DOUBLE BETA DECAY AND NEUTRINO MASSES

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Lecture 1

INTRODUCTION

Double beta decay is a process in which a nucleus (A,Z) decays to another nucleus (A,Z±2) by emitting two electrons or positrons, and, usually, other light particles:

$$(A,Z) \rightarrow (A,Z \pm 2) + 2e^{\mp} + anything$$

The processes where two neutrinos (or antineutrinos) are emitted

$$(A,Z) \rightarrow (A,Z+2) + 2e^{-} + 2\overline{\nu} \qquad (2\nu\beta^{-}\beta^{-})$$

are predicted by the standard model. Indeed, the study of this process was suggested by Maria Goeppert-Meyer[§] in 1935, shortly after the Fermi theory of beta decay appeared (1934). It took however more than 50 years to observe it (Elliott *et al.*, 1987) in view of its very long half-life

$$\tau_{1/2}^{2\nu}({}^{100}Mo) = (7.1 \pm 0.4) \times 10^{18}\, yr$$

[§]M. Goeppert-Meyer, Phys. Rev. 48, 512 (1935)

Now (2014) $2\nu\beta^-\beta^-$ has been observed in 10 nuclei.
[The positron emitting and related processes $2\nu\beta^+\beta^+$, $2\nu\beta^+EC$, $2\nu ECEC$ has been observed only in 1 nucleus (^{130}Ba).]
The measured half-lives are

$$\tau_{1/2}^{2\nu} \sim (10^{18} - 10^{21})\, yr$$

The processes where no neutrinos are emitted

$$(A,Z) \rightarrow (A,Z+2) + 2e^- \qquad (0\nu\beta^-\beta^-)$$

$0\nu\beta^-\beta^-$, and $0\nu\beta^+\beta^+$,$0\nu\beta^+EC$,$0\nu ECEC$, are forbidden by the standard model, and, if observed, will provide evidence for physics beyond the standard model, in particular will determine whether or not the neutrino is a Majorana particle and will measure its mass.

Majorana[§] (1937) suggested that neutral particles could be their own antiparticles and Racah[¶] (1937) pointed out that the neutron cannot be its own antiparticle since it has a magnetic moment, while the neutrino could be such a particle.

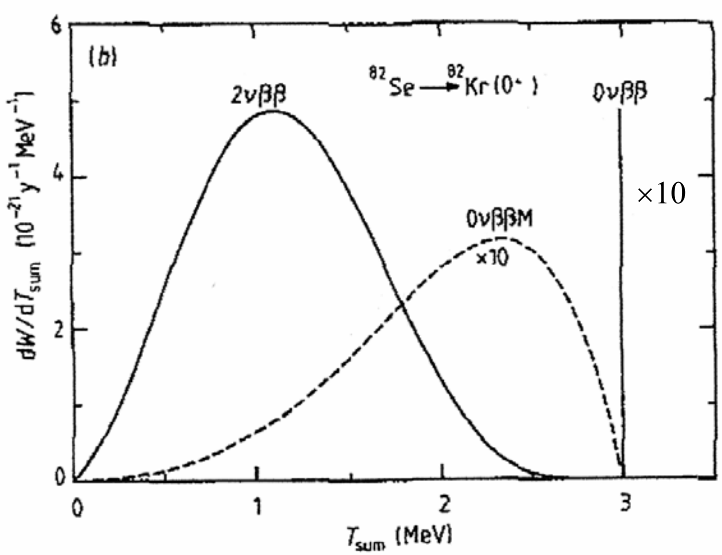
A major experimental effort started a few years ago to detect neutrinoless DBD. All experiments so far have given negative results, with the exception of Klapdor-Kleingrothaus *et al.*, 2004. This result has however been very recently (2013) disproved.

[§] E. Majorana, Nuovo Cimento 14, 171 (1937).
[¶] G. Racah, Nuovo Cimento 14, 322 (1937).

Neutrino less DBD remains therefore one of the most fundamental problems in physics today. Its detection will be crucial for understanding whatever physics is beyond the standard model (SM) and is currently the subject of many experiments.

In addition to the fact that the expected half-life is very long, a major problem is the concomitance of the 2ν process

Summed energy spectra of the two emitted electrons →



In order to be able to extract the neutrino mass if DBD is observed, or to put a limit on its value if it is not observed, one needs a theory of $0\nu\beta\beta$ and of its concomitant process $2\nu\beta\beta$.

For processes allowed by the standard model, the half-life can be, to a good approximation, factorized in the form

$$\left[\tau_{1/2}^{2\nu}\right]^{-1} = G_{2\nu}\left|M_{2\nu}\right|^2$$

For processes not allowed by the standard model, the half-life can be factorized as

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^2\left|f(m_i, U_{ei})\right|^2$$

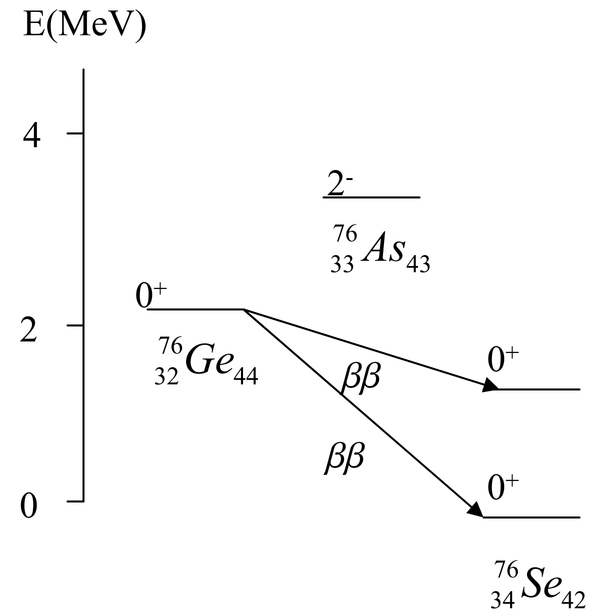
except for 0νECEC, which is forbidden by energy and momentum conservation but can occur under resonance conditions. In this case the inverse half-life is given by

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^2\left|f(m_i, U_{ei})\right|^2 \frac{(m_e c^2)\Gamma}{\Delta^2 + (\Gamma^2 / 4)}$$

For processes not allowed by the standard model one needs to derive the function $f(m_i,U_{ei})$ (lecture 1).
For all processes one needs to calculate the NME (lecture 2) and the PSF (lecture 3).

BRIEF THEORY OF 0νββ

$${}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+2}Y_{N-2} + 2e^{-}$$



Half-life for the process:

$$\left[\tau_{1/2}^{0\nu\beta\beta}(0^{+} \rightarrow 0^{+})\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^{2}\left|f(m_{i},U_{ei})\right|^{2}$$

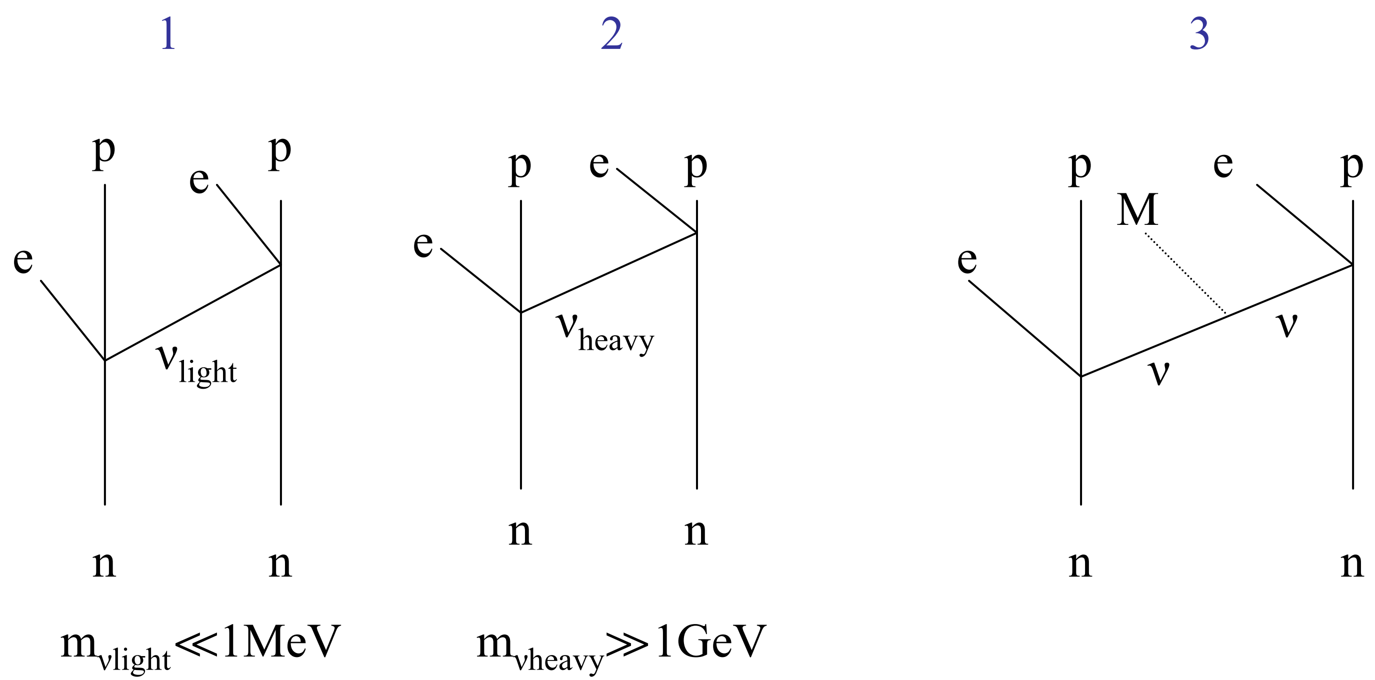
Beyond the standard model
(Particle physics)

Matrix elements
(Nuclear physics)

Phase-space factor
(Atomic physics)

TRANSITION OPERATOR AND F FUNCTION

The transition operator *T(p)* depends on the model of 0νββ decay. Three scenarios have been considered [#,¶,§].



[#] M. Doi *et al*, Prog. Theor. Phys. 66, 1739 (1981); 69, 602 (1983).

[¶] T.Tomoda, Rep. Prog. Phys. 54, 53 (1991).

[§] F.Šimkovic *et al.*, Phys. Rev. C60, 055502 (1999).

To derive the expression for T(p) one starts from the weak interaction Hamiltonian

$$H^{\beta} = \frac{G_F}{\sqrt{2}}\left[\overline{e}\gamma_{\mu}\left(1-\gamma_5\right)\nu_{eL}\right]J_L^{\mu\dagger} + h.c.$$

and the nucleon current §

$$J_L^{\mu\dagger} = \overline{\Psi}\tau^{+}\left[g_V(q^2)\gamma^{\mu} - ig_M(q^2)\frac{q_{\nu}}{2m_p}\sigma^{\mu\nu} - g_A(q^2)\gamma^{\mu}\gamma_5 + g_P(q^2)\frac{q^{\mu}}{2m_p}\gamma_5\right]\Psi$$

vector weak-magnetism axial vector induced pseudo-scalar

HOC

q^{μ} = momentum transferred from hadrons to leptons

§ F. Šimkovic *et al.*, loc.cit.

[Tomoda ¶ also considered right-handed couplings]

¶ T. Tomoda, loc. cit.

From the weak interaction Hamiltonian, H^{β}, and the weak nucleon current, J^{μ}, one finds the transition operator, *T(p)*, which can be written as ($p = |\vec{q}|$)

$$T(p) = H(p) f(m_i, U_{ei})$$

Scenario 1: LIGHT NEUTRINO EXCHANGE

$$f = \frac{\langle m_\nu \rangle}{m_e}$$

$$\langle m_\nu \rangle = \sum_{k=ligth} (U_{ek})^2 m_k$$

In momentum space and including higher order corrections (HOC), *H(p)* can be written as [§]

$$H(p) = \sum_{n,n'} \tau_n^+ \tau_{n'}^+ [-h^F(p) + h^{GT}(p) \vec{\sigma}_n \bullet \vec{\sigma}_{n'} + h^T(p) S_{nn'}^p]$$

[The general formulation of Tomoda [¶] includes more terms, nine in all, 3GT, 3F, 1T, one pseudoscalar (P) and one recoil (R). This formulation is no longer used but it will have to be revisited if a very accurate description of 0νββ is needed.]

[§] F. Šimkovic *et al.*, Phys. Rev. C60, 055502 (1999). [¶] T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).

The form factors $h^{F,GT,T}(p)$ are given by:

$$h^{F,GT,T}(p) = v(p)\tilde{h}^{F,GT,T}(p)$$

with

$$v(p) = \frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}$$

Ã=closure energy=1.12A$^{1/2}$(MeV)

called neutrino "potential", and $\tilde{h}(p)$ listed by Šimkovic *et al.*

This form assumes the closure approximation, which is expected to be very good for 0νββ decay.

The finite nucleon size (FNS) is taken into account by taking the coupling constants, g_V and g_A, momentum dependent

Primitive (V-A)

$$g_V(p^2) = g_V \frac{1}{\left(1+\frac{p^2}{M_V^2}\right)^2}$$

$$g_A(p^2) = g_A \frac{1}{\left(1+\frac{p^2}{M_A^2}\right)^2}$$

$$g_V = 1; M_V^2 = 0.71\left(GeV/c^2\right)^2$$

$$g_A = 1.269; M_A^2 = 1.09\left(GeV/c^2\right)^2$$

Induced (HOC)

$$g_M(p^2) = \left(\mu_p - \mu_n\right) g_V(p^2)$$

$$g_P(p^2) = \left(2m_p\right)^2 g_A(p^2) \frac{\left(1-\frac{m_\pi^2}{M_A^2}\right)}{\left(p^2+m_\pi^2\right)}$$

Short range correlations (SRC) are taken into account by convoluting the "potential" *v(p)* with the Jastrow function *j(p)* parametrized in various forms (Miller-Spencer, MS/ Argonne/CD Bonn) or by other methods (UCOM)

$$u(p) = \int v(p - p')\, j(p')\, dp'$$

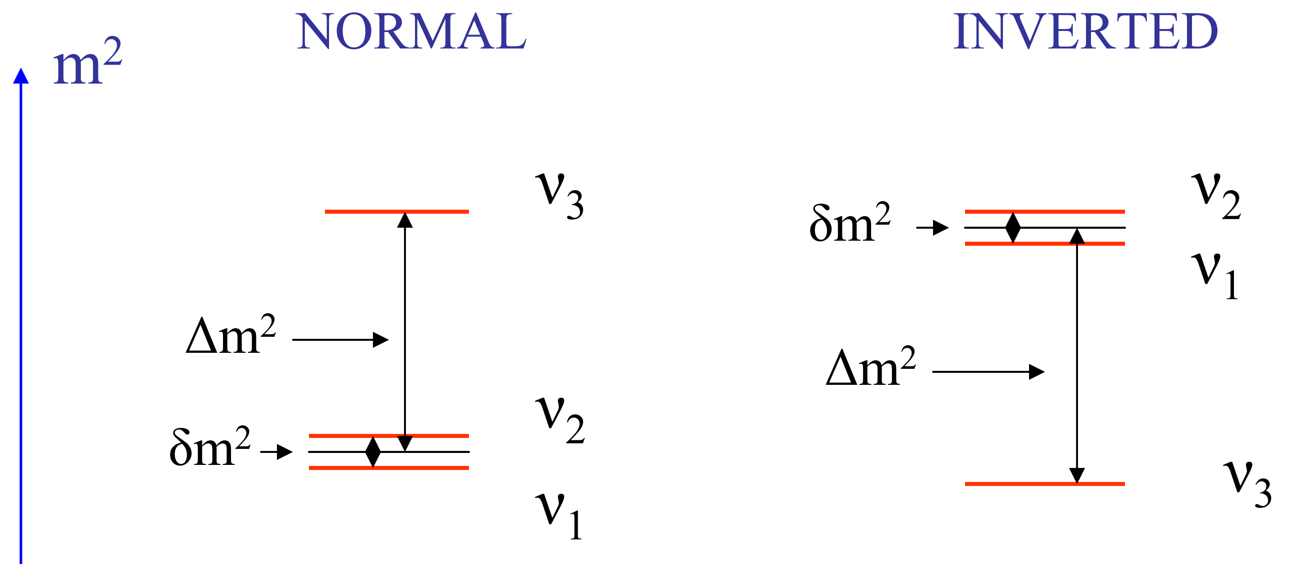
The Jastrow function in configuration space is

$$f_J(r) = 1 - c e^{-ar^2}\left(1 - br^2\right)$$

with

a=1.10 fm^{-2} , b=0.68 fm^{-2} , c=1	MS	soft
a=1.59 fm^{-2} , b=1.45 fm^{-2} , c=0.92	Argonne	hard
a=1.52 fm^{-2} , b=1.88 fm^{-2} , c=0.46	CD Bonn	hard

In the last few years atmospheric, solar, reactor and accelerator neutrino oscillation experiments have provided information on light neutrino mass differences and their mixings. Two possibilities, normal and inverted hierarchy, are consistent with experiment.



The average light neutrino mass can be written as

$$\langle m_\nu \rangle = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\varphi_2} + s_{13}^2 m_3 e^{i\varphi_3} \right|$$

$$c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}, \varphi_{2,3} = [0, 2\pi]$$

$$\left(m_1^2, m_2^2, m_3^2\right) = \frac{m_1^2 + m_2^2}{2} + \left(-\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm\Delta m^2\right)$$

A fit to oscillation experiments gives §

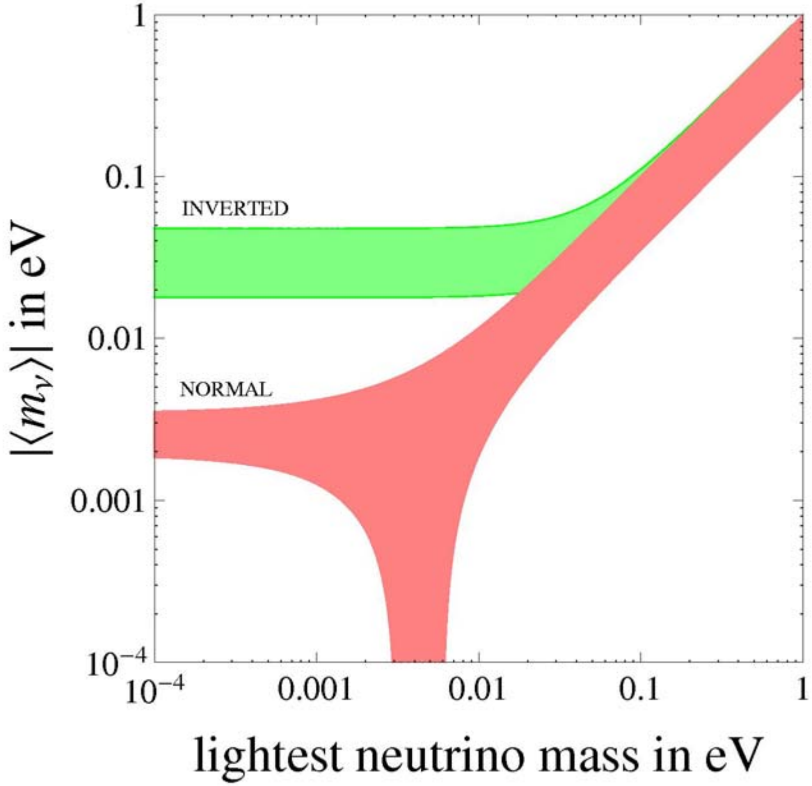
$$\sin^2\theta_{12} = 0.312, \sin^2\theta_{13} = 0.016, \sin^2\theta_{23} = 0.466$$

$$\delta m^2 = 7.67\times10^{-5}\, eV^2, \Delta m^2 = 2.39\times10^{-3}\, eV^2$$

§ G.L. Fogli *et al.*, Phys. Rev. D75, 053001(2007); D78, 033010 (2008).

[A recent result from Daya Bay, Phys. Rev. Lett. 108, 171803 (2012) gives $\sin^2\theta_{13}$=0.024±0.005, which slightly modifies the fit.]

Variation of the phases φ_2 and φ_3 from 0 to 2π gives the values of $<m_\nu>$ consistent with oscillation experiments

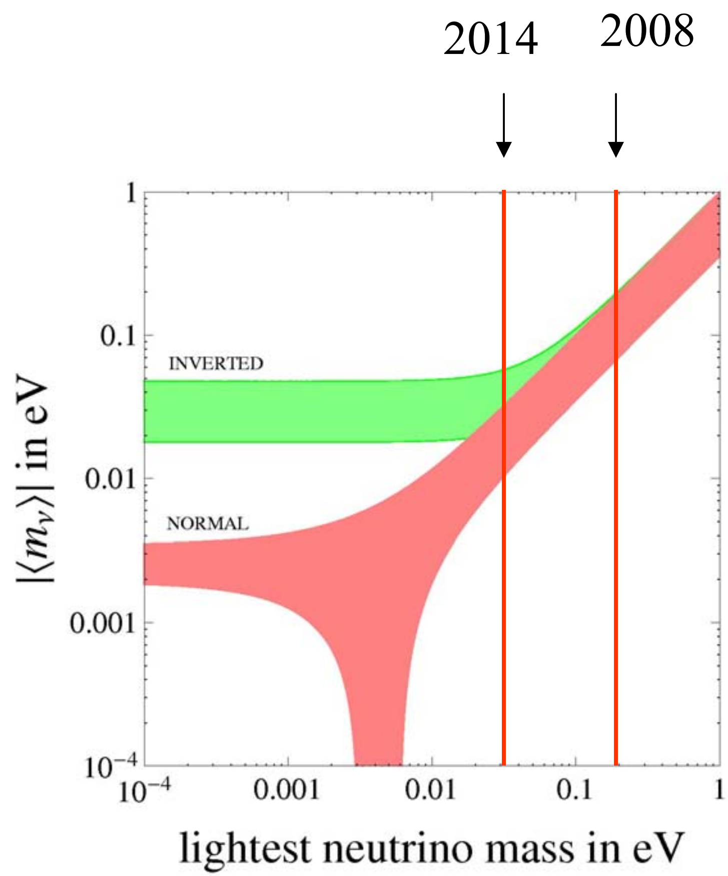


Vissani-Strumia plot

In addition there is a (model dependent) bound from cosmology on the sum of the masses

$$M = \sum_i m_i$$

Cosmological bound



Scenario 2: HEAVY NEUTRINO EXCHANGE

In recent years, scenario 2 has again become of interest. The transition operator for this scenario is the same as for 1, but with

$$T_h(p) = H_h(p) m_p \left\langle m_{\nu_h}^{-1} \right\rangle$$

$$f = m_p \left\langle \frac{1}{m_{\nu_h}} \right\rangle$$

$$\left\langle m_{\nu_h}^{-1} \right\rangle = \sum_{k=heavy} \left(U_{ek_h}\right)^2 \frac{1}{m_{k_h}}$$

and neutrino "potential"

$$v(p) = \frac{2}{\pi} \frac{1}{m_p m_e}$$

Constraints on the average inverse heavy neutrino mass are model dependent. V. Tello *et* al. ¶ have recently (2011) worked out constraints from lepton flavor violating processes and (potentially LHC experiments). In this model

$$f \equiv \eta = \frac{M_W^4}{M_{WR}^4} \sum_{k=heavy} \left(V_{ek_h}\right)^2 \frac{m_p}{m_{k_h}} \equiv \frac{M_W^4}{M_{WR}^4} \frac{m_p}{\left\langle m_{\nu_h} \right\rangle}$$

$$M_W = 80.41 \pm 0.10 GeV; M_{WR} = 3.5 TeV$$

η=lepton violating parameter.

Constraints on η can then be converted into constraints on the average heavy neutrino mass as

$$\left\langle m_{\nu_h} \right\rangle = m_p \left(\frac{M_W}{M_{WR}} \right)^4 \frac{1}{\eta}$$

¶ V. Tello, M. Nemevšek, F. Nesti, G. Senjanović, and F. Vissani, Phys. Rev. Lett. 106, 151801 (2011).

If both light and heavy neutrino exchange contribute, the half-lives are given by

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\left|M_{0\nu}\frac{\langle m_\nu\rangle}{m_e} + M_{0\nu_h}\eta\right|^2$$

The two contributions could add or subtract depending on their relative phase.

Scenario 3: MAJORON EMISSION

This scenario ($0\nu\beta\beta\varphi$ decay) was very much of interest a few years ago, but it is not much studied today. The transition operator for this scenario can be written as ¶

$$T(p) = H(p)\langle g \rangle$$

← effective Majoron coupling constant

The inverse half-life is given by

$$\left[\tau_{1/2}^{0\nu\beta\beta\varphi}\left(0^+ \rightarrow 0^+\right)\right]^{-1} = G_{0\nu\varphi}\left|M_{0\nu}\right|^2 \langle g \rangle^2$$

¶ F. Šimkovic *et al., loc.cit.*

Scenario 4: STERILE NEUTRINOS

In addition, another scenario is currently being discussed, namely the mixing of two or three additional "sterile" neutrinos, 4, 5 and 6, with masses in the keV-GeV range. [The question on whether or not "sterile" neutrinos exist is an active areas of research at the present time with experiments planned at FERMILAB and CERN-LHC.]

This scenario can be investigated by using a transition operator as in scenario 1 and 2 but with

$$f = \frac{m_{\nu I}}{m_e}$$

$$v(p) = \frac{2}{\pi} \frac{1}{\sqrt{p^2 + m_{\nu I}^2}\left(\sqrt{p^2 + m_{\nu I}^2} + \tilde{A}\right)}$$

These formulas apply for a single additional neutrino with mass $m_{\nu I}$

The product fv(p)

$$fv(p) = \frac{m_{\nu I}}{m_e}\frac{2}{\pi}\frac{1}{\sqrt{p^2+m_{\nu I}^2}\sqrt{p^2+m_{\nu I}^2}+\tilde{A}}$$

has the limits:

$$m_{\nu I} \to 0 \qquad fv = \frac{m_{\nu I}}{m_e}\frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}$$

$$m_{\nu I} \to \infty \qquad fv = \frac{m_{\nu I}}{m_e}\frac{2}{\pi}\frac{1}{m_{\nu I}^2} = \frac{2}{\pi}\frac{1}{m_e m_{\nu I}}$$

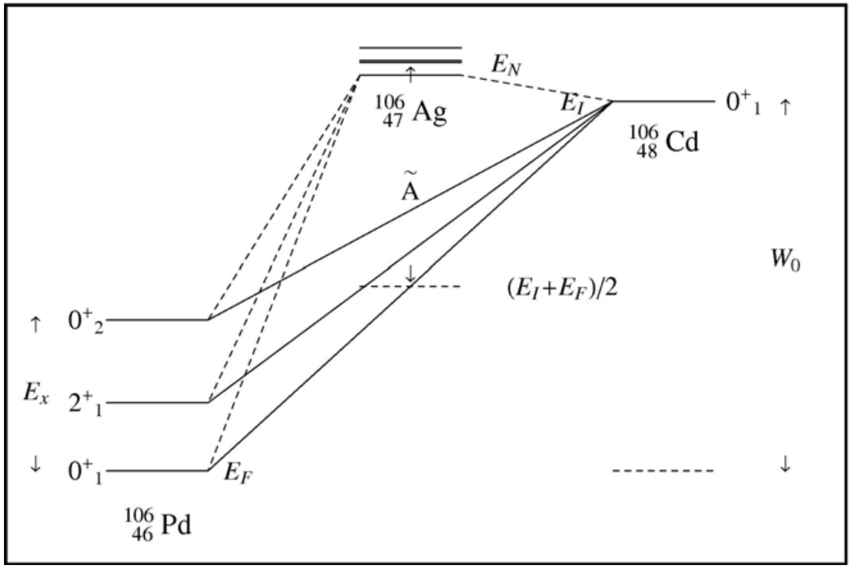
as in scenarios 1 and 2.

BRIEF THEORY OF $0\nu\beta^+\beta^+$ AND $0\nu EC\beta^+$

These processes are:

$$(A,Z) \rightarrow (A,Z-2) + 2e^+$$

$$(A,Z) + e^- \rightarrow (A,Z-2) + e^+$$



The theory for these processes is identical to that of $0\nu\beta^-\beta^-$ with half lives still given by

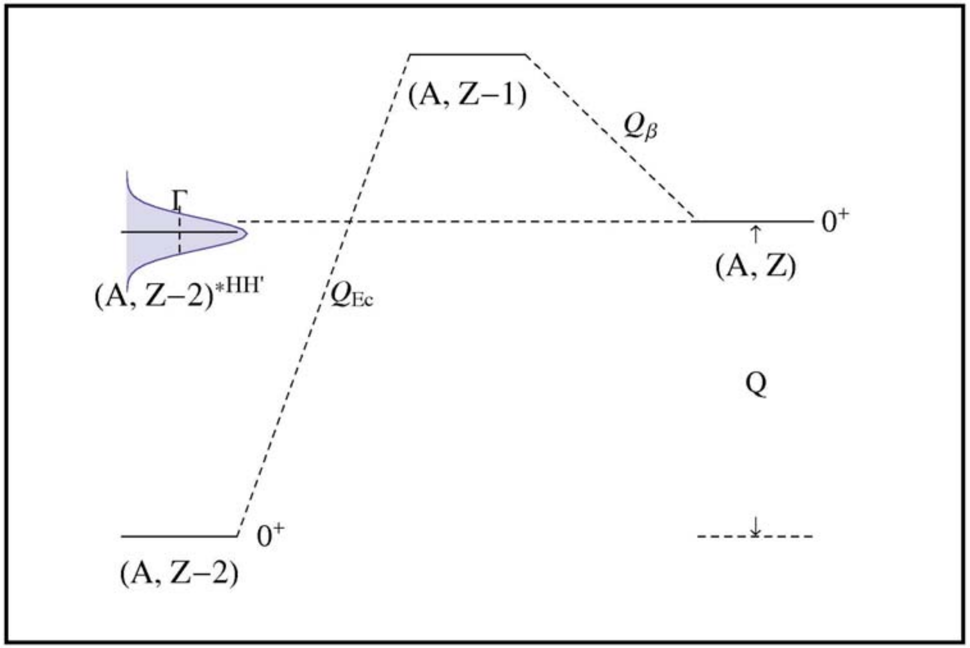
$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^2\left|f\left(m_i, U_{ei}\right)\right|^2$$

but with PSF appropriate for the process, $G_{0\nu}^{\beta^+\beta^+}$ $G_{0\nu}^{\beta^+EC}$

BRIEF THEORY OF 0νECEC

The process

$$(A, Z) + 2e^- \rightarrow (A, Z-2)$$



cannot in general occur because of energy and momentum conservation.

If however the energy of the initial state matches precisely the energy of the final state the process can occur and is termed resonant double electron capture or R0νECEC

For this process the half-life can be factorized as

$$\left[\tau_{1/2}^{ECEC}(0^+ \to 0^+)\right]^{-1} = g_A^4 G_{0\nu}^{ECEC} \left|M_{ECEC}^{0\nu}\right|^2 \left|f(m_i, U_{ei})\right|^2 \frac{\left(m_e c^2\right)\Gamma}{\Delta^2 + \Gamma^2/4}$$

Here Δ is the degeneracy parameter

$$\Delta = \left|Q - B_{2h} - E\right|$$

Energy of the two-holes in the daughter atom

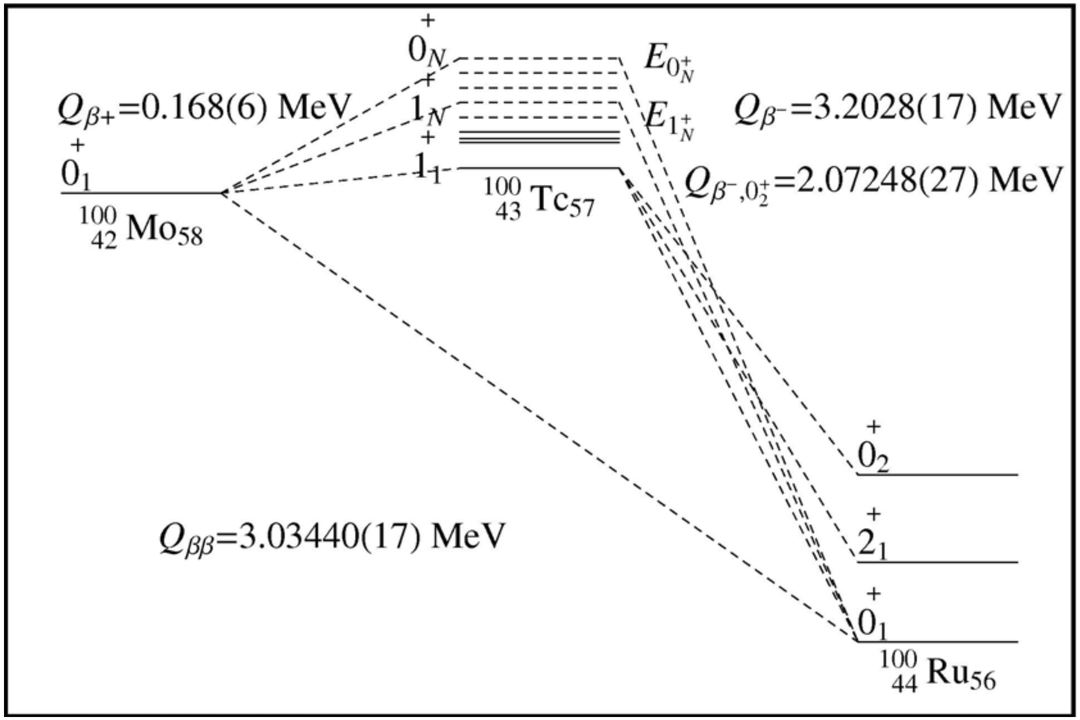
and Γ is the two-hole width in the daughter atom

Calculation of this process heavily relies on atomic physics and on nuclear physics

BRIEF THEORY OF 2νββ

The theory of 2νββ is more complicated than that of 0νββ because in general the closure approximation may not be good and the separation between PSF and NME may not be good.

One needs therefore to calculate NME and PSF for each individual state and sum over them.



To apply this procedure, one needs to calculate states in the intermediate odd-odd nucleus and then

$$M_{GT,N}^{(2\nu)} = \frac{\left\langle 0_F^+ \left\| \tau^\dagger \sigma \right\| 1_N^+ \right\rangle \left\langle 1_N^+ \left\| \tau^\dagger \sigma \right\| 0_I^+ \right\rangle}{\frac{1}{2}\left(Q_{\beta\beta} + 2m_e c^2\right) + E_{1_N^+} - E_I}$$

$$M_{F,N}^{(2\nu)} = \frac{\left\langle 0_F^+ \left\| \tau^\dagger \right\| 0_N^+ \right\rangle \left\langle 0_N^+ \left\| \tau^\dagger \right\| 0_I^+ \right\rangle}{\frac{1}{2}\left(Q_{\beta\beta} + 2m_e c^2\right) + E_{0_N^+} - E_I}$$

This calculation is daunting and has been done only in a selected number of cases

The separation between PSF and NME can be done in two cases:
(i) Closure approximation (CA)
(ii) Single state dominance (SSD)

In both cases the inverse half-life can be written as

$$\left[\tau_{1/2}^{2\nu}\right]^{-1} = G_{2\nu}\left|m_e c^2 M_{2\nu}\right|^2$$

For these cases, the calculation of the NME in IBM-2 is done in the same way as for 0νββ, except that the neutrino potential is

$$v_{2\nu}(p) = \frac{\delta(p)}{p^2}$$

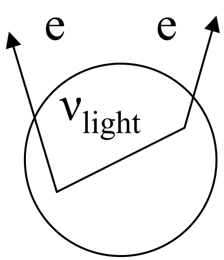
which is the Fourier-Bessel transform of V(r)=1.

Most calculation that attempt a simultaneous calculation of 0νββ and 2νββ are done in this way to avoid possible sources of systematic or accidental errors.

All processes light-neutrino exchange, heavy-neutrino exchange, 2ν decay, Majoron emission, sterile-neutrino exchange, …., can then calculated simultaneously by just changing the neutrino potential.

Light-neutrino

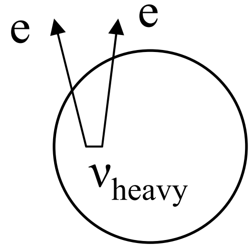
$$v(p)=\frac{2}{\pi}\frac{1}{p(p+\tilde{A})}$$



Long-range

Heavy-neutrino

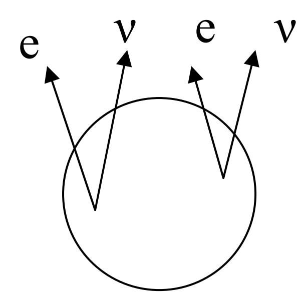
$$v(p)=\frac{2}{\pi}\frac{1}{m_p m_e}$$



Short-range

2ν

$$v(p)=\frac{\delta(p)}{p^2}$$



Constant