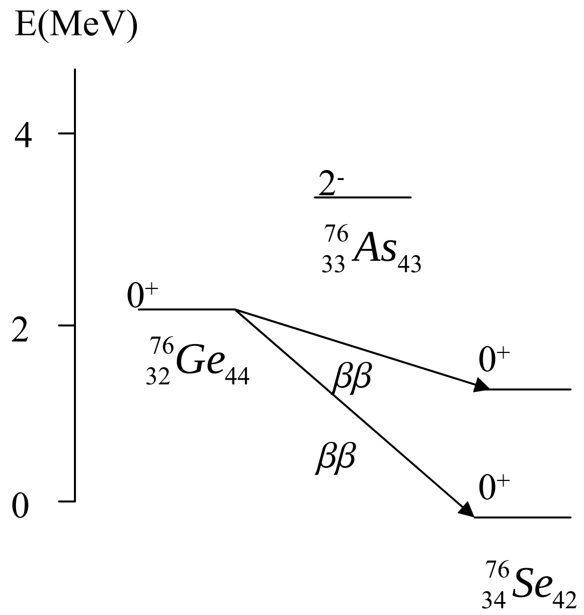
DOUBLE BETA DECAY AND NEUTRINO MASSES

Francesco Iachello
Yale University

Lecture 2

RESULTS: 0νββ

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z\pm2}Y_{N\mp2} + 2e^{\mp} $$



Half-life for the process:

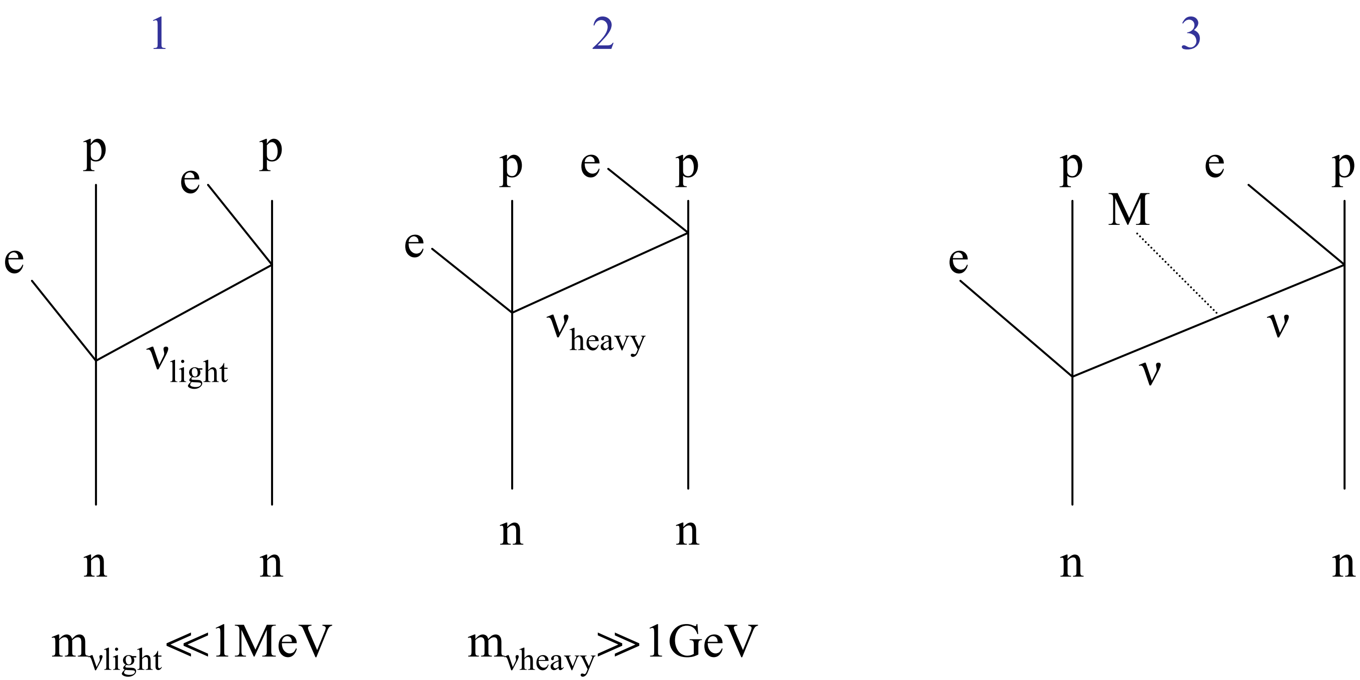
$$ \left[\tau_{1/2}^{0\nu\beta\beta}(0^+ \rightarrow 0^+)\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^2 \left|f(m_i, U_{ei})\right|^2 $$

Beyond the standard model
(Particle physics)

Matrix elements
(Nuclear physics)

Phase-space factor
(Atomic physics)

As discussed in lecture 1, the transition operator *T(p)* depends on the model of 0νββ decay and three scenarios have been considered [#,¶,§].



[#] M. Doi *et al*, Prog. Theor. Phys. 66, 1739 (1981); 69, 602 (1983).

[¶] T.Tomoda, Rep. Prog. Phys. 54, 53 (1991).

[§] F.Šimkovic *et al.*, Phys. Rev. C60, 055502 (1999).

In scenario 3, if the Majoron couples only to light neutrino, the NME needed to calculate the half-life are the same of scenario 1 and will not be considered further.

The appropriate formulas to calculate matrix elements for scenarios 1 and 2 have been given in lecture 1.

EVALUATION OF THE NUCLEAR MATRIX ELEMENTS (NME)

The NME can be written as:

$$M_{0\nu} = g_A^2 M^{(0\nu)}$$

$$M^{(0\nu)} \equiv M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$

Several methods have been used to evaluate $M_{0\nu}$:

QRPA (Quasiparticle Random Phase Approximation)

ISM (Shell Model)

IBM-2 (Interacting Boson Model)

EDF (Density Functional Theory)

EVALUATION OF MATRIX ELEMENTS IN IBM-2 ¶

All matrix elements, F, GT and T, can be calculated at once using the compact expression:

$$V_{s_1,s_2}^{(\lambda)} = \frac{1}{2}\sum_{n,n'} \tau_n^+ \tau_{n'}^+ \left[\Sigma_n^{(s_1)} \times \Sigma_{n'}^{(s_2)}\right]^{(\lambda)} \cdot V(r_{nn'}) C^{(\lambda)}(\Omega_{nn'})$$

$$\lambda = 0, s_1 = s_2 = 0 (F)$$

$$\lambda = 0, s_1 = s_2 = 1 (GT)$$

$$\lambda = 2, s_1 = s_2 = 1 (T)$$

In second quantized form:

$$V_{s_1,s_2}^{(\lambda)} = -\frac{1}{4}\sum_{j_1 j_2}\sum_{j'_1 j'_2}\sum_{J} (-1)^J \sqrt{1+(-1)^J \delta_{j_1 j_2}} \sqrt{1+(-1)^J \delta_{j'_1 j'_2}}$$

$$\times G_{s_1 s_2}^{(\lambda)}(j_1 j_2 j'_1 j'_2; J) \left[\left(\pi_{j_1}^\dagger \times \pi_{j_2}^\dagger\right)^{(J)} \cdot \left(\tilde{\nu}_{j'_1} \times \tilde{\nu}_{j'_2}\right)^{(J)}\right]$$

Creates a pair of protons with angular momentum J

Annihilates a pair of neutrons with angular momentum J

¶ J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009).

The fermion operator V is mapped onto the boson space by using:

$$\left(\pi_j^\dagger \times \pi_j^\dagger\right)^{(0)} \mapsto A_\pi(j) s_\pi^\dagger$$

$$\left(\pi_j^\dagger \times \pi_{j'}^\dagger\right)_M^{(2)} \mapsto B_\pi(j, j') d_{\pi,M}^\dagger$$

$$V_{s_1 s_2}^{(\lambda)} \mapsto -\frac{1}{2} \sum_{j_1} \sum_{j'_1} G_{s_1 s_2}^{(\lambda)}\left(j_1 j_1 j'_1\, j'_1; 0\right) A_\pi(j_1) A_\nu(j'_1) s_\pi^\dagger \cdot \tilde{s}_\nu$$

$$-\frac{1}{4} \sum_{j_1 j_2} \sum_{j'_1 j'_2} \sqrt{1+\delta_{j_1 j_2}} \sqrt{1+\delta_{j'_1 j'_2}}\, G_{s_1 s_2}^{(\lambda)}(j_1 j_2 j'_1\, j'_2; 2) B_\pi(j_1, j_2) B_\nu(j'_1, j'_2) d_\pi^\dagger \cdot \tilde{d}_\nu$$

The coefficients A, B are obtained by equating fermionic matrix elements in the Generalized Seniority (GS) basis with bosonic matrix elements, the so-called OAI mapping procedure ¶.

The basis

$$\left(S^\dagger\right)^{\frac{n-\nu}{2}} \left(D^\dagger\right)^{\frac{\nu}{2}} |0\rangle$$

is constructed with operators:

$$S_\pi^\dagger = \sum_j \alpha_j \sqrt{\frac{j+\frac{1}{2}}{2}} \left(\pi_j^\dagger \times \pi_j^\dagger\right)^{(0)}$$

$$D_\pi^\dagger = \sum_{j \le j'} \beta_{jj'} \frac{1}{\sqrt{1+\delta_{jj'}}} \left(\pi_j^\dagger \times \pi_{j'}^\dagger\right)^{(2)}$$

¶ T. Otsuka, A. Arima and F. Iachello, Nucl. Phys. A309, 1 (1978).

The structure coefficients α_j, $\beta_{jj'}$ are obtained by diagonalizing the surface delta interaction (SDI). The strength of the interaction, A_T, is chosen as to reproduce the 0-2 separation in the two-particle system.

The fermion matrix elements are calculated using the commutator method of Frank and Van Isacker and Lipas *et al.* [¶,§].

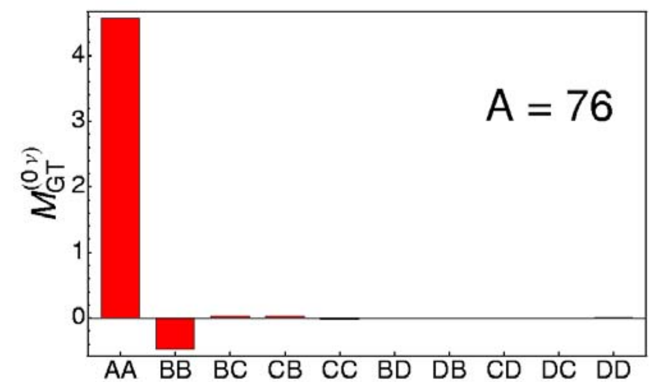
¶ A. Frank and P. Van Isacker, Phys. Rev. C26, 1661 (1982).

§ P.O. Lipas, M. Koskinen, H. Harter, R. Nojarov, and A. Faessler, Nucl. Phys. A508, 509 (1990).

Expansion to next to leading order (NLO) has been considered

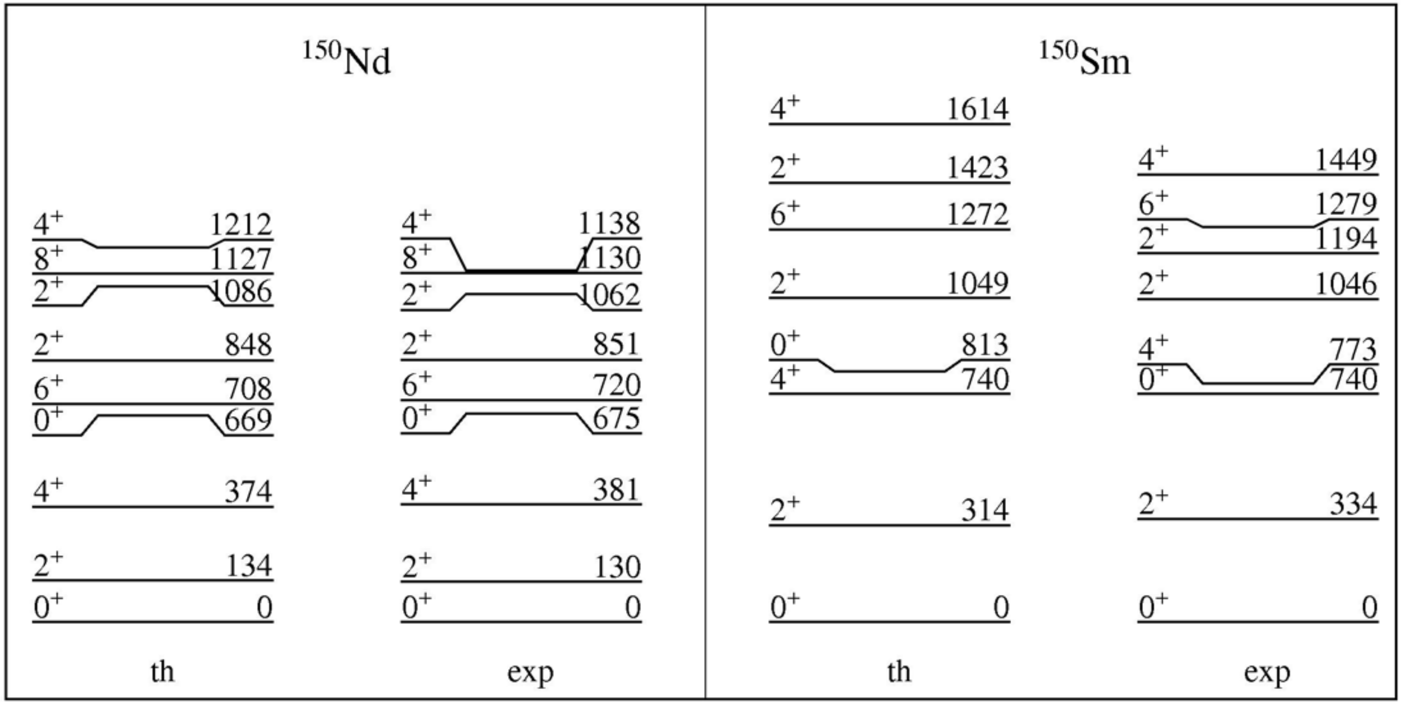
$$(\pi_j^\dagger \times \pi_{j'}^\dagger)_M^{(2)} \mapsto B_\pi(j,j')\left(d_\pi^\dagger\right)_M + C_\pi(j,j')s_\pi^\dagger\left(s_\pi^\dagger \tilde{d}_\pi\right)_M^{(2)} + D_\pi(j,j')s_\pi^\dagger\left(d_\pi^\dagger \tilde{d}_\pi\right)_M^{(2)}$$

Effect small <5%. Will be neglected henceforth.

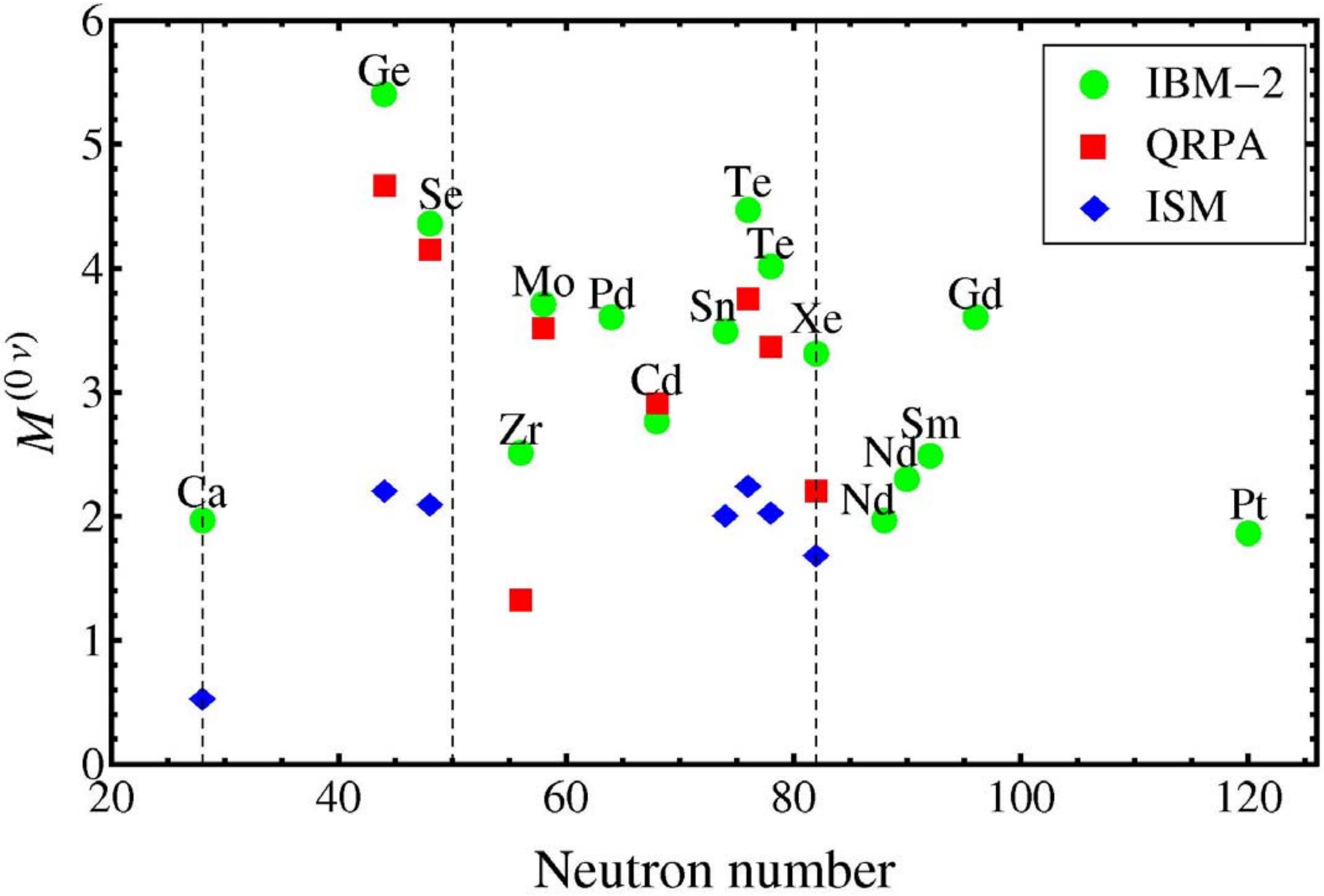


Matrix elements of the mapped operators are then evaluated with realistic wave functions of the initial and final nuclei taken from the literature. They fit all experimental data for excitation energies, B(E2) values and quadrupole moments, B(M1) values and magnetic moments, etc., very well.

Example:



IBM-2 RESULTS (2013)
LIGHT NEUTRINO EXCHANGE

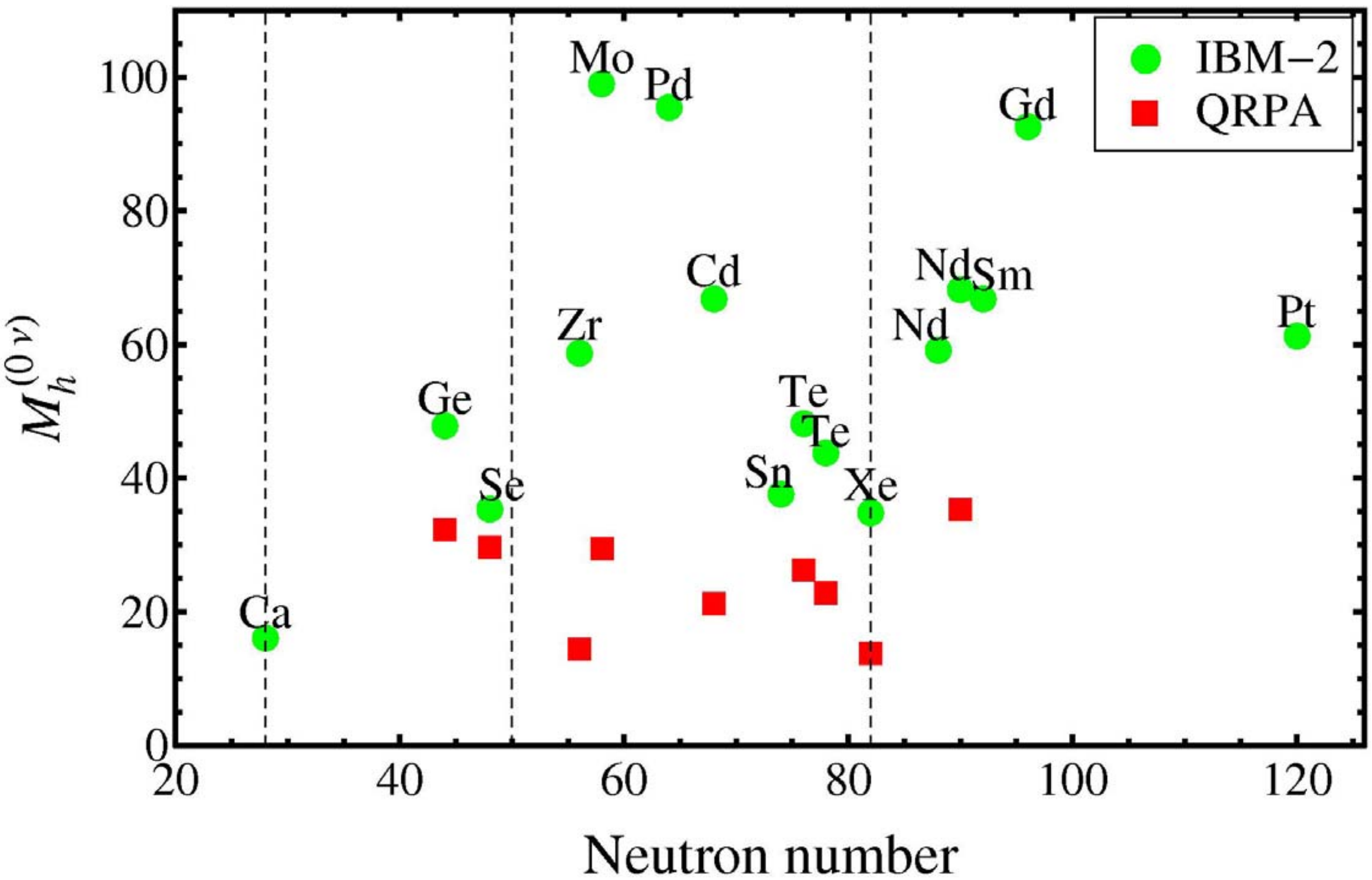


IBM-2 from J. Barea and F. Iachello, Phys. Rev. C 79, 044301 (2009); J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013). MS-SRC.

QRPA from F. Šimkovic *et al*., Phys. Rev. C 77, 045503 (2008).

ISM from E. Caurier *et al*., Phys. Rev. Lett. 100, 052503 (2008).

IBM-2 RESULTS (2013)
HEAVY NEUTRINO EXCHANGE



IBM-2 from J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013). MS-SRC.
QRPA from Šimkovic *et al.*, Phys. Rev. C 60, 055502 (1999). MS-SRC.

NUCLEAR MATRIX ELEMENTS TO 0_1

	$M^{(0\nu)}$		
	IBM-2§	QRPA¶	ISM*
$^{48}Ca \rightarrow ^{48}Ti$	1.98		0.54
$^{76}Ge \rightarrow ^{76}Se$	5.42	4.68	2.22
$^{82}Se \rightarrow ^{82}Kr$	4.37	4.17	2.11
$^{96}Zr \rightarrow ^{96}Mo$	2.53	1.34	
$^{100}Mo \rightarrow ^{100}Ru$	3.73	3.53	
$^{110}Pd \rightarrow ^{110}Cd$	3.62		
$^{116}Cd \rightarrow ^{116}Sn$	2.78	2.93	
$^{124}Sn \rightarrow ^{124}Te$	3.50		2.02
$^{128}Te \rightarrow ^{128}Xe$	4.48	3.77	2.26
$^{130}Te \rightarrow ^{130}Xe$	4.03	3.38	2.04
$^{136}Xe \rightarrow ^{136}Ba$	3.33	2.22	1.70
$^{148}Nd \rightarrow ^{148}Sm$	1.98		
$^{150}Nd \rightarrow ^{150}Sm$	2.32		
$^{154}Sm \rightarrow ^{154}Gd$	2.50		
$^{160}Gd \rightarrow ^{160}Dy$	3.62		
$^{198}Pt \rightarrow ^{198}Hg$	1.88		

§ J. Barea and F. Iachello, Phys. Rev. C79 (2009) 044301; J. Barea, J. Kotila and F. Iachello, Phys. Rev. C87, (2013) 014315. MS-SRC.

¶ F. Šimkovic, A. Faessler, V. Rodin, P. Vogel, and J. Engel, Phys. Rev. C77 (2008) 045503.

* E. Caurier, J. Menendez, F. Nowacki, and A. Poves, Phys. Rev. Lett. 100 (2008) 052503.

SENSITIVITY ANALYSIS (IBM-2): LIGHT NEUTRINO

Estimated sensitivity to input parameter changes:

1. Single-particle energies [¶,§]	10%
2. Strength of surface delta interaction	5%
3. Oscillator parameter	5%
4. Closure energy	5%

Estimated sensitivity to model assumptions:

1. Truncation to S, D space	1% (spherical)-10% (deformed)
2. Isospin purity	1%(GT)-20%(F)-1%(T)

Estimated sensitivity to operator assumptions:

1. Form of the operator	5%
2. Finite nuclear size (FNS)	2%
3. Short range correlations (SRC) [#]	10%

Total: 44%-55% (addition) or 16%-19% (quadrature).

[¶] This point has been emphasized by J. Suhonen and O. Civitarese, Phys. Lett. B668, 277 (2008).

[§] New experiments are being done to check the single particle levels in Ge, Se and Te, J.P. Schiffer *et al.*, Phys. Rev. Lett. **100**, 112501 (2008).

[#] This point is discussed in many articles, for example, M. Kortelainen and J. Suhonen, Phys. Rev. C 75, 051303 (R) (2007).

The sensitivity to SRC has been in recent years the subject of many investigations

For light neutrino exchange going from Miller-Spencer (MS)-soft to Argonne (CCM)-hard correlations introduces a factor of ~1.2.

For heavy neutrino exchange going from MS to CCM has a major effect introducing a factor ~2.5!

From J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013)

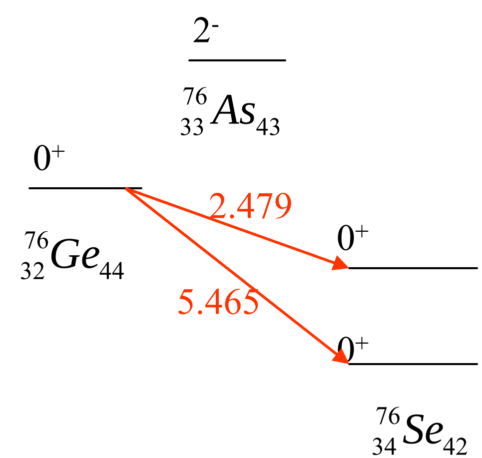
FINAL RESULTS WITH ESTIMATED ERROR (2013)

Decay	Light neutrino exchange	Heavy neutrino exchange
^{48}Ca	1.98(59)	16.3(95)
^{76}Ge	5.42(103)	48.1(255)
^{82}Se	4.37(83)	35.6(189)
^{96}Zr	2.53(40)	59.0(309)
^{100}Mo	3.73(60)	99.3(516)
^{110}Pd	3.62(58)	95.7(498)
^{116}Cd	2.78(44)	67.1(321)
^{124}Sn	3.50(67)	37.8(200)
^{128}Te	4.48(85)	48.4(257)
^{130}Te	4.03(77)	44.0(233)
^{136}Xe	3.33(63)	35.1(186)
^{148}Nd	1.98(32)	59.4(309)
^{150}Nd	2.32(37)	68.4(356)
^{154}Sm	2.50(40)	67.1(349)
^{160}Gd	3.62(58)	92.9(483)
^{198}Pt	1.88(30)	61.5(320)

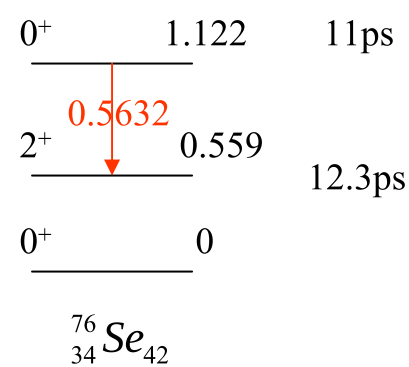
From J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013). MS-SRC.

MATRIX ELEMENTS TO EXCITED STATES

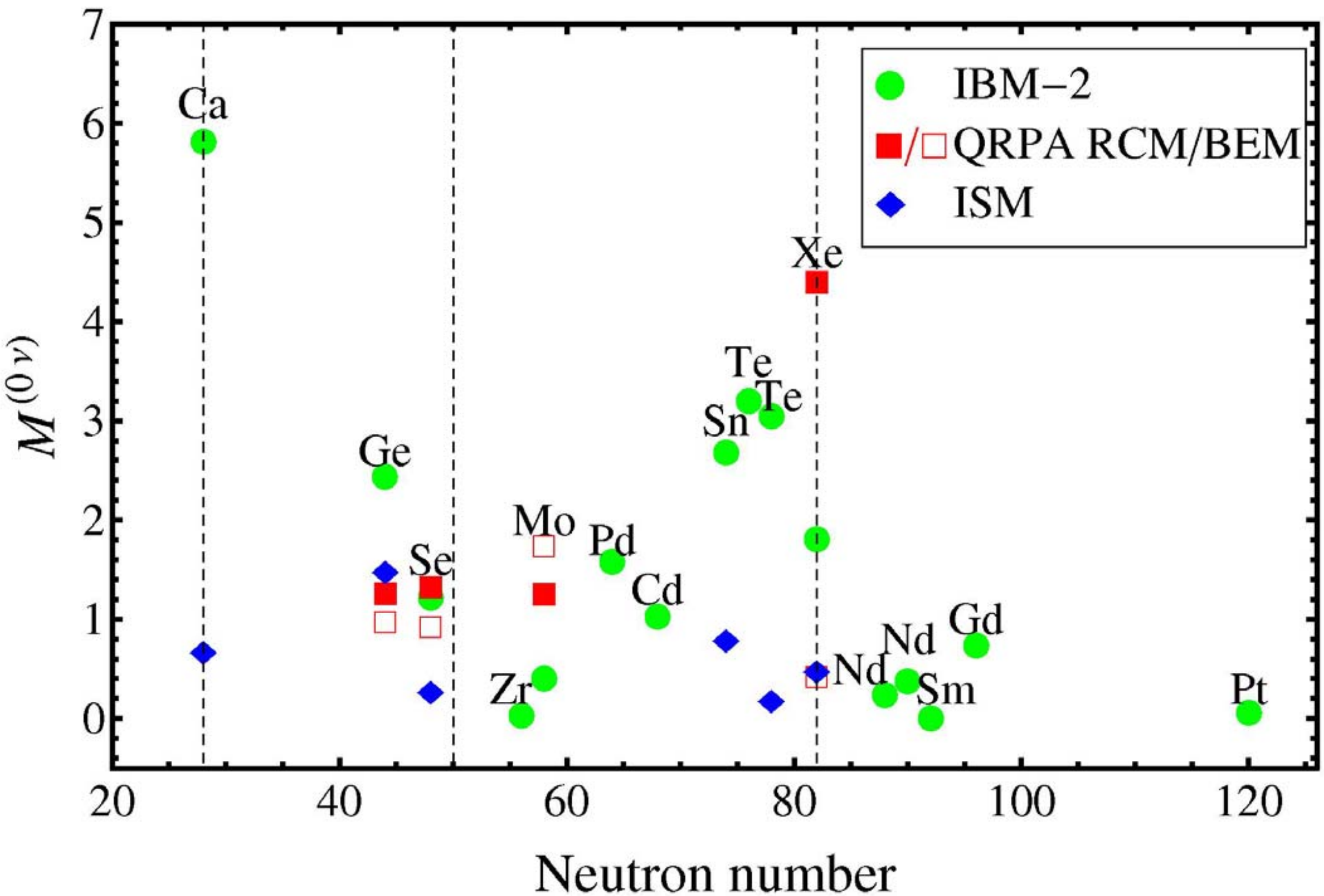
In some cases, the matrix elements to the first excited 0^+ state are large. Although the kinematical factor hinders the decay to the excited state, large matrix elements offer the possibility of a direct detection, by looking at the γ-ray de-exciting the 0^+ level.



[On the contrary, matrix elements to the excited 2^+ state are zero in lowest order since with two leptons in the final state we cannot form angular momentum 2.]



IBM-2 RESULTS (2013)
LIGHT NEUTRINO EXCHANGE
TO FIRST EXCITED 0^+ STATE



NUCLEAR MATRIX ELEMENTS TO 0_2

	$M^{(0\nu)}$		
	IBM-2[§]	QRPA[¶]	ISM[*]
$^{48}Ca \rightarrow ^{48}Ti$	5.83		0.68
$^{76}Ge \rightarrow ^{76}Se$	2.46	1.28	1.49
$^{82}Se \rightarrow ^{82}Kr$	1.23[a]	1.34	0.28
$^{96}Zr \rightarrow ^{96}Mo$	0.04		
$^{100}Mo \rightarrow ^{100}Ru$	0.99	1.27	
$^{110}Pd \rightarrow ^{110}Cd$	0.46		
$^{116}Cd \rightarrow ^{116}Sn$	0.85		
$^{124}Sn \rightarrow ^{124}Te$	2.70		0.80
$^{128}Te \rightarrow ^{128}Xe$	3.22[a]		
$^{130}Te \rightarrow ^{130}Xe$	3.07		0.19
$^{136}Xe \rightarrow ^{136}Ba$	1.82	4.42	0.49
$^{148}Nd \rightarrow ^{148}Sm$	0.25		
$^{150}Nd \rightarrow ^{150}Sm$	0.39		
$^{154}Sm \rightarrow ^{154}Gd$	0.02		
$^{160}Gd \rightarrow ^{160}Dy$	0.75		
$^{198}Pt \rightarrow ^{198}Hg$	0.08[a]		

[a] negative Q-value

[§] J. Barea, J. Kotila and F. Iachello, Phys. Rev. C87 (2013) 014315.
[¶] F. Šimkovic, M. Nowak, W.A. Kaminsky, A.A. Raduta, and A. Faessler, Phys. Rev. C64 (2001) 035501(QRPA-RCM).
[*] J. Menendez, A. Poves, E. Caurier, F. Nowacki, Nucl. Phys. A818 (2009) 139.

DOUBLE POSITRON DECAY

NUCLEAR MATRIX ELEMENTS

Nuclear matrix elements for $0\nu\beta^+\beta^+/0\nu\beta^+EC$ decay have been calculated. The matrix elements are of the same order of magnitude of $0\nu\beta^-\beta^-$ decay.

Decay	0_1^+			0_2^+	
	IBM-2	QRPA		IBM-2	QRPA
^{58}Ni	2.31(37)	1.55		2.24(36)	
^{64}Zn	6.13(116)			0.84(16)	
^{78}Kr	4.48(85)	4.19		1.01(19)	
^{96}Ru	2.48(40)	3.25	3.22-5.83	0.05(1)	1.28-2.26
^{106}Cd	3.11(50)	4.12	5.94-9.08	1.54(25)	0.66-0.91[c]
^{124}Xe	5.16(98)	4.78		0.84(16)	
^{130}Ba	5.04(96)	4.98		0.37(7)	
^{136}Ce	4.90(93)	3.09		0.41(8)	

IBM-2: J. Barea, J. Kotila and F. Iachello, Phys. Rev. C87, 057301 (2013).

QRPA: M. Hirsch, K. Muto, T. Oda, and H.V. Klapdor-Kleingrothaus, Z. Phys. A347, 151 (1994).

QRPA-Jy: J. Suhonen, Phys. Rev. C86, 024301 (2012); J. Suhonen, Phys. Lett. B701, 490 (2011).

RESONANT DOUBLE ELECTRON CAPTURE

NUCLEAR MATRIX ELEMENTS

Nuclear matrix elements for double electron capture 0νECEC have been calculated

TABLE I. Comparison between IBM-2 matrix elements with Argonne SRC for $0\nu ECEC$ decay and QRPA and EDF.

Decay	$M^{(0\nu)}$ (light)			
	IBM-2	spherical QRPA[a]	deformed QRPA[a]	EDF[b]
${}^{152}_{64}Gd_{88} \rightarrow {}^{152}_{62}Sm_{90}$	2.44	7.59	3.23-2.67	1.07-0.89
${}^{164}_{68}Er_{96} \rightarrow {}^{164}_{66}Dy_{98}$	3.95	6.12	2.64-1.79	0.64-0.50
${}^{180}_{74}W_{106} \rightarrow {}^{180}_{72}Hf_{108}$	4.67	5.79	2.05-1.79	0.58-0.38

[a] D.-L Fang *et al.*, Phys. Rev. C **85**, 035503 (2012).
[b] T. R. Rodríguez and G. Martínez-Pinedo, Phys. Rev. C **85**, 044310 (2012).

RENORMALIZATION OF g_A

Results in the previous slides are obtained with $g_A=1.269$.

It is well-known from single β-decay/EC and from 2νββ that g_A is renormalized in models of nuclei. Two reasons:

(i) Limited model space

(ii) Omission of non-nucleonic degrees of freedom (Δ, N^*,…)

For each model (ISM/QRPA/IBM-2) one can define an effective $g_{A,eff}$ by writing

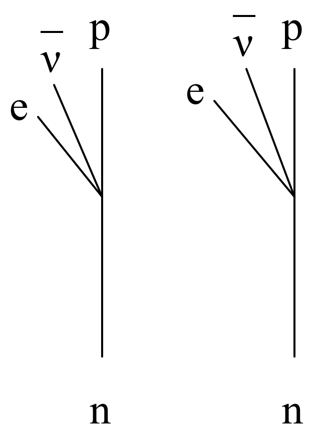
$$M_{2\nu}^{eff} = \left(\frac{g_{A,eff}}{g_A}\right)^2 M_{2\nu}$$

$$M_{\beta/EC}^{eff} = \left(\frac{g_{A,eff}}{g_A}\right) M_{\beta/EC}$$

The value of $g_{A,eff}$ in each nucleus can then be obtained by comparing the calculated and measured half-lives for β/EC and for 2νββ.

APPROXIMATE ESTIMATE FROM 2νββ IN THE CLOSURE APPROXIMATION

An *approximate* estimate of the renormalization effect can be done from a study of 2νββ in the *closure* approximation.
To this end, a simultaneous calculation of NME in 2νββ decay in the closure approximation has been completed, as well as a novel calculation of 2νββ phase space factors.



The calculation of 2νββ in the closure approximation is similar to that of 0νββ except that the neutrino "potential" is different

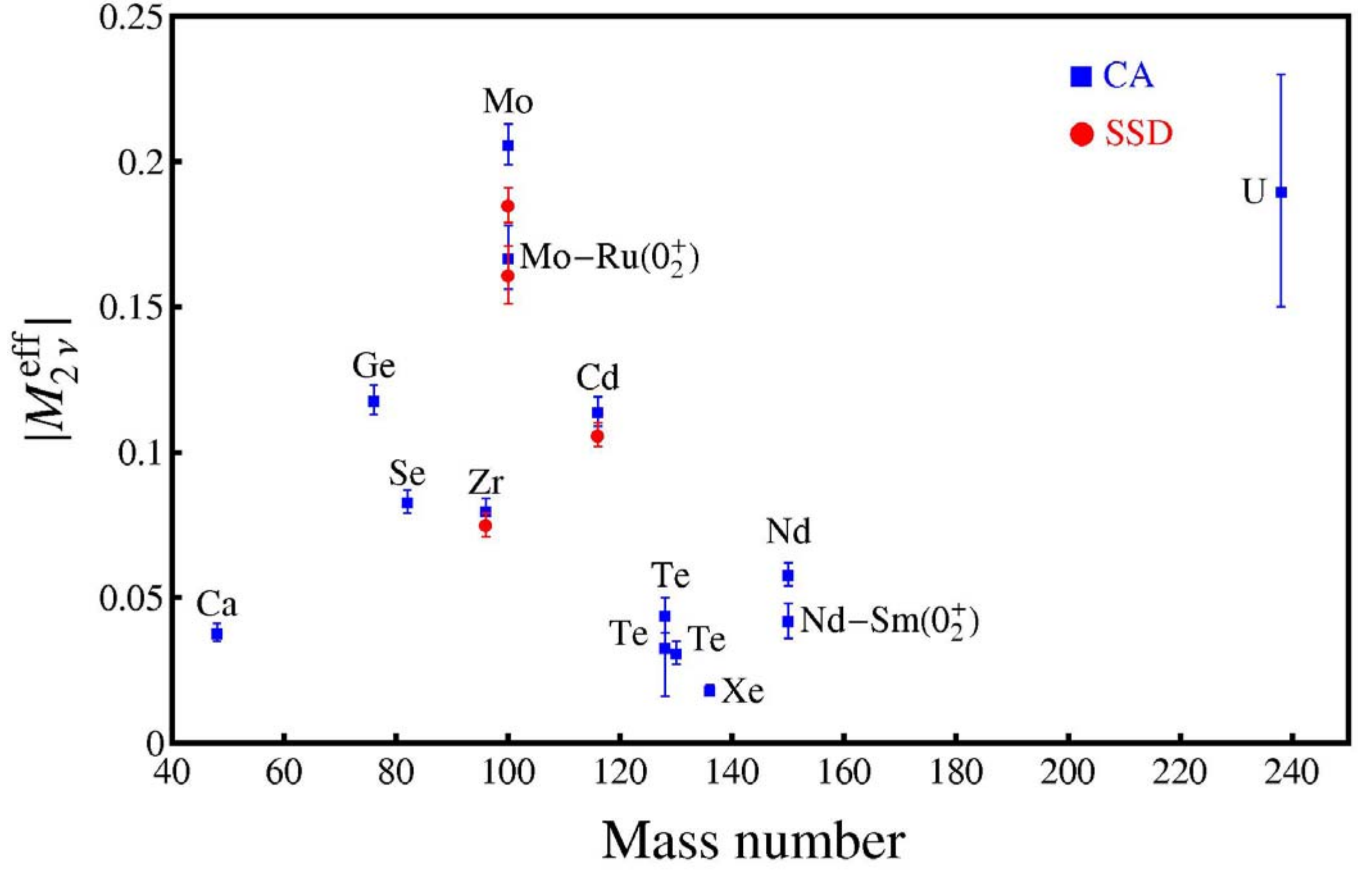
$$v^{(2\nu)}(p) = \frac{\delta(p)}{p^2}$$

In the IBM-2 approach in momentum space all calculations, 0ν-light-neutrino, heavy-neutrino, (Majoron, sterile neutrinos, …) and 2ν can be done at the same time, by changing the neutrino "potential", and thus eliminating possible sources of systematic and accidental error.

The half-lives for 2νββ are calculated using

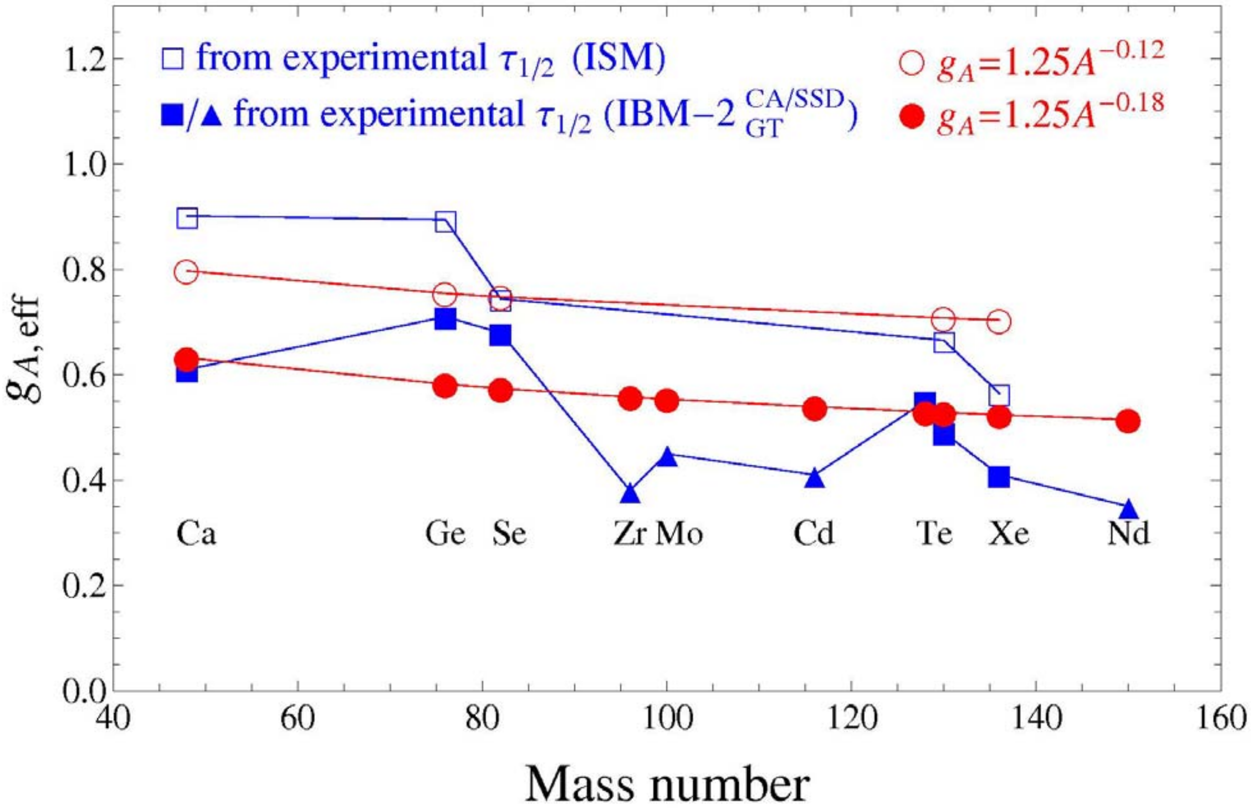
$$\left[\tau_{1/2}^{2\nu}\right]^{-1} = G_{2\nu}\left|m_e c^2 M_{2\nu}\right|^2$$

By comparing the values of $|M_{2\nu}^{eff}|$ obtained from experimental half-lives



with those calculated in IBM-2 (or other models), one can obtain the values of $g_{A,eff}$.

Effective axial vector coupling constant in nuclei from 2νββ ¶



One obtains $g_{A,eff}^{IBM\text{-}2}$~0.6-0.5.

The extracted values can be parametrized as

$$g_{A,eff}^{IBM2} = 1.269 A^{-0.18}$$

A similar analysis can be done for the ISM for which $g_{A,eff}^{ISM}$~0.8-0.7.

$$g_{A,eff}^{ISM} = 1.269 A^{-0.12}$$

¶ J. Barea, J. Kotila and F. Iachello, Phys. Rev. C 87, 014315 (2013).

$g_{A,eff}$, has been extracted also from single β/EC in QRPA, very recently by Suhonen and Civitarese (QRPA-Jy), $g_{A,eff}^{QRPA} \sim 0.8-0.4$ [§], and a few years ago by Faessler *et al.* (QRPA-Tü) ~ 0.7 [*].

[In some earlier QRPA papers[¶] , it is claimed that no renormalization of g_A is needed. However, this claim is based on results where the renormalization of g_A is transferred to a renormalization of the free parameter g_{pp} used in the calculation and adjusted to the experimental 2νββ half-life.]

§ J. Suhonen and O. Civitarese, Phys. Lett. B 725, 153 (2013).
* A Faessler, G.L. Fogli, E. Lisi, V. Rodin, A.M. Rotunno, and F. Šimkovic, J. Phys. G: Nucl. Part. Phys. 35, 075104 (2008).

¶ K. Muto, E. Bender, H.V. Klapdor, Z. Phys. A334, 177 (1989); 187 (1989), as quoted by M. Hirsch (2014).

An “exact” extraction of $g_{A,eff}$ has also recently been done¶ in IBFM-2 both from single β/EC and from 2νββ decay and is given in Appendix B. The extracted values of g_A are ~0.4!

¶ N. Yoshida and F. Iachello, Prog. Theor. Exp. Phys. 2013, 043D01 (2013).

IMPACT OF THE RENORMALIZATION

The axial vector coupling constant, g_A, appears to the second power in the NME

$$M_{2\nu} = g_A^2 M^{(2\nu)}$$

$$M_{0\nu} = g_A^2 M^{(0\nu)}$$

$$M^{(0\nu)} = M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$

and hence to the fourth power in the half-life!

Therefore, the results of the previous slides should be multiplied by 4-16 to have realistic estimates of expected half-lives. [See also, H. Robertson ¶, and S. Dell'Oro, S. Marcocci, F. Vissani#.]

¶ R.G.H. Robertson, Modern Phys. Lett. A 28, 1350021 (2013).

S. Dell'Oro, S. Marcocci, and F. Vissani, Phys. Rev. D90, 033005 (2014).

The question of whether or not g_A in 0νββ is renormalized as much as in 2νββ is of much debate. In 2νββ only the 1^+ (GT) multipole contributes. In 0νββ all multipoles 1^+, 0^+, 2^-, 1^- … contribute. Some of these could be unquenched. However, even in 0νββ, 1^+ intermediate states dominate. Hence, our current understanding is that g_A is renormalized in 0νββ as much as in 2νββ.

This problem is currently being addressed from various sides. Experimentally by measuring the matrix elements to and from the intermediate odd-odd nucleus in 2νββ decay [§]. Theoretically, by using effective field theory (EFT) to estimate the effect of non-nucleonic degrees of freedom (two-body currents) [¶].

[§] P. Puppe *et al.*, Phys. Rev. C 86, 044603 (2012).

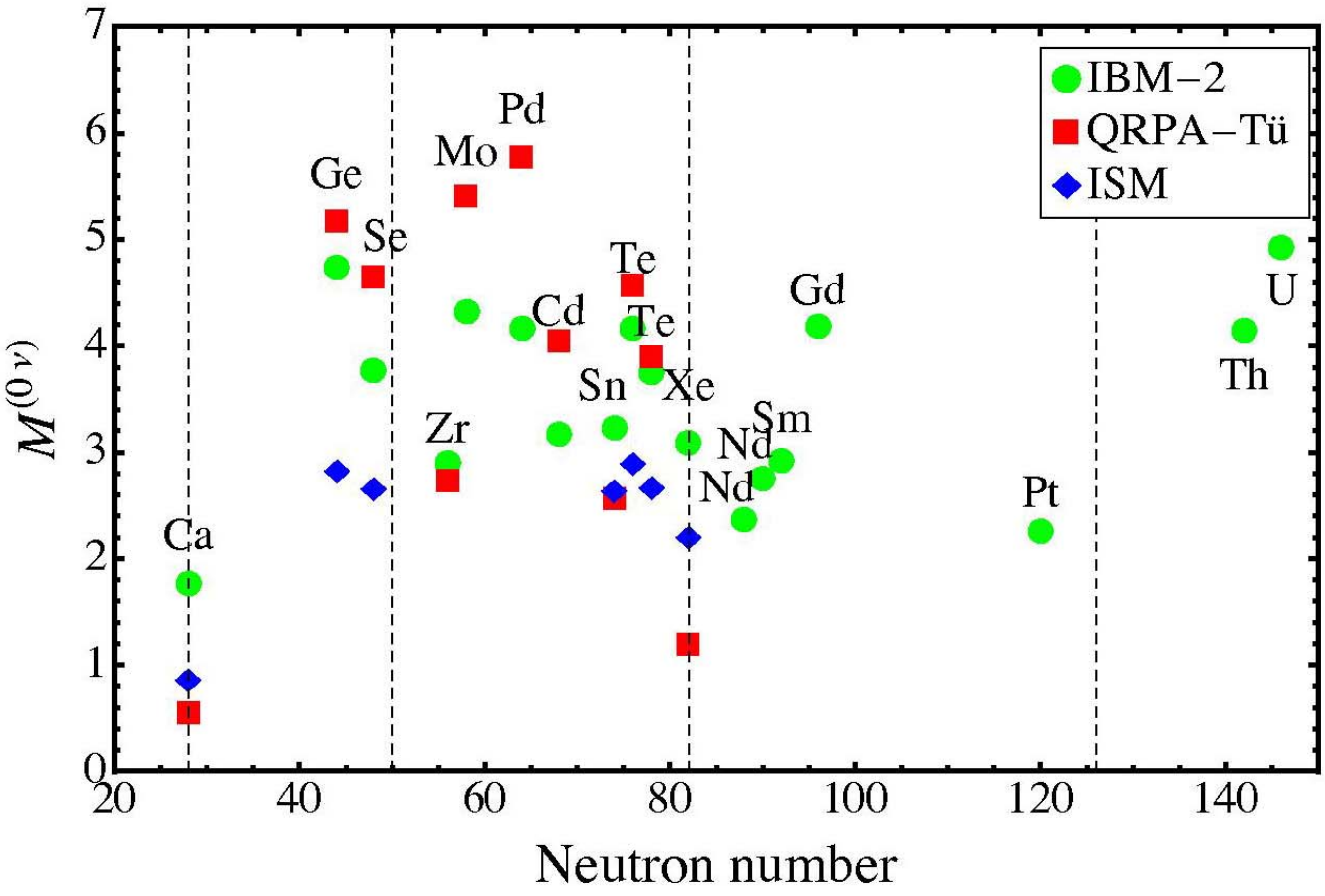
[¶] J. Menendez, D. Gazit, and A. Schwenk, Phys. Rev. Lett. 107, 062501 (2011).

Another question is whether or not the vector coupling constant, g_V, is renormalized in nuclei.
Because of CVC, the mechanism (ii) omission of non-nucleonic degrees of freedom cannot contribute.
However, the mechanism (i), limited model space, can contribute, and, if so, the ratio g_V/g_A may remain the same as the non-renormalized ratio 1/1.269.
No experimental information is available, but is could be obtained by measuring, with (^{3}He,t) and (d,^{2}He) reactions, the F matrix elements to and from the intermediate odd-odd nucleus.
Also measurements of double charge exchange reactions with heavy ions at LNS (Catania) could help understanding this question.

NME: LATEST RESULTS (2015)

In view of the fact that heavy neutrino exchange has become again of interest, and that a reanalysis of other experiments indicates that hard SRC correlations describe best the data, new calculations (2014) have been done both in QRPA and in IBM-2 with Argonne SRC.
Also a problem with isospin projection has been corrected both in QRPA and IBM-2.

IBM-2 RESULTS (2015): LIGHT NEUTRINO EXCHANGE



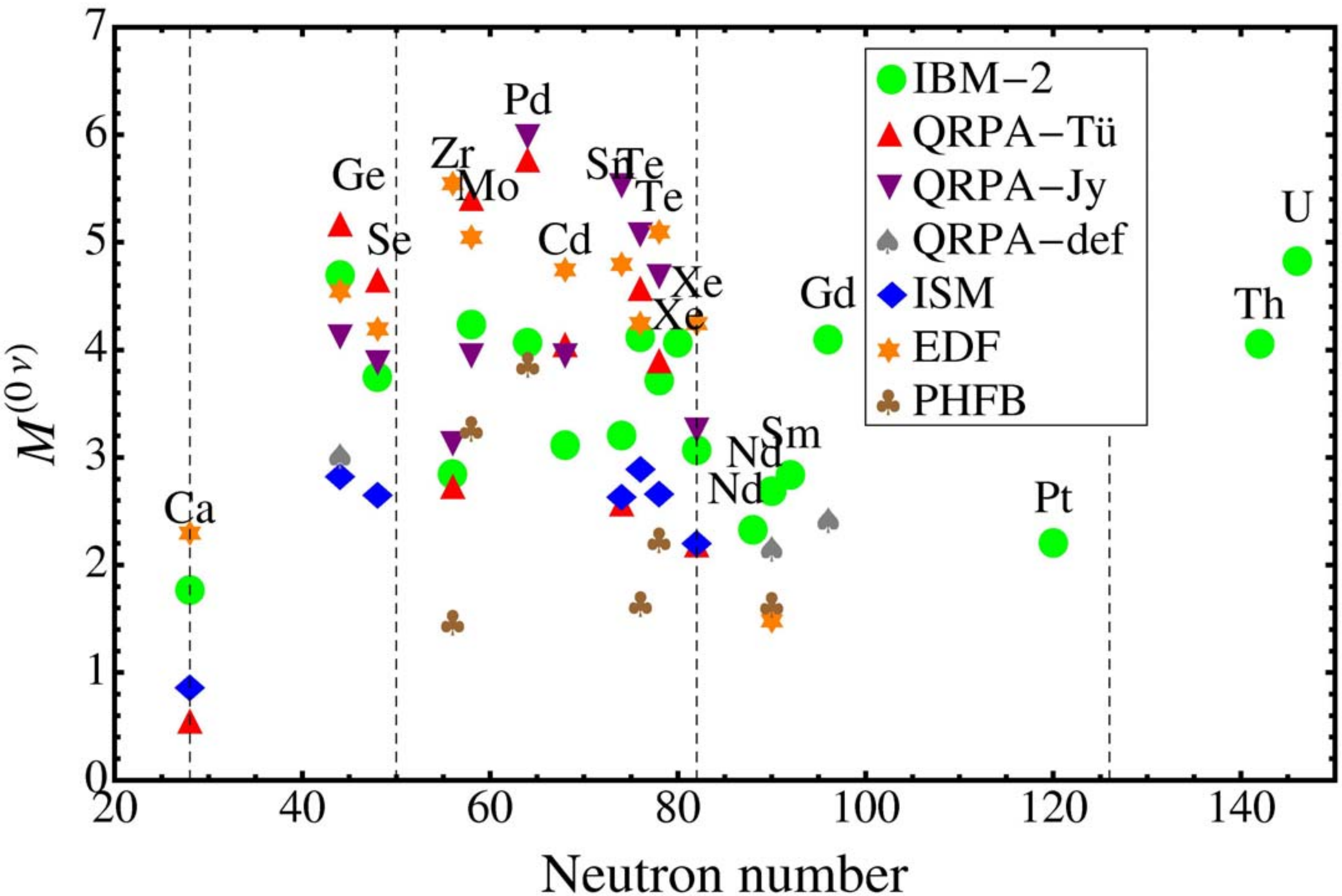
¶ J. Barea, J. Kotila and F. Iachello, unpublished (2015).

\# F. Šimkovic, V. Rodin, A. Faessler, and P. Vogel, Phys. Rev. C 87, 045501 (2013). QRPA with isospin restoration and Argonne SRC.

FINAL IBM-2 RESULTS WITH ERROR (2015)

Decay	Light neutrino exchange	Heavy neutrino exchange
^{48}Ca	1.75(28)	47(13)
^{76}Ge	4.68(75)	104(29)
^{82}Se	3.73(60)	83(23)
^{96}Zr	2.83(45)	99(28)
^{100}Mo	4.22(68)	164(46)
^{110}Pd	4.05(65)	154(43)
^{116}Cd	3.10(50)	110(31)
^{124}Sn	3.19(51)	79(22)
^{128}Te	4.10(66)	101(28)
^{130}Te	3.70(59)	92(26)
^{134}Xe	4.05(65)	91(26)
^{136}Xe	3.05(59)	73(20)
^{148}Nd	2.31(37)	103(29)
^{150}Nd	2.67(43)	116(32)
^{154}Sm	2.82(45)	113(32)
^{160}Gd	4.08(65)	155(43)
^{198}Pt	2.19(35)	104(29)
^{232}Th	4.04(65)	159(45)
^{238}U	4.81(77)	189(53)

APPENDIX A: SUMMARY OF MATRIX ELEMENTS (2015)



APPENDIX B: ESTIMATE FROM $2\nu\beta\beta$ IN THE "EXACT" NON-CLOSURE CALCULATION

A program has been written to calculate $2\nu\beta\beta$ "exactly" in IBFFM-2 by summing over intermediate states in the odd-odd nucleus (Yoshida, 2012).

Steps in this calculation are:

1. Calculation of spectra of the initial and the final even-even nuclei, in IBM-2.
2. (Calculation of spectra of adjacent odd-even and even-odd nuclei, in IBFM-2, to determine the strength of the boson-fermion interaction).
3. Calculation of spectra of the intermediate odd-odd nuclei, in IBFFM-2.
4. Calculation of GT and F matrix elements from even-even to odd-odd and from odd-odd to even-even.
5. Sum of product with PSF over states in the intermediate nucleus. Approximately 150 states are included.

Results for ^{128}Te→^{128}Xe and ^{130}Te→^{130}Xe decay¶ .

Te→Xe

Te→I

I→Xe

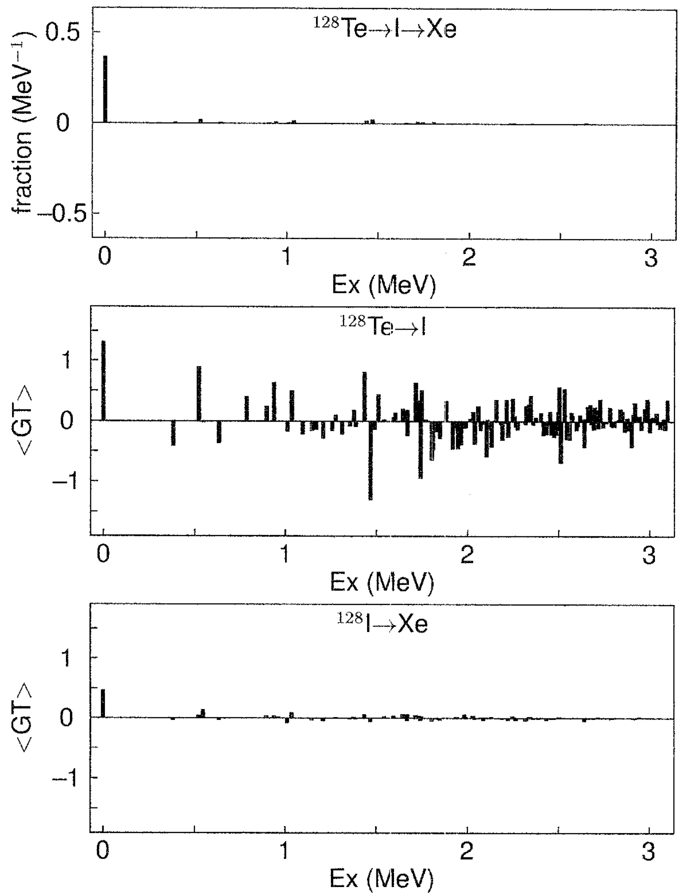


Figure 2: The values of $\langle 0_1^+||t^+\sigma||1_N^+\rangle\langle 1_N^+||t^+\sigma||0_1^+\rangle/(\frac{1}{2}W_0 + E_N - E_I)$ (top), $\langle 1_N^+||t^+\sigma||0_1^+\rangle$ (center), and $\langle 0_1^+||t^+\sigma||1_N^+\rangle$ (bottom), for the double β decay from the lowest 0^+ in ^{128}Te to the lowest 0^+ in ^{128}Xe through the intermediate 1^+ in ^{128}I, plotted as a function of the excitation energy of 1^+.

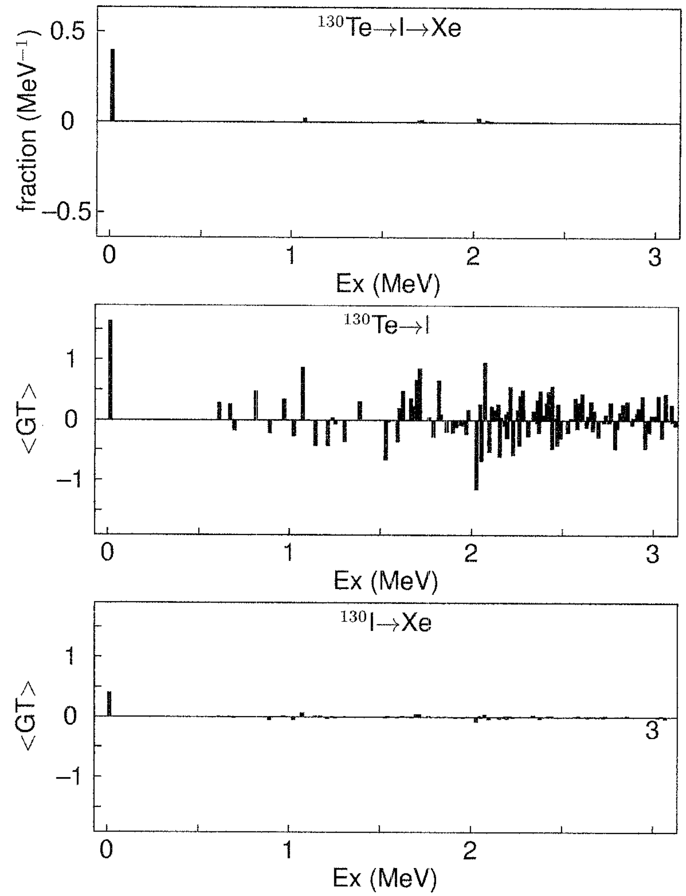
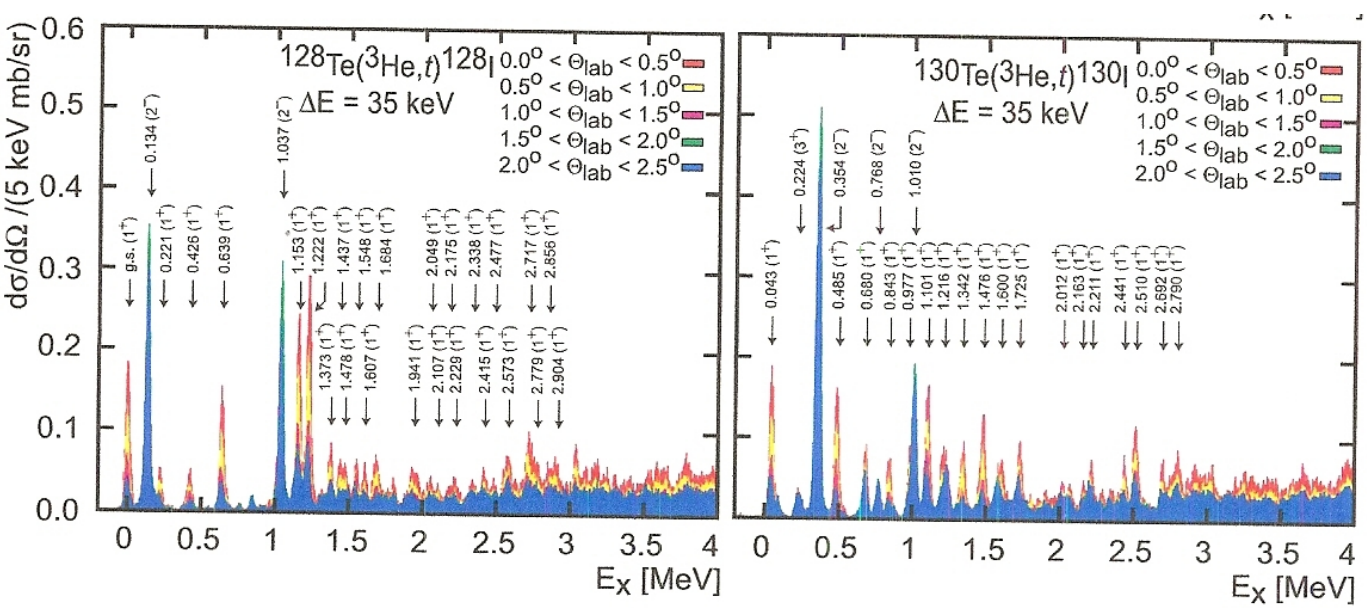


Figure 3: The same plots as Fig. 2 for the decay ^{130}Te→^{130}Xe through ^{130}I.

¶ N. Yoshida and F. Iachello, Prog. Theor. Exp. Phys. 2013, 043D01 (2013).

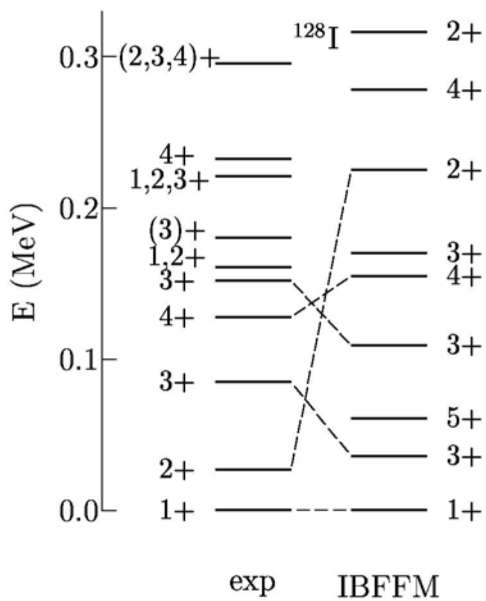
The calculation Te→I can be compared with recent experiment [§]



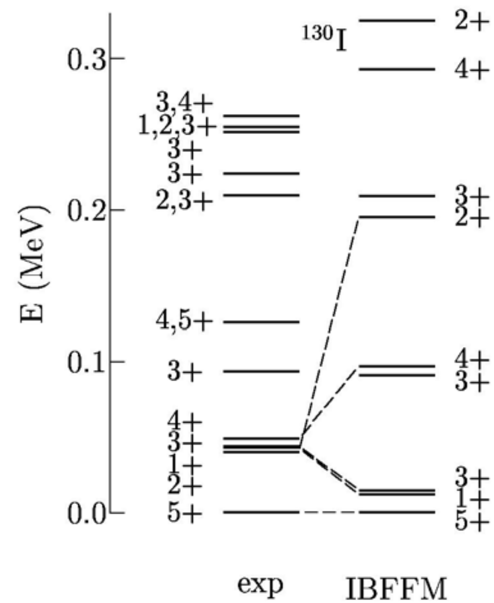
[§] P. Puppe *et al.*, Phys. Rev. C 86, 044603 (2012); D. Frekers, private communication

Properties of the strength distribution are “robust”, but its details depend on the actual values of the single particle energies and of the strength of the interactions. The calculated odd-odd spectra are in fair agreement with experiment.

128



130



Note that Yoshida correctly calculates the g.s. of ^{130}I to be 5^+. He also calculates correctly its magnetic moment.

$\mu(5_1^+)_{th} = 3.12$

$\mu(5_1^+)_{\exp} = 3.349(7)$

The extracted values of $g_{A,eff}$ are of order ~0.4.