Theoretical cross sections for intermediate mass nuclei (in Astrophysics?)

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MCAS COLLABORATION

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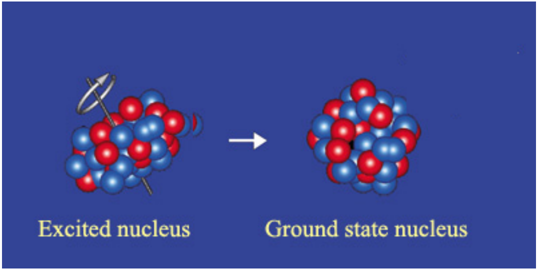
INTRODUCTION

The existing radioactive ion beam (RIB) facilities, and the construction of new ones, allow the formation and investigation of new species, particularly at or beyond the drip lines.

This talk is a review of the efforts by the MCAS collaboration to investigate exotic and non-exotic systems via a coupled-channel method, in the low-energy domain (scattering, bound states, resonances).

Treating scattering on light-medium nuclei

COUPLED-CHANNEL dynamics: including the collective low-energy excitations of the core.



C13 (n-C12), N13 (p-C12), C15 (n-C14), F15 (p-O14), He7, B7, Be7, Li7, Be9, B9, C17 (n-C16), Na-17 (p-Ne16) C-19 (n-C18), ${}^{9}{}_{\Lambda}Be$, ${}^{13}{}_{\Lambda}C$,
current ... Ne23 (n-Ne22), Mn23, Na23 (p-Ne22), Al23, O17 (n-O16), (p-F17), O19 ... O16 (alpha-C12), Be10 (alpha-He6)

Model of nuclear interaction

Current description: nucleon-nucleus scattering (light-medium nuclei with 0^+ g.s.) including first core excitations of collective nature (quadrupole, octupole, etc).

$$V_{cc'}(r) = \sum_{n=C,LS,LL,SI} V_n < (\ell s)jI; J^\pi | \mathcal{O}_n f_n(r, R, \theta_{\mathbf{r},\mathbf{R}}) | (\ell' s)j' I'; J^\pi >$$

For all operators, the functional forms are expanded to second order in the core-deformation parameter ($R = R_0(1 + \beta_2 P_2(\theta))$)

$$f_n(r, R, \theta) = f_n^{(0)}(r) - \beta_2 R_0 P_2(\theta) \frac{d}{dr} f_n^{(0)}(r)$$
$$+ \frac{\beta_2^2 R_0^2}{2\sqrt{\pi}} \left(P_0 - \frac{2\sqrt{5}}{7} P_2(\theta) + \frac{2}{7} P_4(\theta) \right) \frac{d^2}{dr^2} f_n^{(0)}(r)$$

MCAS: A low-energy scattering tool

we want to determine S-matrices to evaluate:
Total elastic scattering cross section

$$\sigma_{EL} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} \left\{ (\ell+1) \left| S_\ell^+(k) - 1 \right|^2 + \ell \left| S_\ell^-(k) - 1 \right|^2 \right\}$$

Total reaction cross section

$$\sigma_R = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} \left\{ (\ell+1) \left(1 - |S_\ell^+(k)|^2 \right) + \ell \left(1 - |S_\ell^-(k)|^2 \right) \right\}$$

The MCAS approach is constructed via:

- Finite-rank separable representations of realistic interactions;
- Scattering matrices for finite-rank expansion of interactions;
- Sturmian functions (aka Weinberg states) to define the representation.

Multichannel T matrices

Solution of Lippmann-Schwinger coupled-channel equation

$$T^{J^\pi}_{cc'}(p,q;E) = V^{J^\pi}_{cc'}(p,q) + \mu \sum_{c''=1}^{\text{closed}} \int_0^\infty V^{J^\pi}_{cc''}(p,x) \frac{x^2}{h^2_{c''} + x^2} T^{J^\pi}_{c''c'}(x,q;E)\, dx$$
$$-\mu \sum_{c''=1}^{\text{open}} \int_0^\infty V^{J^\pi}_{cc''}(p,x) \frac{x^2}{k^2_{c''} - x^2 + i\epsilon} T^{J^\pi}_{c''c'}(x,q;E)\, dx$$

Finte-Rank expansion of realistic CC interaction

$$V_{cc'}(p,q) \sim V^N_{cc'}(p,q) = \sum_{n=1}^{N} \hat{\chi}_{cn}(p)\, \eta_n^{-1}\, \hat{\chi}_{c'n}(q) \; .$$

Optimal representations of $\hat{\chi}_{cn}(p)$ in terms of Sturmians $|\Phi_{cn}\rangle$

$$\sum_{c'} G^0_c V_{cc'} |\Phi_{c'n}\rangle = -\eta_n |\Phi_{cn}\rangle$$

with $\hat{\chi}_{cn}(p)$ defined as $|\hat{\chi}_{cn}\rangle = V_{cc'}|\Phi_{c'n}\rangle$

Multichannel S matrices

Sturmian expansion of $V_{cc'}$ leads to algebraic form for S

$$
\begin{aligned}
S_{cc'} &= \delta_{cc'} \; - \; i\pi\mu\sqrt{k_c k_{c'}} \; T_{cc'} \\
&= \delta_{cc'} \; - \; i^{\phi}\pi\mu \sum_{n,n'=1}^{N} \sqrt{k_c} \; \hat{\chi}_{cn}(k_c) \; \left([\boldsymbol{\eta} - \mathbf{G}_0]^{-1} \right)_{nn'} \; \hat{\chi}_{c'n'}(k_{c'}) \sqrt{k_{c'}} \; ,
\end{aligned}
$$

with matrix elements

$$
\begin{aligned}
[\mathbf{G}_0]_{nn'} \; = \; \mu \Bigg[& \sum_{c=1}^{\text{open}} \int_0^{\infty} \hat{\chi}_{cn}(x) \frac{x^2}{k_c^2 - x^2 + i\epsilon} \hat{\chi}_{cn'}(x) \; dx \\
& - \sum_{c=1}^{\text{closed}} \int_0^{\infty} \hat{\chi}_{cn}(x) \frac{x^2}{h_c^2 + x^2} \hat{\chi}_{cn'}(x) \; dx \Bigg] \\
[\boldsymbol{\eta} \;]_{nn'} = \; & \eta_n \; \delta_{nn'} \; .
\end{aligned}
$$

MCAS and the Pauli Principle

The Pauli Principle can be satisfied in the CC calculations by:

Either generating an $V_{cc'}(r, r')$ defined using shell model wave functions;

Or using an Orthogonalizing Pseudo-Potential to specify the Sturmians:

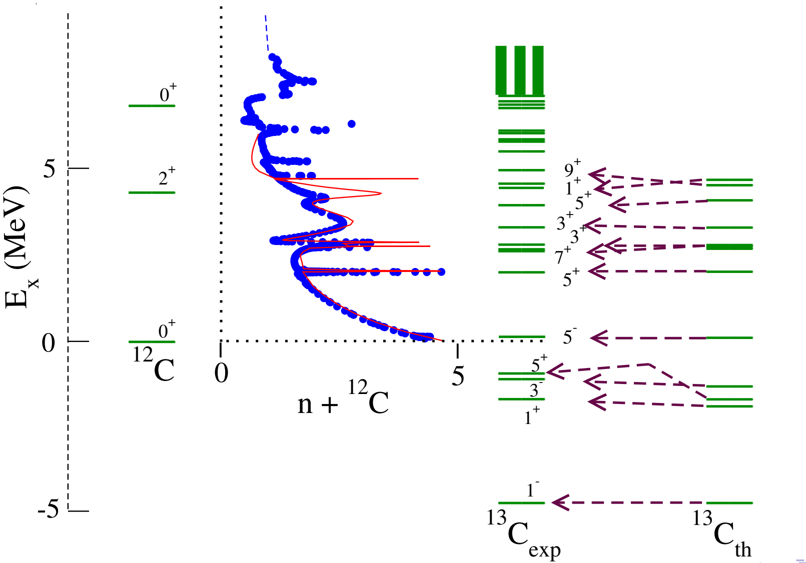
$$\sum_{c'} G_c^0 V_{cc'} |\Phi_{c'n}\rangle \quad = \quad -\eta_n |\Phi_{cn}\rangle$$

But use instead of $V_{cc'}(r)$

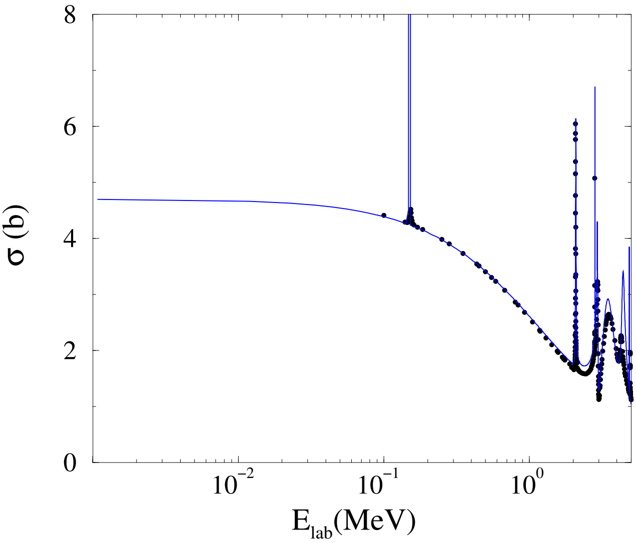
$$\mathcal{V}_{cc'}(r, r') = V_{cc'}(r)\delta(r - r') + \delta_{cc'}\lambda_c A_c(r) A_c(r')$$

for forbidden (and partially forbidden) orbits. The Sturmians $\Phi_{cn}(r)$ are then orthogonal to the Pauli-forbidden orbits $A_c(r)$

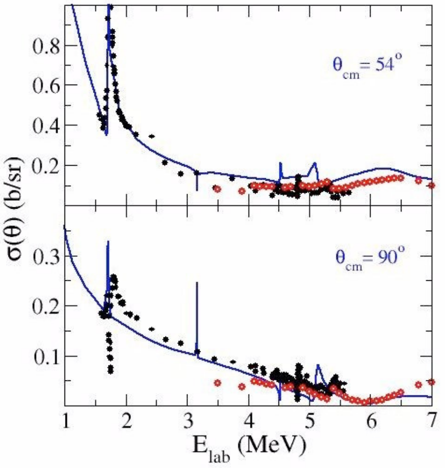
OPP correction for Pauli
L.C. et al. Phys. Rev. Lett. **94**, 122503 (2005)



n-^{12}C: Low energy details

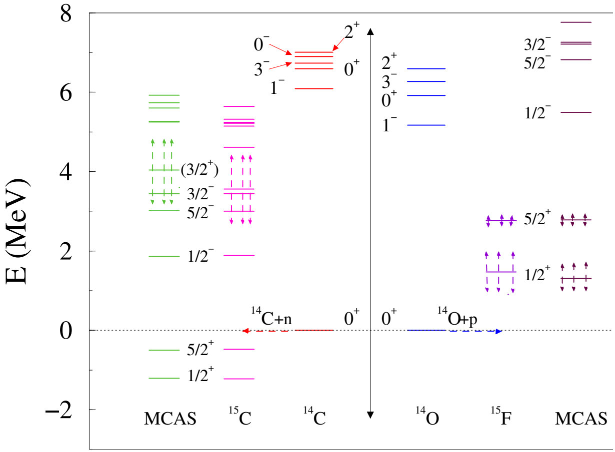


proton-C12 scattering/cross-sections



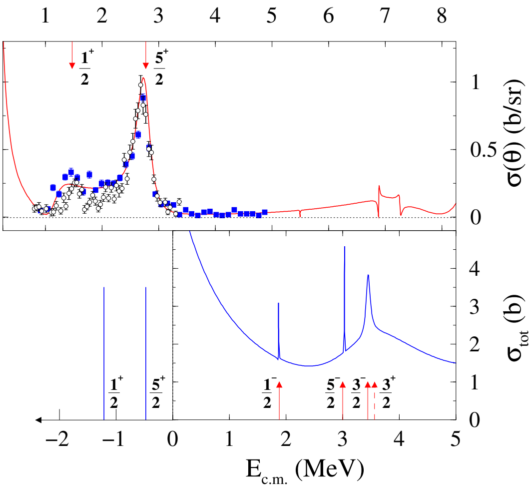
Nuclear interaction (Wood-Saxon form)
RADIUS R=3.05, diffuseness a=0.65
Charge distribution (3p Fermi function)
R= 2.355, a=0.522, w=-0.149

Structure beyond the drip lines: 15F, and mirror 15C
L.C., et al. Phys. Rev. Lett. **94**, 072502 (2006)



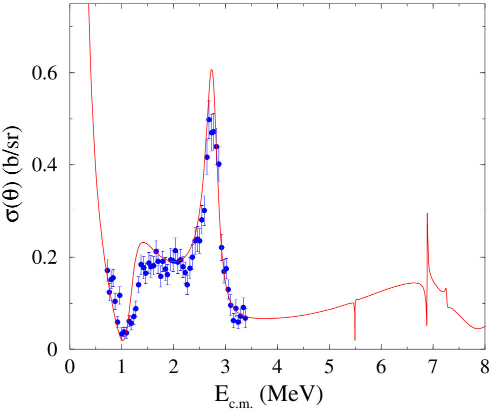
Mass-15 spectra

Structure beyond the drip lines: 15F, and mirror 15C
L.C., et al. Phys. Rev. Lett. **94**, 072502 (2006)



Mass-15 cross-sections

Structure beyond the drip lines: 15F, and mirror 15C
L.C., et al. Phys. Rev. Lett. **94**, 072502 (2006)



Cross section at $\theta = 147^o$ in ^{15}F

Narrow resonances found in 2009-2010 @ GSI

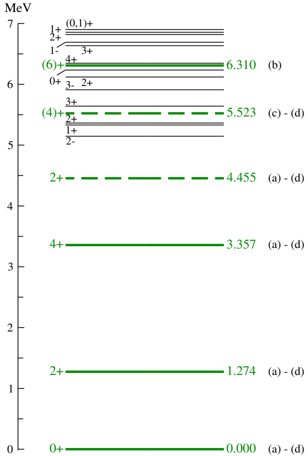
and confirmed in 2011 @ SPIRAL1

Structure @ mass 23: scattering of neutrons off ^{22}Ne P.R.Fraser, et al. Phys. Rev. C **90**, 024616 (2014)

Mass 23 interesting for stellar nucleogenesis (NeNa cycle, ^{23}Ne-Na etc.)

need basic test: neutron - ^{22}Ne scattering

Structure @ mass 23: scattering of neutrons off ^{22}Ne P.R.Fraser, et al. Phys. Rev. C **90**, 024616 (2014)



The low-energy experimental 22Ne spectrum

Structure @ mass 23: scattering of neutrons off ^{22}Ne P.R.Fraser, et al. Phys. Rev. C **90**, 024616 (2014)

PHYSICAL REVIEW C **90**, 024616 (2014)

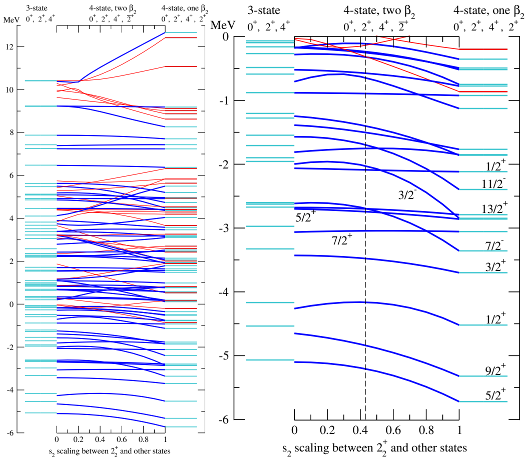
TABLE IV. Parameter values defining the $n+^{22}$Ne interaction. $\lambda^{(OPP)}$ are blocking strengths of occupied shells, in MeV.

	Odd parity	Even parity		
V_0 (MeV)	–65.20	–51.30		
V_{ll} (MeV)	–1.01	–0.30		
V_{ls} (MeV)	7.00	7.00		
V_{ss} (MeV)	–0.20	–1.45		
R_0	a	β_2	$\overline{\beta_2}$[a]	β_4
3.1 fm	0.75 fm	0.22	0.1034	–0.08
	$1s_{1/2}$	$1p_{3/2}$	$1p_{1/2}$	$1d_{5/2}$
0_1^+ $\lambda^{(OPP)}$	10^6	10^6	10^6	0.0
2_1^+ $\lambda^{(OPP)}$	10^6	10^6	10^6	0.0
4_1^+ $\lambda^{(OPP)}$	10^6	10^6	10^6	0.0
2_2^+ $\lambda^{(OPP)}$	10^6	10^6	10^6	0.0

[a] $\overline{\beta_2}$ for linking 2_2^+ to other states; 43% of 0.22. See Sec. IV A.

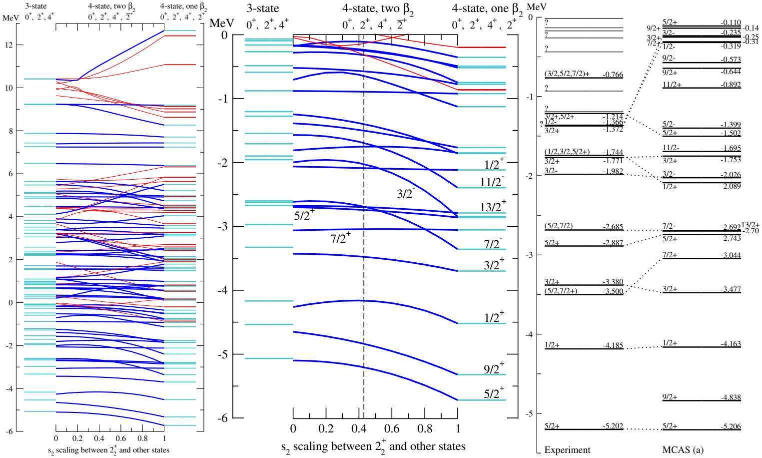
The parameters of the MCAS calculation

Structure @ mass 23: scattering of neutrons off ^{22}Ne
P.R.Fraser, et al. Phys. Rev. C **90**, 024616 (2014)



The calculated 23Ne spectrum with varing the 2nd β. (LOW ENERGY)

Structure @ mass 23: scattering of neutrons off ^{22}Ne
P.R.Fraser, et al. Phys. Rev. C **90**, 024616 (2014)



The calculated 23Ne spectrum with varing the 2nd β and VS experiments

Structure @ mass 23: scattering of neutrons off ^{22}Ne P.R.Fraser, et al. Phys. Rev. C **90**, 024616 (2014)

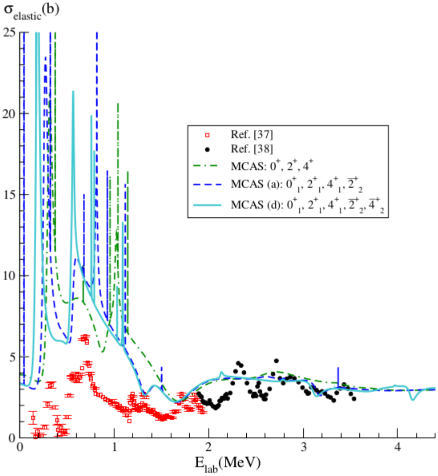


Figure : Experimental data of S. Sikkema et al (1958) and of S. R. Salisbury et al (1966).

The neutron-^{16}O system
J.P.Svenne, et al. in progress (2015)

Up to now investigated nuclides with permanent deformation (rotational)

n-^{16}O system with spherical core: first time we consider a vibrational CC set-up in MCAS

The neutron-^{16}O system
J.P.Svenne, et al. in progress (2015)

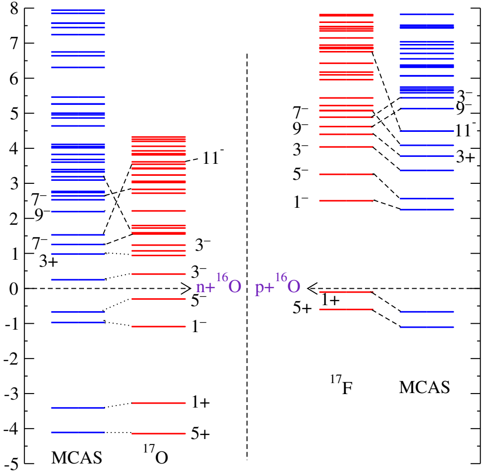
The parameters

MeV	parity -	parity +	geometry	value
V_0	– 47.5	– 50.5	R_0	3.15 fm
V_{LL}	2.55	0.0	a	0.65 fm
V_{Ls}	6.9	7.2	β_2	0.21
V_{ss}	2.5	– 2.0	β_3	0.42

The parameters of the CC interaction

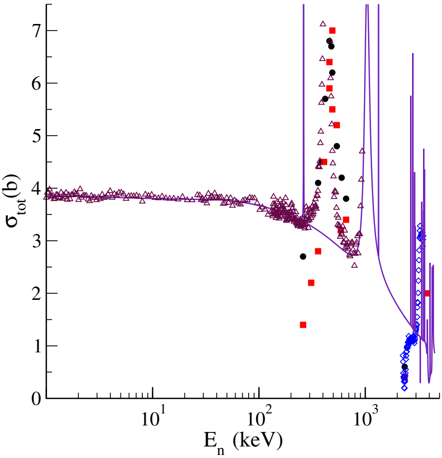
states of ^{16}O included in CC dynamics: 0_1^+ (g.s), 0_2^+ (6.049), 3_1^- (6.13), 2_1^+ (6.92), 1_1^- (7.12)

The neutron-^{16}O system
J.P.Svenne, et al. in progress (2015)



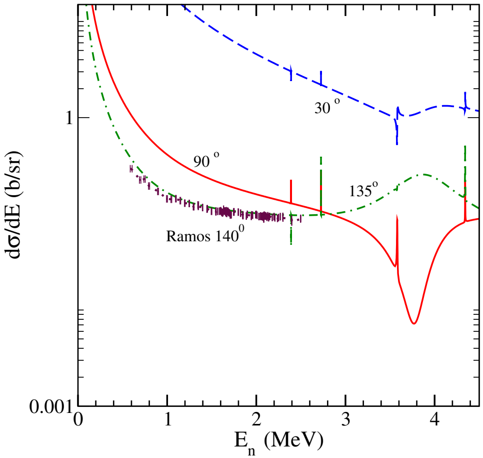
The spectrum for ^{17}O and ^{17}F

The neutron-^{16}O system
J.P.Svenne, et al. in progress (2015)



^{16}O neutron elastic scattering cross section

The proton-^{16}O system
J.P.Svenne, et al. in progress (2015)



^{16}O proton elastic scattering cross section (Data Ramos et al 1993)

α-He6 scattering/cross-sections

$$V_{cc'}(r) = \langle \ell I \mid W(r) \mid \ell' I' \rangle = \left[V_0 \delta_{c'c} f(r) + V_{\ell\ell} f(r) [\ell \cdot \ell] + V_{II} f(r) [\mathbf{I} \cdot \mathbf{I}] + V_{\ell I} g(r) [\ell \cdot \mathbf{I}] \right]_{cc'}$$

Table : The states of ^{6}He used in the coupled-channel evaluations All energies are in units of MeV.

state	Centroid	Width
$0^+_{\mathrm{g.s.}}$	0.000	0.00
2^+_1	1.797	0.113
2^+_2	5.60	10.0

Treatment of Pauli principle using the OPP technique. Method discussed within the Cluster Approach (Analytical RGM) in Yu.A. Lashko, G.F. Filippov, L. Canton Ukr. J. Phys. 2015, to be published.

α-He6 scattering/cross-sections

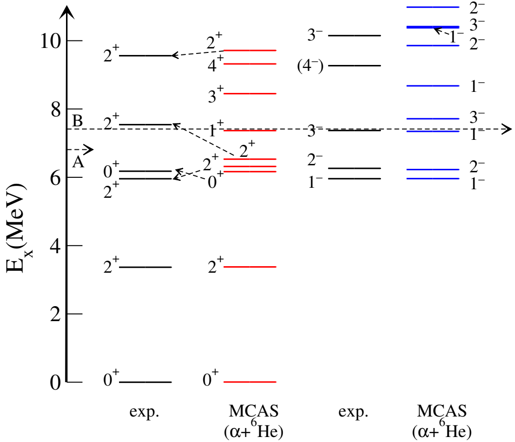


Figure : The spectrum of low-excitation states in ^{10}Be. To aid distinction the positive and negative parity states are shown on the left and right separately. The lines labelled 'A' and 'B' indicate the n+^{9}Be and the α+^{6}He thresholds respectively.

α-He6 scattering/cross-sections

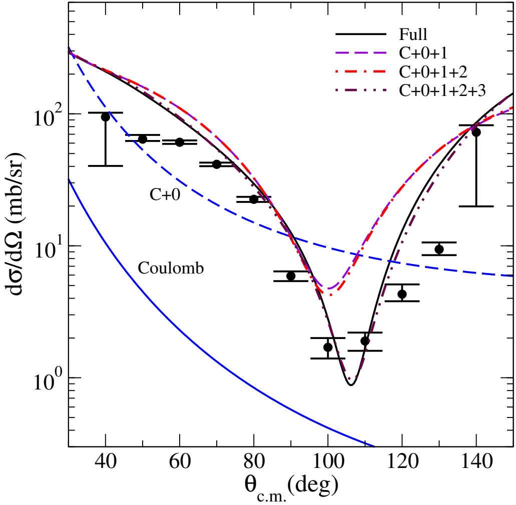


Figure : The differential cross section measured at 3.8 MeV (c.m.) as partial waves are added to the evaluations.

CONCLUSIONS

- Structures of regular/exotic nuclei can be studied from low–energy scattering, particularly looking at the formation of the compound systems.
- MCAS has been extended to calculate **radiative capture** from Coupled-Channel wavefunctions.
- Handling of Pauli principle in CC dynamics is a fundamental requisite.
- For Oxygen (vibrational model), for Neon (rotational model), and for alpha scattering we have first output showing promising results, but also problems due to the complexities of the spectra involved.

Radiative Capture with MCAS: $^3He +^4 He \rightarrow^7 Be + \gamma$

We have chosen the reaction $^3He(\alpha, \gamma)^7Be$ as a test.
Aim was not to achieve quantitative agreement, but to introduce the theoretical ingredients, in view to applications to a variety of more complex CC cases ($^{15}N(p, \gamma)^{16}O$, $^{12}C(\alpha, \gamma)^{16}O$ ....).
Standard Woods-Saxon form with parameter values

$$V_0 = [-76.8^{(-)}, -70.0^{(+)}] \ MeV \quad V_{ll} = 0.6 \ MeV \quad V_{ls} = 1.7 \ MeV$$
$$R_0 = 2.39 \ fm \quad a = 0.68 \ fm \quad R_c = 2.39 \ fm$$

Radiative Capture with MCAS: $^3He +^4 He \rightarrow^7 Be + \gamma$

MCAS evaluation of ^{7}Li, ^{7}Be in terms of a single channel description of ^{3}H- , ^{3}He- α. (A Potential model gave results of Table 2.)

Table : Spectral properties of ^{7}Li from (α+^{3}H) and ^{7}Be from (α+^{3}He)

	^{7}Li		^{7}Be	
J^π	Exp.	Theory	Exp.	Theory
$\frac{3}{2}^-$	-2.47	-2.47	-1.59	-1.53
$\frac{1}{2}^-$	-1.99	-1.75	-1.16	-0.84
$\frac{7}{2}^-$	2.2 (0.06)	2.1 (0.08)	3.0 (0.18)	3.1 (0.18)
$\frac{5}{2}^-$	4.1 (0.92)	4.2 (0.83)	5.1 (1.2)	5.1 (1.19)

Radiative Capture with MCAS: $^{3}He +^{4} He \rightarrow^{7} Be + \gamma$

MCAS elastic scattering cross-sections at three center of mass scattering angles $(54.7^\circ, 90.0^\circ, 125.2^\circ)$ at which data has been taken.

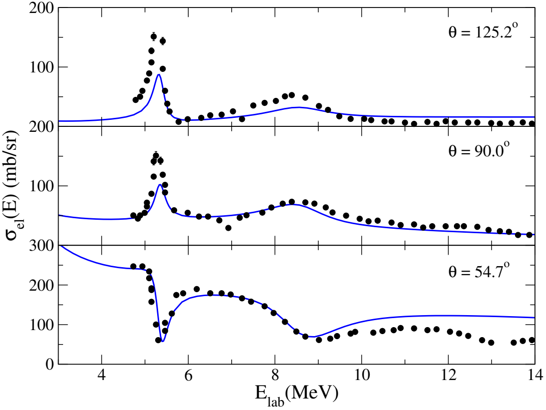


Figure : Cross sections from $\alpha(^{3}\text{He},^{3}\text{He})\alpha$ at the center of mass scattering angles listed.

The scattering matrix WF writes (except inessential factors)

$$\Psi^{J\,(+)}_{a''a}(R) = \exp[i\sigma_L(P)] \left[F_L(PR)\ \delta_{a''a}\ -\ \Phi^{J\,(+)}_{a''a}(R)\right]\ , \qquad (1)$$

$$\Phi^{J\,(+)}_{a''a}(R) = \sum_{n,n'=1}^{N} \left[\lambda - \mathbf{G}_0^{(+)}\right]^{-1}_{nn'} \widehat{\chi}_{an'}(P) \times$$

$$\left(F_{L''}(P''R)\ \chi^G_{a''n}(R)\ - G_{L''}(P''R)\ \chi^F_{a''n}(R) + O^{R(+)}_{L''}(R)\ \widehat{\chi}_{a''n}(P'')\ \right)$$

where we have introduced the notations

$$\chi^G_{an}(R) \equiv \sqrt{\frac{2}{\pi}} \frac{1}{P''} \int_R^\infty G_L(P''r)\ \chi_{an}(r)\mathrm{d}r\ ,$$

$$\chi^F_{an}(R) \equiv \sqrt{\frac{2}{\pi}} \frac{1}{P''} \int_R^\infty F_L(P''r)\ \chi_{an}(r)\mathrm{d}r\ , \qquad (2)$$

$$\widehat{\chi}_{an}(P'') \equiv \chi^F_{an}(0)\ .$$

Construction of generalized E and M dipole operators via 3D formalism based on extension of the Siegert theorem and explicit inclusion of Gauge Independence (Levchuk-Shebeko, Phys.At.Nucl. 1993)

Harmonization with MCAS:

Luciano Canton, Leonid Levchuck, Nucl. Phys. A808 (2008), 192

$$\langle f \mid D_\xi(\mathbf{k}) \mid i\rangle = \frac{E_i^{\rm int} - E_f^{\rm int}}{E_i - E_f}\Big\{ eC_{E1}\left[R_\xi\right]_{if} - \frac{i}{6}(-1)^\kappa k_{-\kappa}\left(\left[t^{\rm A}_{\xi\kappa}\right]_{if} + \left[t^{\rm B}_{\xi\kappa}\right]_{if} + eC_{E2}\left[T^{\rm AB}_{\xi\kappa}\right]_{if}\right)\Big\},$$

$$C_{E1} = \frac{M_B Z_A - M_A Z_B}{M}$$

$$C_{E2} = \frac{M_B^2 Z_A + M_A^2 Z_B}{M^2}$$

$$\langle f \mid M_\xi(\mathbf{k}) \mid i\rangle = \left[\mu^{\mathrm{A}}_\xi\right]_{if} + \left[\mu^{\mathrm{B}}_\xi\right]_{if} + \mu_{\mathrm{N}} C_{M1} \left[L_\xi\right]_{if}$$

$$-i(-1)^\kappa k_{-\kappa} \left(\frac{M_B}{M} \left[\mu^{\mathrm{A}}_\xi R_\kappa\right]_{if} - \frac{M_A}{M} \left[\mu^{\mathrm{B}}_\xi R_\kappa\right]_{if} + \mu_{\mathrm{N}} C_{M2} \left[\{L_\xi, R_\kappa\}\right]_{if} \right)$$

$$(\xi, \kappa = -1, 0, 1)$$

$$C_{M1} = \frac{1}{A_{tot}} \frac{M_B Z_A + M_A Z_B}{M}$$

$$C_{M2} = \frac{1}{3A_{tot} M} \frac{M_B^2 Z_A - M_A^2 Z_B}{M}$$

Radiative Capture with MCAS: $^3He +^4 He \rightarrow^7 Be + \gamma$

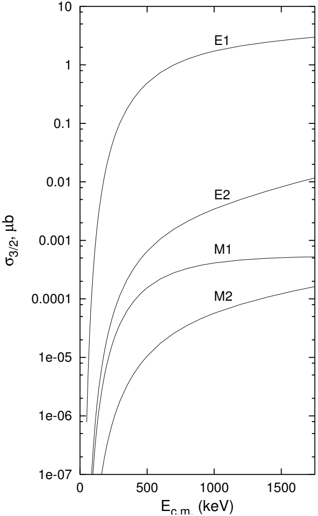


Figure : Multipole contributions to the total cross section of transition to the ^{7}Be ground state.

Radiative Capture with MCAS: $^3He +^4 He \rightarrow^7 Be + \gamma$

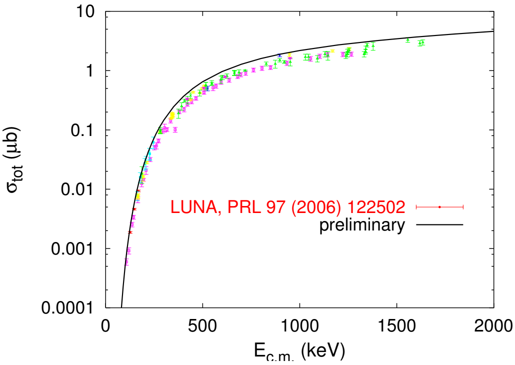


Figure : Total cross section of $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction. Full circles at $E_{\mathrm{c.m.}} =$ 127, 148 and 169 keV refer to recent LUNA data, and other experimental points obtained previously.

$$S(E_{\rm c.m.}) = E_{\rm c.m.}\sigma_{\rm tot}(E_{\rm c.m.})\exp\left[2\pi\eta(P)\right] , \qquad (3)$$

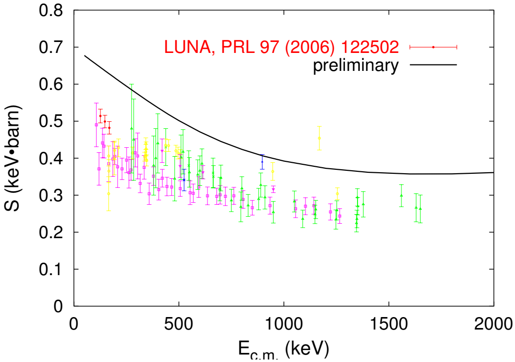


Figure : Astrophysical S factor for reaction $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$.

Radiative Capture with MCAS: $^3He +^4 He \rightarrow^7 Be + \gamma$

Conclusions:

- Calculation of EM transition required the explicit construction of the scattering radial wavefunction based on the MCAS formalism.
- Construction is general, and possible for a two-cluster nuclear potential with non-central forces, in presence of inelastic channels, of Coulomb distortions, and non-local effects due to exchange phenomena.
- Extension of the MCAS approach to cover nuclear radiative capture, including reactions of astrophysical interest.