Dark Matter Annihilation in Small Scale Clumps

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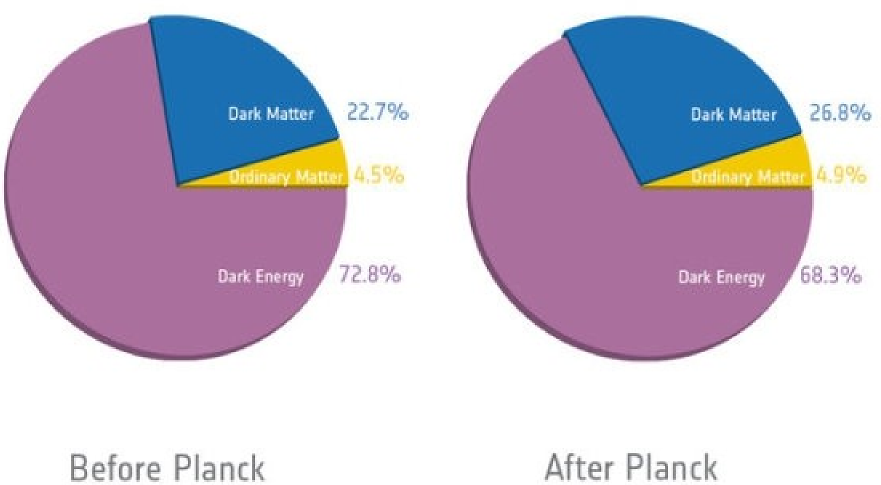
Dark matter clumps

We see directly the galaxies and clusters of galaxies, and we know about their dark halos. But that do we know or that can we say about smaller DM structure at substellar mass scales?



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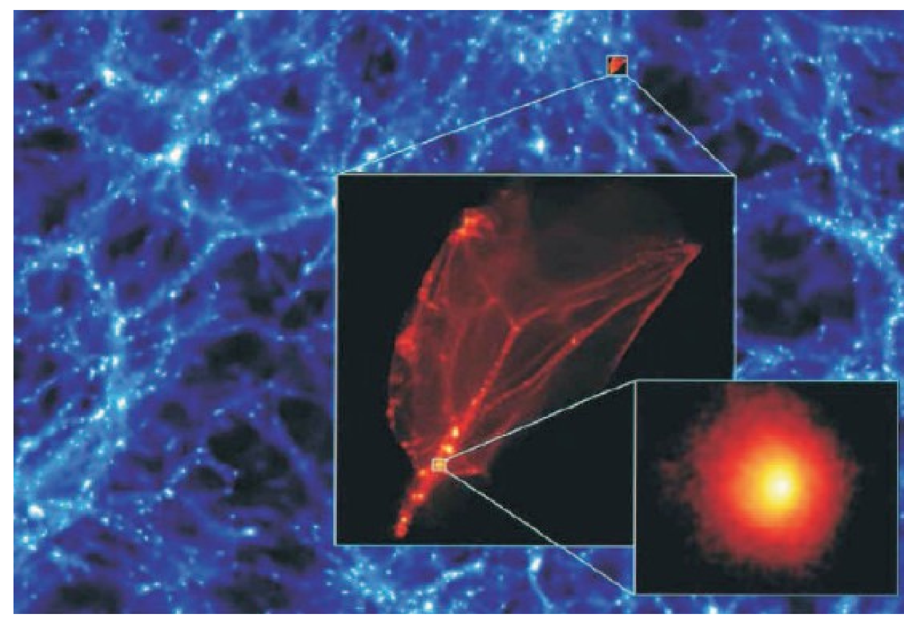


(Diemand, Moore, Stadel 2005)

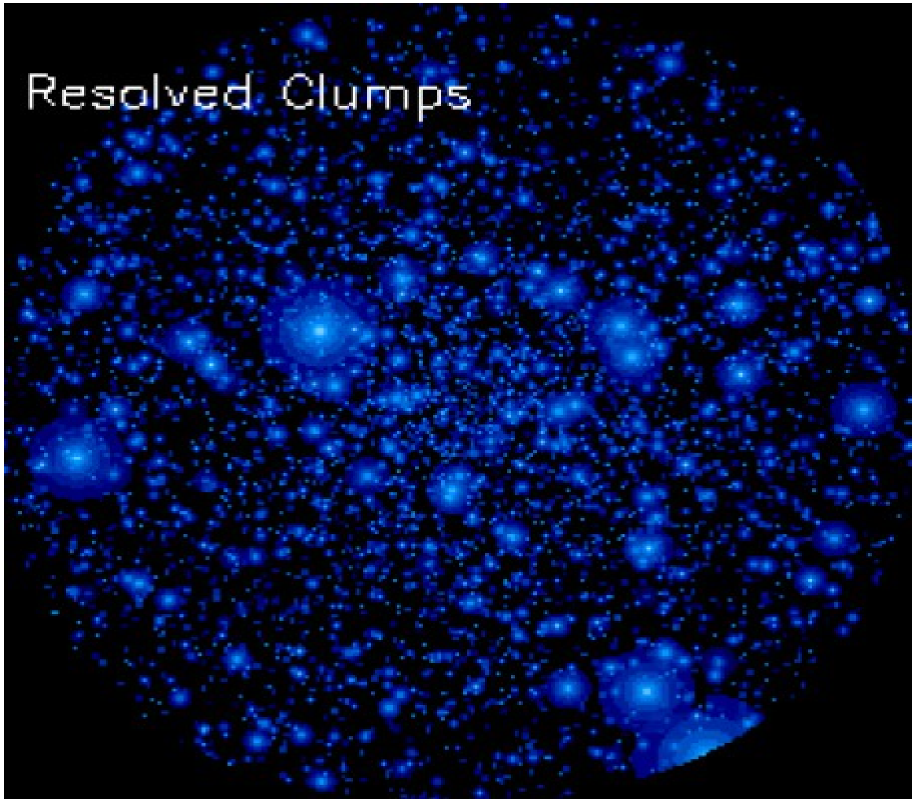
Dark matter clumps

Plan

- Small-scale spectrum of density perturbations
- Clump formation scenarios and models
- Internal structure of clumps
- Clumps with minimal mass
- Formation of the mass function in early hierarchical clustering processes
- Destruction of clumps in the Galaxy
- Particle annihilations in clumps
- Charge particle fluxes in PAMELA and other experiments
- Other possible observational manifestations of clumps
- Conclusions

Dark matter clumps

Galaxy in gamma-rays (modeling):



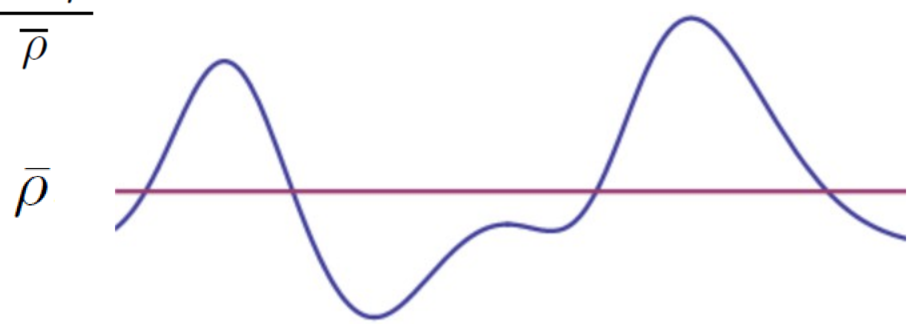
$M_{min}=10^5$-$10^8\,M_\odot$ (Lidia et al. 2002)

Spectrum of perturbations at small scales

Generation of adiabatic perturbations at the inflation stage

Galaxies and other structures form from density perturbations, which generally can be adiabatic perturbations, entropy perturbations, or a mixture of both.

$$\delta \equiv \frac{\delta\rho}{\overline{\rho}} = \frac{\rho - \overline{\rho}}{\overline{\rho}}$$



$$|\delta\phi| = H(\phi)/2\pi$$

$$\delta_{\rm H} \sim M_{\rm Pl}^{-3} V^{3/2}/(dV/d\phi)$$

Statistics: $\delta_{\vec{k}} = \int \delta(\vec{r}) e^{i\vec{k}\vec{x}}\, d^3x$

$$\langle \delta^*_{\vec{k}} \delta_{\vec{k}'} \rangle = (2\pi)^3 P(k) \delta^{(3)}_{\rm D}(\vec{k} - \vec{k}')$$

P(t,k) **- power spectrum**

$$P(t,k) = P_p(k) T^2(k) D^2(t)$$

T(t,k) **- transfer function**

$$\sigma(R) = \frac{1}{2\pi^2} \int k^2\, dk\, P(k) W^2(k,R)$$

- **counting the number of peaks**
- **Press-Schechter formalism**

Standard spectrum from inflation with tilt $P(k) \equiv \delta_k^2 \propto k^{n_s}$ $\quad n_s = 1 - 6\varepsilon + 2\eta$

$$\varepsilon = (V'/V)^2/(16\pi G)$$
$$\eta = (V''/V)/(8\pi G)$$

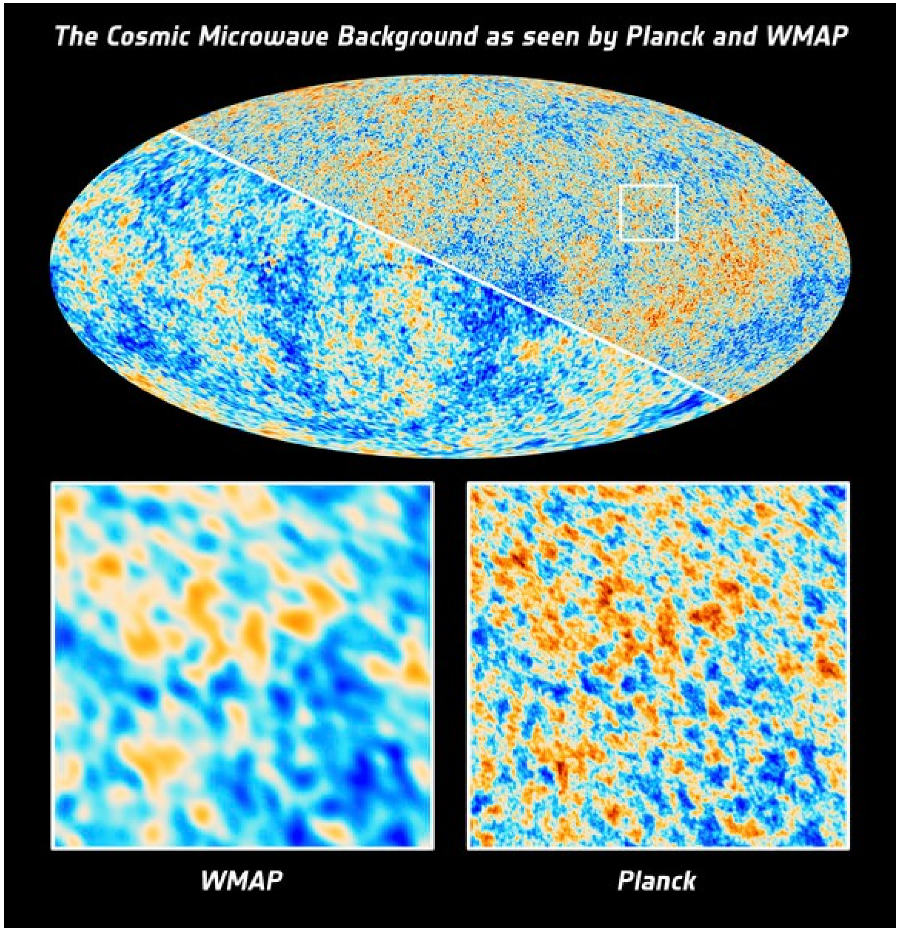
R^2-model by Starobinsky, $n_s \simeq 1 - 2/N$

Linear growth at the radiation-dominated stage $\delta_k \propto \ln(t/t_i) + const$

at dust-like stage ($t > t_{\rm eq}$) $\quad \delta_k \propto t^{2/3}$

Spectrum of perturbations at small scales

Normalization of the perturbation spectrum from observational data



Normalization at:

- **LSS data** $\sigma_8 \simeq 0.82$ at $8h^{-1}$ Mpc scale
- **CMB data** $\mathcal{P}_\mathcal{R} = A_\mathcal{R}\left(\frac{k}{k_*}\right)^{n_s-1}$

where $k_*/a_0 = 0.002 \text{ Mpc}^{-1}$

Planck:

$$A_\mathcal{R} = (2.46 \pm 0.09) \times 10^{-9}$$

$$n_s = 0.9608 \pm 0.0054$$

$dn_s/d\ln k = -0.0134 \pm 0.0090$ at a level of 1.5σ

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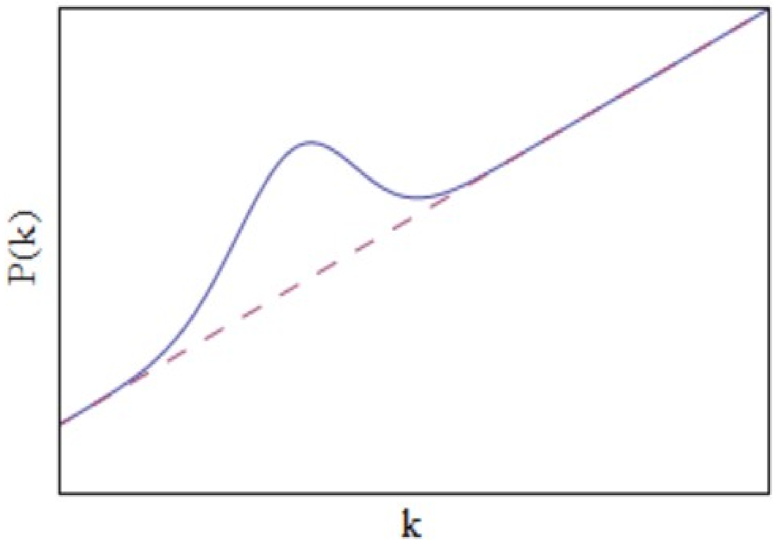
CMB → clumps: ~10^{15} extrapolation!

Perturbation on the horizon scale $\sigma_H(M) \simeq 9.5 \times 10^{-5}\left(\frac{M}{10^{56}\text{ g}}\right)^{\frac{1-n_s}{4}}$

Spectrum of perturbations at small scales

Perturbation spectra with peaks

$$\delta_H \sim M_{Pl}^{-3} V^{3/2} / (dV/d\phi)$$



$$dV(\phi)/d\phi \to 0$$

(Starobinskii 1992)
(Ivanov, Naselsky, Novikov 1994)

A peak at the minimal scale produces clumps with the highest dark matter density in the Universe. The discovery of such clumps could provide invaluable information on the inflation potential.

Constraints from primordial black holes:

$$\beta = \int_{\delta_{th}}^{1} \frac{d\delta_H}{\sqrt{2\pi}\Delta_H} \exp(-\frac{\delta_H^2}{2\Delta_H^2}) \simeq \frac{\Delta_H}{\delta_{th}\sqrt{2\pi}} \exp(-\frac{\delta_{th}^2}{2\Delta_H^2})$$

$$\Omega_{BH} \simeq \beta a(t_{eq})/a(t_H)$$

$\delta_{th} = 1/3$ *(Carr 1975)*

$\delta_{th} \simeq (0.65 \div 0.7)$ **- critical collapse**

Cosmological phase transitions also produce spectra with peaks.

Spectrum of perturbations at small scales

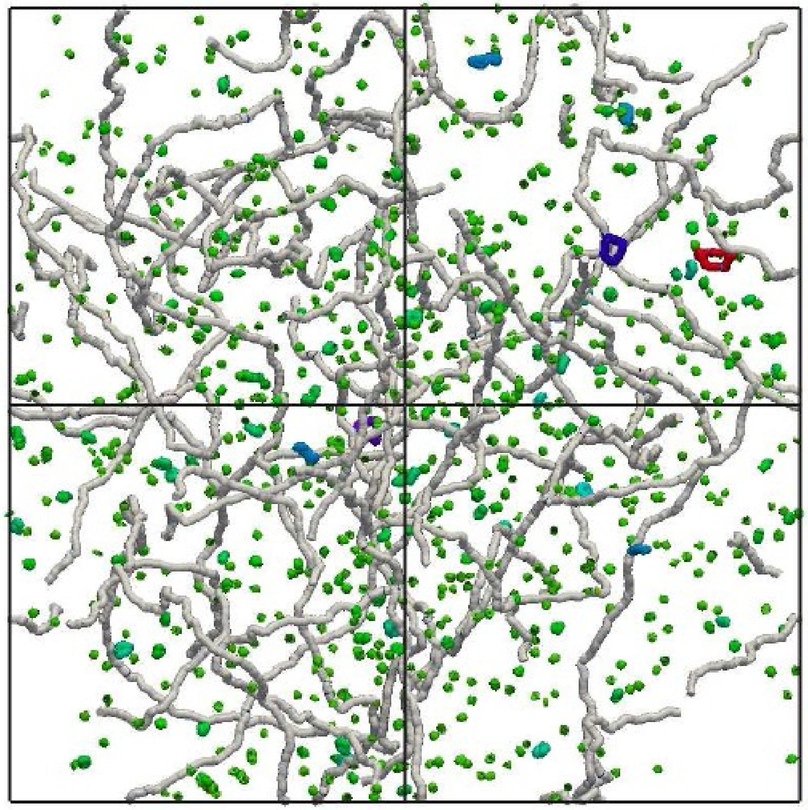
Entropy perturbations

Cosmic strings, loops *(Vilenkin et al.)*

Primordial black holes

$$\delta = M_c/M$$

Axion miniclusters
(Kolb, Tkachev 1993, 1994)



A picture of the string network

(Blanco-Pillado, Olum, Shlaer 2011; arXiv:1101.5173 [astro-ph.CO])

Clump formation scenarios and models

Spherical model of the evolution of perturbations

Equation: $$\frac{d^2r}{dt^2} = -\frac{G(M_{\rm BH}+M)}{r^2} - \frac{8\pi G\rho_r r}{3} + \frac{8\pi G\rho_\Lambda r}{3}$$

$$\rho \to \rho + 3p/c^2$$

$$\varepsilon_r + 3p_r = 2\varepsilon_r$$

$$\varepsilon_\Lambda + 3p_\Lambda = -2\varepsilon_\Lambda$$

Initial conditions from linear theory:

$$\delta(k,z) \simeq \frac{27}{2}\Phi_i(k)\frac{1+z_{\rm eq}}{1+z}\ln(0,2k\eta_{\rm eq}) \qquad z \gg 1 \ (t \ll t_\Lambda)$$

$t \gg t_{\rm eq}$ **Solution for dust-like matter:** $r = r_s\cos^2 p, \quad p + \frac{1}{2}\sin(2p) = \frac{2}{3}\left(\frac{5\delta_i}{3}\right)^{3/2}\frac{t-t_s}{t_i}$

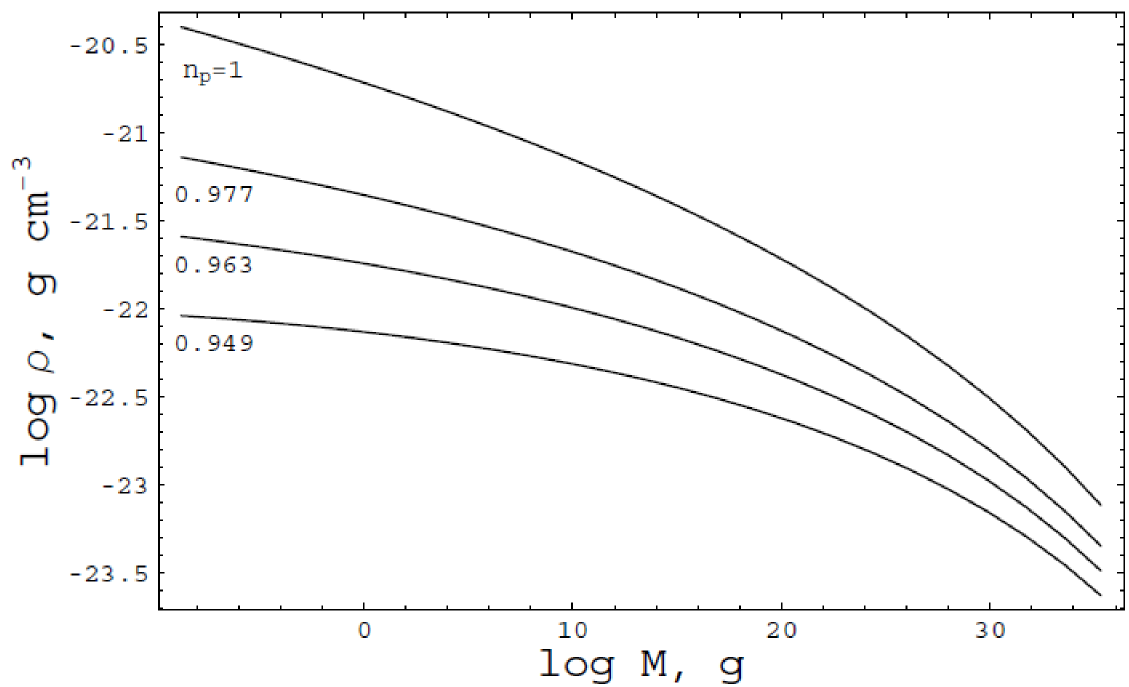
$$t_s = t_i\left[1+\frac{3\pi}{4}\left(\frac{5\delta_i}{3}\right)^{-3/2}\right] \qquad r_s = r_i\left(\frac{3}{5\delta_i}\right)$$

Virialization is mainly completed by the time $t \simeq 3t_s$ *(Knobel 2012)*

$\delta_c = 3(12\pi)^{2/3}/20 \simeq$ 1.686

$\delta(t_c) = \delta_c$ - **Press-Schechter formalism**

$$\kappa = 18\pi^2 \simeq 178 \qquad R = \left(\frac{3M}{4\pi\bar\rho_{\rm int}}\right)^{1/3}$$
$$\bar\rho_{\rm int} = \kappa\bar\rho(z_c)$$



Clump formation scenarios and models

Spherical model for entropy perturbations

$$r = a(\eta)b(\eta)\xi$$

$$y(y+1)\frac{d^2b}{dy^2} + \left[1 + \frac{3}{2}y\right]\frac{db}{dy} + \frac{1}{2}\left[\frac{1+\delta_i}{b^2} - b\right] = 0$$

(Padmanabhan, Subramanian 1993)
(Kolb, Tkachev 1994)

$$y = a(\eta)/a_{\rm eq},\ d\eta = dt/a(t)$$

$$dr/dt = 0 \qquad \rho_{\max} = \rho_{\rm eq} y_{\max}^{-3} b_{\max}^{-3} \qquad R_{\max} = \left(\frac{3M_x}{4\pi\rho_{\max}}\right)^{1/3}$$

$$\rho \simeq 140\delta_i^3(\delta_i + 1)\rho_{\rm eq}$$

$$\delta_i \simeq 1 \div 10^4$$

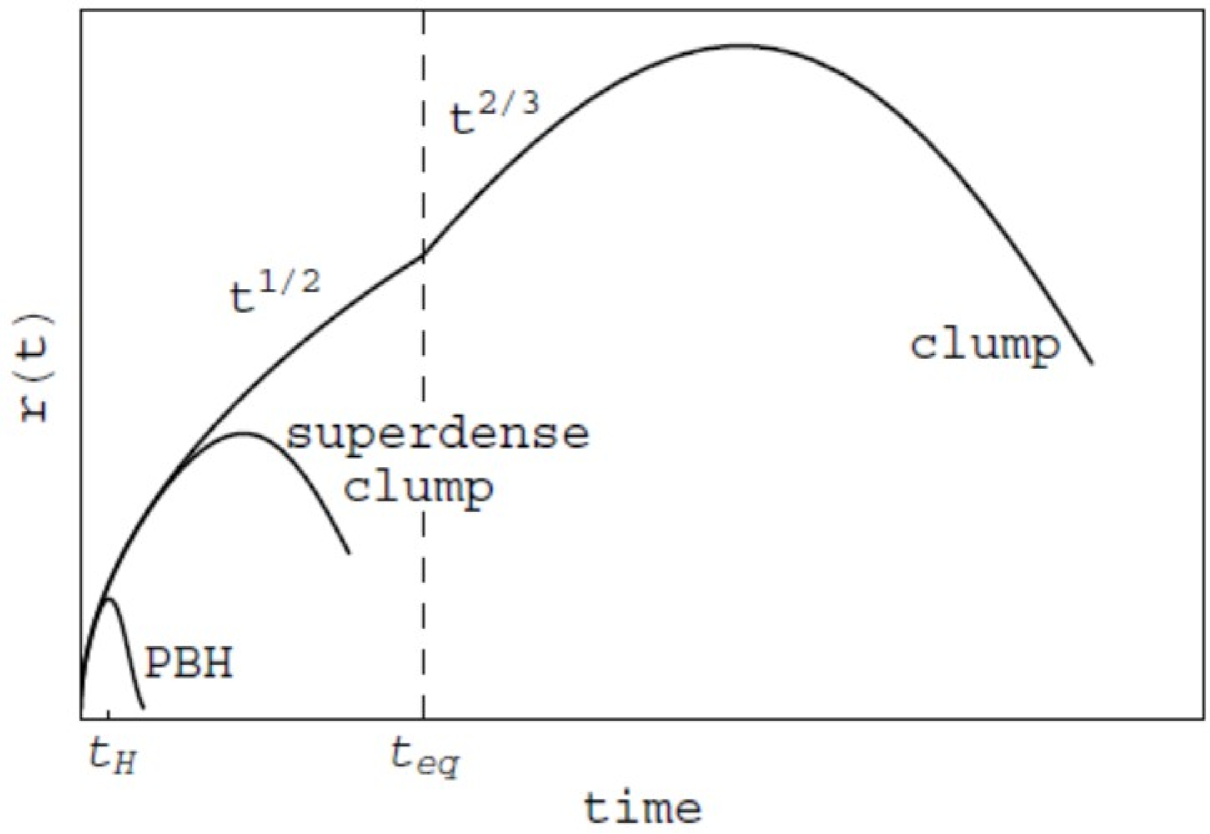
$$\sim (10^{-13} \div 0.1)M_\odot$$

Axion miniclusters *(Kolb, Tkachev 1993, 1994)*

Fast radio bursts can arise from collisions between axion miniclusters and neutron stars *(Iwazaki 2014), (Tkachev 2014)*.

Clump formation scenarios and models

Spherical model for adiabatic perturbations

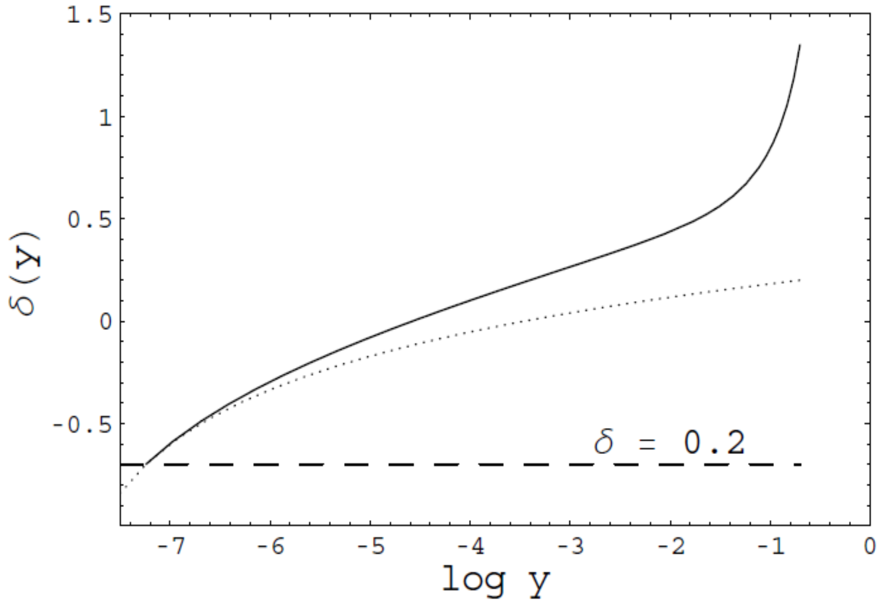


Different evolutions of a density perturbation as a function of its amplitude.

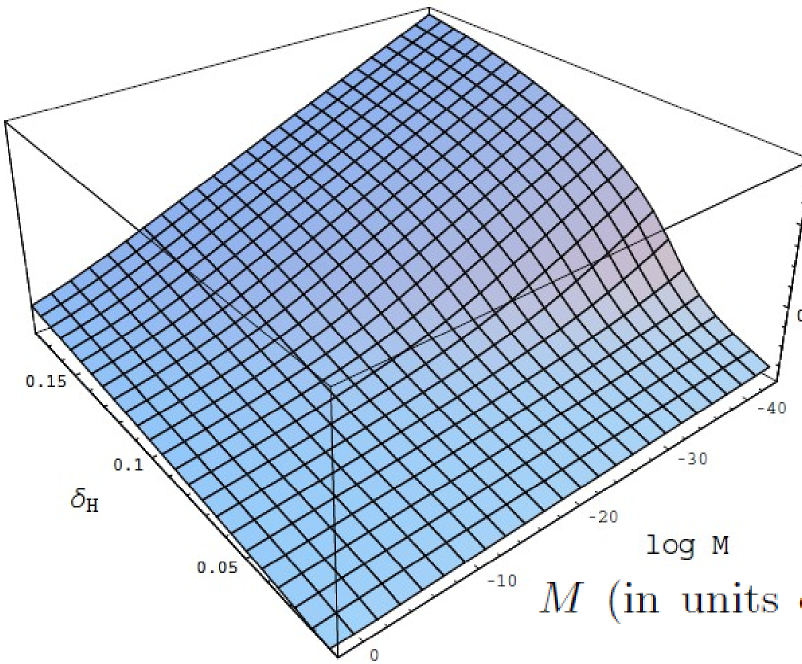
Initial conditions from linear theory:

$$\delta = \frac{3A_{\rm in}}{2}\left[\ln\left(\frac{k\eta}{\sqrt{3}}\right) + \gamma_E - \frac{1}{2}\right] \qquad \text{for } \delta \ll 1$$

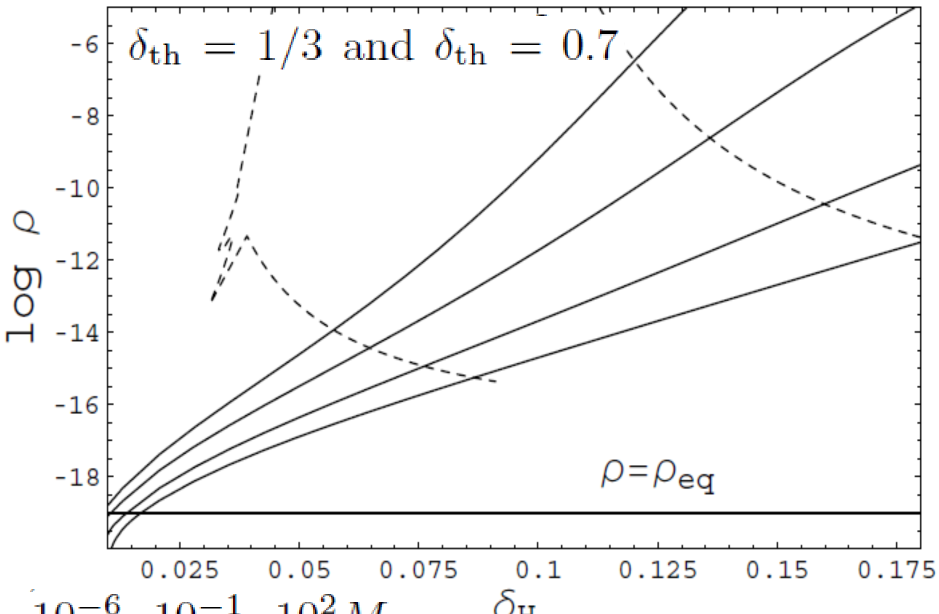
$$b = (1+\delta)^{-1/3}$$



$$\beta_{\rm cl} = \int_0^{\delta_{\rm th}} \frac{d\delta_{\rm H}}{\sqrt{2\pi}\Delta_{\rm H}} \exp\left(-\frac{\delta_{\rm H}^2}{2\Delta_{\rm H}^2}\right) \approx 1/2$$



M (in units of $M_\odot$)

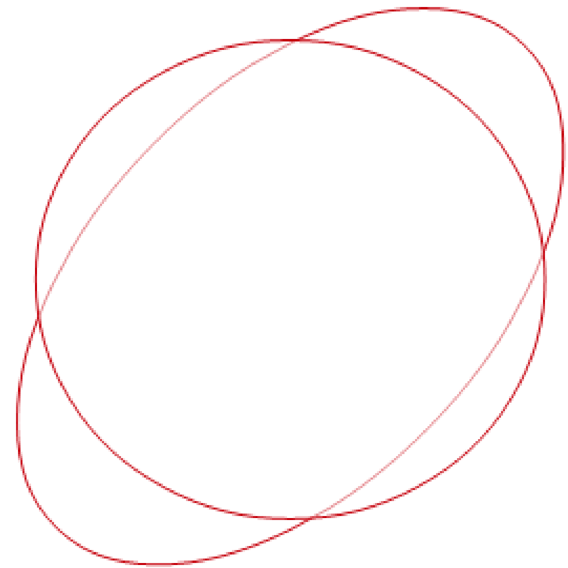


ρ [g cm^{-3}]

$M = 10^{-11},\ 10^{-6},\ 10^{-1},\ 10^{2}\,M_\odot$

Homogeneous ellipsoid:

$$\phi = \frac{1}{2}\Phi_{\alpha\beta}(t)r^{\alpha}r^{\beta}$$



$$\Phi = \Phi_{el} + \Phi_{bg} + \Phi_{sh}, \quad \Phi_{bg} = 4\pi G\bar{\rho}(t)I/3$$

$$S = \begin{Vmatrix} a & & \\ & b & \\ & & c \end{Vmatrix} = Ir + \sigma, \;\; \Phi_{el} = 2\pi G\rho_e \begin{Vmatrix} A_1 & & \\ & A_2 & \\ & & A_3 \end{Vmatrix}$$

$$\frac{d^2 S^{\alpha\beta}}{dt^2} = -\Phi^{\alpha\gamma}S^{\gamma\beta}$$

$$A_1 = abc\int_0^{\infty}\frac{d\lambda}{(a^2+\lambda)[(a^2+\lambda)(b^2+\lambda)(c^2+\lambda)]^{1/2}}$$

$$\frac{d^2\sigma}{dt^2} = \frac{4\pi}{15}G\rho_e\sigma - \frac{4\pi}{3}G(2\rho_r+\rho_m)\sigma$$

$$\rho_e \equiv M_e/V \qquad \rho_e = \rho_m\left(\frac{1+\Phi}{b^3}-1\right)$$

$$\sigma = a(y)s(y)\xi$$

$$y(y+1)s'' + \left(1+\frac{3}{2}y\right)s' - \frac{1}{10}\left(\frac{1}{b^3}-1\right)s = 0$$

Initial conditions:

Conformal Newton system, superhorizon scales:

$$r \gg ct \qquad \delta_r = -2\Phi = const$$

$$\Phi(\eta, \vec{k}) = \Phi_i(\vec{k}) \frac{3\pi^{1/2}}{2^{1/2}(u_s k\eta)^{3/2}} J_{3/2}(u_s k\eta)$$

$$\delta_i = (3/4)\delta_{r,i} = -(3/2)\Phi_i = \delta_H \phi/4$$

$$\delta_i(\vec{x}) = \delta_i = const \text{ if } (x/a)^2 + (y/b)^2 + (z/c)^2 \leq 1$$

$$\delta_i(\vec{k}) = \delta_i (2\pi)^3 abc \left(\frac{\sin(\tilde{k}) - \tilde{k}\cos(\tilde{k})}{2\pi^2 \tilde{k}} \right)$$

where

$$\tilde{k} = ((ak_x)^2 + (bk_y)^2 + (ck_z)^2)^{1/2}$$

$$\tilde{\vec{x}} = (x/a, y/b, z/c)$$

Peculiar velocity: $v_j = \partial v / \partial x_j$

$$v(\tilde{\vec{x}}) = \frac{1}{abc} \int \frac{d^3\tilde{k}}{(2\pi)^3} \frac{-9\Phi_i(\vec{k}) e^{-i\tilde{\vec{x}}\tilde{\vec{k}}}}{\eta \left[(\tilde{k}_x/a)^2 + (\tilde{k}_y/b)^2 + (\tilde{k}_z/c)^2 \right]}$$

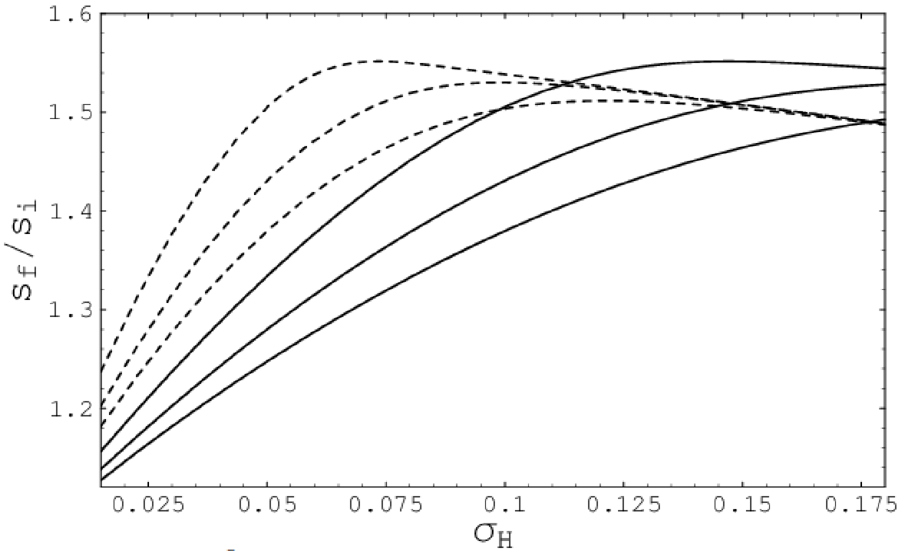
$$s = s_i$$

$$s'|_{y_i} = \frac{3\delta_H b_i^3 s_i}{10 y_i \phi}$$

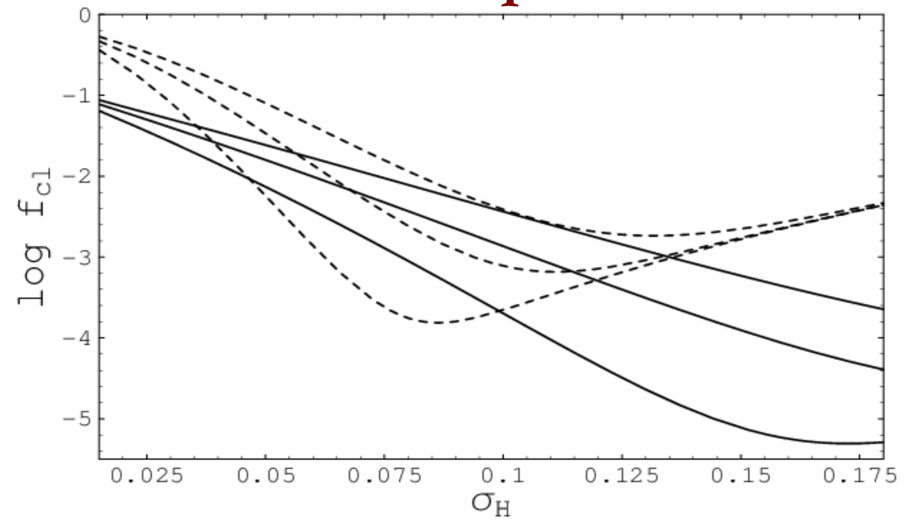
Clump formation scenarios and models

Nonspherical models

Number of clumps:



$M = 10^{-6},\ 10^{-1},\ 10^{2} M_\odot$ (from up to down)



$s_f/b_f < 1$ - **condition of formation**

⇩

$s_i/b_i < (b_f/b_i)(s_i/s_f)$ - **initial nonsphericity**

$$\lambda_i \propto a_i^{-2} \qquad e = \frac{\lambda_1 - \lambda_2}{2\sum \lambda_i} \simeq \frac{2}{3}\frac{s_i}{b_i}$$

$$p = \frac{\lambda_1 + \lambda_3 - 2\lambda_2}{\sum \lambda_i}$$

Gaussian random fields:

(Doroshkevich, 1970), (Bardeen et al., 1986)

$$p(\lambda_1, \lambda_2, \lambda_3) = \frac{15^3}{8\pi\sqrt{5}\,\sigma^6} \exp\left(-\frac{3I_1^2}{\sigma^2} + \frac{15 I_2}{2\sigma^2}\right) \times$$
$$\times (\lambda_1 - \lambda_2)(\lambda_2 - \lambda_3)(\lambda_1 - \lambda_3)$$

$\lambda_1 \geq \lambda_2 \geq \lambda_3$ *(Doroshkevich, 1970)*

$I_2 = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_1\lambda_3 \qquad I_1 = \lambda_1 + \lambda_2 + \lambda_3$

Ellipticity distribution:

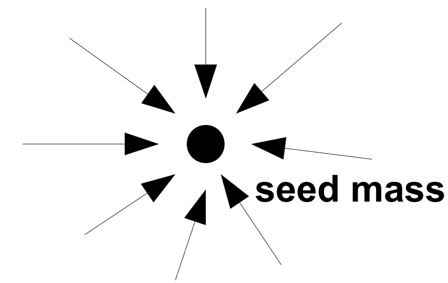
$$g(e, p|\nu) = \frac{1125}{\sqrt{10\pi}} e\,(e^2 - p^2)\,\nu^5 e^{-\frac{5}{2}\nu^2(3e^2 + p^2)}$$

$$-e < p < e$$

(Sheth, Mo, Tormen, 2001)

Internal structure of clumps

- **Physics of violent relaxation and virialization**
  (Lynden-Bell 1967)



- **Secondary accretion, self-similar solutions, and ultra-compact minihalos** *(Gott 1975), (Gunn 1977), (Bertschinger 1985)*

- **Nondissipative gravitational singularity (Gurevich-Zybin theory)**
  (Gurevich, Zybin 1988)

$$\rho_{\rm int}(r) = \begin{cases} \rho_c, & r < R_c \\ \rho_c \left(\dfrac{r}{R_c}\right)^{-\beta}, & R_c < r < R \\ 0, & r > R \end{cases} \qquad \beta \simeq 1.8$$

- **Entropy theory**
  (Mikheeva, Doroshkevich, Lukash 2007)

- **Tidal effects on the density profile**
  (Berezinsky, Dokuchaev, Eroshenko 2003)

- **Gravothermal catastrophe for superheavy particles** ($m>10^{11}$GeV)
  $\rho \propto r^{-2}$ with a very small core

Internal structure of clumps

Constraints on the core radius or maximal central density in the clumps

Example	$M/M_\odot$	$\bar{\rho}$, g cm^{-3}	δ	x_c, thermal velocities*	x_c, peculiar velocities**	x_c, annihilation***
1	10^{-6}	3×10^{-23}	$\delta_{\rm eq}=0.009$	4×10^{-3}	6×10^{-12}	2.6×10^{-5}
2	10^{-6}	4.2×10^{-16}	$\delta_H=0.05$	0.24	0.1	0.1
3	0.1	2.5×10^{-17}	$\delta_{\rm eq}\simeq1$	4×10^{-4}	0.01	2.5×10^{-2}

- **Annihilation limit** *(Berezinsky, Gurevich, Zybin 1992), (Berezinsky, Bottino, Mignola 1997)*
- **Tidal forces** $x_c=\dfrac{R_c}{R}\simeq0.3\nu^{-2}f^2(\delta_{\rm eq})$
- **Energy limit (Gurevich, Zybin)** $R_c/R\simeq\delta_{\rm eq}^3$

$$\rho(r_{\rm min})\simeq\frac{m}{\langle\sigma v\rangle(t_0-t_f)}$$

(Ullio et al. 2002)

- **Gravothermal catastrophe for superheavy particles** ($m>10^{11}$GeV)
- **Liouville theorem (thermal velocities)**
- **Liouville theorem (peculiar velocities)** $R_c/R\simeq0.01\delta_{\rm eq}^{9/2}$

$$f_p(p)d^3rd^3p=\frac{\rho_m}{m(2\pi mkT)^{3/2}}e^{-\frac{p^2}{2mkT}}d^3rd^3p$$

- **Liouville theorem:**

$$f_c<f_p(p=0)$$

$$\frac{R_c}{R}>\frac{2\pi^{1/2}\bar{\rho}^{1/4}T_d^{3/4}}{3^{1/4}G^{3/4}M^{1/2}m^{3/4}\rho_m^{1/2}(t_d)}$$

Internal structure of clumps
Numerical *N*-body simulations

down to $\sim 10^6 M_\odot$

Navarro-Frenk-White profile:

$$\rho_{\rm H}(r) = \frac{\rho_0}{(r/R_s)\,(1 + r/R_s)^2}$$

$$R_s = 20 \ {\rm kpc}$$

$$\rho_{\rm h}(r_\odot) = 0.3\,{\rm GeV/cm^3}$$

Moore et al. profile:

$$\rho_{\rm H}(r) \propto r^{-1.5}$$

Einasto profile:

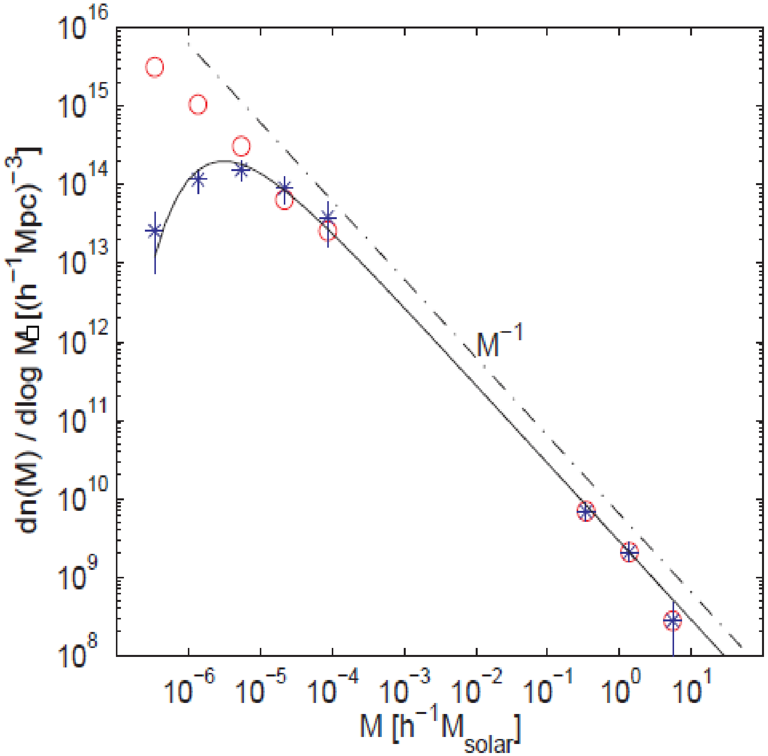
$$\rho_{\rm H}(r) = \rho_0 \exp\left[-\frac{2}{\alpha}\left(\left(\frac{r}{r_s}\right)^{\alpha} - 1\right)\right]$$

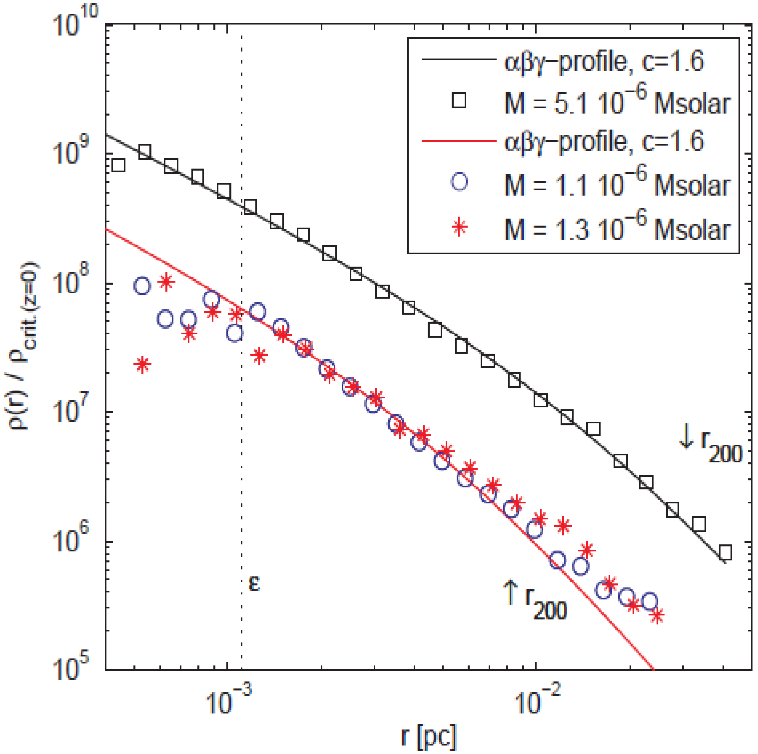
$\alpha = 0.16 - 0.3$ and $r_s \simeq 20$ kpc

Mass function of substructures: $\propto M^{-1.9}$ for $M > 10^6 M_\odot$

Internal structure of clumps

Numerical *N*-body simulations





(Diemand, Moore, Stadel 2005)

Low-mass clumps modelling:

(Diemand, Moore, Stadel 2005)

(Diemand, Kuhlen, Madau 2006)

(Ishiyama, Makino, Ebisuzaki 2010)

(Anderhalden, Diemand 2013)

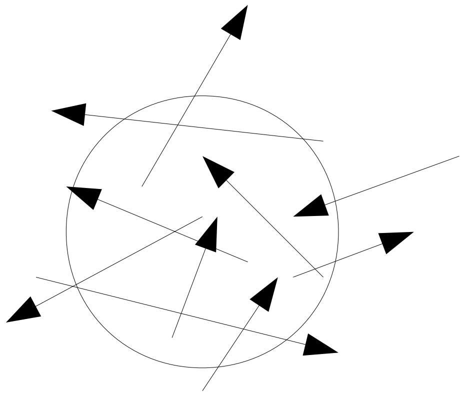
Clumps with minimal mass

Estimates

$T > T_f \sim 0.05 m_\chi$ **- chemical equilibrium**

$\frac{1}{\tau_{\rm rel}} \simeq H(t)$ ⇒ **time and the temperature of the kinetic decoupling**

Free streaming:



$$\lambda_{\rm fs} = a(t_0)\int_{t_d}^{t_0} \frac{v(t')dt'}{a(t')}$$

$$M_{\rm fs} = \frac{4\pi}{3}\rho_\chi(t_0)\lambda_{\rm fs}^3$$

$$\frac{1}{\tau_{\rm rel}} = \frac{1}{E_k}\frac{dE_k}{dt} = \frac{g_f}{2E_k m}\int d\Omega \int d\omega\, n_0(\omega)\left(\frac{d\sigma_{\rm el}}{d\Omega}\right)_{f_L\chi}(\delta p)^2$$

$$\frac{1}{\tau_{\rm rel}} = \frac{40\Gamma(7)\alpha^2_{\rm e.m.}}{9\pi\cos^4\theta_{\rm W}}\frac{T^6}{\tilde{M}^4 m}$$

$$t = \frac{2,42}{\sqrt{g_*}}\left(\frac{T}{1\ {\rm MeV}}\right)^{-2}\ {\rm s}$$

$$t_d \simeq 10^{-3}\left(\frac{m}{100{\rm GeV}}\right)^{-1/2}\left(\frac{\tilde{M}}{0.2\ {\rm TeV}}\right)^{-2}\left(\frac{g_*}{10}\right)^{-3/4}\ {\rm s}$$

$$T_d = 30\left(\frac{m}{100{\rm GeV}}\right)^{1/4}\left(\frac{\tilde{M}}{0.2\ {\rm TeV}}\right)\left(\frac{g_*}{10}\right)^{1/8}\ {\rm MeV}$$

Clumps with minimal mass

Neutralino-lepton scattering cross-section

$f_L + \chi \to f_L + \chi$

We consider the neutralino to be a pure bino $\chi = \tilde{B}$

$$\left(\frac{d\sigma_{\rm el}}{d\Omega}\right)_{f_L\chi} = \frac{\alpha^2_{\rm e.m.}}{8\cos^4\theta_{\rm W}} \frac{\omega^2(1+\cos\theta_{12})}{(m^2-\tilde{m}^2_L)^2}$$

$$\sigma \approx T^2/M_\sigma^4$$

$$\left(\frac{d\sigma_{\rm el}}{d\Omega}\right)_{f_R\chi} = 16\left(\frac{d\sigma_{\rm el}}{d\Omega}\right)_{f_L\chi}$$

- $M_{\rm min} \sim 10^{-12} M_\odot$ (Zybin, Vysotsky, Gurevich, 1999)
- $M_{\rm min} \sim (10^{-7} - 10^{-6}) M_\odot$ (Schwarz, Hofmann, Stocker, 2001)
- $M_{\rm min} \sim 10^{-4} M_\odot$ (Loeb, Zaldarriaga, 2005)
- $M_{\rm min} \sim (10^{-5} - 10^{-4}) M_\odot$ (Bertschinger, 2006)

Clumps with minimal mass

Kinetic decoupling

$$\rho(x,t) = \frac{m}{a^3}\int d^3p f(x,p,t) = \bar{\rho}_\chi(t)(1+\delta(x,t))$$

$$\frac{\partial f}{\partial t} + \frac{p_i}{ma^2}\frac{\partial f}{\partial x_i} - m\frac{\partial \phi}{\partial x_i}\frac{\partial f}{\partial p_i} = D_p(t)\frac{\partial}{\partial p_i}\left(\frac{p_i}{mTa^2}f + \frac{\partial f}{\partial p_i}\right)$$

$$D_p(t) = \frac{g_f}{3}\int d\Omega \int d\omega\, n_0(\omega)\left(\frac{d\sigma_{el}}{d\Omega}\right)_{f_L\chi}(\delta p)^2$$

- diffusion coefficient

$$\int p_i p_j f d^3p = \bar{\rho}_\chi a^5 T_\chi(t)\delta_{ij}$$

$$\frac{dT_\chi}{dt} + 2\frac{\dot{a}}{a}T_\chi - \frac{2D_p(t)}{ma^2}\left(1 - \frac{T_\chi(t)}{T(t)}\right) = 0$$

$$\frac{T_\chi(t)}{T_d} = \frac{1}{\tau}\left(\tau_i^{-1/2}e^{1/4\tau^2 - 1/4\tau_i^2} + \frac{1}{2}e^{1/4\tau^2}\int_{\tau_i}^{\tau} d^3x\, x^{-5/2}e^{1/4x^2}\right)$$

The transition from the kinetic equilibrium of neutralinos with relativistic fermions to the nonequilibrium regime occurs very rapidly. Therefore, the treatment of diffusion separately from free streaming seems to be justified.

Clumps with minimal mass

Diffusion cut-off of the perturbation spectrum

$$\frac{\partial^2\delta}{\partial^2 t} + 2\frac{\dot{a}}{a}\frac{\partial\delta}{\partial t} + D_p(t)\frac{1}{mTa^2}\frac{\partial\delta}{\partial t} = \frac{k_i k_j}{\bar{\rho}_\chi a^7 m}\int p_i p_j f d^3 p$$

$$\frac{\partial\delta(\vec{x},t)}{\partial t} = \frac{D(t)}{a^2(t)}\Delta_{\vec{x}}\delta(\vec{x},t)$$

$$D = \frac{3\pi\cos^4\theta_{\rm W}\tilde{M}^4}{40\Gamma(6)\alpha^2_{\rm e.m.}T^5}$$

$$\delta_{\vec{k}}(t) = \delta_{\vec{k}}(t_f)\exp\left\{-k^2 C g_*^{5/4}\tilde{M}^4\left(t^{5/2} - t_f^{5/2}\right)\right\}$$

$$M_{\rm D} = \frac{4\pi}{3}\rho_\chi(t_d)\lambda^3_{\rm D}(t_d) = 5\times10^{-12}\left(\frac{m}{100\ {\rm GeV}}\right)^{-15/8}$$

$$\times\left(\frac{\tilde{M}}{0.2\ {\rm TeV}}\right)^{-3/2}\left(\frac{g_*}{10}\right)^{-15/16}M_\odot$$

- **coincides with** *(Zybin, Vvysotsky, Gurevich 1999)*

Clumps with minimal mass

Free streaming

After kinetic decoupling: $$\frac{\partial f}{\partial t} + \frac{p_i}{ma^2}\frac{\partial f}{\partial x_i} = 0$$

$$f \propto \exp\left[\frac{ik_j p_j}{ma_d} g(t)\right]$$, where $$g(t) = a(t_d)\int_{t_d}^{t}\frac{dt'}{a^2(t')}$$

$$n_{\vec{k}}(t) = n_{\vec{k}}(t_d)e^{-(1/2)k^2 g^2(t) T_d/m} \qquad \lambda_{\rm fs}(t) = a(t)g(t)\left(\frac{T_d}{m_\chi}\right)^{1/2} \qquad M_{\rm fs}(t) = \frac{4\pi}{3}\rho_m(t)\lambda_{\rm fs}^3(t)$$

$$M_{\rm min} = \frac{\pi^{1/4}}{2^{19/4}3^{1/4}}\frac{\rho_{\rm eq}^{1/4} t_d^{3/2}}{G^{3/4}}\left(\frac{T_d}{m}\right)^{3/2}\ln^3\left\{\frac{24}{\pi G \rho_{\rm eq} t_d^2}\right\}$$

$$M_{\rm min} \simeq 2\times 10^{-7}\left(\frac{m}{100{\rm GeV}}\right)^{-15/8}\left(\frac{\tilde{M}}{0.2\ {\rm TeV}}\right)^{-3/2} \times \left(\frac{g_*}{10}\right)^{-15/16}\left(\frac{\Lambda^*}{83}\right)^3 M_\odot$$

Earth's mass clumps $\sim 10^{-6} M_\odot$

$M \sim 10^{-6} M_\odot$, $n_s = 0.96$:
$z \sim 60$ (for 2σ-perturbations), $\bar{\rho} = 2.6 \times 10^{-23}$ g cm^{-3},
$R = 8.6 \times 10^{-3}$ pc, $v = 71$ cm s^{-1}

Clumps with minimal mass

Cosmological horizon and acoustic oscillation effects

The evolutions of perturbations with masses $M \ll M_d$ and $M \gg M_d$ differ greatly after the horizon crossing.

$$T_d = 7.65\, C^{-1/4} g_*^{1/8} \left(\frac{m}{100 \text{ GeV}} \right)^{5/4} \text{ MeV},$$

$$M_{\min} = 7.59 \times 10^{-3}\, C^{3/4} \left(\frac{m\sqrt{g_*}}{100 \text{ GeV}} \right)^{-15/4} M_\odot$$

$$C = 256\, (G_F m_W^2)^2 \left(\frac{\tilde{m}^2}{m^2} - 1 \right)^{-2} \sum_L (b_L^4 + c_L^4)$$

(Bertschinger 2006)

Clumps with minimal mass

The M_{min} mass for superheavy neutralinos

The free-streaming scale and mass for superheavy dark matter particles are very small. In the case of the bino, the decoupling time is $t_d = 7 \times 10^{-30}$ s and $M_{fs} = 4.6 \times 10^{-11}$ g. The latter quantity is larger than the particle mass by only a factor of 260, and all masses of clumps starting from $M \sim 260\, m$ are possible. In the case of the Higgsino, $M_{fs} < m$, and free streaming plays no role in the perturbation evolution.

Hence, two mass scales, M_d and M_{fs}, could play the role of the minimal clump mass M_{min}. In the case of the bino, $M_{fs} > M_d$, and the cutoff in the mass function starts at $M_{min} \sim M_{fs}$. In the case of the Higgsino, M_{fs} is very small and $M_{min} \sim M_d$.

Formation of clump in early hierarchical clustering

Press-Schechter formalism

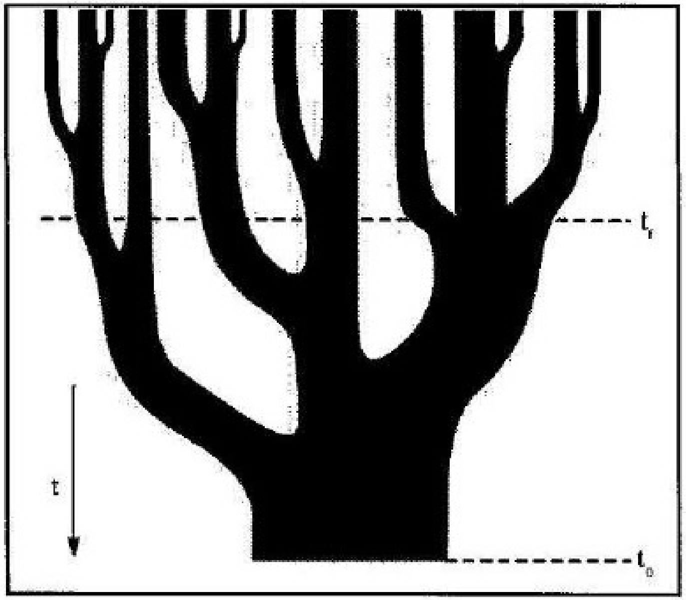
The Press-Schechter theory is based on the spherical model.

$$\delta(t) \geqslant \delta_c$$

$$P(M) = \frac{1}{\sqrt{2\pi}\sigma(M)} \int_{\delta_c}^{\infty} d\delta' \exp\left(-\frac{\delta'^2}{2\sigma^2(M)}\right)$$

$$dn(t, M) = -2\frac{\bar{\rho}_0}{M}\frac{dP(M)}{dM}dM$$
$$= -\left(\frac{2}{\pi}\right)^{1/2} \frac{\bar{\rho}_0}{M\sigma(M)}\frac{d\sigma(M)}{dM}\nu e^{-\nu^2/2}dM$$

Merging tree



(Press, Schechter 1974)

Formation of clump in early hierarchical clustering

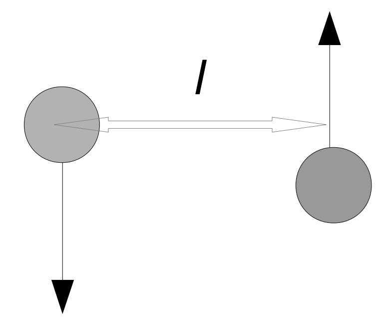
Tidal processes

$$\Delta E = \frac{1}{2}\int d^3r\,\rho_{\rm int}(r)(v_x - \tilde{v}_x)^2$$

- energy increase

$$v_x = \frac{2GM'}{v_{\rm rel}R'}g(y) \qquad y = l/R'$$

- velocity boost

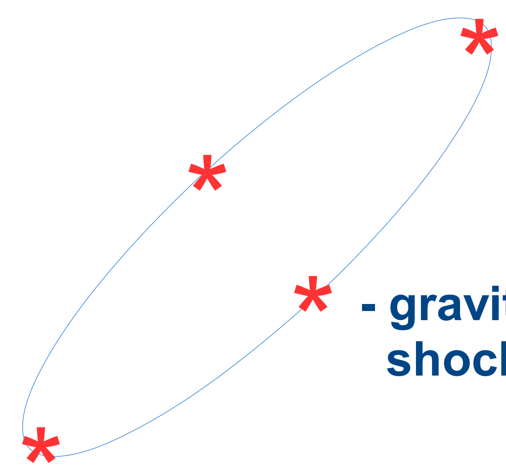


$$\dot{E} = \int 2\pi l v_{\rm rel} dl \int dM' \psi(M', t)\Delta E$$

$$\langle E_p \rangle = \frac{GM_h}{R_h^3} r^2 \left(\frac{R_h}{R_p}\right)^{\beta} \chi_{\rm ecc}(e) A(\omega\tau)$$

$$A(x) = (1 + x^2)^{-\gamma},\ \gamma \simeq 2.5 - 3$$

(Gnedin, Hernquist, Ostriker 1999)



- gravitational shocks

$$\Delta E = \int \langle E_p \rangle \rho_{\rm int}(r) d^3r$$

$$\dot{E} = \frac{2\Delta E}{T_{\rm orb}} \qquad \Delta E \sim \frac{4\pi}{3} G\rho_h M R^2$$

Formation of clump in early hierarchical clustering

Hierarchical clustering taking destructions into account

Distribution function of clumps:

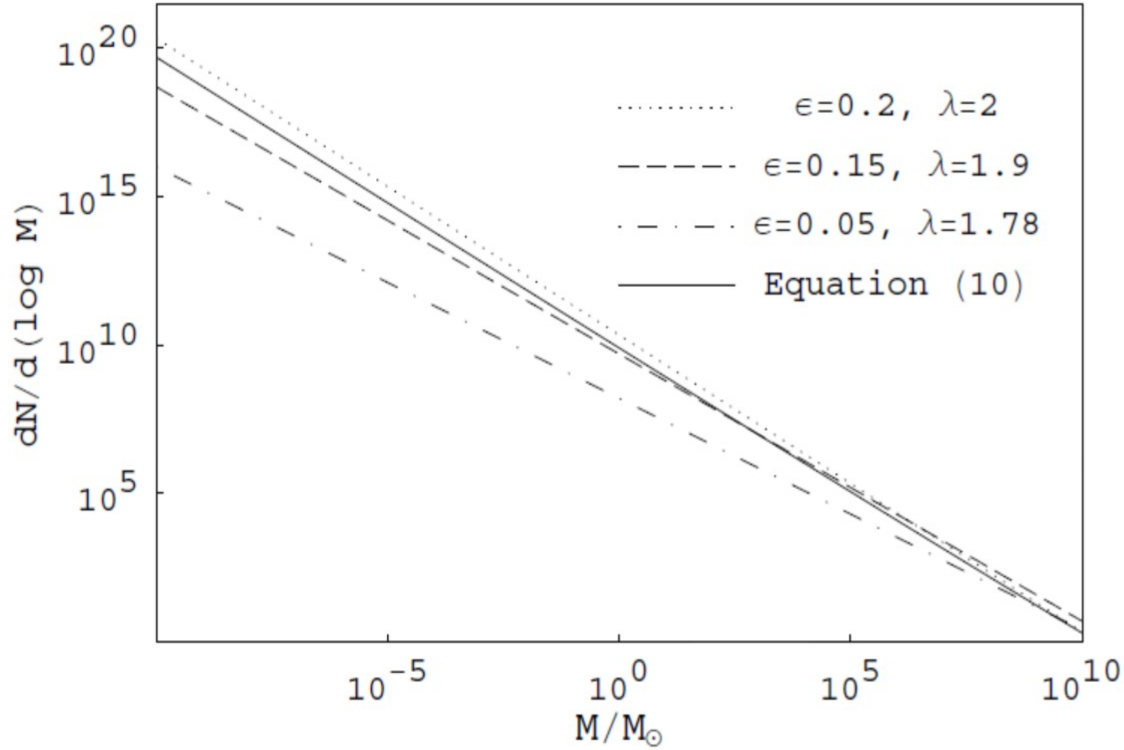
$$\xi \frac{dM}{M} d\nu \simeq \frac{\nu\, d\nu}{\sqrt{2\pi}} e^{-\nu^2/2} f_1 \frac{d\log \sigma_{\rm eq}(M)}{dM} dM \qquad f_1 \simeq 0.2-0.3$$

$$\xi_{\rm int} \frac{dM}{M} \simeq 0,02(n+3)\frac{dM}{M}$$

(Berezinsky, Dokuchaev, Eroshenko 2003)

$$n = -3(1 + 2\partial \ln \sigma_{\rm eq}(M)/\partial \ln M)$$

$$n(M)\, dM \propto dM/M^2$$



Destruction of clumps in the Galaxy

Clump destruction by the disc field

$$\sum_j (\Delta E)_j \sim |E|$$

rough criterion of the clumps tidal destruction

$$\frac{1}{M}\left(\frac{dM}{dt}\right)_d \simeq \frac{1}{\Delta T}\sum\left(\frac{\delta M}{M}\right)_d$$

gradual mass loss

Disc's surface density: $\sigma_s(r) = \frac{M_d}{2\pi r_0^2} e^{-r/r_0}$ $M_d = 8\times 10^{10} M_\odot$ and $r_0 = 4.5$ kpc.

$$\delta E = \frac{4 g_m^2 (\Delta z)^2 m}{v_{z,c}^2} A(a) \qquad A(a) = (1+a^2)^{-3/2}$$

(Ostriker, Spitzer, Chevalier 1972)

(Gnedin, Ostriker 1999)

$$\rho_{\rm int}(r) = 2^{5/2}\pi \int_{\psi(r)}^{0} \sqrt{\varepsilon - \psi(r)}\, f_{\rm cl}(\varepsilon)\, d\varepsilon$$

$$-\delta\varepsilon < \varepsilon < 0$$

$$\delta\rho(r) = 2^{5/2}\pi \int_{-\delta\varepsilon}^{0} \sqrt{\varepsilon - \psi(r)}\, f_{\rm cl}(\varepsilon)\, d\varepsilon.$$

$$\delta M = -4\pi \int_0^R r^2 \delta\rho(r)\, dr.$$

Destruction of clumps in the Galaxy

Clump destruction by stars

$$\Delta E = \frac{2(3-\beta)}{3(5-\beta)} \frac{G^2 M R^2 m_*^2}{v_{\rm rel}^2 l^4}$$

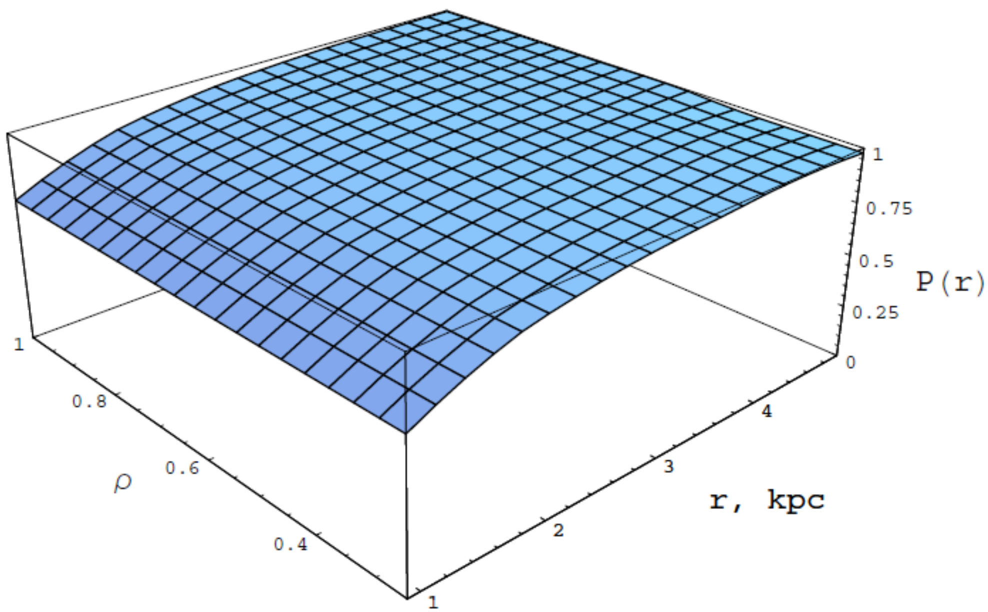
$$\dot{E} = 2\pi \int \Delta E(l)\, n_* v_{\rm rel}\, l\, dl$$

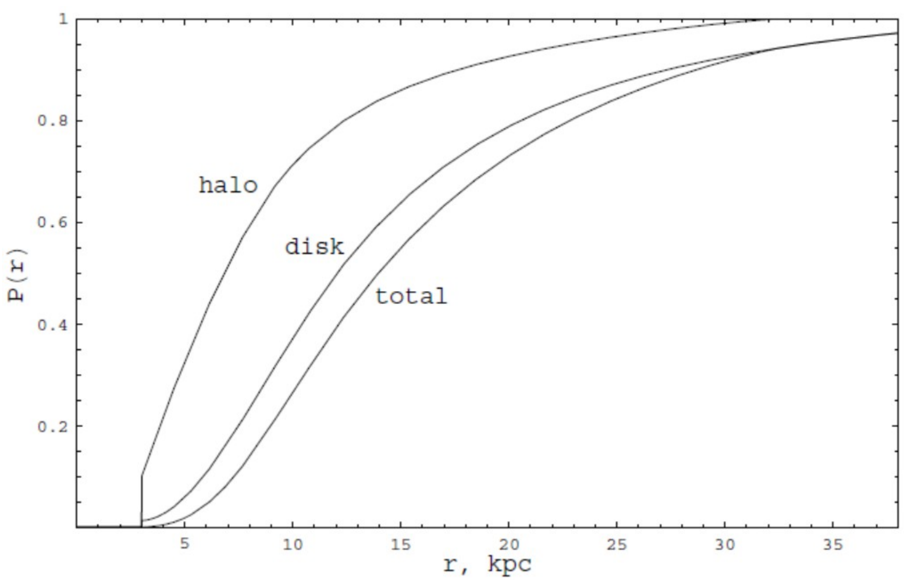
$$\frac{dM}{dt} = \left(\frac{dM}{dt}\right)_d + \left(\frac{dM}{dt}\right)_s$$

$$n_{b,*}(r) = (\rho_b/m_*) \exp\left[-(r/r_b)^{1,6}\right]$$

$$n_{h,*}(r) = (\rho_h/m_*)(r_\odot/r)^3$$

$$\frac{\Delta E^{\rm stars}}{\Delta E^{\rm disk}} \sim 0.6 \left(\frac{R}{0.015\ {\rm pc}}\right)^{-2} e^{(r-r_\odot)/r_0} \cos\theta$$

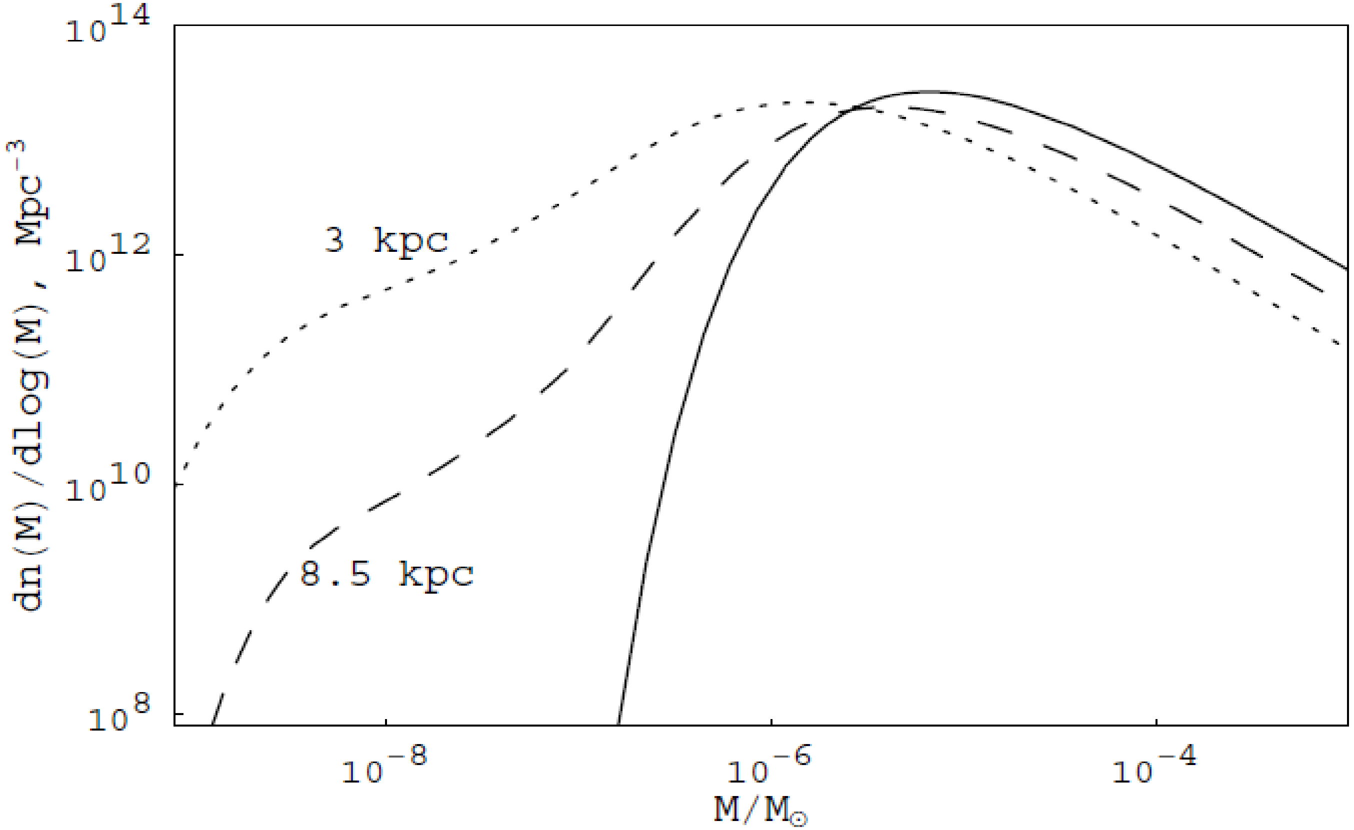




Survival probability $P(r,\rho)$ as a function of the distance to the galactic center r and the mean inner clump density ρ for $x_c = R_c/R = 0.05$.

Destruction of clumps in the Galaxy

Remnants of clumps



The modified clump remnants mass function at the galactocentric distances 3 and 8.5 kpc. The solid curve shows the initial mass function.

Particles annihilation in clumps

Cross sections and spectra of annihilation products

- **Local annihilation rate**

$$\dot{N}_{\rm cl} = 4\pi \int_0^\infty r^2 dr \rho_{\rm int}^2(r) m^{-2} \langle \sigma_{\rm ann} v \rangle$$

- **Cross section**

$$\langle \sigma_{\rm ann} v \rangle = a + bv^2 + cv^4 + \ldots$$

$$< \sigma v > \simeq 3 \times 10^{-26} \ {\rm cm}^3 \ {\rm s}^{-1}$$

$$x + \bar{x} \to \pi^0 + {\rm all}, \quad \pi^0 \to \gamma + \gamma$$

Dark-SUSY

- **Possible excess of 1−3 GeV gamma rays from the inner few degrees of the Galaxy center observed by Fermi-LAT telescope**

- **130 GeV line ?**

- **Sommerfeld enhancement**

$$\langle \sigma v \rangle = \mathcal{R} \langle \sigma v \rangle_0$$

$$\mathcal{R} = \frac{\pi \alpha}{\beta} (1 - e^{-\pi\alpha/\beta})^{-1}$$

(Lattanzi, Silk 2009)

- **Astrophysical backgrounds that are not connected with annihilation**

Particles annihilation in clumps

Parameterization of the annihilation signal

$$J_\gamma(E,\psi,\Delta\Omega) = 9.4\times 10^{-11}\frac{dS}{dE}\langle J(\psi)\rangle_{\Delta\Omega}$$

(Bergstrom et al. 1999)

$$\frac{dS}{dE} = \left(\frac{100\ \mathrm{GeV}}{m}\right)^2 \sum_F \frac{\langle\sigma_F v\rangle}{10^{-26}\ \mathrm{cm^3 s^{-1}}}\frac{dN_\gamma^F}{dE}$$

Astrophysical factor :

$$\langle J(\psi)\rangle_{\Delta\Omega} = \frac{1}{8.5\ \mathrm{kpc}}\frac{1}{\Delta\Omega}\int d\Omega' \int dL \left(\frac{\rho(r)}{0.3\ \mathrm{GeV\ cm^{-3}}}\right)^2$$

Gamma-ray photons from a clump: $2\eta_{\pi^0}\dot{N}_{\mathrm{cl}}$, where $\eta_{\pi^0}\sim 10$

$$\pi^0 \to 2\gamma$$

$$J_\gamma(E>m_{\pi^0}/2,\psi) = 1.9\times 10^{-10}\left(\frac{m}{100\ \mathrm{GeV}}\right)^{-2}\frac{\langle\sigma v\rangle}{10^{-26}\ \mathrm{cm^3 s^{-1}}}\langle J(\psi)\rangle_{\Delta\Omega}$$

$$\langle J(\psi)\rangle_{\Delta\Omega} = \int d\xi_{\mathrm{cl}}\left(\frac{\rho_{cl}}{0.3\ \mathrm{GeV\ cm^{-3}}}\right)\int_{l.o.s.}\frac{dL}{8.5\ \mathrm{kpc}}\left(\frac{\rho_H(r)}{0.3\ \mathrm{GeV\ cm^{-3}}}\right)$$

Particles annihilation in clumps

Enhancement of the annihilation signal

Boost factor: $B(\psi) = \dfrac{J^{\rm cl}(\psi) + J^{\rm hom}(\psi)}{J^{\rm hom}(\psi)}$

Total signal from all directions: $\int\limits_0^{\pi} d\zeta \sin\zeta \int\limits_0^{2\pi} d\phi \; \ldots$

$$l(\zeta, r) = (r^2 + r_\odot^2 - 2 r r_\odot \cos\zeta)^{1/2}$$

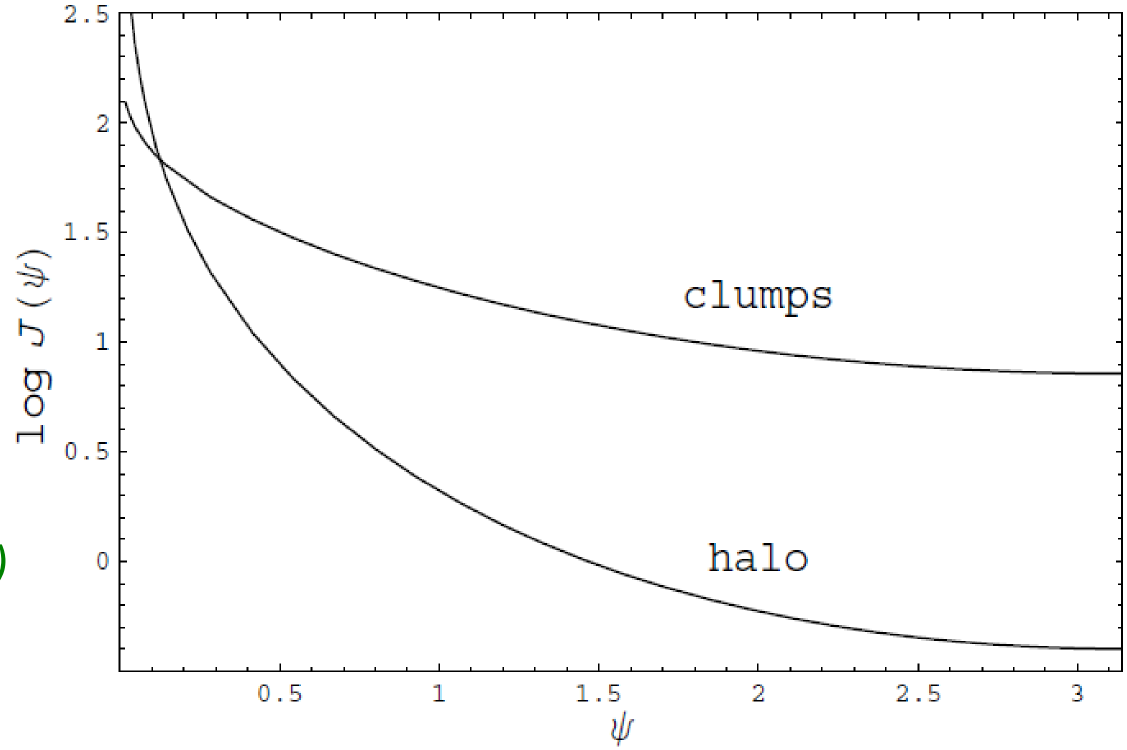
$$r_{\max}(\zeta) = (R_H^2 - r_\odot^2 \sin^2\zeta)^{1/2}$$

$$n_{\rm cl}(l) \;=\; \xi \rho_{\rm DM}(l)/M$$

$$B \approx 1 + \xi S(x_c, \beta) \frac{\bar{\rho}_{\rm int}}{\tilde{\rho}_{\rm DM}} \quad \sim 3$$

(Berezinsky, Dokuchaev, Eroshenko 2003)

$B \sim 4$ *(Anderhalden, Diemand 2013)*

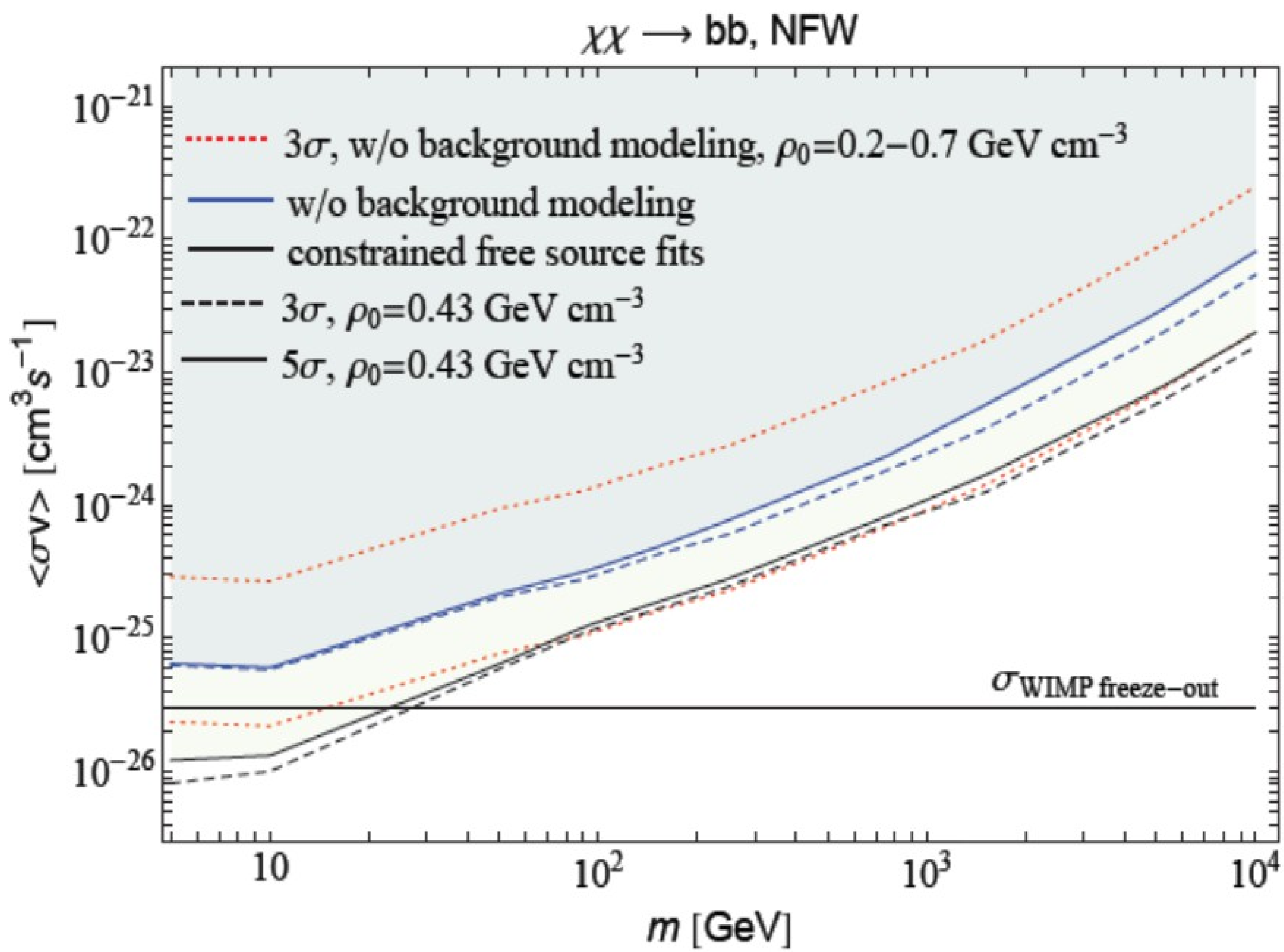


The astrophysical factor as a function of the angle c from the line of sight and the direction to the Galactic center.

Particles annihilation in clumps

Fermi-LAT constraints on the annihilation in Galaxy

Constraints from Fermi-LAT diffuse measurements:



Fermi-LAT collaboration, arXiv:1205.6474 [astro-ph.CO]

Particles annihilation in clumps

Annihilation in galaxies and galaxy clusters

Without clumps

$\delta\psi = 1^\circ$	$\Delta\Omega = 4\pi$ - Galaxy $\Delta\Omega = 2\pi(1-\cos\psi_{\max}) = 0.067$ - Virgo
$\langle J(\psi)\rangle_{\Delta\Omega}(\text{ Milky Way}) \simeq 1.4\times 10^3,$ $\langle J(\psi)\rangle_{\Delta\Omega}(\text{ Virgo}) \simeq 5\times 10^{-2}.$	$\langle J(\psi)\rangle_{\Delta\Omega}(\text{ Milky Way}) \simeq 3,$ $\langle J(\psi)\rangle_{\Delta\Omega}(\text{ Virgo}) \simeq 9\times 10^{-4}.$

With clumps

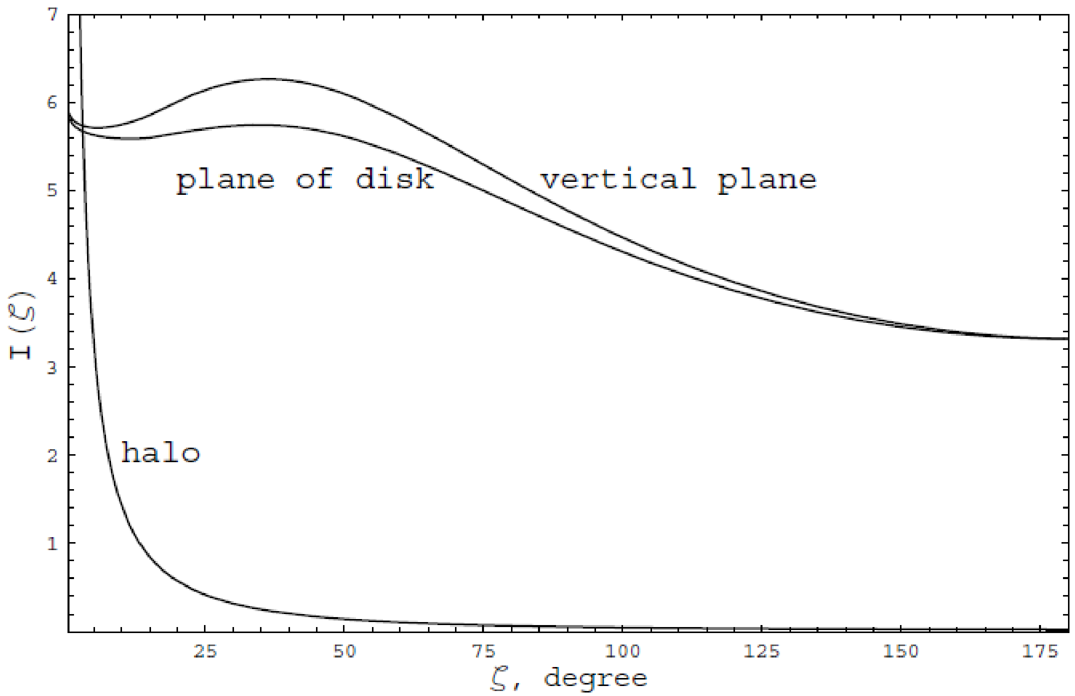
$\langle J(\psi)\rangle_{\Delta\Omega}(\text{ Milky Way}) \simeq 1.4\times 10^2,$ $\langle J(\psi)\rangle_{\Delta\Omega}(\text{ Virgo}) \simeq 13$	$\langle J(\psi)\rangle_{\Delta\Omega}(\text{ Milky Way}) \simeq 15,$ $\langle J(\psi)\rangle_{\Delta\Omega}(\text{ Virgo}) \simeq 1.3.$

$$\langle J(\psi)\rangle_{\Delta\Omega} \simeq 7.01\left(\frac{S(x_c,\beta)}{S(0.01;1.8)}\right)\left(\frac{\sigma_{\rm eq}(M_{\min},n_s)}{\sigma_{\rm eq}(10^{-6}M_\odot;0.963)}\right)^3 \times$$
$$\times\frac{1}{\Delta\Omega}\int d\Omega'\int\frac{dL\,\rho(r)}{0.3\text{ GeV cm}^{-3}},$$

Particles annihilation in clumps

Anisotropy of annihilation signals

- **Non-central galactic location of the Sun**
- **Halo nonsphericity**
- **Anisotropic clumps destructions**



The annihilation signal in the galactic disc plane and in the plane normal to the galactic disc as a function of the angle between the line of sight and the direction to the galactic center. For comparison, the annihilation signal from the galactic halo without clumps is also shown.

$$\delta = (I_2 - I_1)/I_1$$

$\delta \simeq 0.09$ at $\zeta \simeq 39^\circ$

(Berezinsky, Dokuchaev, Eroshenko 2007)

- **If there is a density cusp, a bright source in the center of the Galaxy should be present** *(Berezinsky, Gurevich, Zybin 1992)*.

Particles annihilation in clumps
Annihilation in ultra-dense clumps

For the strongly suppressed s-wave annihilation channel $\langle \sigma v \rangle = 1.7 \times 10^{-30} m_{100}^{-2}\ \mathrm{cm}^3/\mathrm{s}$

$\bar{\rho}_{\rm int} = 178 \rho_{eq}$ *(Berezinsky, Bottino, Mignola 1997)*

Even for minimal gamma-ray flux $f_{\rm cl} \ll 1$

Neutralino stars *(Gurevich, Zybin, Sirota 1997)*

Annihilation of superheavy particles in superdense clumps:

$\dot{N}_{\rm ann} \propto m^{-4}$ **Background radiation** $1/E^{\alpha}$ with $\alpha \leq 3$

With gravothermal catastrophe taken into account, the annihilation flux from superheavy DM particles may be at the observable level for all types of superheavy neutralinos *(Berezinsky, Dokuchaev, Eroshenko, Kachelries, Solberg 2010)*.

Annihilation in superdense clumps around cosmic string loops

Neutralino annihilation in superdense clumps around cosmic string loops *(Berezinsky, Dokuchaev, Eroshenko 2011)*.

Cosmological phase transitions → network of infinite strings → interconnections → transition stage → self-similar regime → closed loops

$G\mu/c^2 \leq 2 \times 10^{-7}$ - **CMB**

$G\mu/c^2 \leq 4 \times 10^{-9}$ - **pulsar timing** *(R. van Haasteren et.al., 2011)*

$l \simeq \alpha c t_i$, where $\alpha \simeq 0.1$

$$dn_{\rm loop} = \frac{N dl}{c^{3/2} t^{3/2} l^{5/2}}$$

(Olum, Vilenkin, 2006)

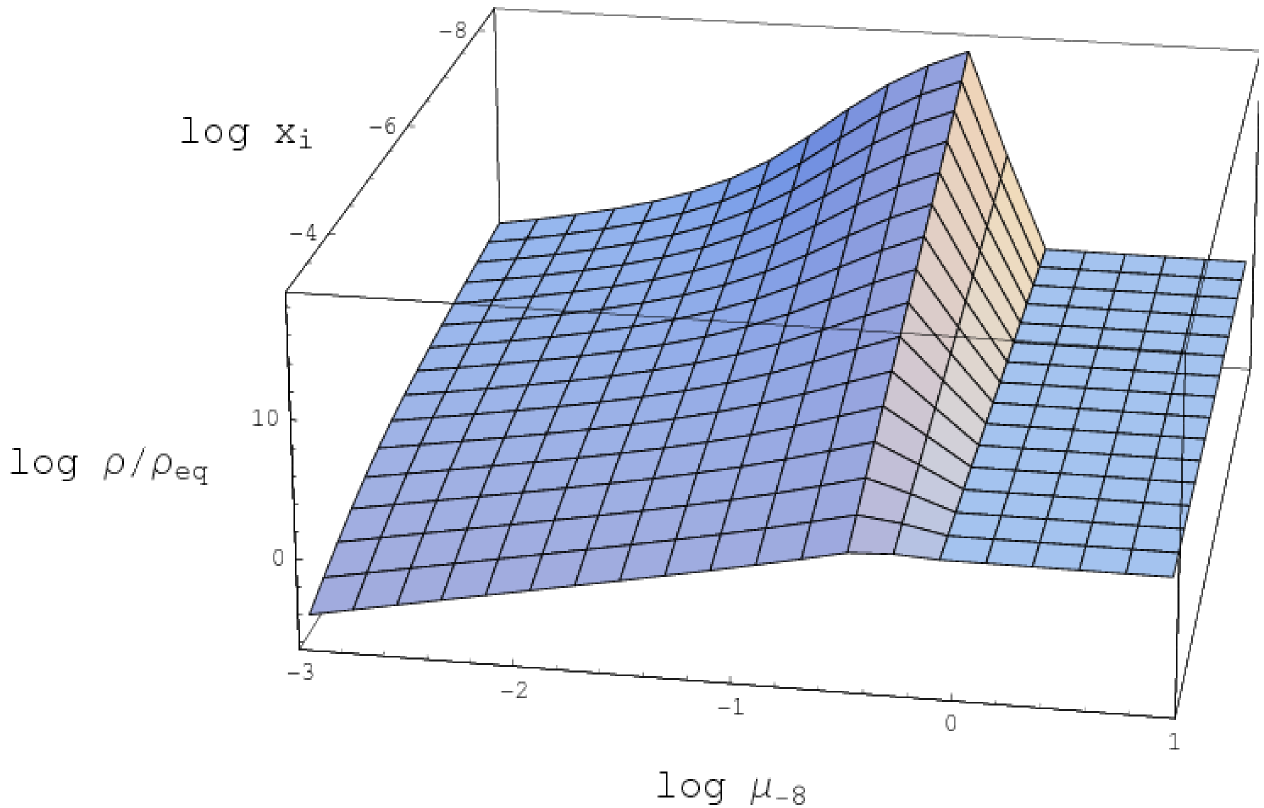
$P_{\rm lv} \sim 2 \times 10^{-7}$ - **Probability of slow loop formation**

Loop's evaporation:

$dM_l/dt = -\Gamma G\mu^2/c \quad \tau \simeq lc/(G\mu\Gamma)$

Adiabatic expansion *(Kolb, Tkachev 1993, 1994)*:

$$M_{\rm tot} R = const$$



Particles annihilation in clumps

Annihilation in superdense clumps around cosmic string loops

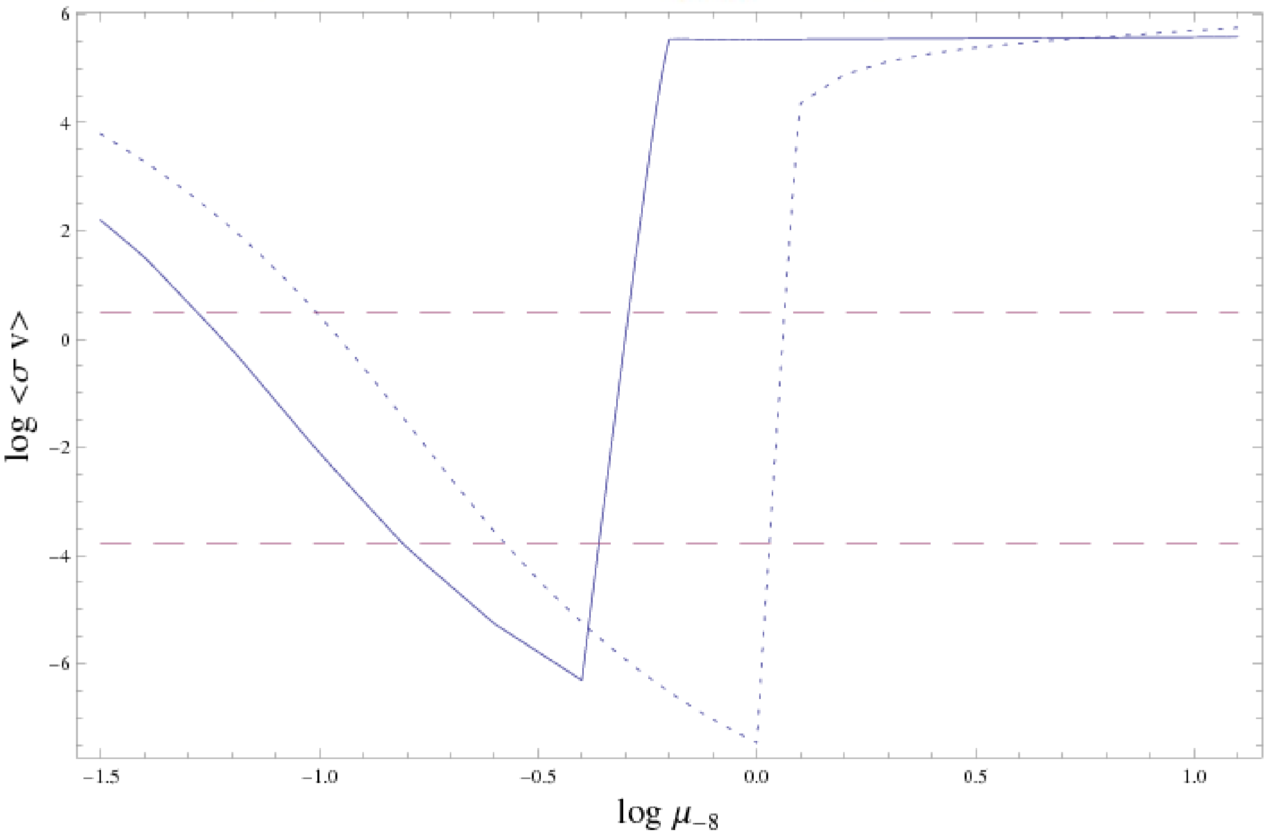
Restrictions on the cross-section from Fermi-LAT limits:

$$J_\gamma(E > m_{\pi^0}/2, \psi) = 1.9 \times 10^{-10} \left(\frac{m_\chi}{100\ \mathrm{GeV}}\right)^{-2} \frac{\langle \sigma v \rangle}{10^{-26}\ \mathrm{cm^3 s^{-1}}} \langle J(\psi) \rangle_{\Delta\Omega},$$

$$\langle J(\psi) \rangle_{\Delta\Omega} = \int d\xi_{\mathrm{cl}} \left(\frac{\rho_{cl}}{0.3\ \mathrm{GeV\ cm^{-3}}}\right) \int_{l.o.s.} \frac{dL}{8.5\ \mathrm{kpc}} \left(\frac{\rho_H(r)}{0.3\ \mathrm{GeV\ cm^{-3}}}\right),$$

$$\mu_{-8} = G\mu/(10^{-8} c^2)$$

$$m_\chi = 100\ \mathrm{GeV}$$



$\langle \sigma v \rangle \simeq 3 \times 10^{-26}\ \mathrm{cm^3 s^{-1}}$

$1.7 \times 10^{-30} m_{100}^{-2}\ \mathrm{cm^3 s^{-1}}$

(minimal)

Excluded: $0.05 < \mu_{-8} < 0.51$ - **decay**

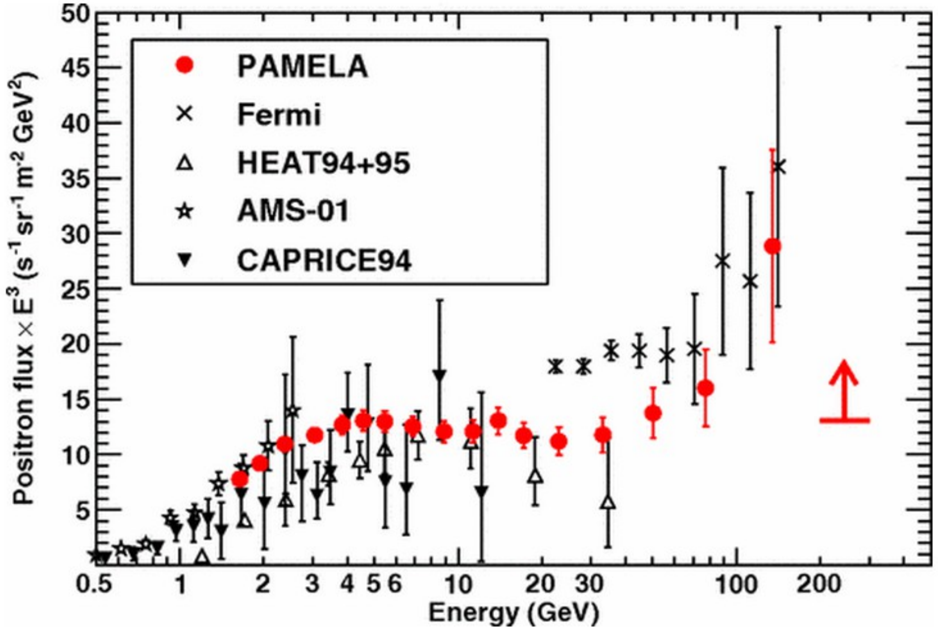
$0.1 < \mu_{-8} < 1.16$ - **evaporation**

(Berezinsky, Dokuchaev, Eroshenko 2011)

Charge particle fluxes in PAMELA, ATIC, ...

- Observational data

  e^+ excess in cosmic rays in comparison with secondary generation model



Phys. Rev. Lett. 111, 081102 (2013)

- Annihilation scenario and its problems

  PAMELA does not show the antiproton excess

  Gamma-rays disagreement with Fermi-LAT observations

- Alternative explanations

  Are the cosmic ray propagation and secondary generation model incomplete?

  Generation of electrons and positrons in pulsars

  Flares on dwarf main-sequence stars *(Stozhkov, Galper 2011)*

  Generated and accelerated by cosmic-ray sources themselves *(Blasi 2009)*

Other possible observational manifestations of clumps

Direct detection of dark matter particles. Ministreams

The probability that Earth is now inside a dark matter clump is estimated to be from 0.0001 % to 0.1%, depending on the clump mass and the assumed perturbation spectrum.

Presently 10^2-10^4 ministreams may be crossing Earth *(Schneider, Krauss, Moore 2010)*.

www.universetoday.com

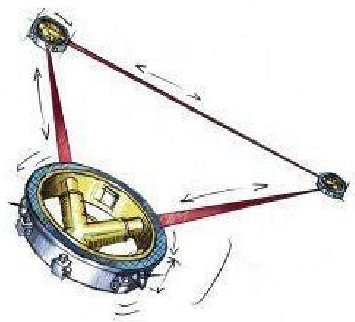
Other possible observational manifestations of clumps

Registration of clumps by gravitational wave detectors

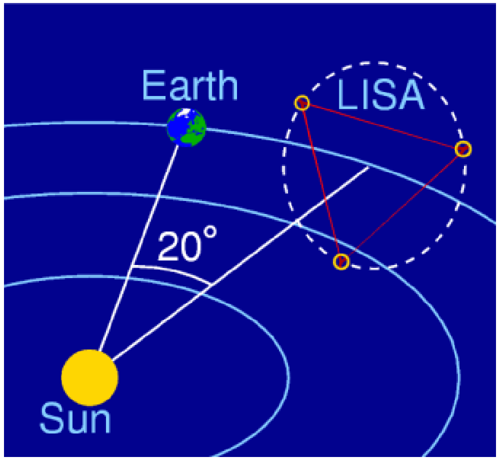
Primordial black holes *(Seto, Cooray 2004)*

Asteroids *(Tricarico 2009)*

Compact dark matter objects of an unknown nature: (Adams, Bloom, 2004, "*Direct Detection of Dark Matter with Space-based Laser Interferometers*", arXiv:astro-ph/0405266v2)



lisa.nasa.gov

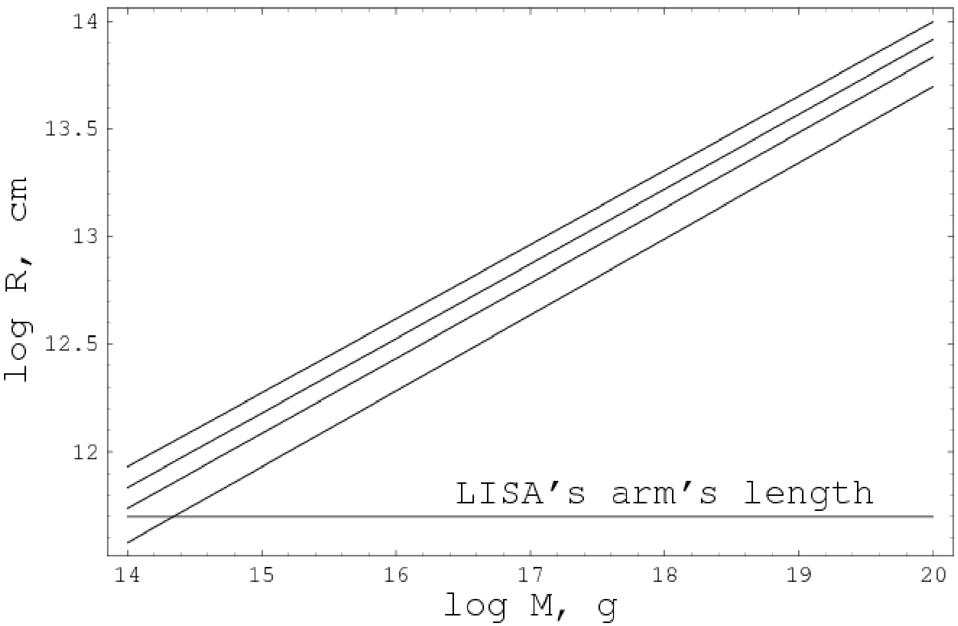


Clumps can be included into this list

0.03 мГц–0.1 Гц

Mass range for LISA 10^{16} g < M < 10^{20} g *(Seto, Cooray 2004)*
10^{14} g < M < 10^{20} g *(Adams, Bloom 2004)*

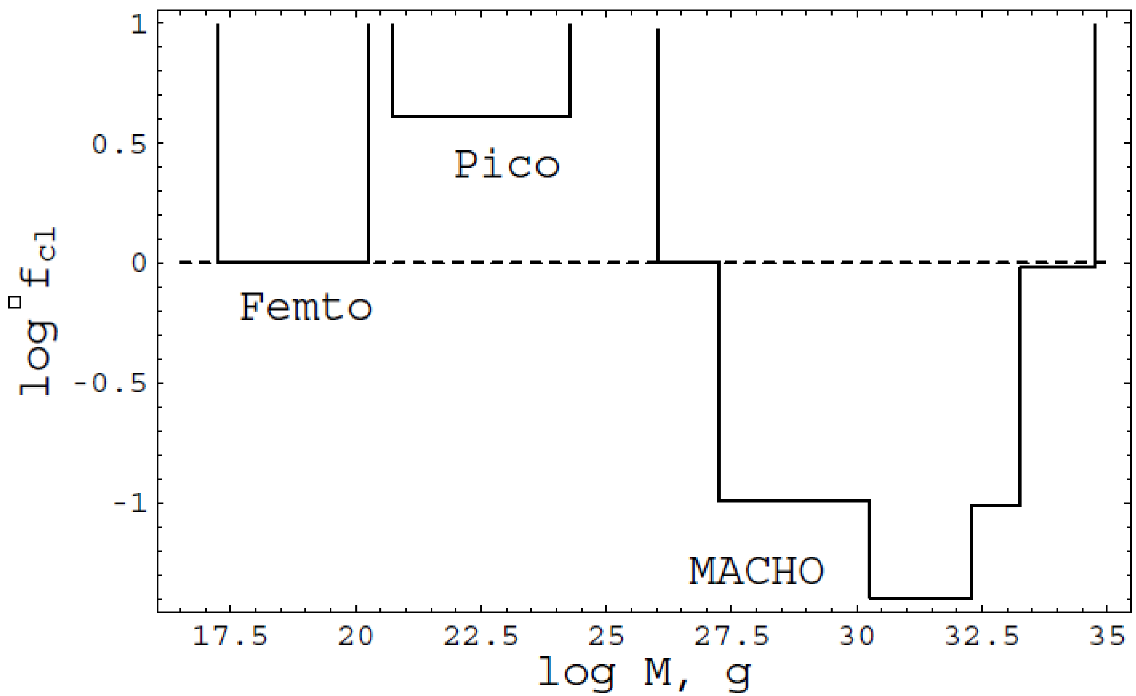
Standard spectra with
n_p=0.949,0.963,0.977 and 1



Other possible observational manifestations of clumps

Neutralino stars and microlensing

(Gurevich, Zybin, Sirota 1997)



(Carr 2010)

Upper bounds on the relative clumped dark matter fraction $f_{\rm cl} = \Omega_{\rm cl}/\Omega_m$ from microlensing MACHO observations, as well as from femto- and picolensing observations of cosmic gamma-ray bursts.

Other possible observational manifestations of clumps

- **Baryons in clumps**

  Baryonic core modifies the microlensing light curve of the clumps

  Potential wells for early population-III stars

  Temperature fluctuations in the baryonic gas, 21 cm observations

- **Motion of clumps on the celestial sphere** *(Koushiappas 2007),*
  (Pieri, Bertone, Branchini 2008),
  (Ando S et al. 2008),
  (Belotsky, Kirillov, Khlopov 2012)

  The possibility of observing proper motions is limited by annihilation gamma-ray constraints from the galactic center and other sources.

Conclusion

The early formed clumps can be the densest dark matter objects in the Universe. Therefore, the dark matter annihilation in these small-scale clumps can be very effective. The clumps enable the annihilation signal to be enhanced in galactic halos by several times or even orders of magnitude.

If a signal from annihilating dark matter is detected, it will be possible to study the dark matter distribution in greater detail and to obtain information on the primordial perturbation spectrum.