# *Ab-initio* calculations of few-nucleon reactions of astrophysical interest

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# Outline

- Thereotical framework
  - *Ab-initio* methods for few-nucleon systems
  - (Standard) phenomenological approach (PhenAp) vs. Chiral effective field theory ($\chi$EFT)
- Results
  - Some tests:
    1. Electromagnetic structure of $A = 2, 3$ nuclei
    2. Muon capture on light nuclei
  - $p + p \rightarrow d + e^+ + \nu_e$
  - $p + d \rightarrow {}^3\mathrm{He} + \gamma$
- Outlook

# Theoretical framework: *ab-initio* studies

$\boxed{A\text{-nucleon system}} \leftrightarrow \boxed{\text{Observable X}}$

$$\boxed{H = T + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} \ \text{ \& nuclear currents } j^{EW}}$$

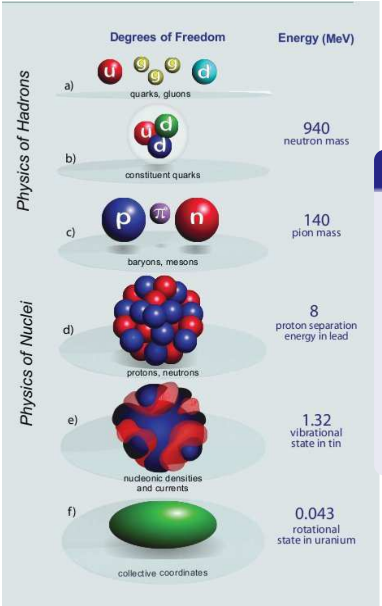
## *Ab-initio* method and *Ab-initio* results

- *Ab-initio* method → obtain X by solving the relevant quantum many-body equations, without any uncontrolled approximation
- controlled approximations are allowed (expansion on a certain basis) → converged results = *ab-initio* results
- comparison of *ab-initio* results obtained with different *ab-initio* methods → benchmark calculations
- comparison of *ab-initio* results with data → test of $H$ and $j^{EW}$

FEW ⇔ *ab-initio* methods

W. Leidemann and G. Orlandini, Progr. Part. Nucl. Phys. **68**, 158 (2013)

# Phenomenological approach



## Nuclear interaction: $V_{NN} + V_{NNN}$

- Until $\simeq$ 15 years ago:
  phenomenological potentials
  - $V_{NN} + V_{NNN}$ semi-phenomenological
  - $V_{NN}$ with $\simeq$ 40 parameters fitted to $A = 2$ data $\rightarrow \chi^2/\text{datum} \simeq 1$
  - $V_{NNN}$ with 2-3 parameters fitted to $B(A = 3, 4)$

$\Rightarrow$ **no simple connection to QCD**

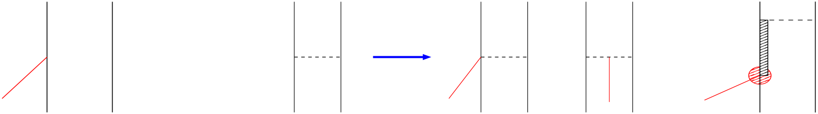
# Electroweak currents in the phenomenological approach

EW operators: $\rho^\gamma, \mathbf{j}^\gamma$; $\rho^{V/A}, \mathbf{j}^{V/A}$ but CVC $\Rightarrow \rho^V/\mathbf{j}^V \rightarrow \rho^\gamma/\mathbf{j}^\gamma$

- $\mathbf{j}^\gamma \rightarrow$ Current Conservation Relation (CCR) $\longrightarrow \mathbf{q}\cdot\mathbf{j}^\gamma \propto [\rho^\gamma, H]$

**Realistic model**

**1b operators** **2b operators**: Meson-exchange currents + $\mathbf{j}^{MD} \perp \mathbf{q}$



CCR with $T$

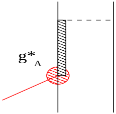
Interplay potential-current: MI

MD

CCR "exact" with AV18/UIX [L.E. Marcucci *et al.*, Phys. Rev. C **72**, 014001 (2005)]

- $\mathbf{j}^A \rightarrow$ no CCR $\Rightarrow$ **MD**

Largest contribution to $\mathbf{j}^A$(MD) from



$g_A^*$ fit to observable: $GT_{exp}$ of tritium $\beta$-decay

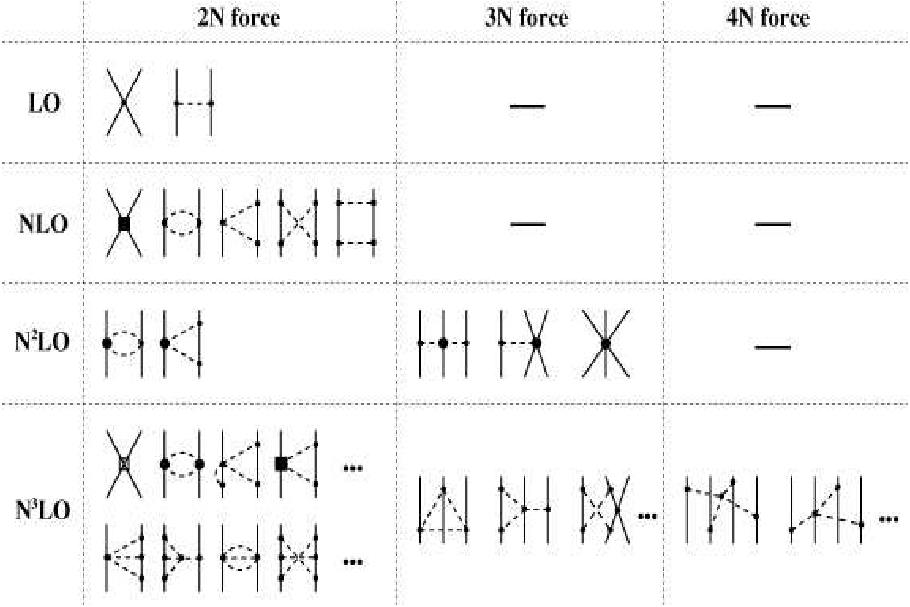
# Chiral Effective Field Theory ($\chi$EFT)

- QCD → quarks and gluons ("high-energy" d.o.f.)
- Nuclear physics → nucleons and pions ("low-energy" d.o.f.)
- EFT → processes with $E \simeq p \simeq m_\pi \ll \Lambda_{\rm QCD} \sim 1$ GeV
  - ⋆ "h-e" d.o.f. integrated out → contact interactions with "l-e" d.o.f. and low-energy constants (LECs) obtained from experiment
  - ⋆ perturbative theory: matrix elements $\propto O(p/\Lambda_{\rm QCD})^\nu$
- **$\chi$EFT → EFT with spontaneous breaking of QCD's $\chi$-symmetry**
- Regularization with cutoff function → $\Lambda \simeq 414, 450, 500, 600$ MeV

Disadvantage: limited to processes with $E \sim 1 - 2\, m_\pi$

## Advantages

- nuclear force "hierarchy" → accurate $V_{NN} + V_{NNN}$ (N3LO/N2LO)
- consistent framework for interactions + currents (just add EW field among the d.o.f.)
- possibility to estimate the theoretical uncertainty



Similar power counting for the EW currents ($\mathbf{j}^\gamma \rightarrow \mathrm{N3LO}$ & $\mathbf{j}^A \rightarrow \mathrm{N2LO}$)

# $A = 2$ results

| | AV18 | N3LO | Exp. |
|---|---|---|---|
| $B_d$ (MeV) | 2.22457 | 2.22456 | 2.224574(9) |
| $a_{n-n}$ (fm) | –18.487 | –18.900 | –18.9(4) |
| $^1a_{n-p}$ (fm) | –23.732 | –23.732 | –23.740(20) |
| $^3a_{n-p}$ (fm) | 5.412 | 5.417 | 5.419(7) |

N3LO $\equiv$ N3LO-Idaho

# $A = 3$ bound and low-energy scattering states

| | $^3$H | | | | $^3$He | | | |
|---|---|---|---|---|---|---|---|---|
| Potential | $B$ (MeV) | $r_p$ (fm) | $< V >$ (MeV) | $P_D$ (%) | $B$ (MeV) | $r_p$ (fm) | $< V >$ (MeV) | $P_D$ (%) |
| AV18 | 7.624 | 1.653 | –54.351 | 8.510 | 6.925 | 1.872 | –52.610 | 8.467 |
| AV18+UIX | 8.479 | 1.582 | –59.754 | 9.301 | 7.750 | 1.771 | –57.961 | 9.248 |
| N3LO | 7.854 | 1.655 | –42.409 | 6.312 | 7.128 | 1.855 | –40.917 | 6.313 |
| N3LO+N2LO | 8.474 | 1.611 | –44.956 | 6.815 | 7.733 | 1.794 | –43.478 | 6.818 |
| Exp. | 8.482 | 1.60 | – | – | 7.718 | 1.77 | – | – |

| | AV18 | AV18+UIX | N3LO | N3LO+N2LO | Exp. |
|---|---|---|---|---|---|
| $^2a_{n-d}$ (fm) | 1.248 | 0.590 | 1.100 | 0.675 | 0.645± 0.010 |
| $^4a_{n-d}$ (fm) | 6.346 | 6.343 | 6.342 | 6.342 | 6.35±0.02 |
| $^2a_{p-d}$ (fm) | 1.134 | -0.089 | 0.876 | 0.072 | – |
| $^4a_{p-d}$ (fm) | 13.662 | 13.662 | 13.646 | 13.647 | – |

A. Kievsky *et al.*, J. Phys. G: Nucl. Part. Phys. **35**, 063101 (2008)
M. Viviani *et al.*, Phys. Rev. C **71**, 024006 (2005)

with the Hyperspherical Harmonics (HH) method - *ab-initio* method

# Electromagnetic structure of $A = 2, 3, 4$ nuclei

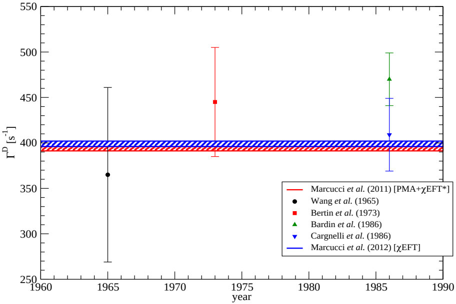
| | PhenAp | $\chi$EFT | Exp. |
|---|---|---|---|
| $r_c(d)$ [fm] | **2.119** | **2.126 ± 0.004** | 2.130 ± 0.010 |
| $\mu(d)$ [n.m.] | 0.847 | 0.8574 | 0.8574 |
| $Q(d)$ [fm$^2$] | **0.280** | **0.2836 ± 0.0016** | 0.2859 ± 0.0003 |
| $r_c(^3\mathrm{He})$ [fm] | **1.928** | **1.962 ± 0.004** | 1.973 ± 0.014 |
| $r_m(^3\mathrm{He})$ [fm] | 1.909 | 1.920 ±0.007 | 1.976 ±0.047 |
| $\mu(^3\mathrm{H})$ [n.m.] | 2.953 | 2.979 | 2.979 |
| $\mu(^3\mathrm{He})$ [n.m.] | –2.125 | –2.128 | –2.128 |
| $r_c(^4\mathrm{He})$ [fm] | **1.639** | **1.663 ± 0.011** | 1.681 ± 0.004 |

L.E. Marcucci *et al.*, arXiv:1504.05063, submitted to J. Phys. G: Nucl. Part. Phys.

# Muon capture on $A = 2, 3$ nuclei

- $\mu^- + d \rightarrow n + n + \nu_\mu$ ⟶ capture rate in the doublet hyperfine state $\Gamma^D$
- $\mu^- + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + \nu_\mu$ ⟶ total capture rate $\Gamma_0$

### $\mu^- + d \rightarrow n + n + \nu_\mu$



### $\mu^- + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + \nu_\mu$

$\boxed{\Gamma_0(\mathrm{PhenAp}) = 1495(11)\ \mathrm{s}^{-1}}$

vs. $\boxed{\Gamma_0(\chi\mathrm{EFT}) = 1494(21)\ \mathrm{s}^{-1}}$

vs. $\Gamma_0(\mathrm{exp}) = 1496(4)\ \mathrm{s}^{-1}$

L.E. Marcucci *et al.*, Phys. Rev. C **83**, 014002 (2011)

L.E. Marcucci *et al.*, Phys. Rev. Lett. **108**, 052502 (2012)

# The proton-proton weak capture reaction

$S(E)$ in $\chi$EFT and PhenAp

- Energy range 2 keV – 100 keV
- PhenAp or $\chi$EFT + FULL EM interaction
- $pp$ $L \leq 1$ partial waves: ${}^1S_0$ + all $P$-waves

$S(0)$ cumulative contributions (in $10^{-23}$ MeV fm$^2$)

| | ${}^1S_0$ | $\cdots + {}^3P_0$ | $\cdots + {}^3P_1$ | $\cdots + {}^3P_2$ |
|---|---|---|---|---|
| PhenAp | 4.000(3) | 4.003(3) | 4.015(3) | 4.033(3) |
| $\chi$EFT | 4.008(6) | 4.011(6) | 4.020(6) | 4.030(6) |

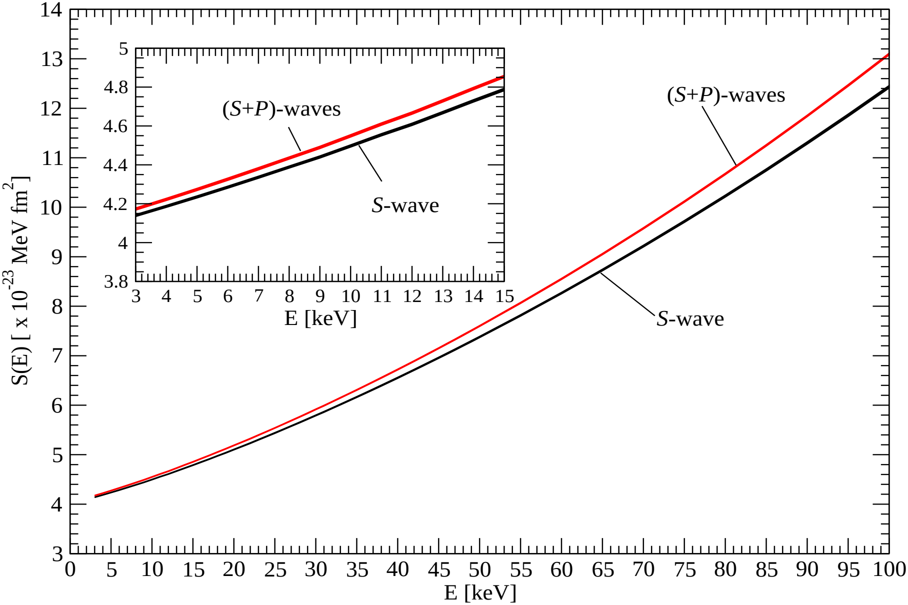
$$\boxed{S(0) = \mathbf{(4.030 \pm 0.006)} \times 10^{-23}\ \mathbf{MeV\ fm^2}}$$

vs.

$$S(0)^{\rm SFII} = (4.01 \pm 0.04) \times 10^{-23}\ \text{MeV fm}^2$$

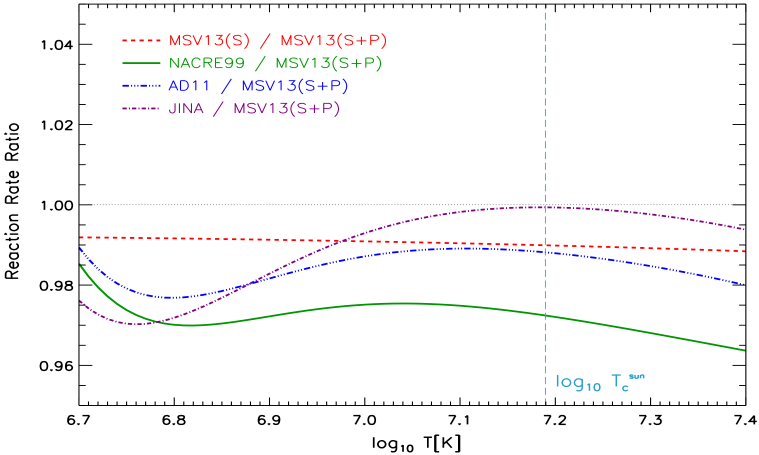
SFII: E.G. Adelberger *et al.*, Rev. Mod. Phys. **83**, 195 (2011)

L.E. Marcucci *et al.*, Phys. Rev. Lett. **110**, 192503 (2013)

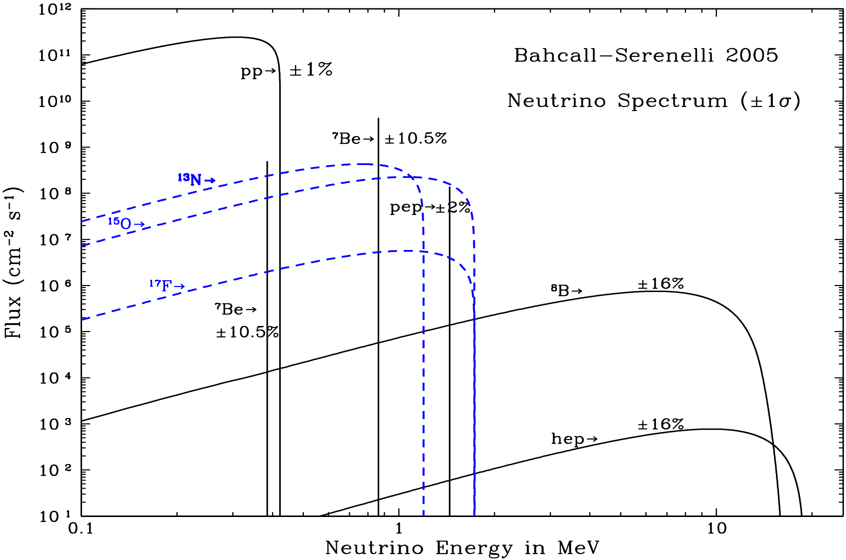

S(E) [ x $10^{-23}$ MeV $fm^2$]
E [keV]
(S+P)-waves
S-wave
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
3
4
5
6
7
8
9
10
11
12
13
14
E [keV]
3
4
5
6
7
8
9
10
11
12
13
14
15
3.8
4
4.2
4.4
4.6
4.8
5
(S+P)-waves
S-wave

Effects on

- age of mid and old stellar clusters (1-12 Gyr)
- standard solar model predictions



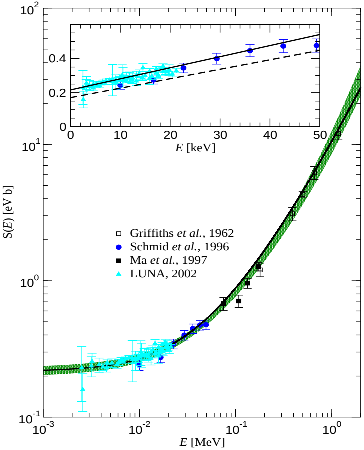
E. Tognelli *et al.*, Phys. Lett. B **742**, 189 (2015)


Bahcall−Serenelli 2005
Neutrino Spectrum (±1σ)
pp→ ±1%
⁷Be→ ±10.5%
¹³N→
¹⁵O→
¹⁷F→
pep→±2%
⁷Be→
±10.5%
⁸B→ ±16%
hep→ ±16%
Flux (cm⁻² s⁻¹)
10¹²
10¹¹
10¹⁰
10 ⁹
10 ⁸
10 ⁷
10 ⁶
10 ⁵
10 ⁴
10 ³
10 ²
10 ¹
0.1
1
10
Neutrino Energy in MeV

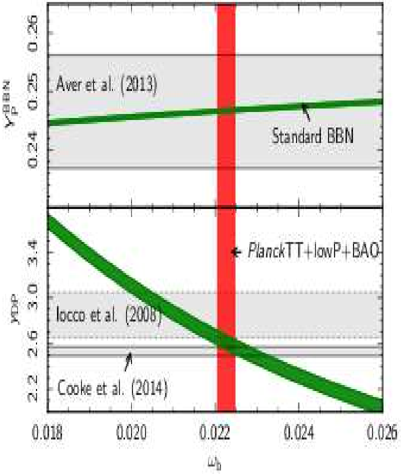
# Neutrino fluxes relative differences

| | `MSV13(S+P)`<br>reference | `MSV13(S)` | `NACRE99` | `AD11` | `JINA` |
|---|---|---|---|---|---|
| | | relative differences | | | |
| $T_c$ $[10^7\ \mathrm{K}]$ | 1.54794 | $-1‰$ | $-3‰$ | $-2‰$ | $-1‰$ |
| $\Phi^\nu_{\mathrm{pp}}$ $[10^{10}]$ | 6.020 | $1‰$ | $2‰$ | $2‰$ | $1‰$ |
| $\Phi^\nu_{\mathrm{pep}}$ $[10^8]$ | 1.446 | $-2‰$ | $-6‰$ | $-2‰$ | $-1‰$ |
| $\Phi^\nu_{\mathrm{hep}}$ $[10^3]$ | 8.584 | $-1‰$ | $-3‰$ | $< 1‰$ | $2‰$ |
| $\Phi^\nu_{\mathrm{Be-7}}$ $[10^9]$ | 4.503 | **−1%** | **−3%** | **−1%** | $-9‰$ |
| $\Phi^\nu_{\mathrm{B-8}}$ $[10^6]$ | 3.694 | **−3%** | **−7%** | **−4%** | **−2%** |
| $\Phi^\nu_{\mathrm{N-13}}$ $[10^8]$ | 2.417 | **−2%** | **−6%** | **−3%** | **−1%** |
| $\Phi^\nu_{\mathrm{O-15}}$ $[10^8]$ | 1.811 | **−3%** | **−8%** | **−4%** | **−2%** |
| $\Phi^\nu_{\mathrm{F-17}}$ $[10^6]$ | 3.373 | **−3%** | **−8%** | **−4%** | **−2%** |

# The $p + d \to {}^3\mathrm{He} + \gamma$ reaction



SFII: E.G. Adelberger *et al.*, Rev. Mod. Phys. **83**, 195 (2011)

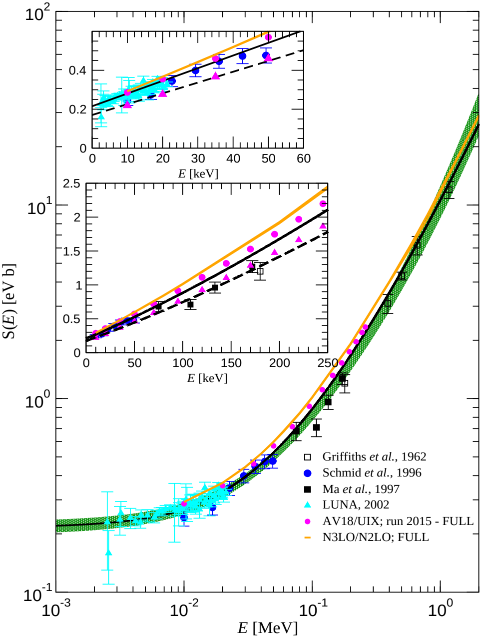


Planck Collab., arXiv:1501.01589 (2015)

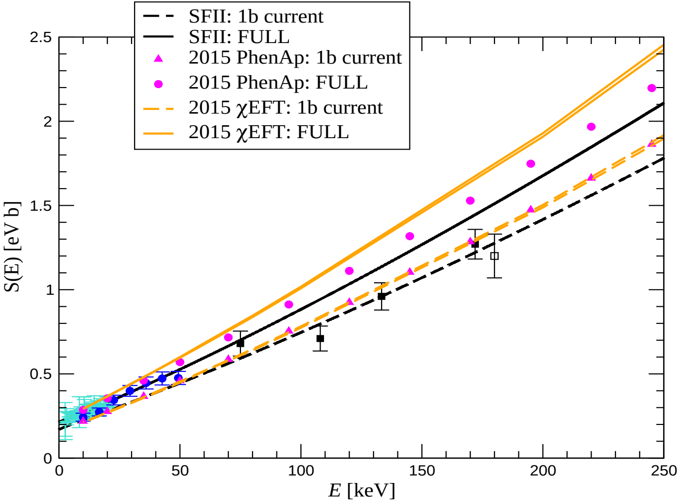
# Latest news on the $p + d \rightarrow {}^3\mathrm{He} + \gamma$ reaction

- New measurement scheduled by the **LUNA Collab.** @ LNGS in the energy range of interest for BBN
- New calculation in PhenAP
- New calculation in $\chi$EFT

PRELIMINARY RESULTS



# PRELIMINARY RESULTS: zoom for $E = 0 - 250$ keV



| $E_{cm}$ [keV] | Δ-PhenAp* | $\mathbf{j}^{\rm MEC}$ | Δ-$\chi$EFT |
|---|---|---|---|
| 35 | +8 % | +24 % | +4 % |
| 70 | +8 % | +20 % | +5 % |
| 95 | +8 % | +18 % | +6 % |
| 120 | +8 % | +17 % | +6 % |
| 145 | +7 % | +16 % | +6 % |
| 170 | +7 % | +16 % | +6 % |
| 195 | +7 % | +16 % | +6 % |
| 220 | +7 % | +15 % | +7 % |
| 245 | +6 % | +15 % | +7 % |

- Δ-PhenAp*≡ |2015-PhenAp – SFII|/SFII
- Δ-$\chi$EFT ≡ |2015-$\chi$EFT – 2015-PhenAp|/2015-PhenAp

LUNA experiment at 3-6 % accuracy can discriminate!!

**WARNING: STILL PRELIMINARY ($\chi$EFT) RESULTS**

# Outlook

## “Reasonably” near future

- $A = 4$ radiative and weak captures: $p + {}^3\mathrm{H} \to {}^4\mathrm{He} + \gamma$ & $p + {}^3\mathrm{He} \to {}^4\mathrm{He} + e^+ + \nu_e$ (the *hep* reaction)
- $\nu$-scattering on light nuclei ($A = 2 - 4$)
- $\mu$-capture on ${}^3$He in other channels: $\mu^- + {}^3\mathrm{He} \to n + d + \nu_\mu$ & $\mu^- + {}^3\mathrm{He} \to n + n + p + \nu_\mu$ (with Faddeev method)
- $A = 6$ as a 3-body system: $\alpha + d \to {}^6\mathrm{Li} + \gamma$
- “Our” *ab-initio* method above breakup-threshold: first steps in E. Garrido *et al.*, Phys. Rev. C **90**, 1 (2014)

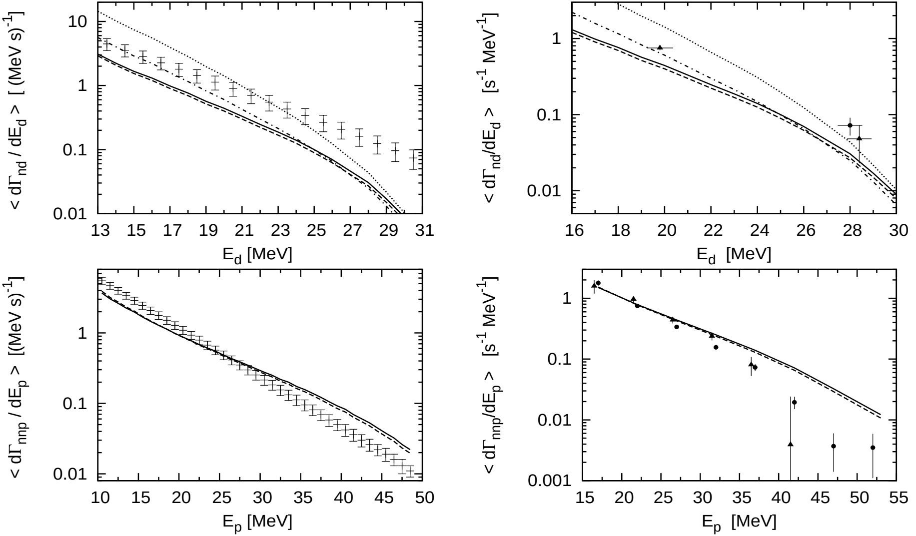
## Less near future

- “Our” *ab-initio* method for $A > 4$ systems (especially $A = 5, 6$) with realist interactions
- Start to work with other methods (for instance GFMC)

# $\mu^- + {}^3\mathrm{He} \rightarrow n + d + \nu_\mu$ & $\mu^- + {}^3\mathrm{He} \rightarrow n + n + p + \nu_\mu$

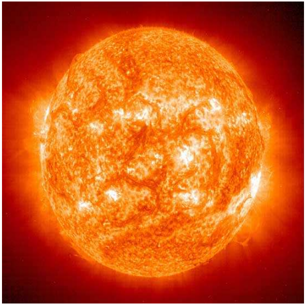
Only 1b + Faddeev equation method [J. Golak *et al.*, Phys. Rev. C **90**, 024001 (2014)]

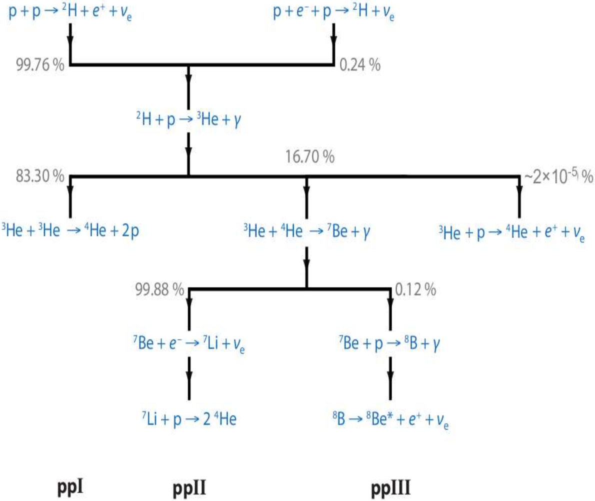
Bystritsky *et al.*, Phys. Rev. A **69**, 012712 (2004)    Kuhn *et al.*, Phys. Rev. C **50**, 1771 (1994)



# SPARES

# The *pp*-chain





$p + p \rightarrow {}^2H + e^+ + \nu_e$

$p + e^- + p \rightarrow {}^2H + \nu_e$

99.76 %

0.24 %

${}^2H + p \rightarrow {}^3He + \gamma$

83.30 %

16.70 %

~$2\times10^{-5}$ %

${}^3He + {}^3He \rightarrow {}^4He + 2p$

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

${}^3He + p \rightarrow {}^4He + e^+ + \nu_e$

99.88 %

0.12 %

${}^7Be + e^- \rightarrow {}^7Li + \nu_e$

${}^7Be + p \rightarrow {}^8B + \gamma$

${}^7Li + p \rightarrow 2\,{}^4He$

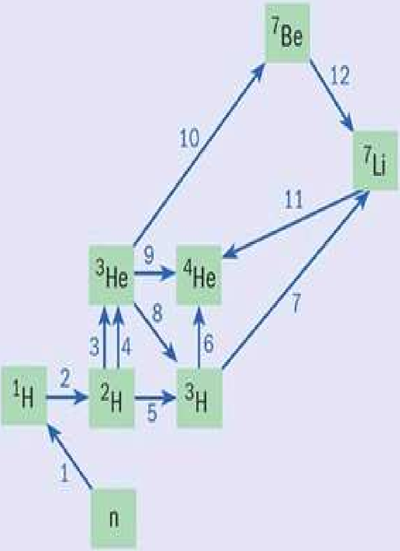
${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

**ppI**

**ppII**

**ppIII**

# The Big-Bang Nucleosynthesis



1 $n \longrightarrow {}^{1}H + e^{-} + \bar{\nu}$

2 ${}^{1}H + n \longrightarrow {}^{2}H + \gamma$

3 ${}^{2}H + {}^{1}H \longrightarrow {}^{3}He + \gamma$

4 ${}^{2}H + {}^{2}H \longrightarrow {}^{3}He + n$

5 ${}^{2}H + {}^{2}H \longrightarrow {}^{3}H + {}^{1}H$

6 ${}^{2}H + {}^{3}H \longrightarrow {}^{4}He + n$

7 ${}^{3}H + {}^{4}He \longrightarrow {}^{7}Li + \gamma$

8 ${}^{3}He + n \longrightarrow {}^{3}H + {}^{1}H$

9 ${}^{3}He + {}^{2}H \longrightarrow {}^{4}He + {}^{1}H$

10 ${}^{3}He + {}^{4}He \longrightarrow {}^{7}Be + \gamma$

11 ${}^{7}Li + {}^{1}H \longrightarrow {}^{4}He + {}^{4}He$

12 ${}^{7}Be + n \longrightarrow {}^{7}Li + {}^{1}H$

# Observables

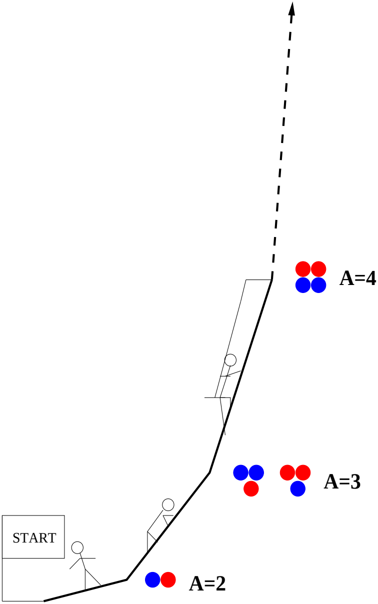
## Total cross-sections $\sigma(E) \rightarrow$ astrophysical $S$-factor $S(E)$

$$\begin{aligned}
S(E) &= E\,\sigma(E)\exp(2\pi\eta); \qquad \eta = \frac{Z_1 Z_2 \alpha}{v_{rel}} \\
r_{12} &= \frac{n_1 n_2}{1+\delta_{12}} \langle \sigma\, v_{rel} \rangle \\
\langle \sigma\, v_{rel} \rangle &\propto \int_0^\infty E\,\sigma(E)\,\exp\left(-\frac{E}{kT}\right) dE \\
&\rightarrow \int_0^\infty S(E)\exp\left(-2\pi\eta - \frac{E}{kT}\right) dE
\end{aligned}$$

Typically:

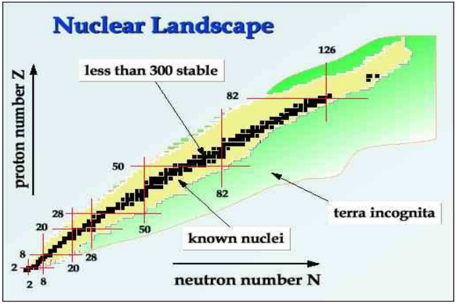
- $E_0$ = Gamow peak energy ($\simeq$ few keV)
- $S(E) \rightarrow$ Taylor expansion $\Rightarrow S(0), S'(0), S''(0),$ etc. etc. etc.

# Why *few*-nucleon and how few are few?



Few-nucleon systems ↔ *ab-initio* methods

$A \leq 12 \rightarrow A \leq 4$



ideal "laboratory" to test $H$

# The Hyperspherical Harmonics (HH) method

## Bound states

$$\Psi^{JJ_z} = \sum_{\mu} c_\mu \Psi_\mu$$

- $\Psi_\mu \rightarrow$ known functions (spin-isospin HH functions)
- Rayleigh-Ritz var. principle: $\delta_c \langle \Psi^{JJ_z} | H - E | \Psi^{JJ_z} \rangle = 0$
  $\Rightarrow$ Solve for $E$ and $c_\mu$

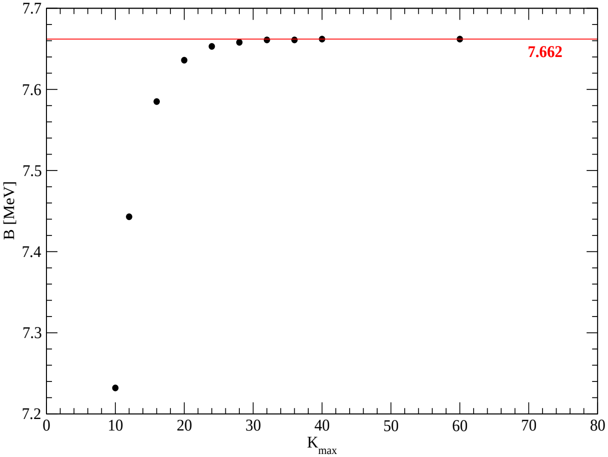
## Scattering states

$$\Psi_{LSJ} = \Psi^{LSJ}_{core} + \Psi^{LSJ}_{asym}$$

- $\Psi^{LSJ}_{core} = \sum_\mu c_\mu \Psi_\mu$
- $\Psi^{LSJ}_{asym} \propto \Omega^R_{LS} + \sum_{L'S'} R_{LL',SS'} \Omega^I_{L'S'}$
- Kohn var. principle: $[R_{LL',SS'}] = R_{LL',SS'} - \langle \Psi_{L'S'J} | H - E | \Psi_{LSJ} \rangle$
  $\Rightarrow R_{LL',SS'} \rightarrow$ phase-shifts and mixing angles $\rightarrow$ scatt. length

# Convergence of the method

B($^3$H) with first 3 spin-isospin channel – N3LO



# Nuclear EW currents in $\chi$EFT

## History

$\mathbf{j}^\gamma$

- Park *et al.* in heavy-baryon $\chi$PT (HB$\chi$PT)
  → since $\simeq$ 1995
- Pastore *et al.* in time-ordered perturbation theory (TOPT)
  → since 2009
- Kölling *et al.* with the unitary transform method
  → in parallel since 2009

$\mathbf{j}^A$

- Park *et al.* in HB$\chi$PT → since $\simeq$ 2000
- Baroni *et al.* in TOPT → work in progress

To be remarked:

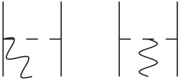
- Park *et al.* currents ready BEFORE the $\chi$EFT potentials
  ⇒ "hybrid" $\chi$EFT
- Park *et al.*: only available FULL set

# Power counting for $\mathbf{j}^\gamma$

$\mathcal{O}(Q^{-2})$

$$\mathbf{j}^{(-2)} \propto [e_N(1)(\mathbf{p}'_1+\mathbf{p}_1)+\mathrm{i}\mu_N(1)\boldsymbol{\sigma}_1\times\mathbf{q}]\times\delta(\mathbf{p}'_2-\mathbf{p}_2)+1\leftrightarrow 2$$

$\mathcal{O}(Q^{-1})$

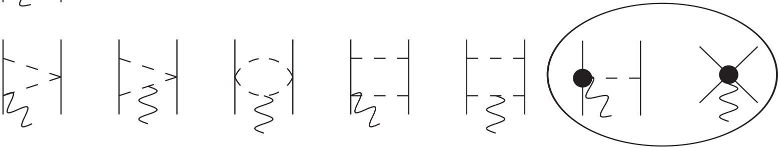


”standard” one − pion − exchange

$\mathcal{O}(Q^{0})$

■ = relativistic corrections

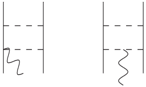
$\mathcal{O}(Q^{1})$



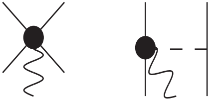
# Comments

- Similar results between Pastore *et al.* and Kölling *et al.*
- Differences with Park *et al.*

for the box-diagrams



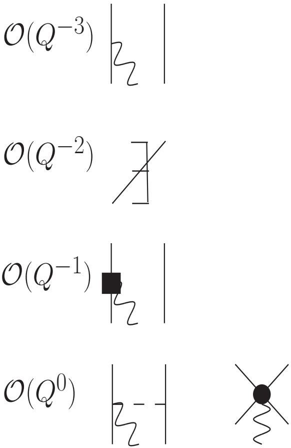
for the terms



Park *et al.* $\chi$EFT EM currents should be reconsidered

- LECs fitted to selected EM observables $[\sigma_{np},\ \mu_d,\ \mu^{S/V}(A=3)]$

# Power counting for $\mathbf{j}^A$



Note:

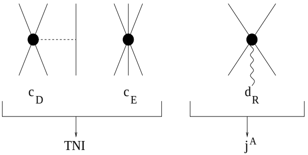
- $\mathcal{O}(Q^1)$: loop and two-pion-exchange contributions (not yet calculated)
- Park *et al.* only available model at $\mathcal{O}(Q^0)$ → one LEC - $d_R$

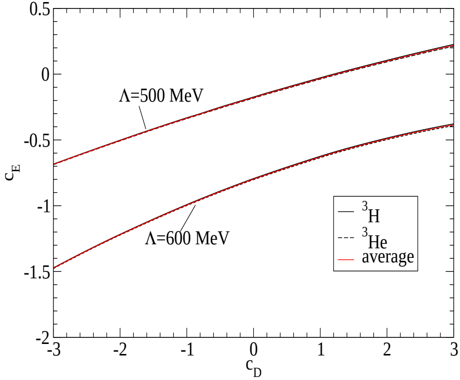
$$\boxed{d_R = \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}}$$

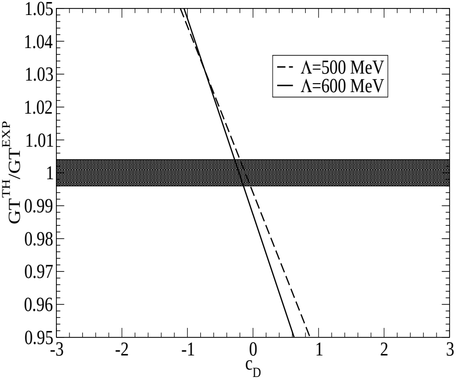
Gårdestig and Phillips, PRL **96**, 232301 (2006)

Gazit *et al.*, PRL **103**, 102502 (2009)

- fit $c_D$ and $c_E$ (in TNI at N2LO) to $B(A=3)$ and $GT_{Exp}$







$\Rightarrow \{c_D;c_E\}_{\mathrm{MAX}}$ and $\{c_D;c_E\}_{\mathrm{MIN}}$

| Model | Λ<br>[MeV] | $c_D$ | $c_E$ | B($^4$He)<br>[MeV] | $^2a_{nd}$<br>[fm] |
|---|---|---|---|---|---|
| N3LO/N2LO* | 500 | 1.0 | −0.029 | 28.36 | 0.675 |
| N3LO/N2LO | 500 | −0.12 | −0.196 | 28.49 | 0.666 |
| N3LO/N2LO | 600 | −0.26 | −0.846 | 28.64 | 0.696 |
| Exp. | | | | 28.30 | 0.645(10) |

Marcucci *et al.*, PRL **108**, 052502 (2012); Viviani *et al.*, PRL **111**, 172302 (2013)

# Nuclear transition operators in $\chi$EFT at N3LO

$$\text{Weak operators: } \rho^{(V/A)}, \mathbf{j}^{(V/A)}$$
$$\text{CVC} \Rightarrow \rho^{V}/\mathbf{j}^{V} \rightarrow \rho^{\gamma}/\mathbf{j}^{\gamma}$$

Park *et al.*, PRC **67**, 055206 (2003)
Song *et al.*, PRC **79**, 064002 (2009)
→ only available $\chi$EFT weak transition operators

- One-body operators: NRR of $j_i^{\mu} \rightarrow O(1/m^2)$
- Two-body $\rho^{(A)}$: soft $\pi$-exchange dominant
- Two-body $\rho^{(V)} = 0$ at N3LO
- Two-body $\mathbf{j}^{(V)}$: CVC → EM current:
  $1\pi + 1\pi C + 2\pi + \text{CT} \rightarrow$ two LECs ($g_{4S}$ & $g_{4V}$) $\Rightarrow$
  from $\mu(^3\mathrm{H} - {}^3\mathrm{He})$
- Two-body $\mathbf{j}^{(A)}$: $1\pi + \text{CT} \rightarrow$ one LEC ($d_R$) $\Rightarrow$
  from $GT_{exp}$ of tritium $\beta$-decay

Gaussian regulator → $\Lambda \simeq 500 - 800$ MeV

# Nuclear transition operators in PMA

- One-body operators: NRR of $j_i^\mu \rightarrow O(1/m^2)$
- Two-body $\rho^{(V)}$ and $\mathbf{j}^{(V)}$: CVC $\rightarrow$ EM operators (MEC+$\underline{\mathbf{j}^{(V)}(\Delta)}$)

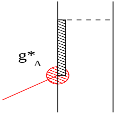
|  | $\mu(^3\mathrm{H})$ | $\mu(^3\mathrm{He})$ |
|---|---|---|
| 1b | 2.5745 | -1.7634 |
| **Full** | **2.9525** | **-2.1299** |
| Exp. | 2.9790 | -2.1276 |

AV18+UIX
$\Rightarrow$ Full=1b+2b+3b [1]

[1] Marcucci *et al.*, PRC **72**, 014001 (2005)

- Two-body $\rho^{(A)}$: PCAC + low-energy theorem $\rightarrow$ $\pi$-exchange and short-range terms
- Two-body $\mathbf{j}^{(A)}$: $\pi$- and $\rho$-exchange, $\pi\rho$ mechanism, and $\underline{\mathbf{j}^{(A)}(\Delta)}$

Largest contribution to $\mathbf{j}^{(A)}(\Delta)$ from



$g_A^*$ fit to observable: $\underline{GT_{exp}\text{ of tritium }\beta\text{-decay}}$

# The proton-proton weak capture reaction

$S(E)$
in $\chi$EFT and
PhenAp

- Energy range 2 keV – 100 keV
- PhenAp [AV18] or $\chi$EFT [N3LO] + FULL EM interaction
- *pp* $L \leq 1$ partial waves: ${}^1S_0$ + all $P$-waves

$S(0) - {}^1S_0$
(in $10^{-23}$ MeV fm$^2$)

| | $V_{nucl} + V_{Coul}$ | $V_{nucl} + V_{EM}$ |
|---|---|---|
| PhenAp-IA | 3.99 | 3.96 |
| PhenAp-FULL | 4.03 | 4.00 |
| $\chi$EFT(500)-IA | 3.96 | 3.94 |
| $\chi$EFT(500)-FULL | 4.03 | 4.01 |
| $\chi$EFT(600)-IA | 3.94 | 3.93 |
| $\chi$EFT(600)-FULL | 4.01 | 4.01 |

- agreement with $S^{\rm SFII}(0) = 4.01(1 \pm 0.009) \times 10^{-23}$ MeV fm$^2$
- $V_{EM}$-$V_{Coul}$ → $\leq$1 % effect
- agreement PMA-$\chi$EFT
- very small cutoff dependence ($\leq$1 %)

SFII: Adelberger *et al.*, RMP **83**, 195 (2011)

# Cumulative contributions to $S(0)$

| | ${}^1S_0$ | $\cdots + {}^3P_0$ | $\cdots + {}^3P_1$ | $\cdots + {}^3P_2$ |
|---|---|---|---|---|
| PMA | 4.000(3) | 4.003(3) | 4.015(3) | 4.033(3) |
| $\chi$EFT(500) | 4.008(5) | 4.011(5) | 4.020(5) | 4.030(5) |
| $\chi$EFT(600) | 4.007(5) | 4.010(5) | 4.019(5) | 4.029(5) |

- $P$-waves contribution to $S(0) \simeq 1$ %
- theoretical uncertainty very small
- very small cutoff dependence ($< 1$ %)

$$\boxed{S(0) = \mathbf{4.03(1 \pm 0.006)} \times 10^{-23}\ \textbf{MeV fm}^2}$$

vs.

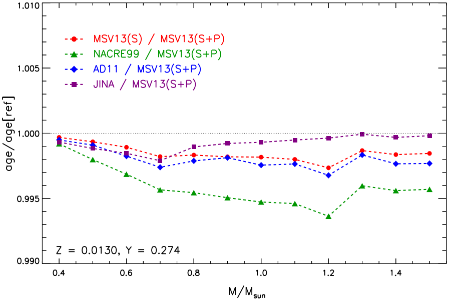
$$S(0)^{\rm SFII} = 4.01(1 \pm 0.009) \times 10^{-23}\ \text{MeV fm}^2$$

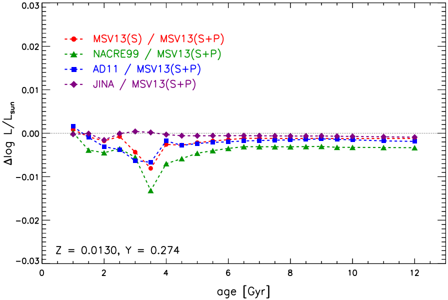
# Age determination in stellar cluster (I)

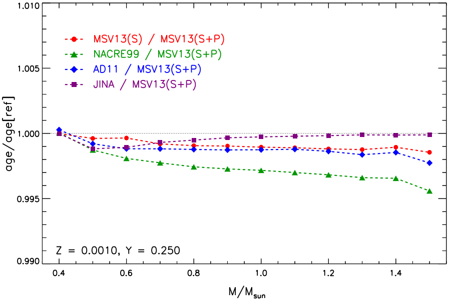
- Stellar cluster: stars with **same** age, **same** chemical composition and **same** distance from Earth, but different masses ⇒ from the same starting nebula
- Age determined through the luminosity of the point corresponding to the central H-exhaustion (`TO` for Sun-like stars or `OC` for larger mass stars)
- Stars in clusters older than ∼10 Gyr → **pp chain** during the `MS` phase
- Stars close to the `TO` phase in younger clusters→ **CNO-cycle** (and small contribution from pp chain)

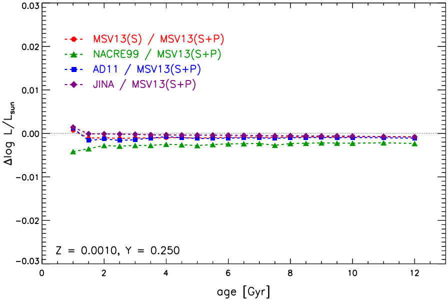
⇒ **pp reaction efficiency is CRUCIAL**

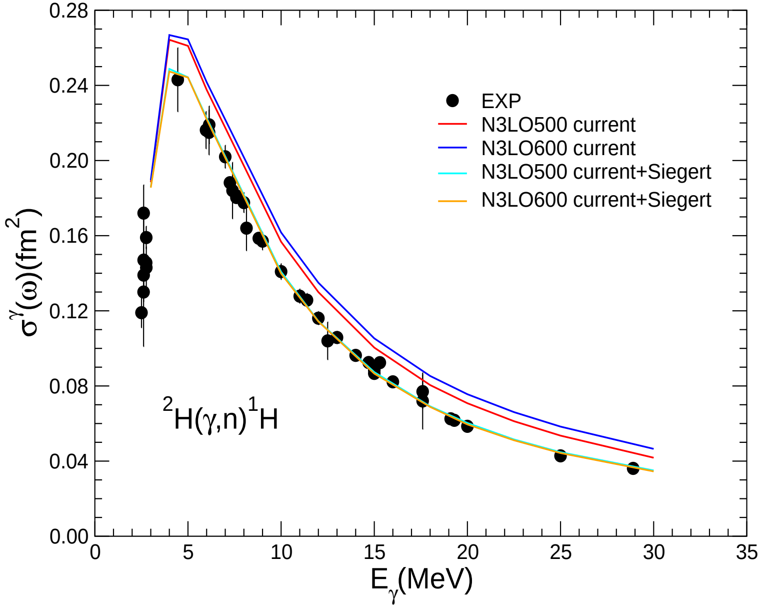
# Age determination in stellar cluster (II)










0.28
0.24
0.20
0.16
0.12
0.08
0.04
0.00
0
5
10
15
20
25
30
35
$\sigma^{\gamma}(\omega)(\mathrm{fm}^2)$
$E_\gamma$(MeV)
● EXP
N3LO500 current
N3LO600 current
N3LO500 current+Siegert
N3LO600 current+Siegert
$^2H(\gamma,n)^1H$