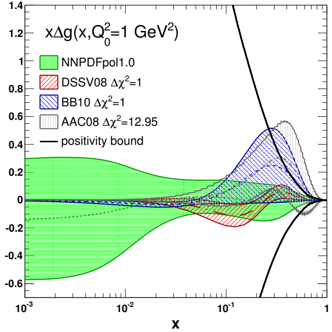


simple functional forms

Neural Networks

Polarized PDFs from DIS: status 2013





$$\int_{0.001}^{1} dx \Delta\Sigma(x, Q_0^2) = +0.25 \pm 0.09 \qquad \int_{0.001}^{1} dx \Delta g(x, Q_0^2) = -0.26 \pm 0.19$$

$$\int_{0}^{1} dx \Delta\Sigma(x, Q_0^2) = +0.22 \pm 0.20 \qquad \int_{0}^{1} dx \Delta g(x, Q_0^2) = -1.2 \pm 4.2$$

Two new analyses in 2014

DSSV++ [1404.9293]
Δg update of DSSV08 [0804.0422]

PRL 113, 012001 (2014) PHYSICAL REVIEW LETTERS week ending 4 JULY 2014

Evidence for Polarization of Gluons in the Proton

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(Received 17 April 2014; published 2 July 2014)

We discuss the impact of recent high-statistics Relativistic Heavy Ion Collider data on the determination of the gluon polarization in the proton in the context of a global QCD analysis of polarized parton distributions. We find evidence for a nonvanishing polarization of gluons in the region of momentum fraction and at the scales mostly probed by the data. Although information from low momentum fractions is presently lacking, this finding is suggestive of a significant contribution of gluon spin to the proton spin, thereby limiting the amount of orbital angular momentum required to balance the proton spin budget.

DOI: 10.1103/PhysRevLett.113.012001 PACS numbers: 13.88.+e, 12.38.Bx, 13.60.Hb, 13.85.Ni

Introduction.—The gluon helicity distribution function $\Delta g(x)$ of the proton has long been recognized as a fundamental quantity characterizing the inner structure of the nucleon. In particular, its integral $\Delta G \equiv \int_0^1 dx \Delta g(x)$ over all gluon momentum fractions x may in $A^+ = 0$ light-cone gauge be interpreted as the gluon spin contribution to the proton spin [1]. As such, ΔG is a key ingredient to the proton helicity sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g, \qquad (1)$$

where $\Delta\Sigma$ denotes the combined quark and antiquark spin contribution and $L_{q,g}$ are the quark and gluon orbital angular momentum contributions. For simplicity, we have omitted the renormalization scale Q and scheme dependence of all quantities.

It is well known that the quark and gluon helicity distributions can be probed in high-energy scattering processes with polarized nucleons, allowing access to $\Delta\Sigma$ and ΔG. Experiments on polarized deep inelastic lepton-nucleon scattering (DIS) performed since the late eighties [2] have shown that relatively little of the proton spin is carried by the quark and antiquark spins, with a typical value $\Delta\Sigma \sim 0.25$ [2–4]. The inclusive DIS measurements have, however, very little sensitivity to gluons. Instead, the best probes of Δg are offered by polarized proton-proton collisions available at the BNL Relativistic Heavy Ion Collider (RHIC) [5]. Several processes in pp collisions, in particular jet or hadron production at high transverse momentum p_T, receive substantial contributions from gluon-induced hard scattering, hence, opening a window on Δg when polarized proton beams are used.

The first round of results produced by RHIC until 2008 [5] were combined with data from inclusive and semi-inclusive DIS in a next-to-leading order (NLO) global QCD analysis [3], hereafter referred to as "DSSV analysis". One of the main results of that analysis was that the RHIC data—within their uncertainties at the time—did not show any evidence of a polarization of gluons inside the proton. In fact, the integral of Δg over the region $0.05 \leq x \leq 0.2$ of momentum fraction primarily accessed by the RHIC experiments was found to be very close to zero. Other recent analyses of nucleon spin structure [4] did not fully include RHIC data; as a result Δg was left largely unconstrained.

Since the analysis [3], the data from RHIC have vastly improved. New results from the 2009 run [6,7] at center-of-mass energy $\sqrt{s} = 200$ GeV have significantly smaller errors across the range of measured p_T. This will naturally put tighter constraints on $\Delta g(x)$ and may extend the range of x over which meaningful constraints can be obtained. A striking feature is that the STAR jet data [6] now exhibit a double-spin asymmetry A_{LL} that is clearly nonvanishing over the whole range $5 \lesssim p_T \lesssim 30$ GeV, in contrast to the previous results. Keeping in mind that, in this regime, jets are primarily produced by gluon-gluon and quark-gluon scattering, this immediately suggests that gluons inside the proton might be polarized. At the same time, new PHENIX data for π^0 production [7] still do not show any significant asymmetry, and it is of course important to reveal whether the two data sets provide compatible information. In this Letter, we assess the impact of the 2009 RHIC data sets on

0031-9007/14/113(1)/012001(5) 012001-1

NNPDFpol1.1 [1406.5539]
Δg, $\Delta\bar{q}$ update of NNPDFpol1.0 [1303.7236]

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A first unbiased global determination of polarized PDFs and their uncertainties

NNPDF Collaboration

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Received 25 June 2014; received in revised form 17 August 2014; accepted 18 August 2014

Available online 22 August 2014

Editor: Tommy Ohlsson

Abstract

We present a first global determination of spin-dependent parton distribution functions (PDFs) and their uncertainties using the NNPDF methodology: NNPDFpol1.1. Longitudinally polarized deep-inelastic scattering data, already used for the previous NNPDFpol1.0 PDF set, are supplemented with the most recent polarized hadron collider data for inclusive jet and W boson production from the STAR and PHENIX experiments at RHIC, and with open-charm production data from the COMPASS experiment, thereby allowing for a separate determination of the polarized quark and antiquark PDFs, and an improved determination of the medium- and large-x polarized gluon PDF. We study the phenomenological implications of the NNPDFpol1.1 set, and we provide predictions for the longitudinal double-spin asymmetry for semi-inclusive pion production at RHIC.

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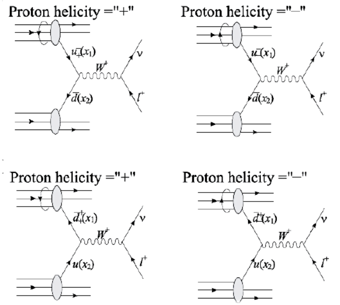
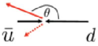
http://dx.doi.org/10.1016/j.nuclphysb.2014.08.008
0550-3213/

New data from RHIC: $W^{\pm}$ production

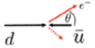


$$A_L^{W^-} \sim$$

$$\frac{\Delta\bar{u}_{x_1} d_{x_2}(1-\cos\theta)^2 - \Delta d_{x_1}\bar{u}_{x_2}(1+\cos\theta)^2}{\bar{u}_{x_1} d_{x_2}(1-\cos\theta)^2 - d_{x_1}\bar{u}_{x_2}(1+\cos\theta)^2}$$



backward lepton rapidity



forward lepton rapidity

Longitudinal single- and double-spin asymmetries

$$A_L = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \qquad A_{LL} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}$$

FEATURES

- quark/antiquark separation at $Q \sim M_W$
- no need of fragmentation functions
- at RHIC, $\langle x_{1,2}\rangle \simeq \frac{M_W}{\sqrt{s}} e^{-\eta_l/2} \approx [0.04, 0.4]$
- for W^+, $d \longleftrightarrow u$ and $\Delta d \longleftrightarrow \Delta u$
- non-trivial positivity bound [arXiv:1104.2920]

$$1 \pm A_{LL}(y_W) > |A_L(y_W) \pm A_L(-y_W)|$$

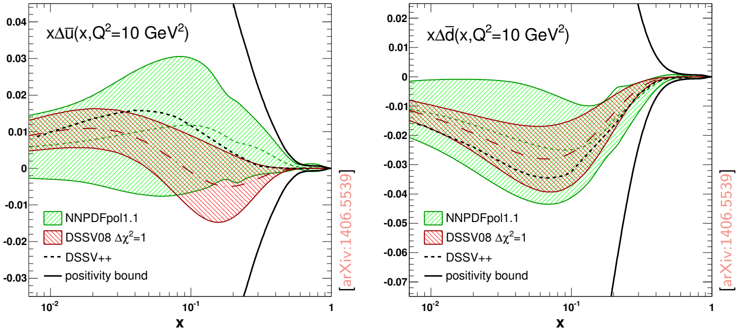
- no access to strangeness ($W^{\pm}$ + c required)

MEASUREMENTS

- STAR [arXiv:1404.6880] + PHENIX [PoS(DIS2014)205]
- much more to come from ongoing RHIC run

New data from RHIC: $W^{\pm}$ production

EFFECTS ON THE LIGHT QUARK SEA POLARIZATION $\Delta\bar{u}$ AND $\Delta\bar{d}$

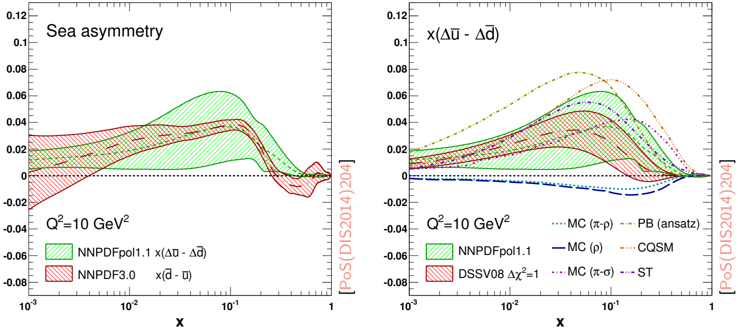


NNPDFpol1.1: SIDIS ☒, $W^{\pm}$ ☑; DSSV08: SIDIS ☑, $W^{\pm}$ ☒; DSSV++: SIDIS ☑, $W^{\pm}$ ☑;

$Q^2 = 10$ GeV^2	$\int_0^1 dx \Delta f(x, Q^2)$		$\int_{10^{-3}}^1 dx \Delta f(x, Q^2)$		
	NNDPFpol1.0	NNPDFpol1.1	NNDPFpol1.0	NNPDFpol1.1	DSSV08
$\Delta\bar{u}$	—	$+0.06 \pm 0.06$	—	$+0.04 \pm 0.05$	$+0.028^{+0.059}_{-0.059}$
$\Delta\bar{d}$	—	-0.11 ± 0.06	—	-0.09 ± 0.05	$-0.089^{+0.090}_{-0.080}$

New data from RHIC: $W^{\pm}$ production

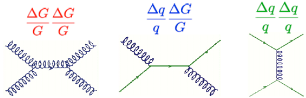
EFFECTS ON THE FLAVOR ASYMMETRY $\Delta\bar{u} - \Delta\bar{d}$



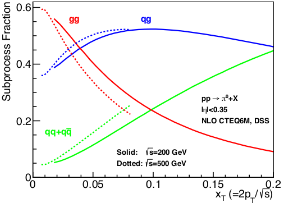
$$I_{\Delta}(Q^2) = \int_0^1 dx \left[\Delta\bar{u}(x, Q^2) - \Delta\bar{d}(x, Q^2)\right], \quad Q^2 = 10 \text{ GeV}^2$$

I_{Δ}	MC (π-meson) $\equiv 0$	MC (ρ-meson) < 0	PB (bag-model) $\simeq 0.09$	PB $\simeq 0.2$	IN $\simeq 0.2$	DSSV08 $\Delta\chi^2 = 1$ **0.14 ± 0.05**
I_{Δ}	MC (π-ρ inter.) $[-4 \cdot 10^{-3}, -0.033]$	MC (π-σ inter.) $\simeq 0.12$	PB (ansatz) $\simeq 0.3$	CQS 0.31	ST > 0.12	NNPDFpol1.1 **0.17 ± 0.08**

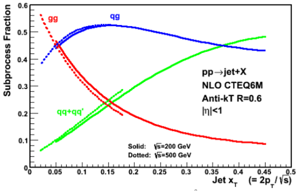
New data from RHIC: jet and π production



π production



Jet production



Longitudinal double-spin asymmetry

$$A_{LL} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}$$

FEATURES

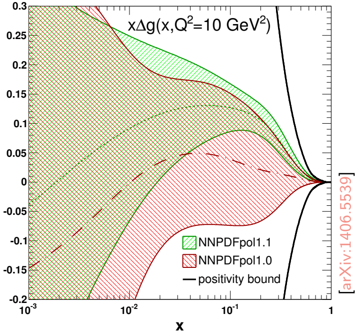
- at RHIC, $\langle x_{1,2} \rangle \simeq \frac{2p_T}{\sqrt{s}} e^{-\eta/2} \approx [0.05, 0.2]$
- *qg* and *gg* initiated subprocesses dominate (for most of the RHIC kinematics)
- A_{LL} sensitive to gluon polarization
- cross sections are well described at NLO in pQCD

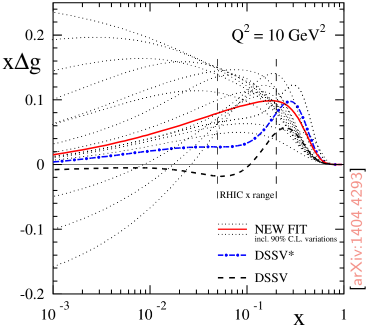
MEASUREMENTS

- STAR (jets) [arXiv:1405.5134]
- PHENIX (π) [arXiv:1402.6296] [arXiv:1409.1907]
- much more to come from ongoing RHIC run
  $\rightarrow$ gaining precision
  $\rightarrow$ di-jet measurements

New data from RHIC: jet and π production

EFFECTS ON THE GLUON POLARIZATION Δg



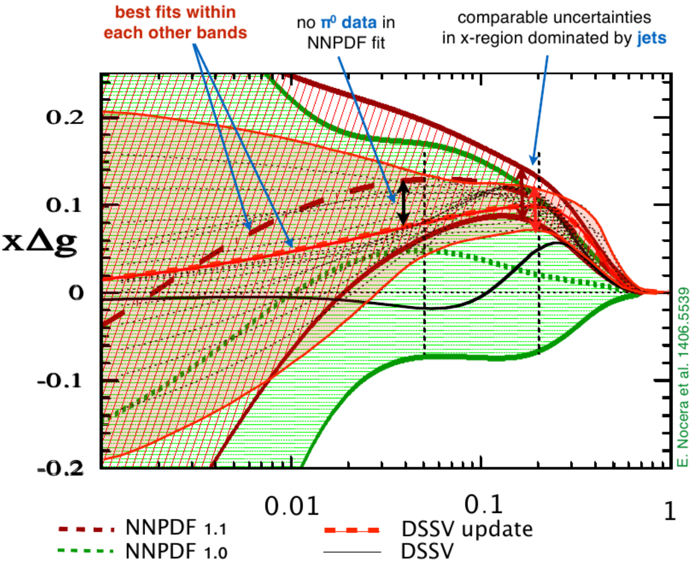


NNPDFpol1.0: jets ☒, π ☒; NNPDFpol1.1: jets ☑, π ☒; DSSV++: jets ☑, π ☑;

$Q^2 = 10$ GeV^2	$\int_0^1 dx\Delta g(x, Q^2)$	$\int_{10^{-3}}^1 dx\Delta g(x, Q^2)$	$\int_{0.05}^{0.2} dx\Delta g(x, Q^2)$
NNPDFpol1.0	-0.95 ± 3.87	-0.06 ± 1.12	$+0.05 \pm 0.15$
NNPDFpol1.1	-0.13 ± 2.60	$+0.31 \pm 0.77$	$+0.15 \pm 0.06$
DSSV08	—	$0.013^{+0.702}_{-0.314}$	$0.005^{+0.129}_{-0.164}$
DSSV++	—	—	$0.10^{+0.06}_{-0.07}$

New data from RHIC: jet and π production

EFFECTS ON THE GLUON POLARIZATION Δg



First evidence of gluon polarization in the proton

First evidence of gluon polarization in the proton

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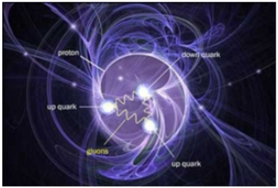
Proton Spin Mystery Gains a New Clue

Physicists long assumed a proton's spin came from its three constituent quarks. New measurements suggest particles called gluons make a significant contribution

July 21, 2014 | By Clara Moskowitz

Protons have a constant spin that is an intrinsic particle property like mass or charge. Yet where this spin comes from is such a mystery it's dubbed the "proton spin crisis." Initially physicists thought a proton's spin was the sum of the spins of its three constituent quarks.



First evidence of gluon polarization in the proton

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Mystery of 'proton spin' solved? Particle collider reveals that quarks AND gluons may hold answer to great subatomic puzzle

- **Researchers using a collider in New York say they have solved 'spin' mystery**
- **Since an experiment in 1987 the origins of proton spin have been unknown**
- **It had once been thought to be cause exclusively by quarks**
- **But this was proved to be wrong in the failed experiment 27 years ago**
- **Now a new study says gluons play an important role in proton spin**
- **Could bring to a close one of the greatest mysteries of subatomic physics**

By JONATHAN O'CALLAGHAN

PUBLISHED: 14:23 GMT, 22 July 2014 | **UPDATED:** 14:37 GMT, 22 July 2014

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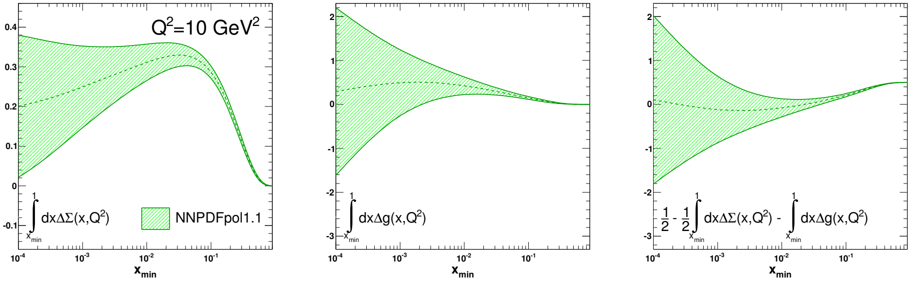
The spin content of the proton

$$\frac{1}{2} = \frac{1}{2}\int_0^1 dx\Delta\Sigma(x,Q^2) + \int_0^1 dx\Delta g(x,Q^2) + \mathcal{L}_q + \mathcal{L}_g$$



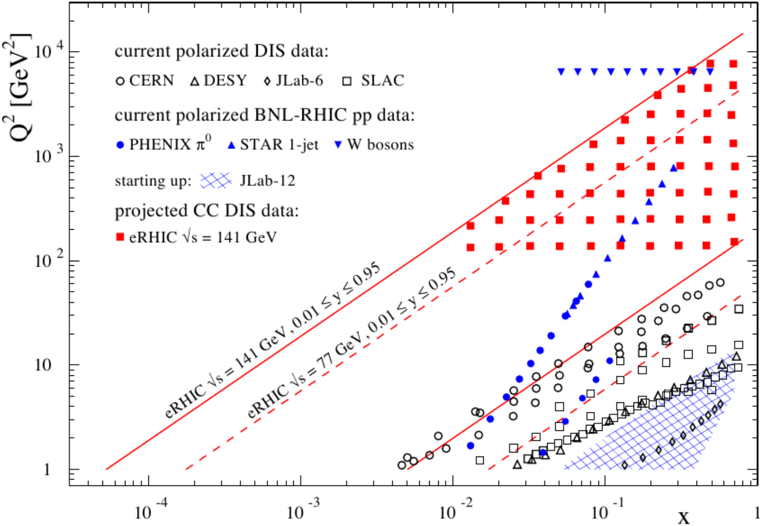
quarks and antiquarks $\sim 20\% - 30\%$ gluons $\sim 70\%$ OAM $\sim 0\%$

BUT LARGE UNCERTAINTIES $\Longleftrightarrow$ NEED FOR DATA

3. The path forward

Luiza Vizali, *Path through the sky*, Acrylic, metallic acrylic, stretched canvas (2008)

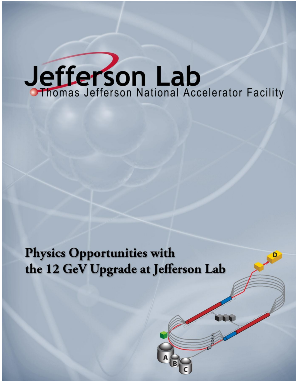
Need for experimental data



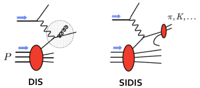
[arXiv:1409.1633]

Ongoing/planned/proposed experimental programs

Jefferson Laboratory



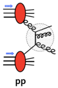
[arXiv:1208.1244]



large-x, high precision

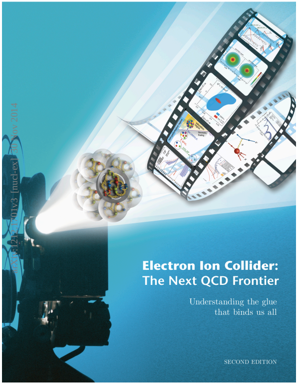
Relativistic
Heavy Ion Collider

[arXiv:1501.01220]

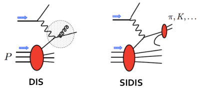


Δg, Δq, $\Delta\bar{q}$

A future
Electron-Ion Collider



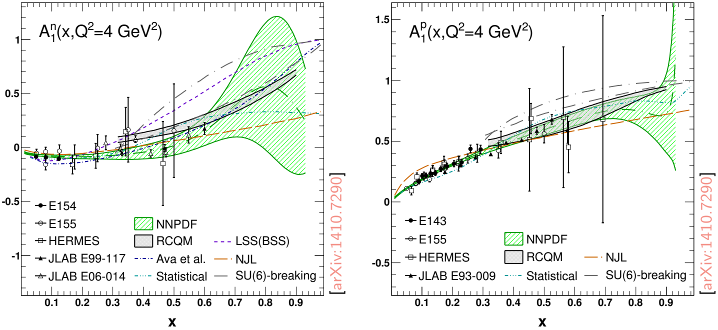
[arXiv:1212.1701]



small-x, high-Q^2

Opportunities at JLAB (towards 12 GeV upgrade)

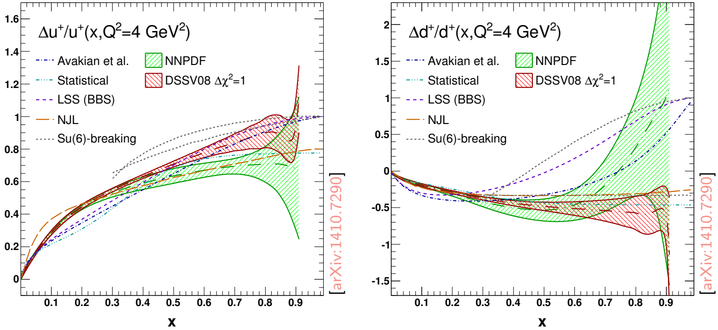
VIRTUAL PHOTOABSORPTION ASYMMETRY IN INCLUSIVE DIS



Model	A_1^n	A_1^p	Model	A_1^n	A_1^p
SU(6)	0	5/9	NJL	0.35	0.77
RCQM	1	1	DSE (*realistic*)	0.17	0.59
QHD ($\sigma_{1/2}$)	1	1	DSE (*contact*)	0.34	0.88
QHD (ψ_ρ)	1	1	pQCD	1	1
NNPDF ($x = 0.7$)	0.41 ± 0.31	0.75 ± 0.07	NNPDF ($x = 0.9$)	0.36 ± 0.61	0.74 ± 0.34

Opportunities at JLAB (towards 12 GeV upgrade)

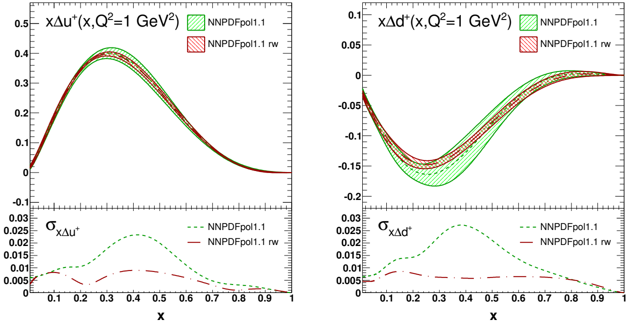
POLARIZED TO UNPOLARIZED PDF RATIOS



Model	$\Delta u^+/u^+$	$\Delta d^+/d^+$	Model	$\Delta u^+/u^+$	$\Delta d^+/d^+$
SU(6)	2/3	$-1/3$	NJL	0.80	-0.25
RCQM	1	$-1/3$	DSE (*realistic*)	0.65	-0.26
QHD ($\sigma_{1/2}$)	1	1	DSE (*contact*)	0.88	-0.33
QHD (ψ_ρ)	1	$-1/3$	pQCD	1	1
NNPDF ($x = 0.7$)	0.07 ± 0.05	-0.19 ± 0.34	NNPDF ($x = 0.9$)	0.61 ± 0.48	$+0.85 \pm 6.55$

Opportunities at JLAB (towards 12 GeV upgrade)

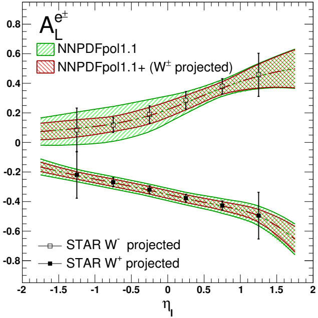
POTENTIAL IMPACT OF JLAB DATA (INCLUSIVE DIS)

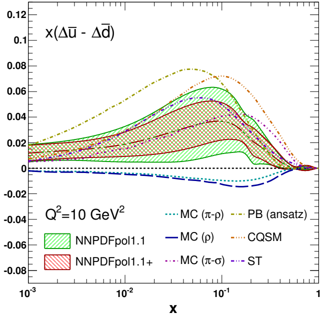


Experiment		Observable	Target	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$	$\chi^2_{\rm rw}/N_{\rm dat}$
JLAB E99-117	[nucl-ex/0405006]	A_1^n	He^3	3	1.22	1.02
JLAB E93-009	[nucl-ex/0605028]	A_1^p	NH^3	9	1.20	1.05
		A_1^d	ND^3	9	1.65	1.05
JLAB E06-014	[arXiv:1406.1207]	A_1^n	He^3	6	4.31	1.32
				27	1.93	1.09

Opportunities at RHIC

PINNING DOWN THE LIGHT POLARIZED SEA ASYMMETRY

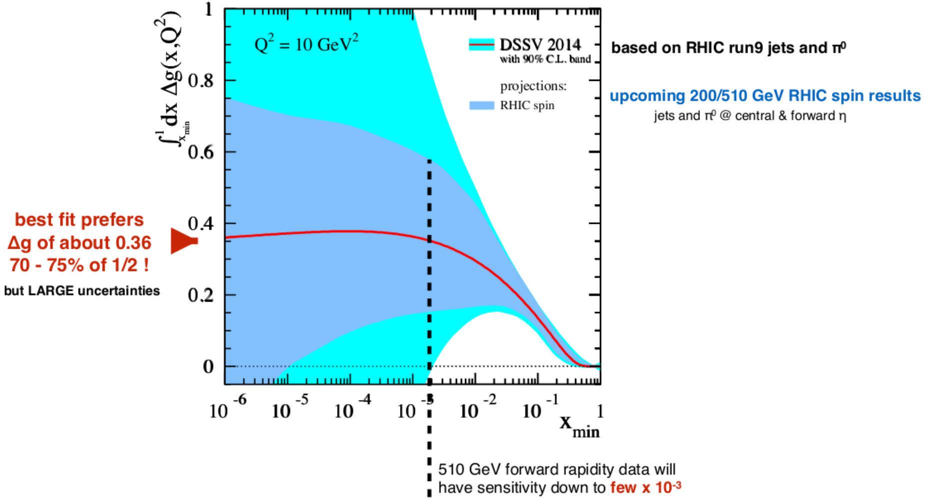




	$\int_{10^{-3}}^{1} dx \Delta f(x, Q^2)$			$\int_{0.05}^{0.4} dx \Delta f(x, Q^2)$		
	cv	*unc* (pol1.1)	*unc* (pol1.1+)	*cv*	*unc* (pol1.1)	*unc* (pol1.1+)
Δu^+	$+0.764$	± 0.035	± 0.034	$+0.523$	± 0.014	± 0.013
Δd^+	-0.407	± 0.037	± 0.036	-0.231	± 0.018	± 0.018
$\Delta\bar{u}$	$+0.044$	± 0.046	± 0.030	$+0.019$	± 0.023	± 0.012
$\Delta\bar{d}$	-0.088	± 0.067	± 0.032	-0.037	± 0.021	± 0.013
$\Delta_{\rm sea}$	$+0.123$	± 0.076	± 0.038	$+0.056$	± 0.030	± 0.016

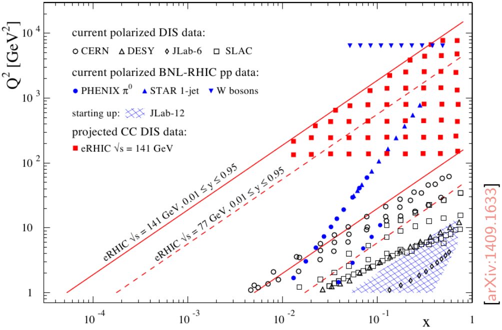
Opportunities at RHIC

PINNING DOWN THE GLUON POLARIZATION



[M. Stratmann, *Talk at HiX2014*]

Opportunities at a future Electron-Ion Collider



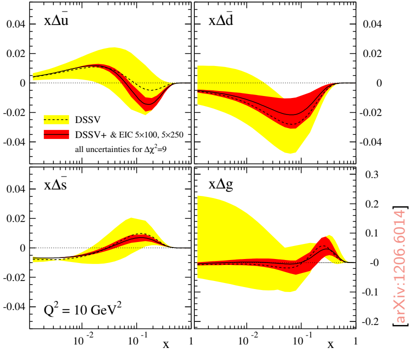
REQUIREMENTS & FEATURES

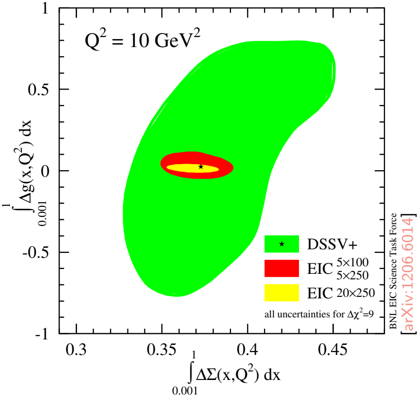
- large kinematic reach
  $\rightarrow$ high-energy collider
- precision of electromagnetic probes
  $\rightarrow$ electron beams
- spin
  $\rightarrow$ polarized hadron beams
- versatility
  $\rightarrow$ heavy ion beams

DELIVERABLES	OBSERVABLES	WHAT WE LEARN
Δg	scaling violations in DIS	gluon contribution to proton spin
Δq, $\Delta\bar{q}$	SIDIS for pions and kaons	quark contribution to proton spin; flavor asymmetry $\Delta\bar{u} - \Delta\bar{d}$; strangeness Δs
$g_1^{W^-}$, $g_5^{W^-}$	inclusive CC DIS at high Q^2	flavor separation at medium x and high Q^2

Opportunities at a future Electron-Ion Collider

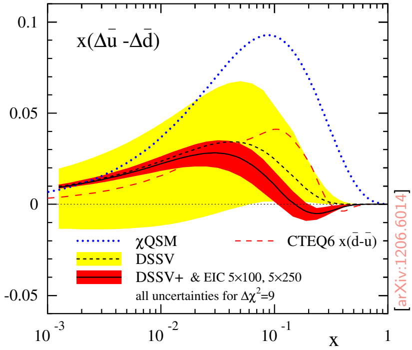
- Dramatic reduction of uncertainties of both PDFs and their moments [arXiv:1206.6014]
- Accurate determination of Δg via scaling violations in DIS [arXiv:1206.6014] [arXiv:1310.0461]
- Accurate determination of $\Delta\bar{u}$, $\Delta\bar{d}$ via SIDIS and CC DIS [arXiv:1309.5327]
- Access to unknown electroweak structure functions [arXiv:1309.5327]

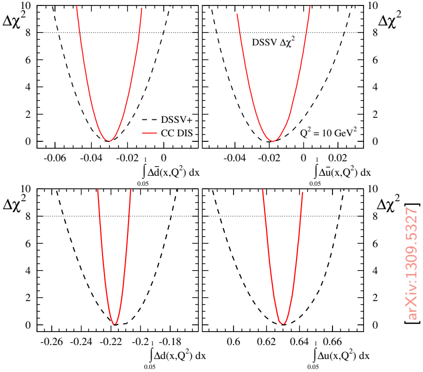




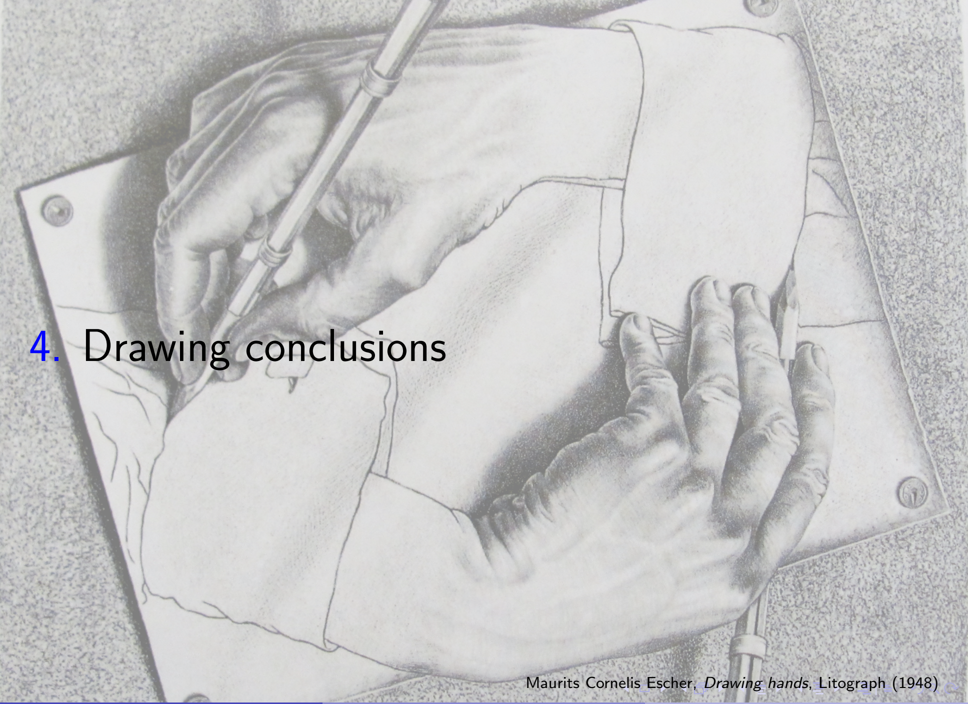
Opportunities at a future Electron-Ion Collider

- Dramatic reduction of uncertainties of both PDFs and their moments [arXiv:1206.6014]
- Accurate determination of Δg via scaling violations in DIS [arXiv:1206.6014] [arXiv:1310.0461]
- Accurate determination of $\Delta\bar{u}$, $\Delta\bar{d}$ via SIDIS and CC DIS [arXiv:1309.5327]
- Access to unknown electroweak structure functions [arXiv:1309.5327]



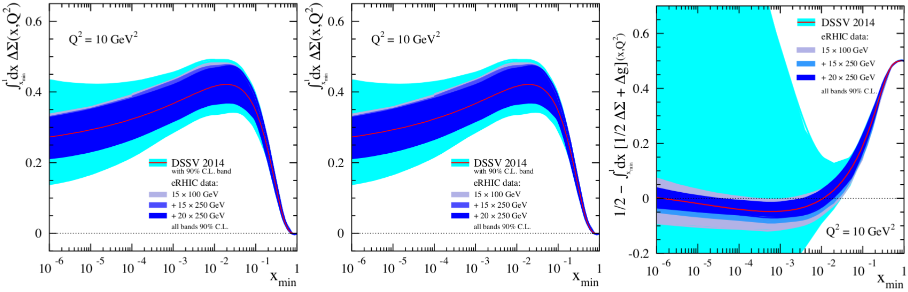


4. Drawing conclusions



Maurits Cornelis Escher, *Drawing hands*, Litograph (1948)

The ultimate spin content of the proton [arXiv:1409.1633]



After three decades of experimental and theoretical activity,
we cannot really say we know $\Delta\Sigma$ and Δg
Main culprit: small-x behavior of polarized PDFs

Spin experiments continue to produce high impact results (RHIC, JLAB, ...)
first evidence of gluon polarization in the proton
Theory efforts and global QCD analyses try to keep up

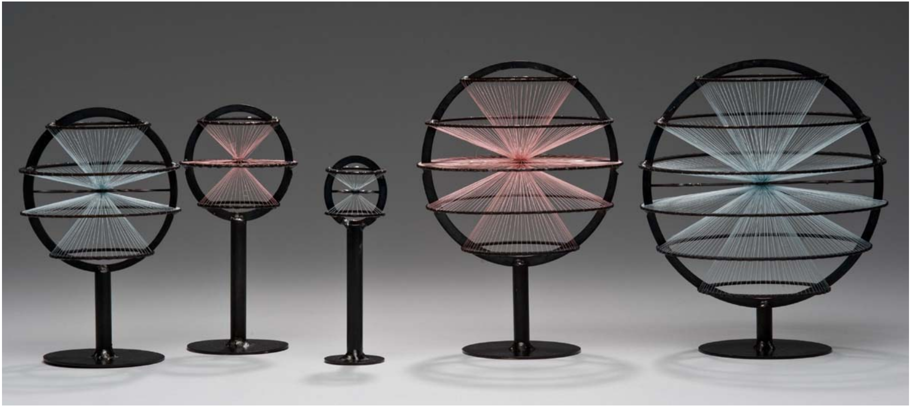
Only an EIC would be able to push forward our knowledge of nucleon spin content
Best fit results allow for zero OAM at 10 GeV^2
(presumably the most precise indirect handle on OAM)

Thank you for your attention

Sin-Itiro Tomonaga, Nobel Prize for Physics, 1965

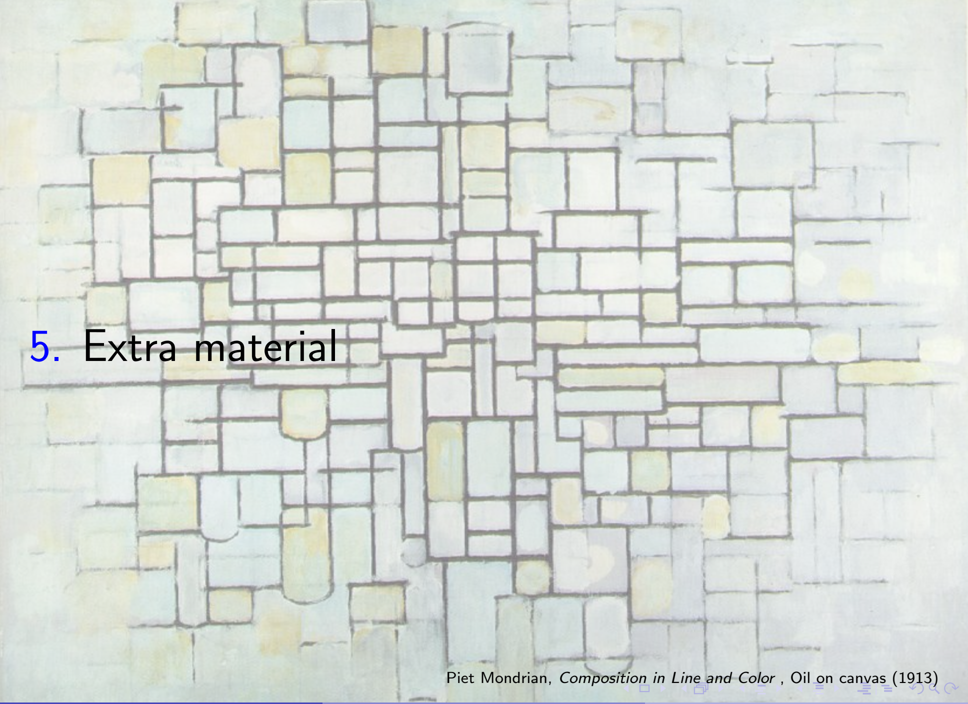
[Spin] is a mysterious beast, and yet its practical effect prevails the whole of science. The existence of spin, and statistics associated with it, is the most subtle and ingenious design of Nature - without it the whole Universe would collapse.

S-I. Tomonaga, *The story of spin* 2nd ed., University of Chicago Press (1998) [from the preface]



Julian Voss-Andreae, *Spin Family (Bosons and Fermions)*, Steel and silk (2009)

5. Extra material



Piet Mondrian, *Composition in Line and Color* , Oil on canvas (1913)

The NNPDF methodology: Monte Carlo sampling

MONTE CARLO SAMPLING

- Sample the probability density $\mathcal{P}[\Delta q]$ in the space of functions assuming multi-Gaussian data probability distribution

$$g_{1,p}^{(\mathrm{art}),(k)}(x, Q^2) = \left[1 + \sum_c r_{c,p}^{(k)}\sigma_{c,p} + r_{s,p}^{(k)}\sigma_{s,p}\right] g_{1,p}^{(\mathrm{exp})}(x, Q^2)$$

$\sigma_{c,p}$: correlated systematics $\sigma_{s,p}$: statistical errors (also uncorrelated systematics)

$r_{c,p}^{(k)}$, $r_{s,p}^{(k)}$: Gaussian random numbers

- Generate MC ensemble of N_{rep} replicas with the data probability distribution

MAIN FEATURES

- Expectation values for observables are Monte Carlo integrals

$$\langle \mathcal{O}[\Delta q] \rangle = \frac{1}{N_{\mathrm{rep}}} \sum_{k=1}^{N_{\mathrm{rep}}} \mathcal{O}[\Delta q_k]$$

...and the same is true for errors, correlations etc.

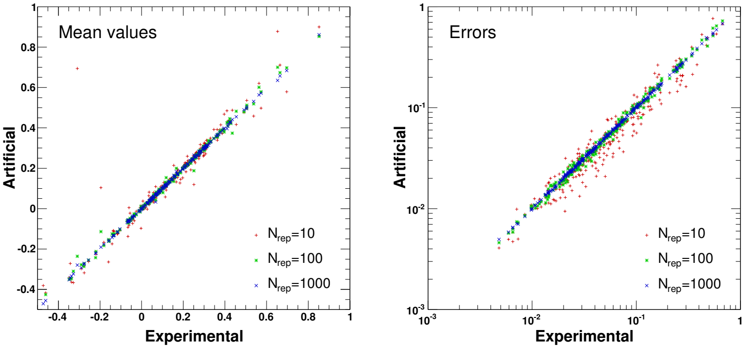
- No need to rely on linear propagation of errors
- Possibility to test for non-Gaussian behaviour in fitted PDFs

The NNPDF methodology: Monte Carlo sampling

DETERMINING THE SAMPLE SIZE

- Require the average over the replicas reproduces central values and errors of the original experimental data to desired accuracy

Qualitative approach: look at the scatter plots



Accuracy of few % requires $\sim$ 100 replicas

The NNPDF methodology: Monte Carlo sampling

DETERMINING THE SAMPLE SIZE

- Require the average over the replicas reproduces central values and errors of the original experimental data to desired accuracy

Quantitative approach: devise proper statistical estimators

	$\left\langle \mathrm{PE}\left[\langle g_1^{(\mathrm{art})}\rangle\right]\right\rangle$ [%]			$r\left[g_1^{(\mathrm{art})}\right]$		
N_{rep}	10	100	1000	10	100	1000
EMC	23.7	3.5	2.9	.76037	.99547	.99712
SMC	19.4	5.6	1.2	.94789	.99908	.99993
...	...	...	...	...	...	...

$$\left\langle PE\left[\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right]\right\rangle_{\mathrm{dat}} = \frac{1}{N_{\mathrm{dat}}}\sum_{i=1}^{N_{\mathrm{dat}}}\left|\frac{\langle F_i^{(\mathrm{art})}\rangle_{\mathrm{rep}} - F_i^{(\mathrm{exp})}}{F_i^{(\mathrm{exp})}}\right|$$

Percentage Error

$$r\left[F^{(\mathrm{art})}\right] = \frac{\left\langle F^{(\mathrm{exp})}\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right\rangle_{\mathrm{dat}} - \langle F^{(\mathrm{exp})}\rangle_{\mathrm{dat}}\left\langle\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right\rangle_{\mathrm{dat}}}{\sigma_s^{(\mathrm{exp})}\sigma_s^{(\mathrm{art})}}$$

Scatter Correlation

Accuracy of few % requires $\sim$ 100 replicas

The NNPDF methodology: Monte Carlo sampling

DETERMINING THE SAMPLE SIZE

- Require the average over the replicas reproduces central values and errors of the original experimental data to desired accuracy

Quantitative approach: devise proper statistical estimators

	$\left\langle \mathrm{PE}\left[\langle\delta g_1^{(\mathrm{art})}\rangle\right]\right\rangle$ [%]			$r\left[\delta g_1^{(\mathrm{art})}\right]$		
N_{rep}	10	100	1000	10	100	1000
EMC	12.8	4.9	2.0	.97397	.99521	.99876
SMC	22.4	5.4	1.7	.96585	.99489	.99980
...	...	...	...	...	...	...

$$\left\langle PE\left[\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right]\right\rangle_{\mathrm{dat}} = \frac{1}{N_{\mathrm{dat}}}\sum_{i=1}^{N_{\mathrm{dat}}}\left|\frac{\langle F_i^{(\mathrm{art})}\rangle_{\mathrm{rep}} - F_i^{(\mathrm{exp})}}{F_i^{(\mathrm{exp})}}\right|$$

Percentage Error

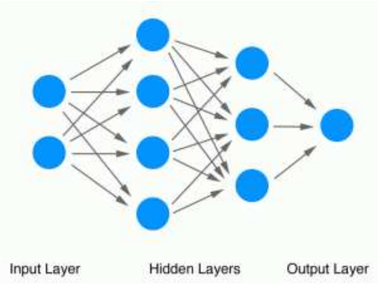
$$r\left[F^{(\mathrm{art})}\right] = \frac{\left\langle F^{(\mathrm{exp})}\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right\rangle_{\mathrm{dat}} - \langle F^{(\mathrm{exp})}\rangle_{\mathrm{dat}}\left\langle\langle F^{(\mathrm{art})}\rangle_{\mathrm{rep}}\right\rangle_{\mathrm{dat}}}{\sigma_s^{(\mathrm{exp})}\sigma_s^{(\mathrm{art})}}$$

Scatter Correlation

Accuracy of few % requires $\sim$ 100 replicas

The NNPDF methodology: neural networks

A convenient functional form
providing redundant and flexible parametrization
used as a generator of random functions in the PDF space



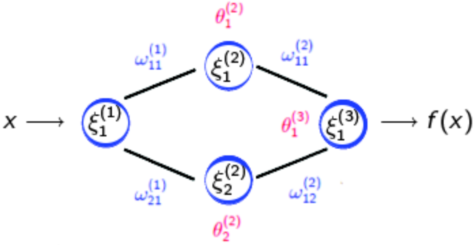
$$\xi_i^{(l)} = g\left(\sum_j^{n_l-1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)}\right)$$

$$g(x) = \frac{1}{1+e^{-x}}$$

- made of neurons grouped into layers (define the architecture)
- each neuron receives input from neurons in preceding layer (feed-forward NN)
- activation determined by parameters (weights and thresholds)
- activation determined according to a non-linear function (except the last layer)

The NNPDF methodology: neural networks

EXAMPLE: THE SIMPLEST 1-2-1 NN



$$f(x) \equiv \xi_1^{(3)} = \left\{ 1 + \exp\left[\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1 + e^{\theta_1^{(2)} - x\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1 + e^{\theta_2^{(2)} - x\omega_{21}^{(1)}}} \right] \right\}^{-1}$$

Recall: $\xi_i^{(l)} = g\left(\sum_j^{n_l - 1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)} \right)$; $g(x) = \frac{1}{1 + e^{-x}}$

The NNPDF methodology: minimization and stopping

GENETIC ALGORITHM

Standard minimization unefficient owing to the large parameter space
and non-local x-dependence of the observables
Genetic algorithm provides better exploration of the whole parameter space

- Set Neural Network parameters randomly
- Make clones of the parameter vector and mutate them
- Define a figure of merit or error function for the k-th replica

$$E^{(k)} = \frac{1}{N_{\text{rep}}} \sum_{i,j=1}^{N_{\text{rep}}} \left(g_{1,i}^{(\text{art})(k)} - g_{1,i}^{(\text{net})(k)} \right) \left((\text{cov})^{-1} \right)_{ij} \left(g_{1,j}^{(\text{art})(k)} - g_{1,j}^{(\text{net})(k)} \right)$$

$g_{1,i}^{(\text{art})(k)}$: generated from Monte Carlo sampling
$g_{1,i}^{(\text{net})(k)}$: computed from Neural Network PDFs

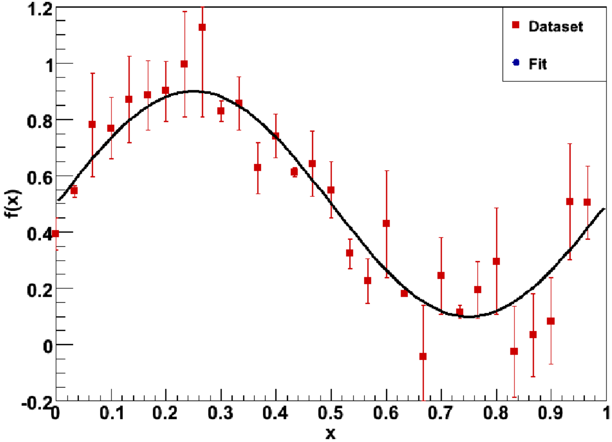
- Select the best set of parameters and perform other manipulations (crossing, mutating, ...) until stability is reached.

The NNPDF methodology: minimization and stopping

DRAWBACK

- NN can learn fluctuations owing to their flexibility

UNDERLYING PHYSICAL LAW

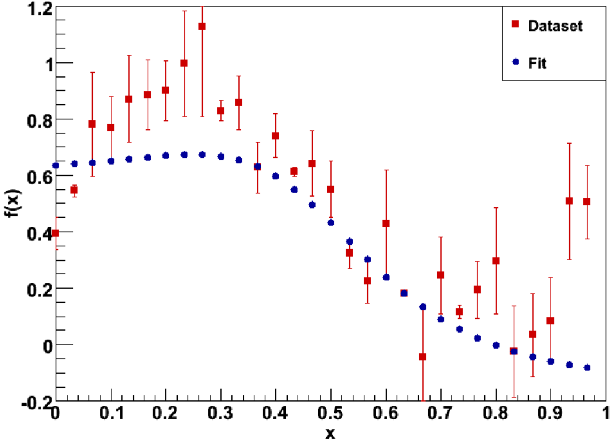


The NNPDF methodology: minimization and stopping

DRAWBACK

- NN can learn fluctuations owing to their flexibility

UNDERLEARNING

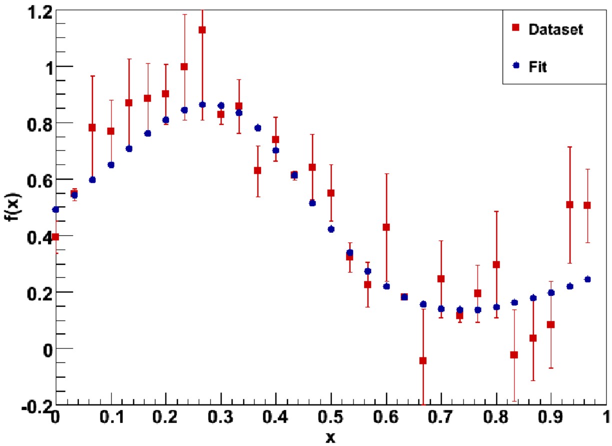


The NNPDF methodology: minimization and stopping

DRAWBACK

- NN can learn fluctuations owing to their flexibility

PROPER LEARNING

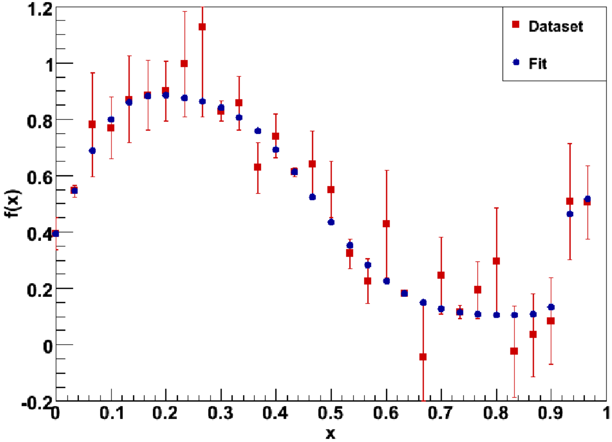


The NNPDF methodology: minimization and stopping

DRAWBACK

- NN can learn fluctuations owing to their flexibility

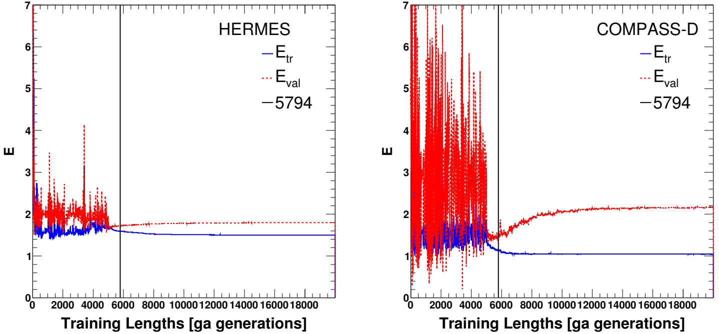
OVERLEARNING



The NNPDF methodology: minimization and stopping

CROSS-VALIDATION METHOD

- divide data into two subsets (training & validation)
- train the NN on training subset and compute χ^2 for each subset
- stop when χ^2 of validation subset no longer decreases (NN are learning noise!)



The best fit does not coincide with the χ^2 absolute minimum

The NNPDF methodology: reweighting [arXiv:1012.0836] [arXiv:1108.1758]

1. We would like to assess the impact of including a new data set $\{y\} = \{y_1, \ldots, y_n\}$ (delivered with σ_{ij}) in a prior ensemble of PDF replicas $\{f_k\}$, $k = 1, \ldots, N_{\rm rep}$
2. We can apply Bayes theorem to determine the conditional probability of PDF upon inclusion of the new data and update the probability density in the space of PDFs

$$\mathcal{P}_{\rm new} = \mathcal{N}_\chi \mathcal{P}(\chi^2_k|\{f_k\})\mathcal{P}_{\rm old}(\{f_k\}) \quad \mathcal{P}(\chi^2_k|\{f_k\}) = [\chi^2_k(\{y\},\{f_k\}]^{\frac{1}{2}(n-1)} e^{-\frac{1}{2}\chi^2_k(\{y\},\{f_k\})}$$

$$\chi^2_k(\{y\},\{f_k\}) = \sum_{i,j}^{n} \{y_i - y_i[f_k]\}\,\sigma_{ij}\,\{y_j - y_j[f_k]\}$$

3. Replicas are no longer equally probable. Expectation values are given by

$$\langle \mathcal{O}[f_i(x,Q^2]\rangle_{\rm new} = \sum_{k=1}^{N_{\rm rep}} w_k \mathcal{O}[f_i^{(k)}(x,Q^2)]$$

$$w_k \propto [\chi^2_k(\{y\},\{f_k\})]^{\frac{1}{2}(n-1)} e^{-\frac{1}{2}\chi^2_k(\{y\},\{f_k\})} \quad \text{with} \quad N_{\rm rep} = \sum_{k=1}^{N_{\rm rep}} w_k$$

Reweighting allows to incorporate new datasets **without** need of **refitting**

The NNPDF methodology: unweighting [arXiv:11081758]

Unweighting allows for constructing an ensemble of equally probable PDFs statistically equivalent to a given reweighted set
Hence, the new set can be given without weights

IDEA

Given a weighted set of $N_{\rm rep}$ replicas, select (possibly more than once) replicas carrying relatively hight weight and discard replicas carrying relatively small weight

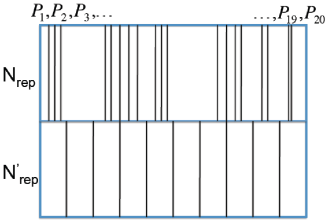
CONSTRUCTION OF THE UNWEIGHTED SET

1. Set the number of replicas $N'_{\rm rep}$ in the unweighted set
(pointless to choose $N'_{\rm rep} > N_{\rm rep}$: no gain of information)
2. Compute, for the k-th replica of the reweighted set, the integer number

$$w'_k = \sum_{j=1}^{N'_{\rm rep}} \theta\left(\frac{j}{N'_{\rm rep}} - P_{k-1}\right)\theta\left(P_k - \frac{j}{N'_{\rm rep}}\right) \, , \quad P_k = \sum_{j=0}^{k} \frac{w_j}{N_{\rm rep}} \, , \quad \sum_{k=1}^{N_{\rm rep}} w'_k = N'_{\rm rep}$$

3. Construct the unweighted set taking w'_k copies of the k-th replica, $k = 1, \ldots, N_{\rm rep}$

The NNPDF methodology: unweighting [arXiv:11081758]



CONSTRUCTION OF THE UNWEIGHTED SET

1. Set the number of replicas N'_{rep} in the unweighted set (pointless to choose $N'_{\mathrm{rep}} > N_{\mathrm{rep}}$: no gain of information)
2. Compute, for the k-th replica of the reweighted set, the integer number

$$w'_k = \sum_{j=1}^{N'_{\mathrm{rep}}} \theta\left(\frac{j}{N'_{\mathrm{rep}}} - P_{k-1}\right)\theta\left(P_k - \frac{j}{N'_{\mathrm{rep}}}\right), \quad P_k = \sum_{j=0}^{k} \frac{w_j}{N_{\mathrm{rep}}}, \quad \sum_{k=1}^{N_{\mathrm{rep}}} w'_k = N'_{\mathrm{rep}}$$

3. Construct the unweighted set taking w'_k copies of the k-th replica, $k = 1, \ldots, N_{\mathrm{rep}}$

The NNPDF methodology: closure test [arXiv:1410.8849]

Validation and optimization of fitting strategy
performed on closure test with known underlying physical law

1. Set the theory (*e.g.* NLO pQCD)
2. Set the fitting methodology (*e.g.* genetic algortihm, cross-validation)
3. Set the underlying *true* physical law (*e.g.* input PDFs from MSTW08)
4. Generate pseudodata and perform the fit with given theory and methodology
5. Validate the resulting PDF set
   → reproduce the underlying law (both central value and uncertainties)
   → check the consistency of expected values of χ^2
   → check that PDF reweighting is equal to PDF refitting (Bayesian inference)

level 0: fluctuations on pseudodata ☒; Monte Carlo replica generation ☒

level 1: fluctuations on pseudodata ☑; Monte Carlo replica generation ☒

level 2: fluctuations on pseudodata ☑; Monte Carlo replica generation ☑

Full control of procedural uncertainties

NNPDFpol1.1: fit quality

Experiment	N_{dat}	χ^2/N_{dat} NNPDFpol1.0	χ^2/N_{dat} NNPDFpol1.1
EMC	10	0.44	0.43
SMC	24	0.93	0.90
SMClowx	16	0.97	0.97
E143	50	0.64	0.67
E154	11	0.40	0.45
E155	40	0.89	0.85
COMPASS-D	15	0.65	0.70
COMPASS-P	15	1.31	1.38
HERMES97	8	0.34	0.34
HERMES	56	0.79	0.82
	245	0.77	0.78
COMPASS (OC)	45	—	1.22
STAR (jets)	41	—	1.05
PHENIX (jets)	6	—	0.24
STAR-A_L	24	—	0.72
STAR-A_{LL}	12	—	0.75
	373	—	1.05

NNPDFpol1.1: open-charm production at COMPASS

$$A^{\gamma N \to D^0 X} = \frac{\Delta g \otimes \Delta\hat{\sigma}_{\gamma g} \otimes D_c^H}{g \otimes \hat{\sigma}_{\gamma g} \otimes D_c^H}$$

Virtual photon-nucleon asymmetry for open-charm production
[arXiv:1212.1319]

FEATURES

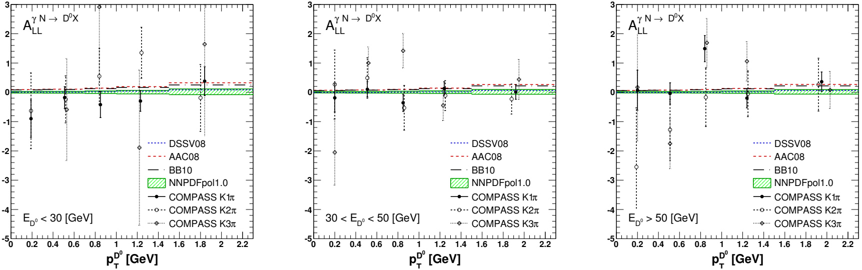
- Δg is probed *directly* through the photon-gluon fusion process
  (in DIS Δg is mostly probed through scaling violations instead)
- the fragmentation functions for heavy quarks are computable in perturbation theory
  (and only introduce a very moderate uncertainty in the fit)

EXPERIMENTAL MEASUREMENT

- COMPASS (2002-2007) [arXiv:1211.6849]

Experiment	Set	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$ NNPDFpol1.0	DSSV08	AAC08	BB10
COMPASS		45	1.23	1.23	1.27	1.25
	COMPASS $K1\pi$	15	1.27	1.27	1.43	1.38
	COMPASS $K2\pi$	15	0.51	0.51	0.56	0.55
	COMPASS $K3\pi$	15	1.90	1.90	1.81	1.82

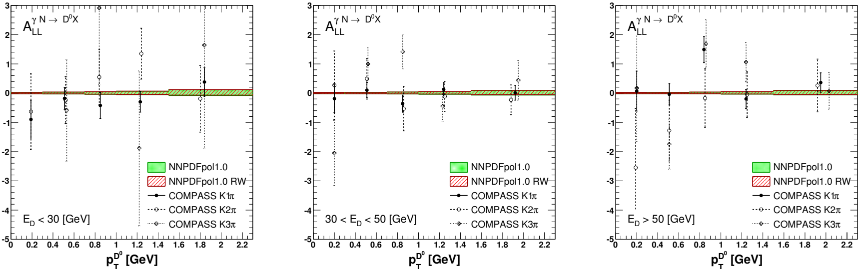
NNPDFpol1.1: open-charm production at COMPASS



Data are affected by large uncertainties w.r.t. the uncertainty due to PDFs
They do not show a clear trend

Experiment	Set	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$ NNPDFpol1.0	DSSV08	AAC08	BB10
COMPASS		45	1.23	1.23	1.27	1.25
	COMPASS $K1\pi$	15	1.27	1.27	1.43	1.38
	COMPASS $K2\pi$	15	0.51	0.51	0.56	0.55
	COMPASS $K3\pi$	15	1.90	1.90	1.81	1.82

NNPDFpol1.1: open-charm production at COMPASS



The impact of open-charm data from COMPASS is mostly negligible, as we notice from the value of the $\chi^2/N_{\rm dat}$ and the reweighted observable

Experiment	Set	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$	$\chi^2_{\rm rw}/N_{\rm dat}$
COMPASS		45	1.23	1.23
	COMPASS $K1\pi$	15	1.27	1.27
	COMPASS $K2\pi$	15	0.51	0.51
	COMPASS $K3\pi$	15	1.90	1.89

NNPDFpol1.1: inclusive jet production at RHIC

$$A_{LL}^{1jet} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}$$

Longitudinal double-spin asymmetry for single-inclusive jet production

[arXiv:hep-ph/9808262] [arXiv:hep-ph/0404057] [arXiv:1209.1785]

FEATURES

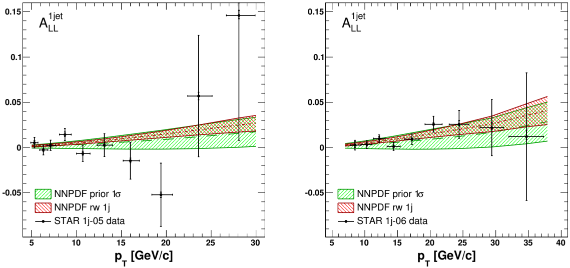
- sensitive to the polarized gluon Δg
  (receives leading contribution from $gq \to qg$ and $qg \to qg$ partonic subrocesses)

EXPERIMENTAL MEASUREMENT

- STAR 2005, 2006 [arXiv:1205.2735], 2009 [arXiv:1405.5134]
- PHENIX [arXiv:1009.4921] at RHIC

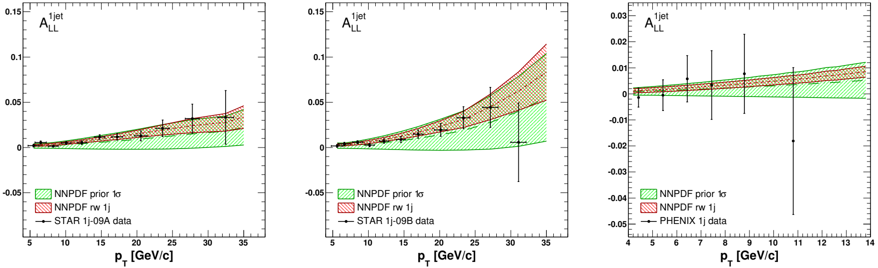
Data set	$N_{\rm dat}$	jet-algorithm	R	$[\eta_{\rm min}, \eta_{\rm max}]$	$\sqrt{s}$ [GeV]	$\mathcal{L}$ [pb^{-1}]
STAR 1j-05	10	midpoint-cone	0.4	$[+0.20, +0.80]$	200	2.1
STAR 1j-06	9	midpoint-cone	0.7	$[-0.70, +0.90]$	200	5.5
STAR 1j-09A	11	anti-k_t	0.6	$[-0.50, +0.50]$	200	25
STAR 1j-09B	11	anti-k_t	0.6	$[-1.00, -0.50]$ $[+0.50, +1.00]$	200	25
PHENIX 1j	6	seeded-cone	0.3	$[-0.35, +0.35]$	200	2.1

NNPDFpol1.1: inclusive jet production at RHIC



Experiment	Set	N_{dat}	χ^2/N_{dat}				χ^2_{rw}/N_{dat}			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR		41	1.50	1.49	1.50	1.50	1.05	1.04	1.04	1.04
	STAR 1j-05	10	1.04	1.05	1.04	1.04	1.01	1.02	1.02	1.02
	STAR 1j-06	9	0.75	0.76	0.76	0.76	0.59	0.58	0.59	0.59
	STAR 1j-09A	11	1.40	1.39	1.39	1.40	0.98	0.99	0.98	0.98
	STAR 1j-09B	11	3.04	3.05	3.03	3.05	1.18	1.17	1.17	1.18
PHENIX										
	PHENIX 1j	6	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
		47	1.35	1.35	1.35	1.36	1.00	1.01	1.01	1.00

NNPDFpol1.1: inclusive jet production at RHIC



Experiment	Set	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$				$\chi^2_{\rm rw}/N_{\rm dat}$			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR		41	1.50	1.49	1.50	1.50	1.05	1.04	1.04	1.04
	STAR 1j-05	10	1.04	1.05	1.04	1.04	1.01	1.02	1.02	1.02
	STAR 1j-06	9	0.75	0.76	0.76	0.76	0.59	0.58	0.59	0.59
	STAR 1j-09A	11	1.40	1.39	1.39	1.40	0.98	0.99	0.98	0.98
	STAR 1j-09B	11	3.04	3.05	3.03	3.05	1.18	1.17	1.17	1.18
PHENIX										
	PHENIX 1j	6	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
		47	1.35	1.35	1.35	1.36	1.00	1.01	1.01	1.00

NNPDFpol1.1: $W^{\pm}$ production at RHIC

$$A_L^{W^\pm} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \qquad A_{LL}^{W^\pm} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}$$

$$A_L^{W^+} \sim \frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)} \qquad A_L^{W^-} \sim \frac{\Delta d(x_1)\bar{u}(x_2) - \Delta\bar{u}(x_1)d(x_2)}{d(x_1)\bar{u}(x_2) + \bar{u}(x_1)d(x_2)}$$

Longitudinal single-spin asymmetry for $W^{\pm}$ boson production

[arXiv:1003.4533]

FEATURES

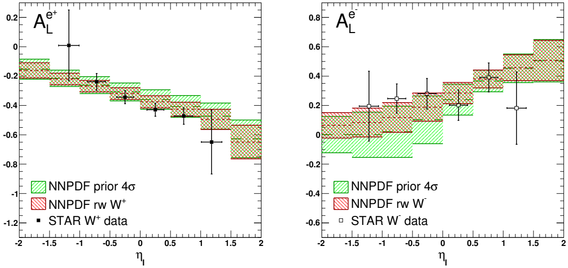
- sensitive to individual quark and antiquark flavours $(\Delta u, \Delta\bar{u}, \Delta d, \Delta\bar{d})$
  (purely weak process coupling q_L with $\bar{q}_R$ at partonic level, $u_L\bar{d}_R \to W^+$ or $d_L\bar{u}_R \to W^-$)
- no need for fragmentation functions (instead of SIDIS)

EXPERIMENTAL MEASUREMENT

- STAR and PHENIX at RHIC [arXiv:1009.0326] [arXiv:1009.0505] [arXiv:1404.6880]

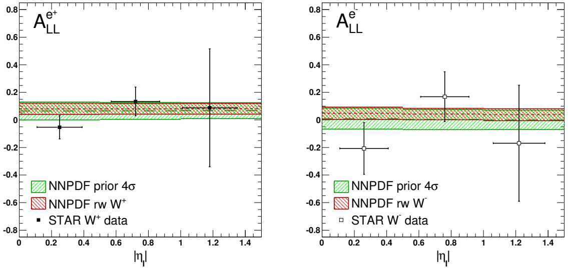
Data set	$N_{\rm dat}$	$[p_{T,\rm min}, p_{T,\rm max}]$ [GeV]	$\sqrt{s}$ [GeV]	$\mathcal{L}$ [pb^{-1}]
STAR-W^+ (prel.)	6	[25, 50]	510	72
STAR-W^- (prel.)	6	[25, 50]	510	72

NNPDFpol1.1: $W^{\pm}$ production at RHIC



Experiment	Set	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$				$\chi^2_{\rm rw}/N_{\rm dat}$			
			1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
STAR-A_L		12	1.38	1.44	1.39	1.33	1.08	0.88	0.74	0.74
	STAR-$A_L^{W^+}$	6	0.75	0.75	0.86	0.90	0.75	0.75	0.68	0.70
	STAR-$A_L^{W^-}$	6	1.92	2.03	1.82	1.67	1.32	1.08	0.83	0.82
STAR-A_{LL}		6	0.82	0.81	0.78	0.78	0.82	0.80	0.76	0.76
	STAR-$A_{LL}^{W^+}$	3	0.92	0.88	0.81	0.80	0.90	0.85	0.77	0.76
	STAR-$A_{LL}^{W^-}$	3	0.73	0.74	0.75	0.76	0.73	0.74	0.75	0.76
		18	1.19	1.20	1.15	1.15	1.00	0.87	0.78	0.77

NNPDFpol1.1: $W^{\pm}$ production at RHIC



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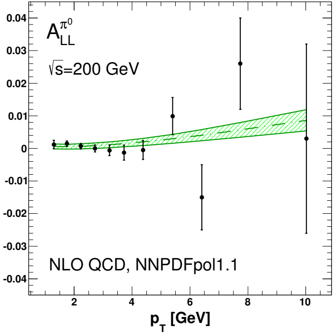
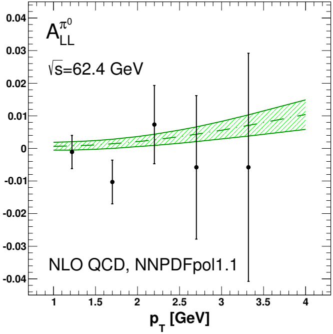
NNPDFpol1.1: single-hadron production at RHIC

$$A_{LL}^{H} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma{+-}} = \frac{\sum_{a,b,c=q,\bar{q},g} f_a \otimes f_b \otimes D_c^H \otimes \Delta\hat{\sigma}_{ab}^c}{\sum_{a,b,c=q,\bar{q},g} f_a \otimes f_b \otimes D_c^H \otimes \hat{\sigma}_{ab}^c}$$

PHENIX [arXiv:0810.0701] [arXiv:0810.0694] [arXiv:1402.6296] STAR [arXiv:1309.1800]





- Good agreement between experimental data and theoretical predictions
- Experimental uncertainties are larger than than those of the corresponding predictions
- We expect a slight impact on the gluon PDF from these data

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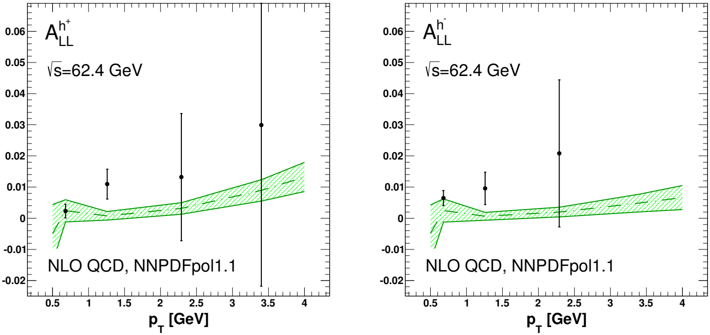

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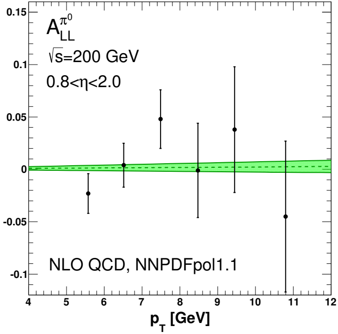

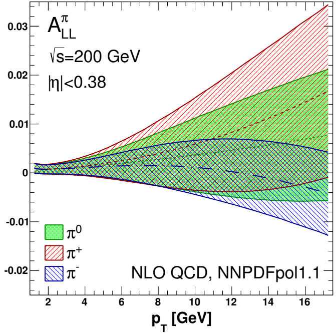

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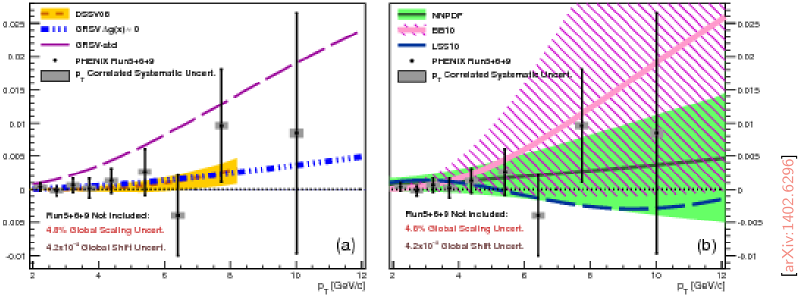

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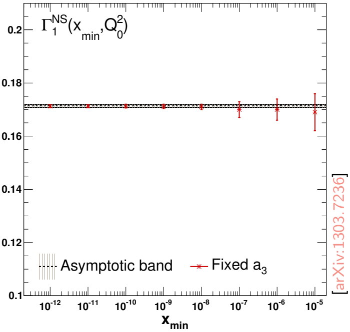
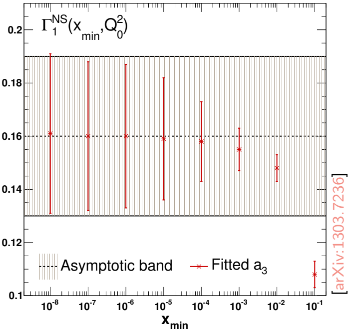
Open issues: the Bjorken sum rule



fixed $a_3 = 1.2701 \pm 0.0025$



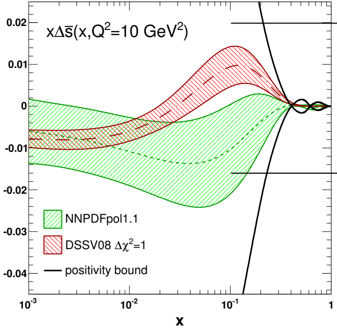
fitted $a_3 = 1.19 \pm 0.22$

$$\Gamma_1^{\rm NS}(x_{\rm min}, Q^2) \equiv \int_{x_{\rm min}}^1 dx [g_1^p(x,Q^2) - g_1^n(x,Q^2)] \xrightarrow{x_{\rm min}=0} \frac{1}{6} a_3(Q^2) \Delta C_{\rm NS}[\alpha_s(Q^2)]$$

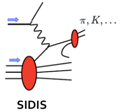
$$a_3(Q^2) = \int_0^1 dx \left[\Delta u(x,Q^2) + \Delta\bar{u}(x,Q^2) - \Delta d(x,Q^2) - \Delta\bar{d}(x,Q^2)\right]$$

Open issues: the strange content of the proton

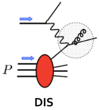
$\Delta\bar{s}$ (assuming $\Delta s = \Delta\bar{s}$, which may not be true [hep-ph/0505153])



directly from SIDIS Kaon data



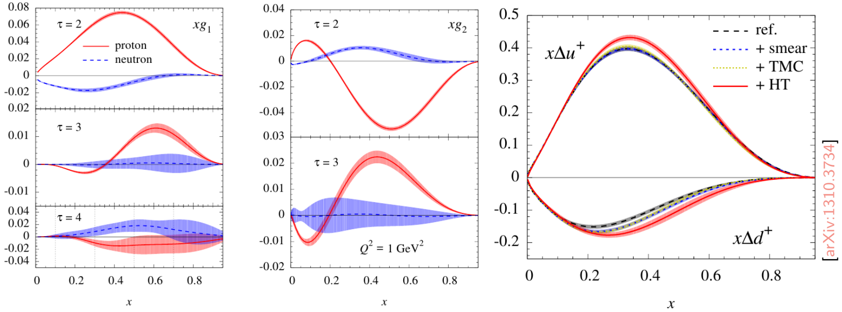
indirectly from DIS scaling violations



NNPDFpol1.1: DIS ☑, SIDIS ($K^{\pm}$) ☒; DSSV08: DIS ☑, SIDIS ($K^{\pm}$) ☑;

- DIS data ⇒ negative $x\Delta\bar{s}$; SIDIS data ⇒ changing-sing $x\Delta\bar{s}$
- New, very precise, JLAB data (DIS) point to negative $x\Delta s$ [arXiv:1410.1657]
- Is there mounting tension between DIS and SIDIS data?
- How well do we know K fragmentation functions? [arXiv:1103.5979]

Progress on inclusion of higher-twist corrections: JAM13



- leading-twist factorization of g_1 and g_2 receives contributions from higher-twist terms

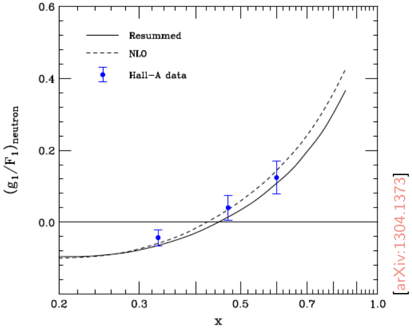
$$g_1 = g_1^{\tau=2} + g_1^{\tau=3} + g_1^{\tau=4} \qquad g_2 = g_2^{\tau=2} + g_2^{\tau=3}$$

→ $g_2^{\tau=2}$ can be related to $g_1^{\tau=2}$ via Wandzura-Wilckzek relation [PLB,72,195]
→ $g_2^{\tau=3}$ can be related to $g_1^{\tau=3}$ via Blümlein-Tkablaze identity [arXiv:hep-ph/9812478]
→ $g_2^{\tau=3}$ can be parametrized (using *e.g.* the form by Braun *et al.*) [arXiv:1103.1269]
→ $g_1^{\tau=4}$ can be parametrized as $g_1^{\tau=4}(x, Q^2) = h(x)/Q^2$ (D. Hui)

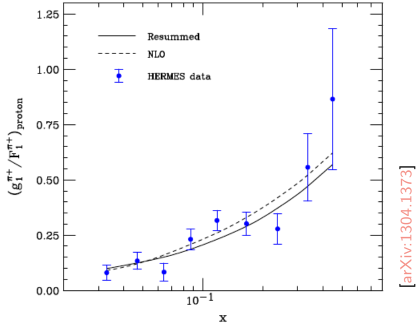
- higher twists to both g_1 and g_2 are included in JAM13
- higher twist contributions are sizable and are needed for describing JLAB data properly

Progress on all-order resummation

DIS: Hall A data [arXiv:nucl-ex/0405006]

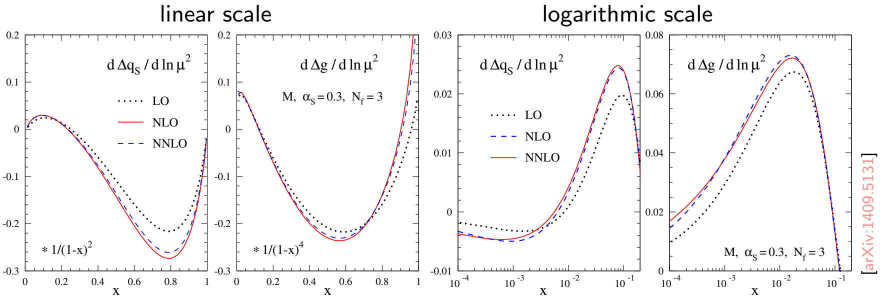


SIDIS: G. Karayan data [arXiv:hep-ex/0407032]



- resummation of large logarithm corrections to spin asymmetries in DIS and SIDIS
- asymmetries are rather insensitive to the inclusion of resummed higher-order terms
- modest decrease of spin asymmetries at fairly high x values, more pronounced for SIDIS
- most relevant for JLAB kinematics, important for future high statistic JLAB12

Progress on higher-order computations ($\overline{\text{MS}}$ scheme)



- NNLO (three-loop) corrections to spin-dependent splitting functions have been computed
- NNLO corrections to the splitting functions are small outside the region of small x
- corrections to the evolution of the PDFs can be unproblematic down to $x \approx 10^{-4}$
- QCD analyses of polarized PDFs are now feasible up to NNLO accuracy
  $\rightarrow$ only in a FFN scheme (VFN would require non-trivial unknown matching conditions)
  $\rightarrow$ only including DIS data (coefficient functions are know at NNLO only for DIS)