Must space-time be singular?

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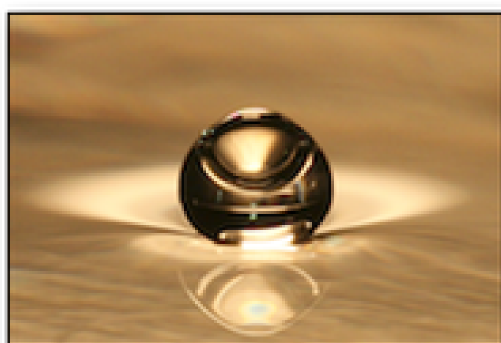

Einstein's general relativity:

space-time singularities (such as Big Bang) are unavoidable (under some mild assumptions)

$\rightarrow$ Signals breakdown of the theory?

Does quantum gravity eliminate the singularities?

$\rightarrow$ Depends on approach to quantum gravity.

E.g. Wheeler-DeWitt quantization, loop quantum gravity, ...

$\rightarrow$ Depends also on approach to quantum theory.

E.g. Collapse theory, Everett, Bohmian mechanics, ...

Some recent results for mini-superspace:

- Standard quantum theory (Ashtekar, Corichi, Pawloski, Singh)
  - Wheeler-DeWitt quantization: singularities for generic states
  - Loop quantum gravity: no singularities for generic states
- Consistent histories (Craig, Singh)
  - Wheeler-DeWitt quantization: singularities for generic states
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In this talk:

- Bohmian mechanics (with F. Falciano and N. Pinto-Neto, PRD 91, 043524, 2015)
  - Wheeler-DeWitt quantization: *sometimes* singularities
  - Loop quantum gravity: no singularities

I. INTRODUCTIONS TO BOHMIAN MECHANICS

(a.k.a. pilot-wave theory, de Broglie-Bohm theory, . . .)

- De Broglie (1927), Bohm (1952)

- Particles moving under influence of the wave function.

- Dynamics:

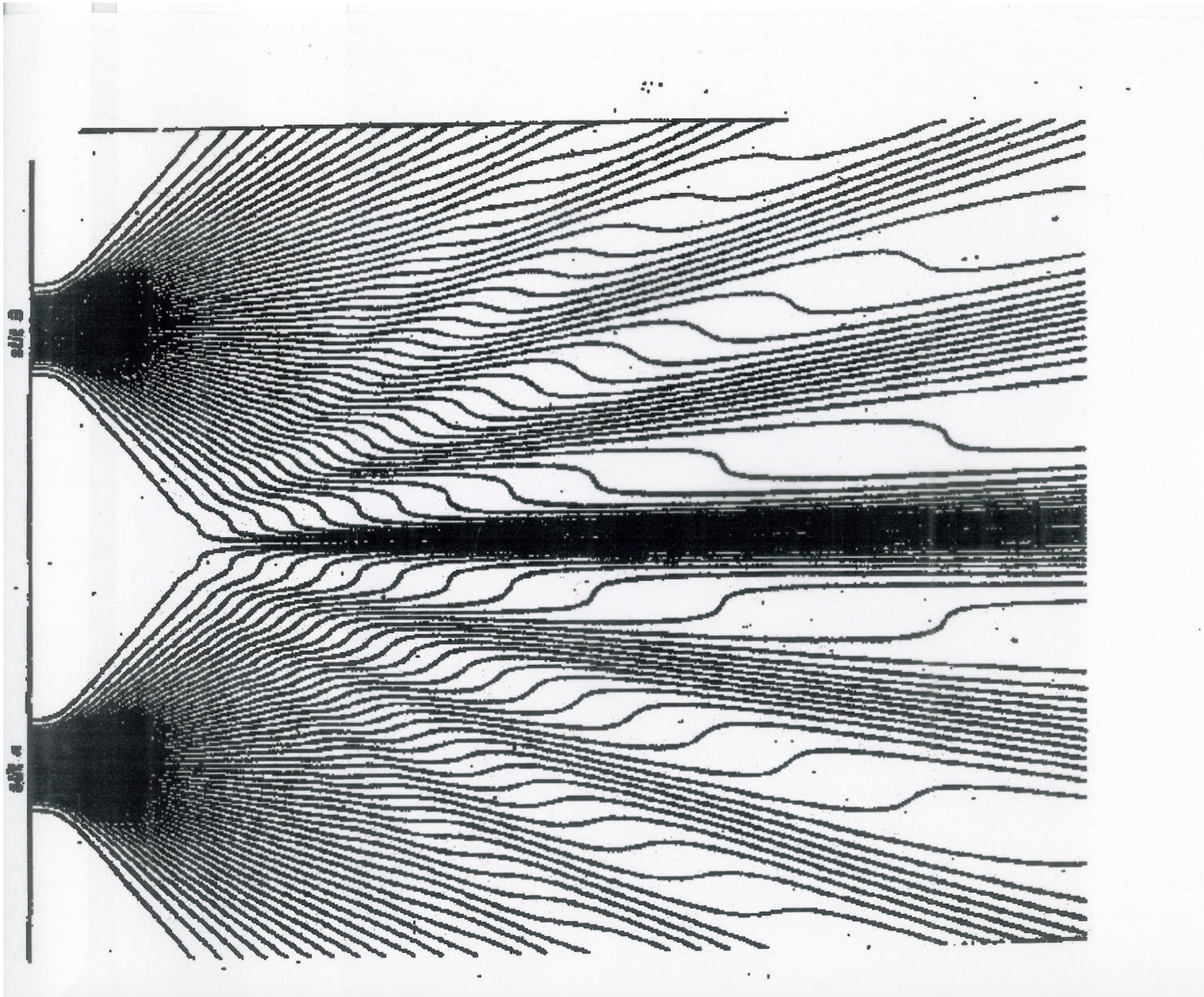
$$\frac{d\mathbf{X}_k(t)}{dt} = \mathbf{v}_k^{\psi_t}(X_1(t), \ldots, X_N(t))$$

where

$$\mathbf{v}_k^{\psi} = \frac{\hbar}{m_k}\mathrm{Im}\frac{\boldsymbol{\nabla}_k\psi}{\psi} = \frac{1}{m_k}\boldsymbol{\nabla}_k S, \qquad \psi = |\psi|e^{\mathrm{i}S/\hbar}$$

$$\mathrm{i}\hbar\partial_t\psi_t(x) = \left(-\sum_{k=1}^{N}\frac{\hbar^2}{2m_k}\nabla_k^2 + V(x)\right)\psi_t(x)\,, \qquad x = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$$

• Double Slit experiment:



- **Quantum equilibrium**:

  - for an ensemble of systems with wave function ψ
  - distribution of particle positions $\rho(x) = |\psi(x)|^2$

Quantum equilibrium is preserved by the particle motion ($=$ equivariance), i.e.

$$\rho(x, t_0) = |\psi(x, t_0)|^2 \qquad \Rightarrow \qquad \rho(x, t) = |\psi(x, t)|^2 \qquad \forall t$$

Agreement with quantum theory in quantum equilibrium.

- **Classical limit:**

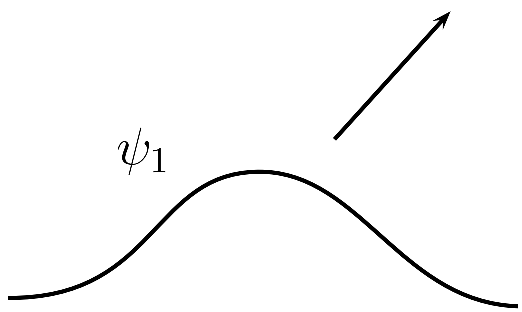
$$\dot{\mathbf{x}} = \frac{1}{m}\boldsymbol{\nabla} S \qquad \Rightarrow \qquad m\ddot{\mathbf{x}} = -\boldsymbol{\nabla}(V + Q)$$

$$\psi = |\psi| e^{\mathrm{i}S/\hbar}, \qquad Q = -\frac{\hbar^2}{2m}\frac{\nabla^2|\psi|}{|\psi|} = \text{quantum potential}$$

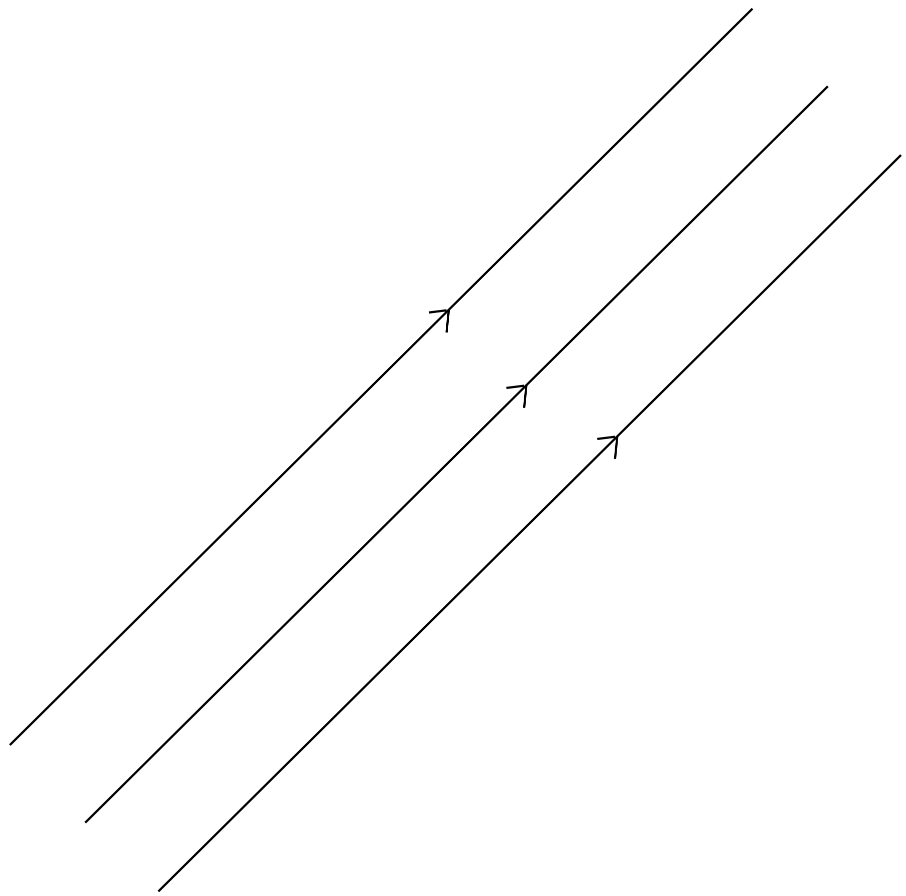
Classical trajectories when $|\boldsymbol{\nabla} Q| \ll |\boldsymbol{\nabla} V|$.

- **"Surreal" trajectories**

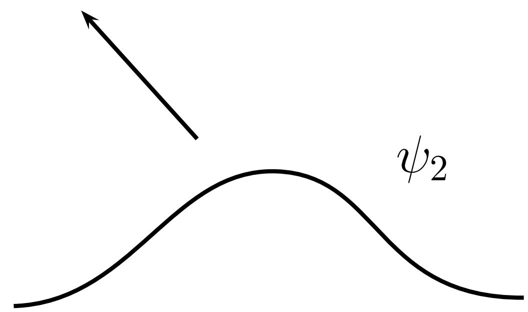
  Suppose $\psi = \psi_1$



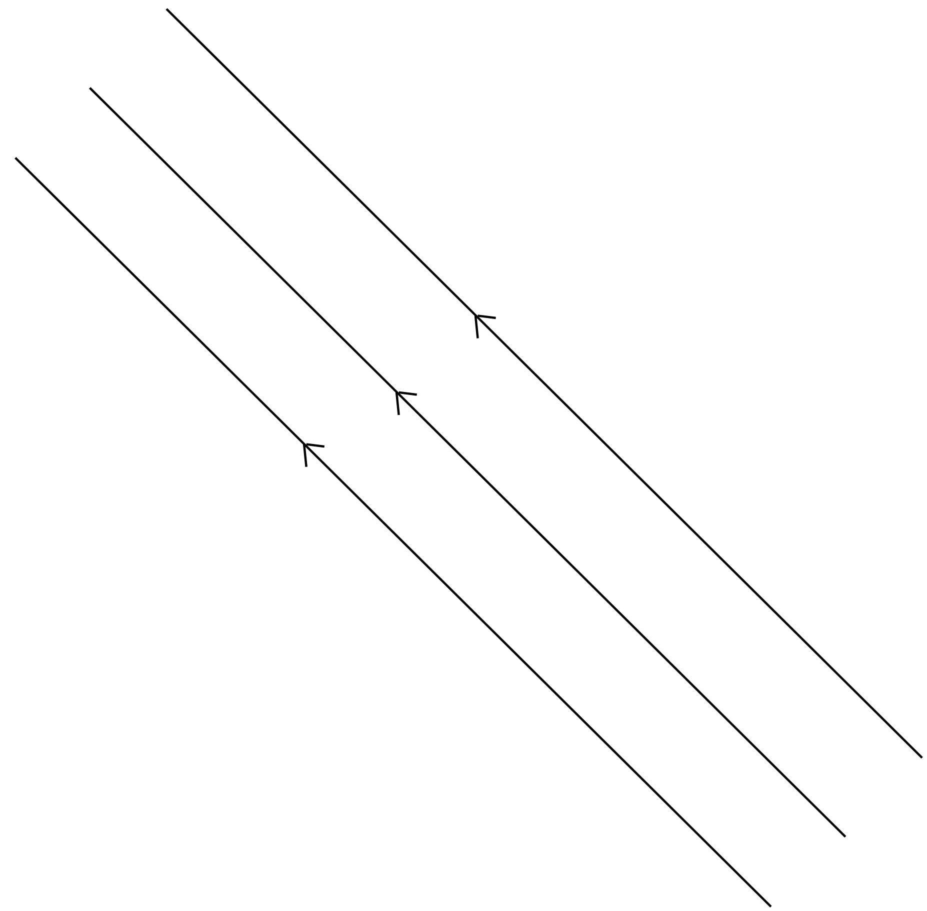
Bohmian trajectories:



Suppose $\psi = \psi_2$

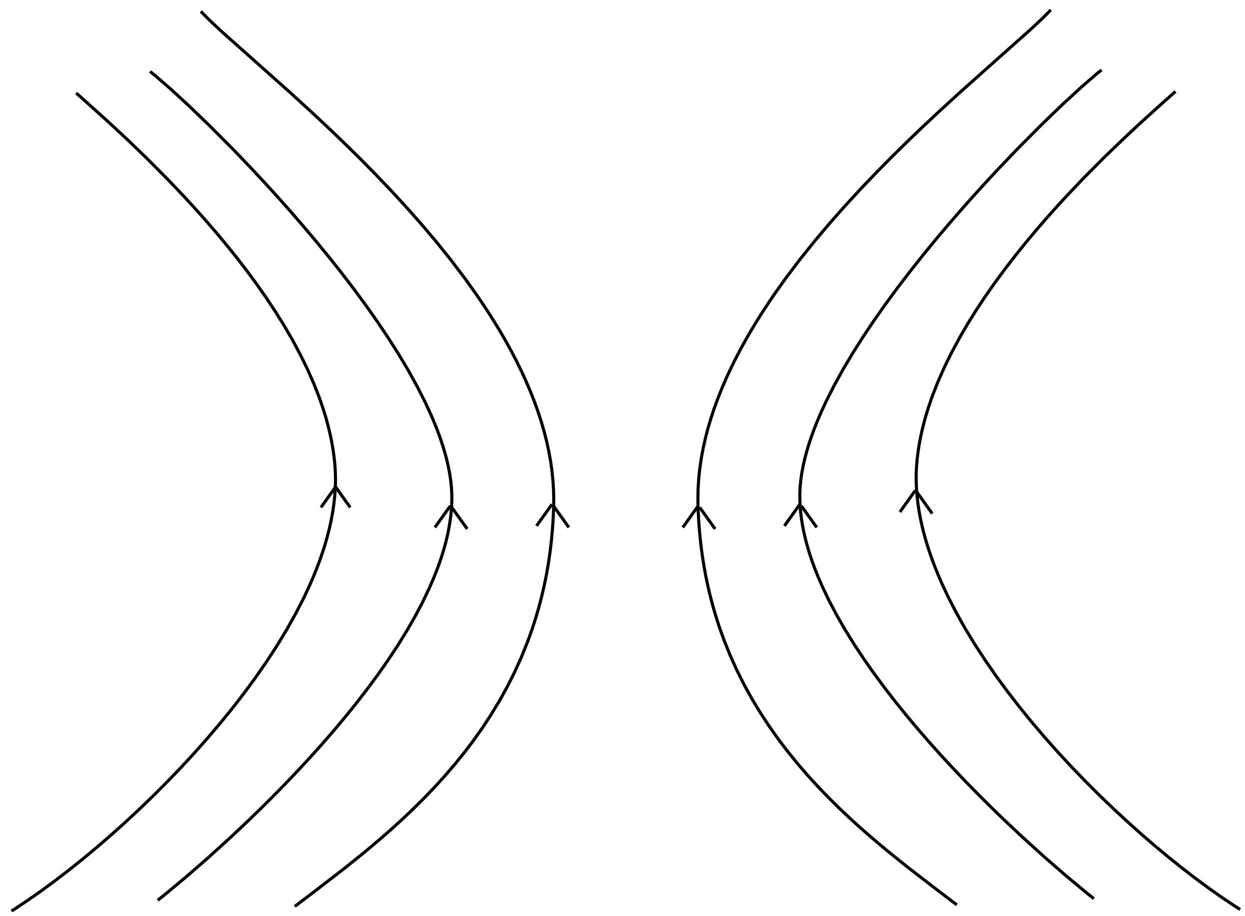


Bohmian trajectories:

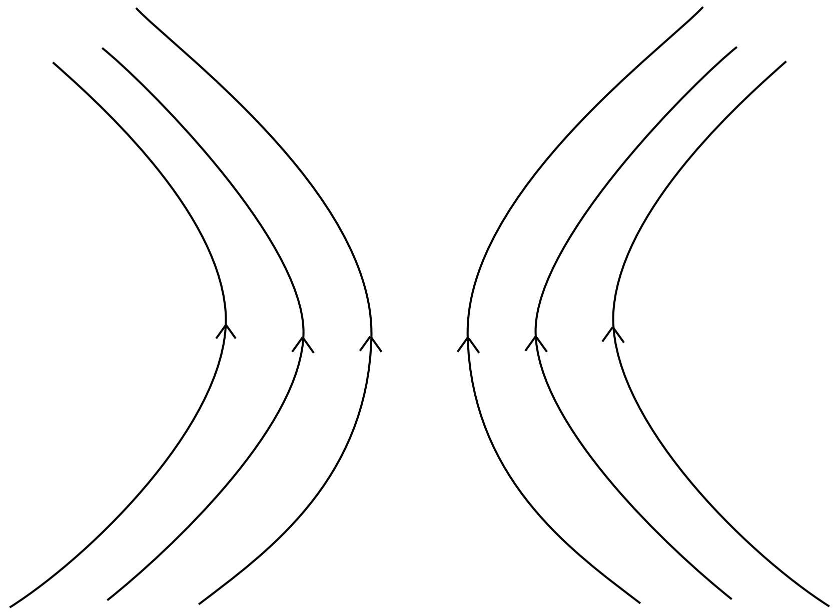


Suppose $\psi = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$

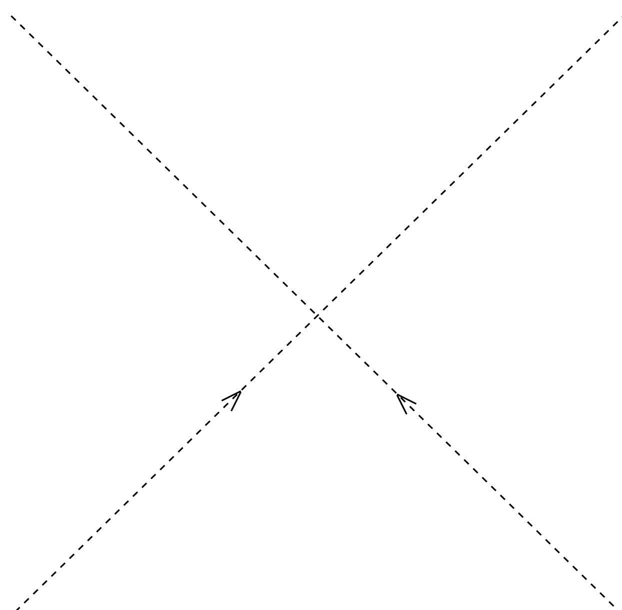
Bohmian trajectories:



Bohmian trajectories:



Consistent histories:



Bouncing droplets:

with Boris Filoux, Nicolas Vandewalle (quandrops Liege, Belgium)

II. SINGULARITIES

- **Quantum gravity**

  Canonical quantization of Einstein's theory for gravity:

$$g^{(3)}(x) \to \widehat{g}^{(3)}(x)$$

  In functional Schrödinger picture:

$$\Psi = \Psi(g^{(3)})$$

  Satisfies the Wheeler-De Witt equation:

$$\mathrm{i}\frac{\partial \Psi}{\partial t} = \widehat{H}\Psi = 0$$

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- **Conceptual problems**:

  1. Measurement problem: We are considering the whole universe. There are no outside observers or measurement devices.
  2. Problem of time: There is no time evolution, the wave function is static. (How can we tell the universe is expanding or contracting?)

- **Bohmian approach**

In a Bohmian approach we have an actual 3-metric $g^{(3)}$ which satisfies:

$$\dot{g}^{(3)} = v^{\Psi}(g^{(3)})$$

This solves problems 1.

It also solves problem 2:

We can tell whether the universe is expanding or not, whether it goes into a singularity or not, etc.: It depends on the actual metric.

- **Singularities**

  What does it mean to have a space-time singularity in quantum gravity?

  – Ψ has support on singular metrics?

  – Ψ is peaked around singular metrics?

  – $\langle \Psi | \widehat{g} | \Psi \rangle$ is singular?

  In the Bohmian approach: singularities if the actual metric is singular.

Mini-superspace

Friedman-Lemaître-Robertson-Walker space-time.

Restriction to homogeneous and isotropic metrics and fields:

– Gravity: $\mathrm{d}s^2 = \mathrm{d}t^2 - a(t)^2\mathrm{d}x^2$

– Matter: $\phi = \phi(t)$

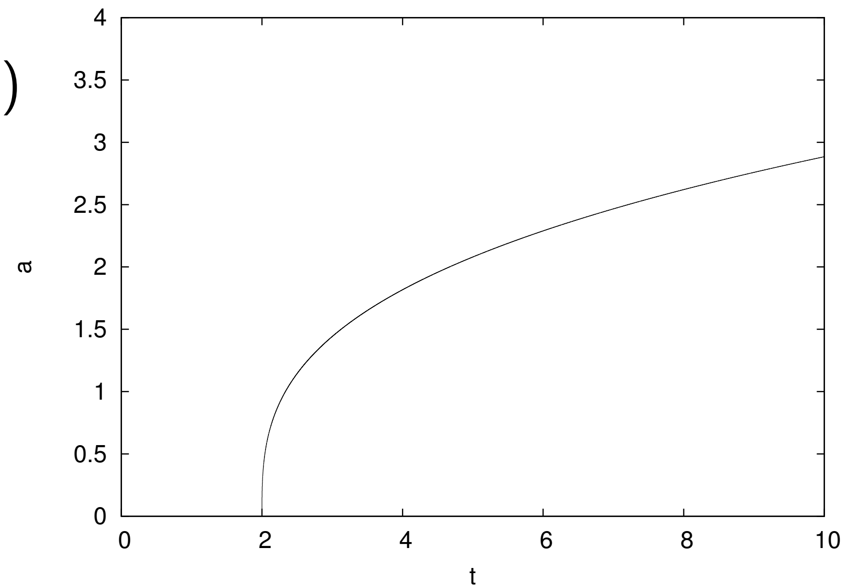
Singularity if $a = 0$

Classical theory $(4\pi G/3 = 1)$

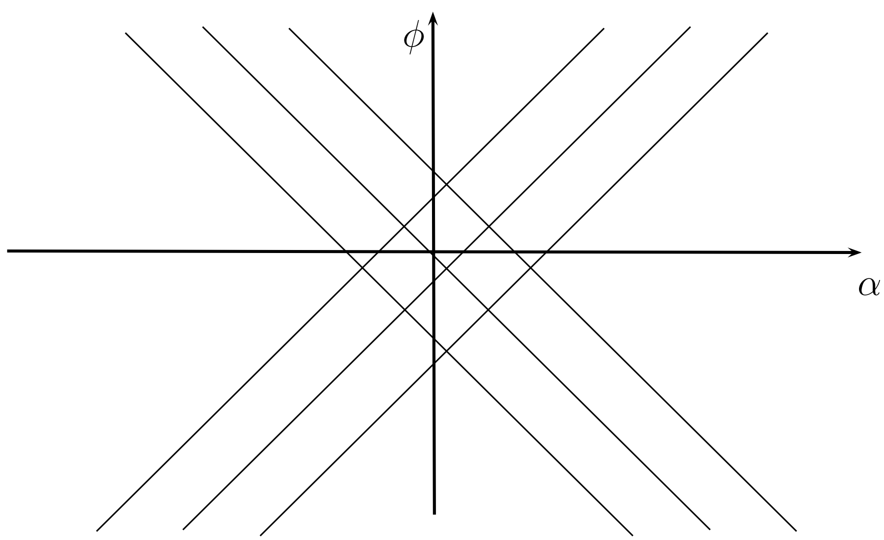
$$\dot\phi = \pm\frac{c}{e^{3\alpha}}, \quad \dot\alpha = \frac{c}{e^{3\alpha}} \qquad \text{with} \quad a = e^{\alpha} \qquad c \text{ constant}$$

Always singularity

($a = 0$ or $\alpha \to -\infty$)



In (α, ϕ)-space

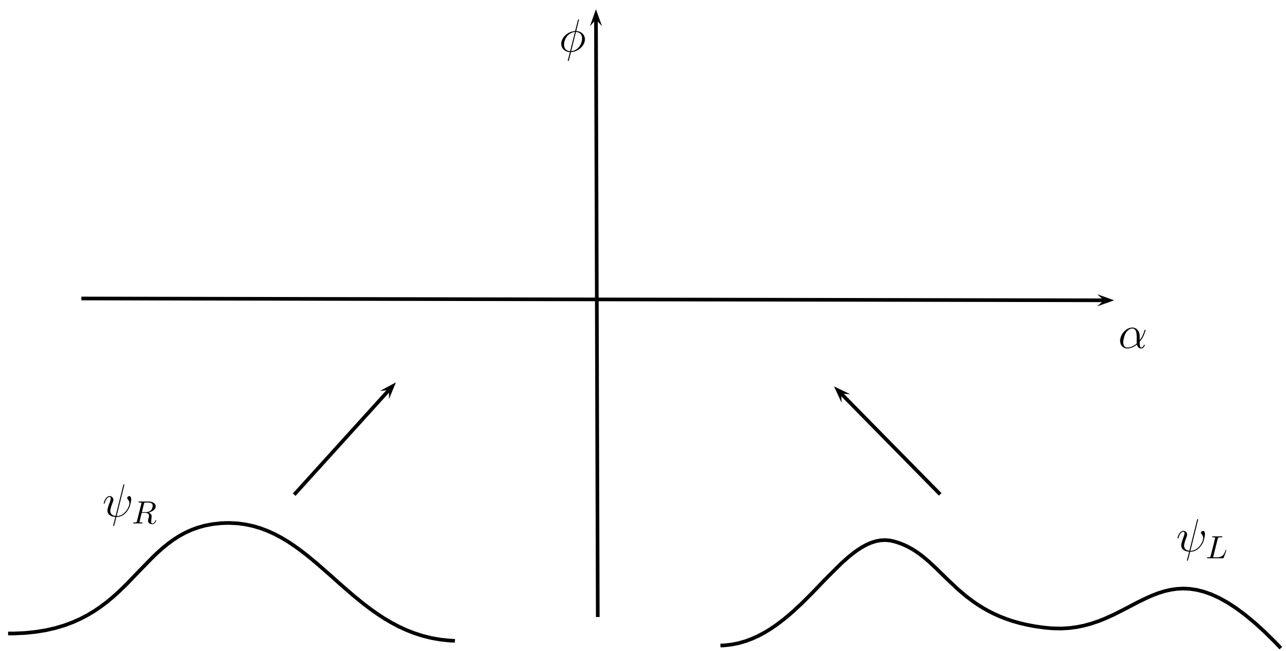


Quantum theory

Wheeler-DeWitt equation

$$(\partial_\alpha^2 - \partial_\phi^2)\Psi = 0\,, \qquad a = e^\alpha$$

Solutions $\Psi = \Psi_R + \Psi_L$; $\qquad \Psi_R = \Psi_R(\alpha - \phi)$, $\quad \Psi_L = \Psi_L(\alpha + \phi)$:



Bohmian equations:

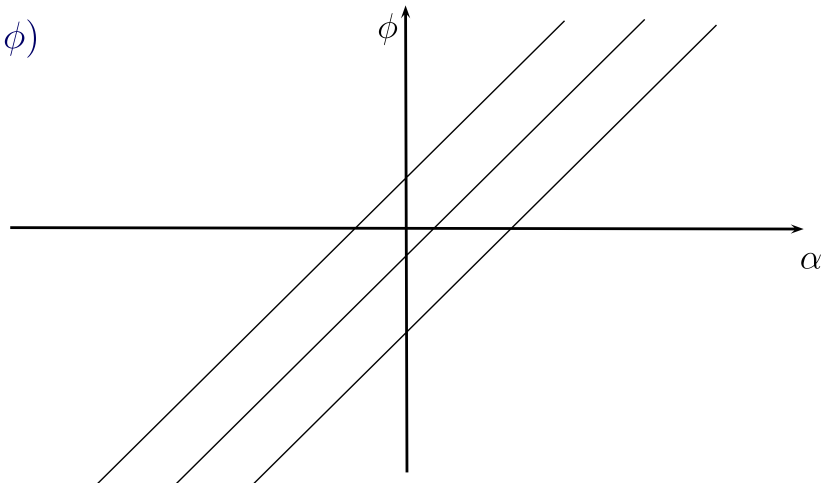
$$\dot{\phi} = \frac{1}{e^{3\alpha}}\partial_\phi S\,, \quad \dot{\alpha} = -\frac{1}{e^{3\alpha}}\partial_\alpha S\,, \qquad \psi = |\psi|\mathrm{e}^{\mathrm{i}S}$$

Examples.

- Ψ real, i.e., $S = 0$: α is constant, i.e. Minkowski space-time; no singularities.
- $\Psi = \Psi_R(\alpha - \phi) = \mathrm{e}^{-(\alpha-\phi)^2-\mathrm{i}(\alpha-\phi)}$

  classical trajectories

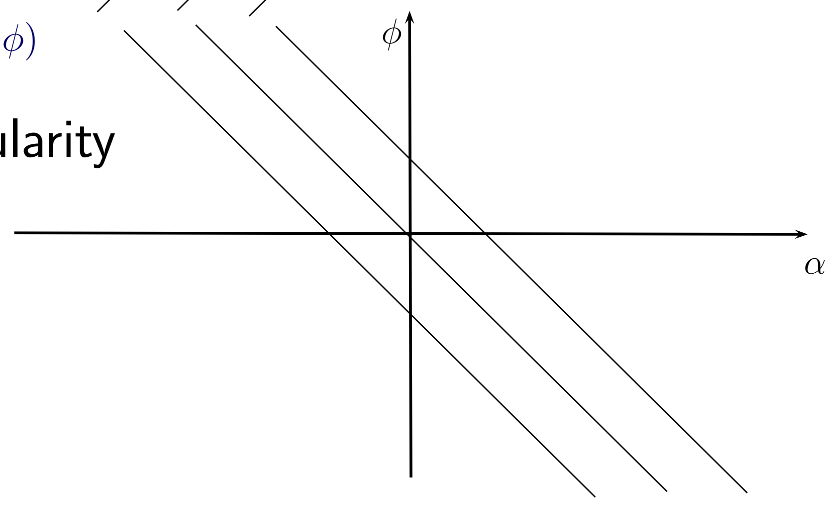
  always singular (big bang)



- $\Psi = \Psi_L(\alpha - \phi) = \mathrm{e}^{-(\alpha+\phi)^2+\mathrm{i}(\alpha+\phi)}$

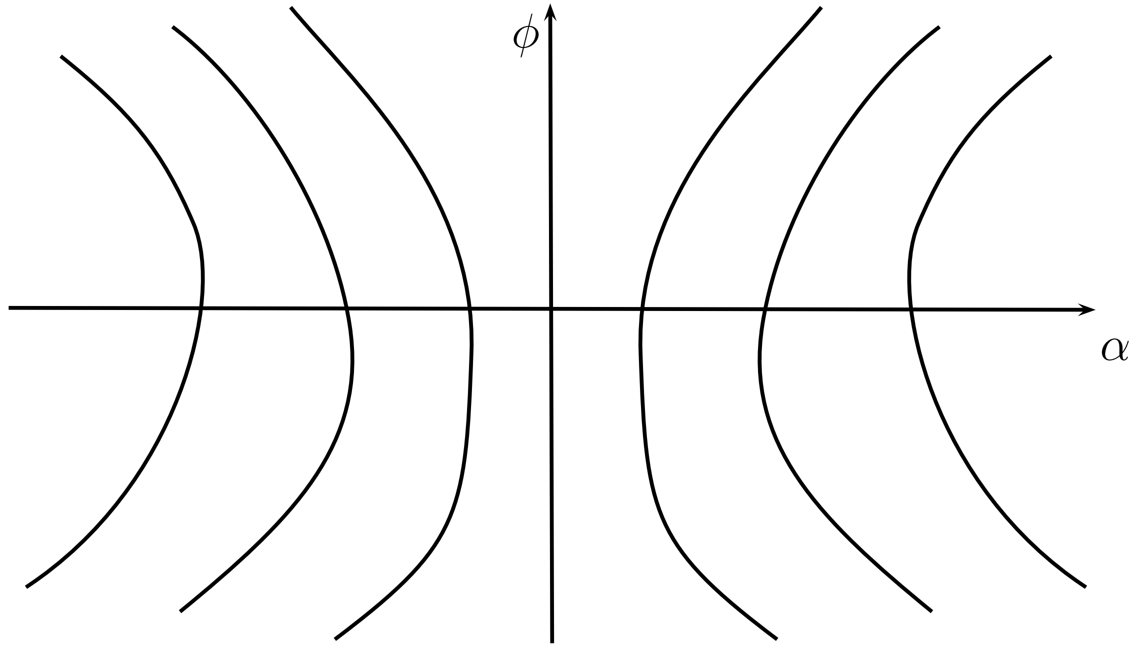
  classical trajectories, hence singularity

  always singular (big crunch)



- Superposition $\Psi = \mathrm{e}^{-(\alpha-\phi)^2+\mathrm{i}(\alpha-\phi)} + \mathrm{e}^{-(\alpha+\phi)^2+\mathrm{i}(\alpha+\phi)}$

  Is symmetric: $\Psi(\phi, \alpha) = \Psi(\phi, -\alpha)$



  Big bang and big crunch for trajectories on the left; bounce for trajectories on the right

- Note: no probability distribution

Regard ϕ as time variable

"Square root" of Wheeler-DeWitt equation:

$$\mathrm{i}\partial_\phi \psi_\pm = \widehat{H}_\pm \psi_\pm = \mp\sqrt{-\partial_\alpha^2}\psi_\pm$$

$$\Psi = (\psi_+, \psi_-)$$

– Probability distribution for α:

$$\rho = |\psi_+|^2 + |\psi_-|^2$$

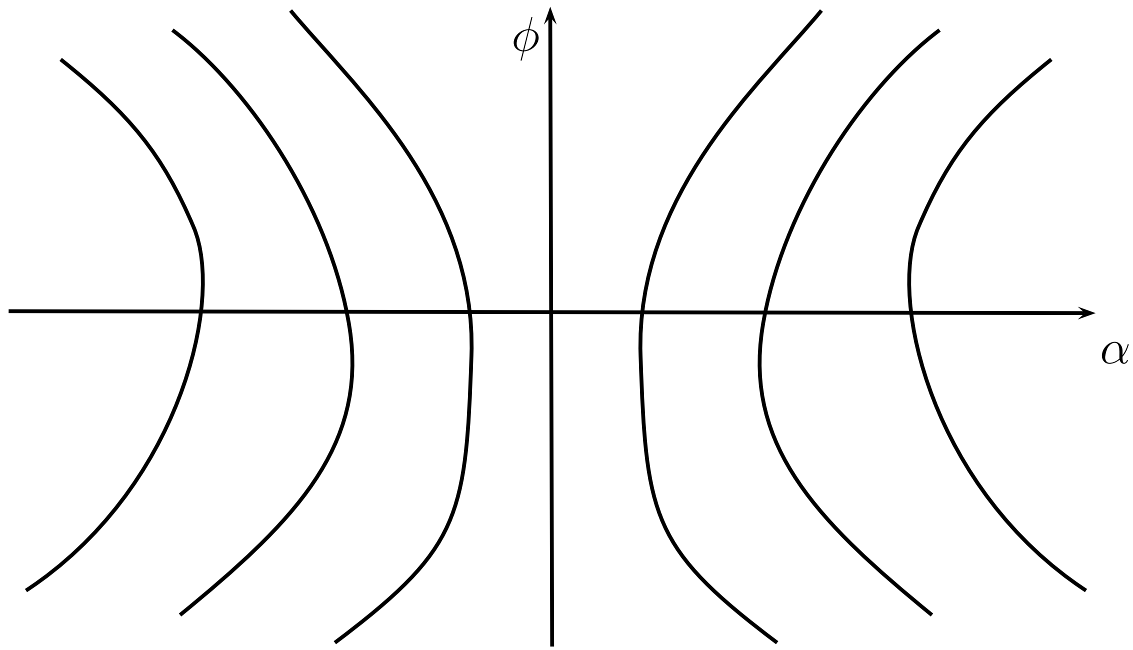
– Bohmian dynamics preserves this distrubtion

– For $\Psi = \Psi_R$ or $\Psi = \Psi_L$: classical trajectories, hence singularity

– For superposition $\Psi = \Psi_R + \Psi_L$:

$$\frac{1}{2} \leqslant P_{\text{singularity}} < 1$$

For example, symmetric state $\Psi(\phi, \alpha) = \Psi(\phi, -\alpha)$



Big bang and big crunch for trajectories on the left; bounce for trajectories on the right

Probability for singularity is $1/2$.

Probability for a bouncing universe is $1/2$.

– Singularity according to

  * Standard quantum theory, for generic states (Ashtekar, Corichi, Pawloski, Singh)
  * Consistent histories (Craig, Singh)

Loop quantum cosmology

(Application of loop quantum gravity ideas to mini-superspace)

- Different from Wheeler-DeWitt quantization
- Scale factor a takes discrete values.

  States $\psi(\nu, \phi)$, where $\nu \sim a^3$ and $\nu = 4\lambda n$, $n \in \mathbb{N}$
- Wave equation becomes difference equation:

$$\mathrm{i}\partial_\phi \psi(\nu, \phi) = -\sqrt{\Theta}\psi(\nu, \phi)$$

$$\begin{aligned}\Theta\psi(\nu, \phi) \sim \sqrt{|\nu(\nu + 4\lambda)|}|\nu + 2\lambda|\psi(\nu + 4\lambda, \phi) - 2\nu^2\psi(\nu, \phi)& \\ +\sqrt{|\nu(\nu - 4\lambda)|}|\nu - 2\lambda|\psi(\nu - 4\lambda, \phi)&\end{aligned}$$

- No singularities according to
  - Standard quantum theory, for generic states (Ashtekar, Corichi, Pawloski, Singh)
  - Consistent histories, for generic states (Craig, Singh)

  Main reason. $\psi(0, \phi) = 0 \quad \Rightarrow \quad P(\nu = 0, \phi) = |\psi(0, \phi)|^2 = 0$

- Bohmian approach.
  - There is an actual value for the scale factor.
  - Takes discrete values $a^3 \sim n \in \mathbb{N}$
  - Bohmian dynamics is stochastic. (Proposed by Bell for QFT.)
  - Probability for $a^3 \sim \nu$ at time ϕ is $|\psi(\nu, \phi)|^2$

    $|\psi(0, \phi)|^2 = 0$, so probability to have a singularity is zero

Conclusions:

- We need a precise version of quantum theory to address the issue of singularities in quantum gravity
- Different versions may give different answers
- According to Bohmian mechanics:
  - Wheeler-DeWitt quantization: Depends on the wave function. For a generic wave function there is a non-zero probability for absence of singularities
  - Loop quantum gravity: No singularities

Question:

Results are only for mini-superspace. What happens in the more general quantum space-times?