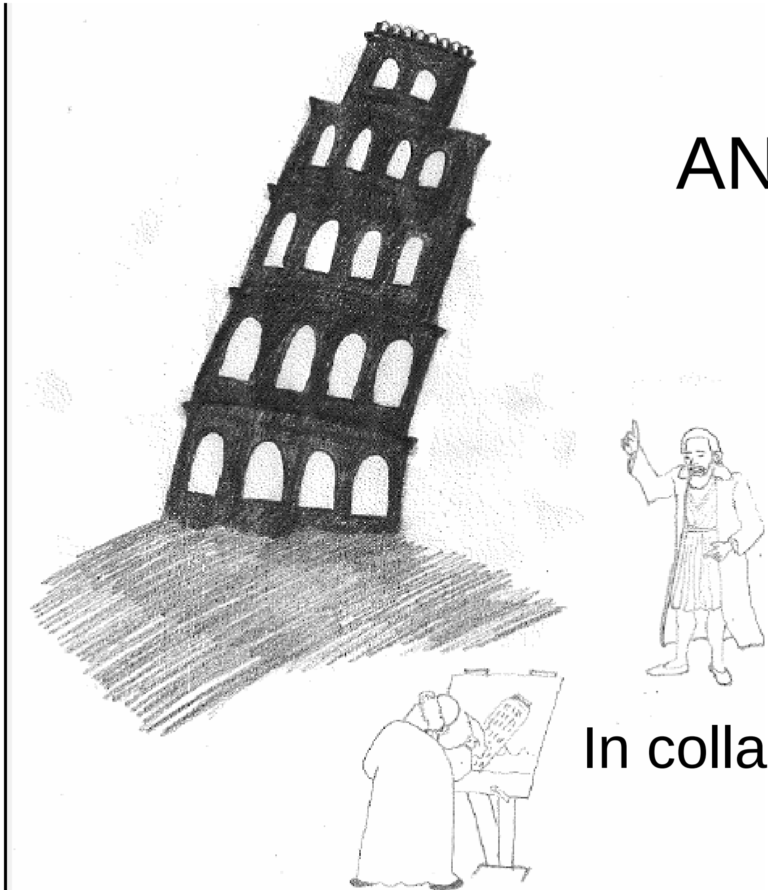
FUNDAMENTAL PROBLEMS IN QUANTUM PHYSICS

DISSIPATIVE MODELS OF SPONTANEOUS WAVE-FUNCTION COLLAPSE



"They should not have saved the money on the foundations"

ANDREA SMIRNE & ANGELO BASSI

UNIVERSITÀ DEGLI STUDI DI TRIESTE

ulm university universität ulm

In collaboration with Bassano Vacchini, University of Milan

Erice, 24 March 2015

1. MOTIVATION:

- Brief introduction to collapse models
- Energy divergence in GRW and CSL models

2. DISSIPATIVE COLLAPSE MODELS

- Momentum-dependent localization operators
- Energy relaxation
- Possible experimental effects

1. MOTIVATION: COLLAPSE MODELS AND THE ENERGY DIVERGENCE

Collapse models: challenging the superposition principle

- Unified description of microscopic and macroscopic systems

- 

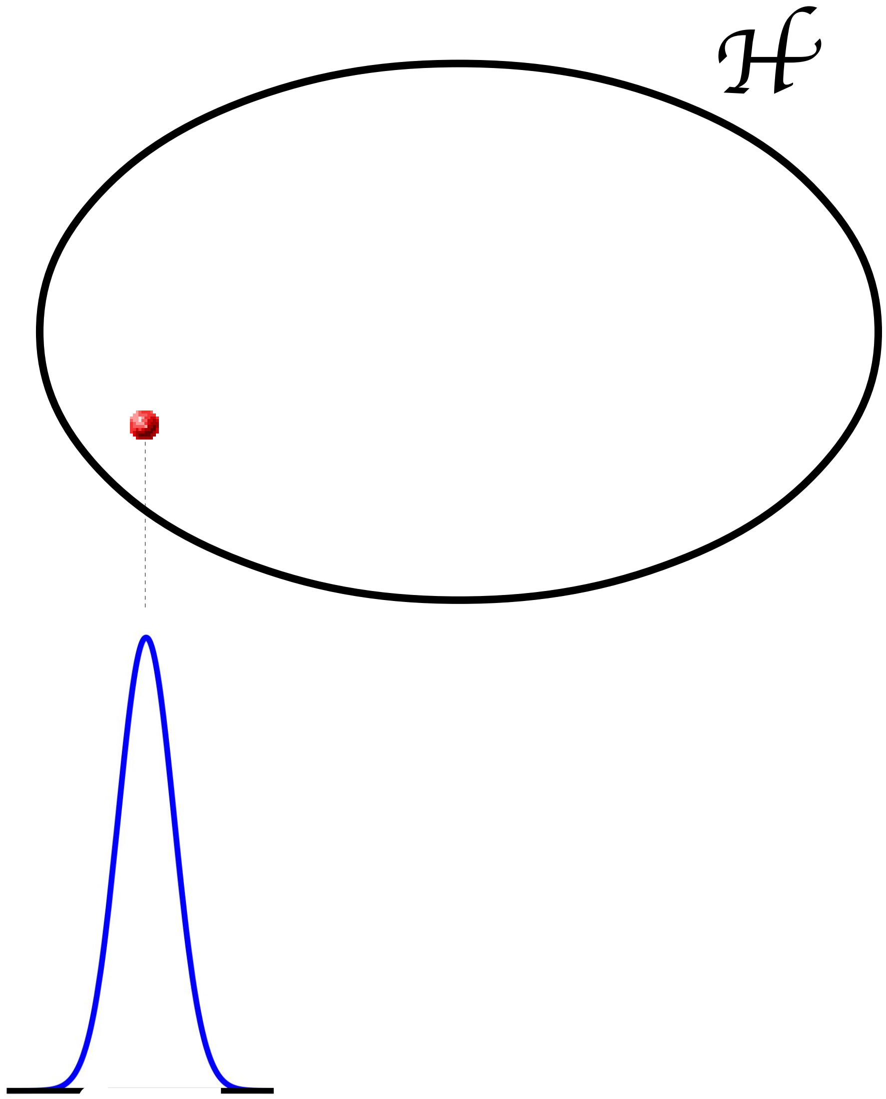
  No superposition of **macroscopic** systems

- 

  Testable predictions different from standard QM

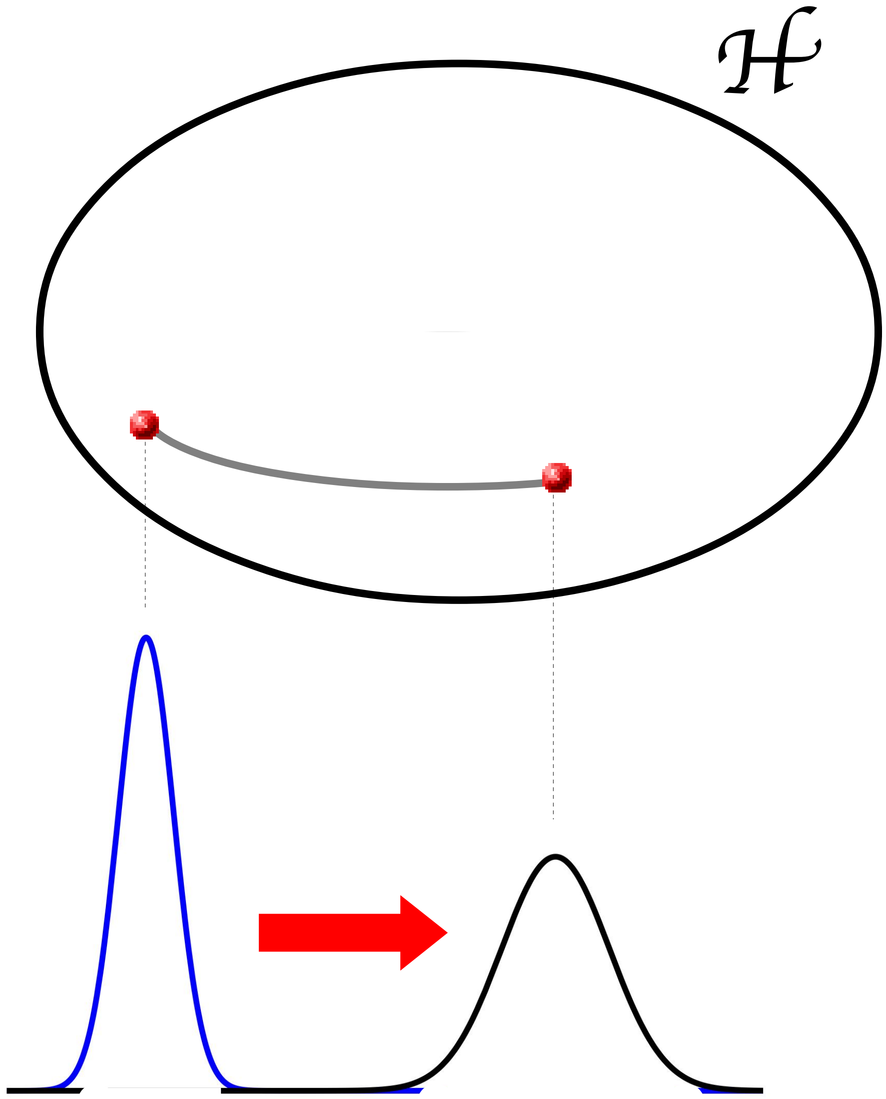
A. Bassi & G.C. Ghirardi, Phys. Rep. 2003
A. Bassi, K. Lochan, S. Satin, T.P. Singh & H. Ulbricht, Rev. Mod. Phys. 2013
M. Arndt & K. Hornberger, Nat. Phys. 2014

GRW model: wave-function localization



- Usual Schrödinger evolution

GRW model: wave-function localization



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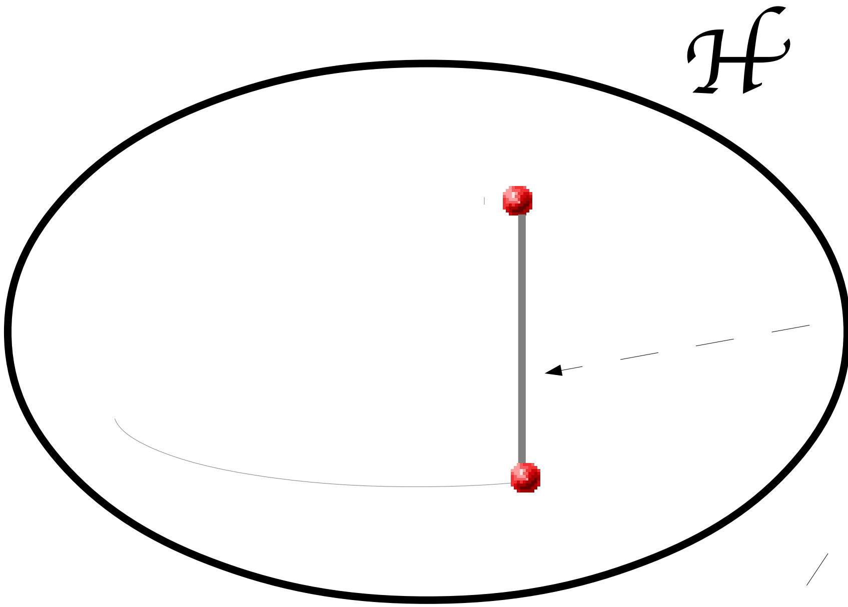
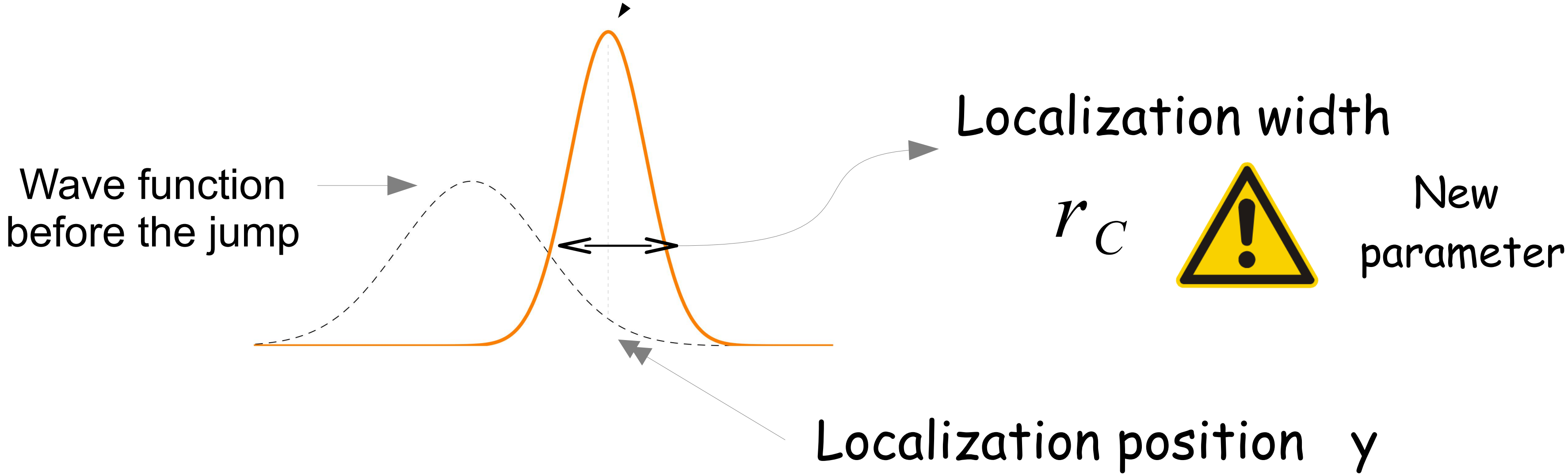
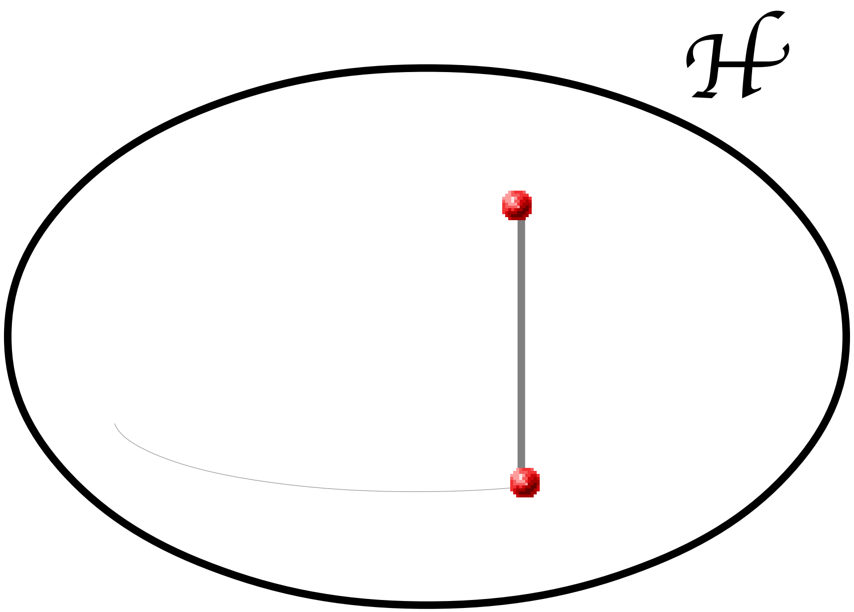


- Usual Schrödinger evolution
- ***Instantaneous*** jump

$$L_y(\hat{X}) = (\pi r_c^2)^{-1/4} e^{-\frac{(\hat{X}-y)^2}{2r_c^2}}$$



GRW model: wave-function localization



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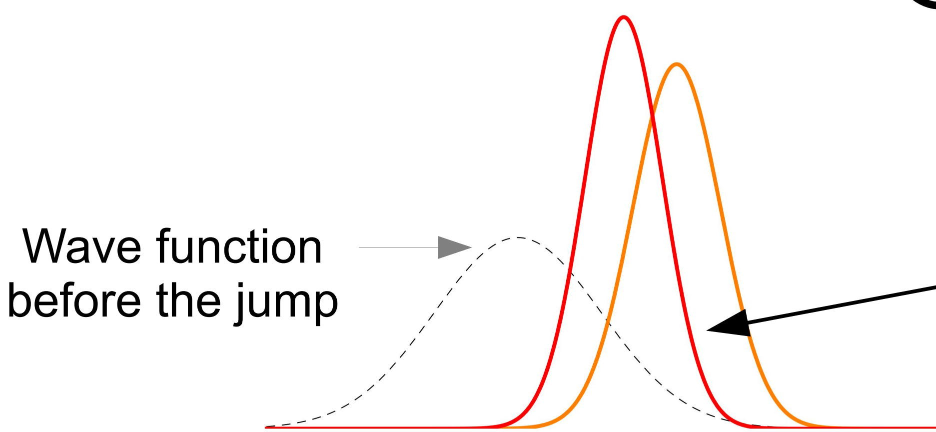
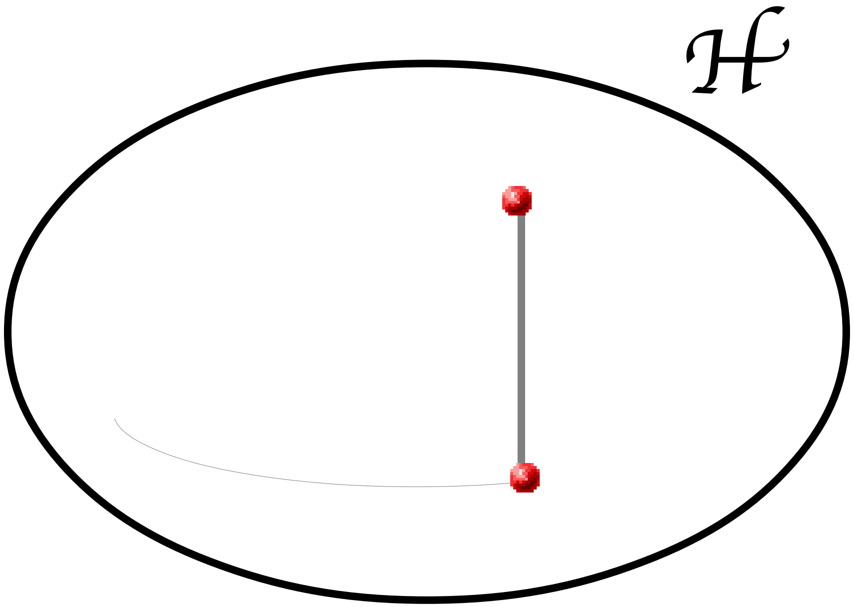


Wave function localization!!

$$|\psi(t)\rangle \longrightarrow |\psi_y(t)\rangle \equiv \frac{L_y(\hat{X})|\psi(t)\rangle}{\|L_y(\hat{X})|\psi(t)\rangle\|}$$

G.C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. A 1986

GRW model: wave-function localization



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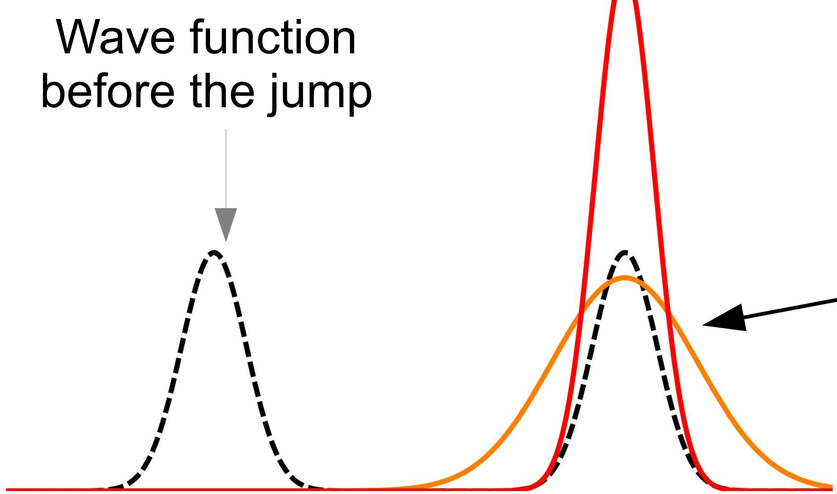


Destruction of the superposition

$$|\psi(t)\rangle \longrightarrow |\psi_y(t)\rangle \equiv \frac{L_y(\hat{X})|\psi(t)\rangle}{\|L_y(\hat{X})|\psi(t)\rangle\|}$$

G.C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. A 1986

Distribution of the jumps

- Probability distribution of the localization position:

$$p(y) = \|L_y(\widehat{X})|\psi(t)\rangle\|^2$$

→ Dynamical **derivation** of the Born's rule

- Poisson time-distribution of the jumps

- Localization rate for one nucleon $\boxed{\lambda = 10^{-16} s^{-1}}$

New parameter

Microscopic systems are NOT affected by the localization mechanism!

- Localization rate for an N-particle system c.o.m. $\boxed{\lambda_{macro} = N \lambda}$

Amplification mechanism: macroscopic systems are strongly affected !

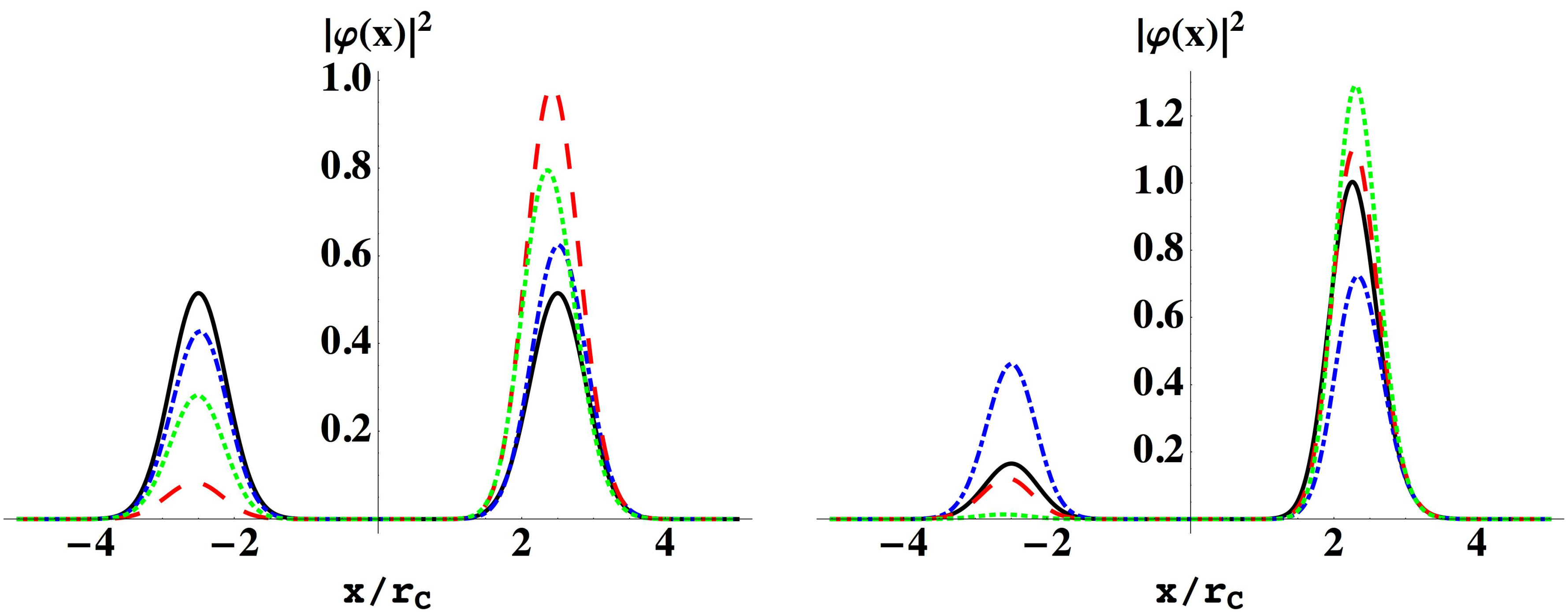
CSL model: SDE and localization

$$\mathrm{d}|\varphi_t\rangle = \left[-\frac{i}{\hbar}\hat{H}\mathrm{d}t + \frac{\sqrt{\gamma}}{m_0}\int \mathrm{d}\mathbf{y}[\hat{M}(\mathbf{y}) - \langle M(\mathbf{y})\rangle_t]\mathrm{d}W_t(\mathbf{y}) - \frac{\gamma}{2m_0^2}\int \mathrm{d}\mathbf{y}[\hat{M}(\mathbf{y}) - \langle M(\mathbf{y})\rangle_t]^2\mathrm{d}t\right]|\varphi_t\rangle$$

G.C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A 1990

- Smeared mass density operator $\hat{M}(\mathbf{y}) = \sum_j m_j \int \frac{\mathrm{d}\mathbf{x}}{(\sqrt{2\pi}r_C)^3} e^{-\frac{|\mathbf{y}-\mathbf{x}|^2}{2r_C^2}} \hat{\psi}_j^\dagger(\mathbf{x})\hat{\psi}_j(\mathbf{x})$

It also applies to identical particles !!



CSL model: SDE and localization

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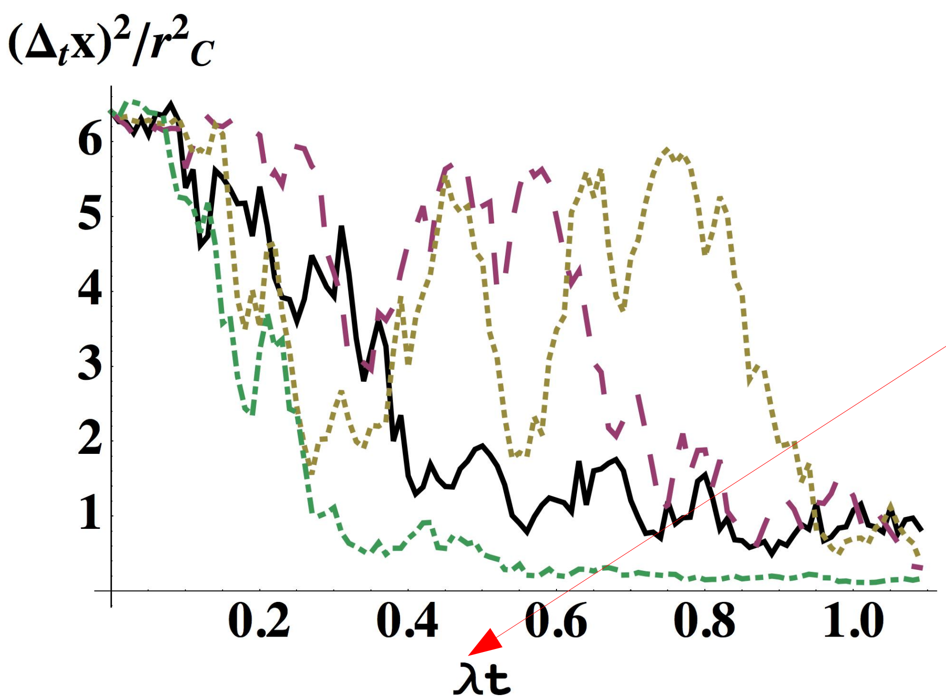
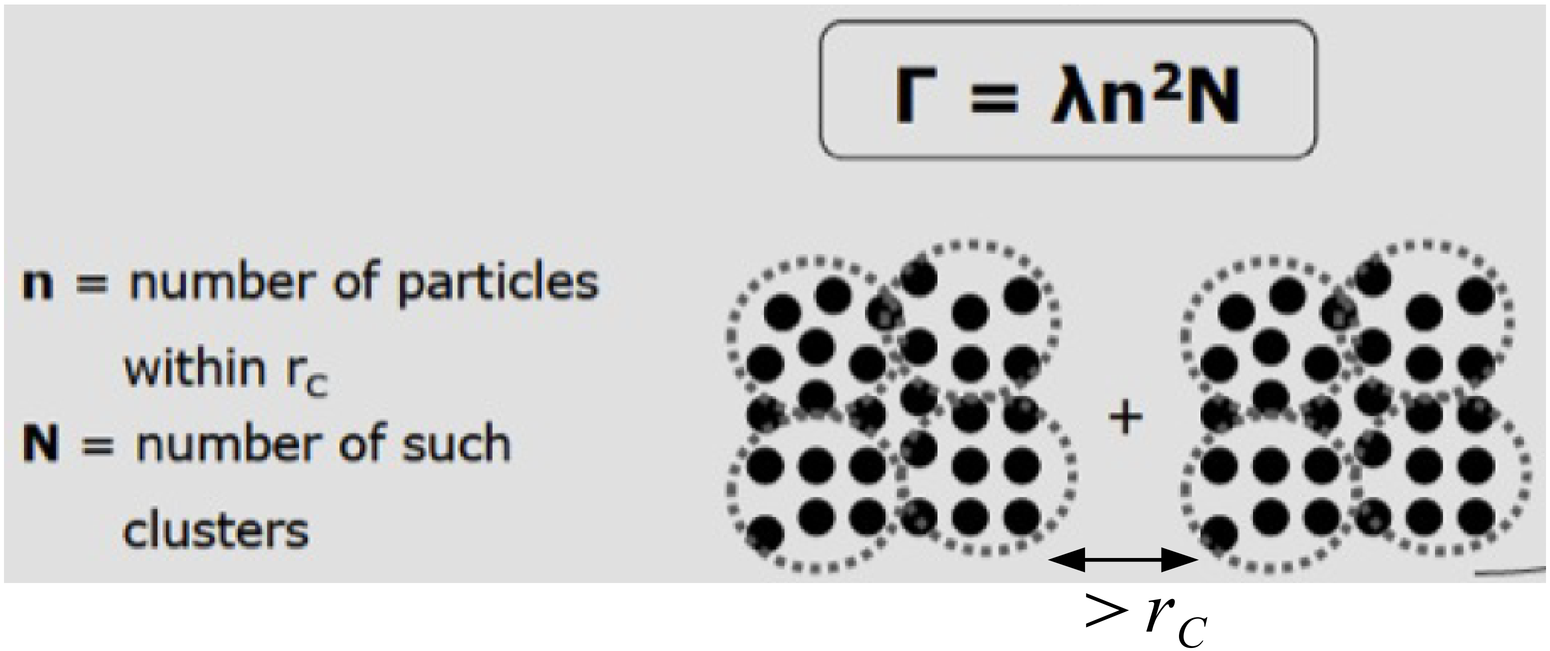


Localization rate

$$\lambda_{\mathrm{CSL}} = \frac{\gamma}{(4\pi r_C^2)^{3/2}}$$

Amplification mechanism in CSL

- Collapse rate for the center of mass of a composite object



Quantum properties of microscopic systems (few constituents)

Classical properties of macroscopic systems (many constituents)

$$\boxed{\textit{Quadratic increase with the number of constituents}}$$

Specific feature of the action of the noise on identical particles !!

Energy divergence

The average energy of the system diverges linearly in time, with a rate

$$\xi = \frac{\hbar^2 \lambda}{4 M r_c^2} \approx 10^{-25} eV\, s^{-1}$$

- S. Adler & A. Bassi, Science, 325 (2009)
- S. Adler JPA, 40 (2007)

Upper bounds on λ	
Laboratory experiments	**Decades above the conventional value**
Fullerene diffraction experiments	13
Decay of supercurrents	14
Spontaneous x-ray emission from Ge	6
Proton decay	18
Mirror cantilever interferometric experiment	9
Cosmological data	**Decades above the conventional value**
Dissociation of cosmic hydrogen	17
Heating of intergalactic medium (IGM)	8
Heating of interstellar dust grains	15

2[nd] strongest bound on λ !!

Secular energy increase compatible with **experimental data**

2. DISSIPATIVE COLLAPSE MODELS

A. Smirne, B. Vacchini & A. Bassi, Phys. Rev. A 90, 062135 (2014)

A. Smirne & A. Bassi, arXiv:1408.6446 (2014)

Master equation

- **Statistical operator** $\hat{\rho}(t) \equiv \mathbb{E}[|\psi(t)\rangle\langle\psi(t)|]$

Lindblad equation:
Markovian dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -\frac{i}{\hbar}\left[\widehat{H}, \hat{\rho}(t)\right] + \lambda\left(\int \mathrm{d}y\, L_y(\widehat{X})\hat{\rho}(t)L_y(\widehat{X}) - \hat{\rho}(t)\right)$$

$$= -\frac{i}{\hbar}\left[\widehat{H}, \hat{\rho}(t)\right] + \underline{\lambda}\left(\int d\boldsymbol{Q}\, e^{\frac{i}{\hbar}\boldsymbol{Q}\cdot\widehat{\boldsymbol{X}}} G(\boldsymbol{Q})\, \rho(t)\, G(\boldsymbol{Q})\, e^{-\frac{i}{\hbar}\boldsymbol{Q}\cdot\widehat{\boldsymbol{X}}} - \rho(t)\right)$$

Fourier Transform: $L_y(\widehat{\boldsymbol{X}}) = \int \frac{d\boldsymbol{Q}}{(2\pi\hbar)^{3/2}}\, e^{\frac{i}{\hbar}\boldsymbol{Q}\cdot(\widehat{\boldsymbol{X}}-\boldsymbol{y})} G(\boldsymbol{Q})$

Describes also recoil-free collisional decoherence

M $\longrightarrow$ ∞

- $G(\boldsymbol{Q}) = \left(\frac{r_C}{\sqrt{\pi}\,\hbar}\right)^{3/2} \exp\left(-\frac{r_C^2|\boldsymbol{Q}|^2}{2\hbar^2}\right)$

➡ Pure position-decoherence dynamics   NO DISSIPATION

- **To include dissipation** $\boxed{G(\boldsymbol{Q}) \longrightarrow G(\boldsymbol{Q}, \widehat{\boldsymbol{P}})}$

Dissipative localization operators

$$L_y(\widehat{\boldsymbol{X}}, \widehat{\boldsymbol{P}}) = \int \frac{d\boldsymbol{Q}}{(2\pi\hbar)^{3/2}}\; e^{\frac{i}{\hbar}\boldsymbol{Q}\cdot(\widehat{\boldsymbol{X}}-\boldsymbol{y})}\, G(\boldsymbol{Q}, \widehat{\boldsymbol{P}})$$

$$\frac{\hbar}{2mv_\eta r_C}$$

$$G(\boldsymbol{Q}, \widehat{\boldsymbol{P}}) \propto \exp\left(-\frac{r_C^2}{2\hbar^2}\left|(1+k)\mathbf{Q} + 2k\hat{\mathbf{P}}\right|^2\right)$$

New parameter

$$\boxed{v_\eta = \frac{4\,k_B T\, r_C}{\hbar}}$$

Dissipative localization operators

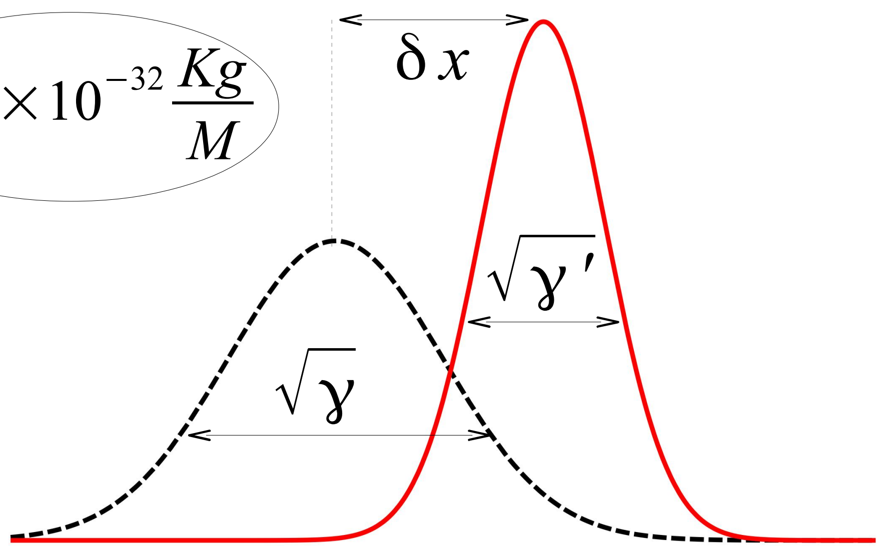
$$L_y(\hat{X},\hat{P}) = \int \frac{d\boldsymbol{Q}}{(2\pi\hbar)^{3/2}}\, e^{\frac{i}{\hbar}\boldsymbol{Q}\cdot(\hat{X}-y)}\, G(\boldsymbol{Q},\hat{P})$$

$$\frac{\hbar}{2mv_\eta r_C}$$

$$G(\boldsymbol{Q},\hat{\boldsymbol{P}}) \propto \exp\left(-\frac{r_C^2}{2\hbar^2}\left|(1+k)\mathbf{Q}+2k\hat{\mathbf{P}}\right|^2\right)$$

New parameter $v_\eta = \dfrac{4k_B T r_C}{\hbar}$

$$k = 5\times10^{-32}\,\frac{Kg}{M}$$



- $\delta x = (1-f_\gamma)(y-x)$

$$\left(\frac{\gamma}{r_c^2}+1\right)^{-1} \qquad \left(\frac{\gamma}{r_c^2(1-k^2)}+\frac{1-k}{1+k}\right)^{-1}$$

- $\gamma' = g_\gamma^{-1}$

$$\frac{1}{\gamma}+\frac{1}{r_c^2} \qquad \frac{(1-k)^2}{\gamma(1+k)^2}+\frac{1}{r_c^2(1+k)^2}$$

Dissipative localization operators

$$L_y(\widehat{\boldsymbol{X}},\widehat{\boldsymbol{P}}) = \int \frac{d\boldsymbol{Q}}{(2\pi\hbar)^{3/2}}\, e^{\frac{i}{\hbar}\boldsymbol{Q}\cdot(\widehat{\boldsymbol{X}}-\boldsymbol{y})}\, G(\boldsymbol{Q},\widehat{\boldsymbol{P}})$$

$$\frac{\hbar}{2 m v_\eta r_C}$$

$$G(\boldsymbol{Q},\widehat{\boldsymbol{P}}) \propto \exp\left(-\frac{r_C^2}{2\hbar^2}\left|(1+k)\mathbf{Q} + 2k\hat{\mathbf{P}}\right|^2\right)$$

New parameter $\quad v_\eta = \frac{4 k_B T r_C}{\hbar}$

$$\boxed{\sum_i \frac{m_j}{(2\pi\hbar)^3}\int \mathrm{d}\mathbf{P}\mathrm{d}\mathbf{Q}\, \hat{a}_j^\dagger(\mathbf{P}+\mathbf{Q})\, e^{-\frac{i}{\hbar}\mathbf{Q}\cdot\mathbf{y}} \exp\left(-\frac{r_C^2}{2\hbar^2}\left|(1+k_j)\mathbf{Q}+2k_j\mathbf{P}\right|^2\right)\hat{a}_j(\mathbf{P})}$$

- Gaussian distribution centered around $\frac{-2P_i k_j}{1+k_j}$ $\quad i = x, y, z$
- The action of the noise suppresses high momenta

Approach to equilibrium

- Stationary solution in canonical form $\varrho(\hat{\mathbf{P}}) = \left(\frac{\beta}{2m\pi}\right)^{3/2} e^{-\frac{\beta|\hat{\mathbf{P}}|^2}{2m}}$

- Spohn's theorem H.Spohn, Rep. Math. Phys. 10, 189 (1976) &Lett. Math. Phys. 2, 33 (1977)

  - The set $\{L_k,\ k \in I\}$ is self-adjoint
  - $[A, L_k] = 0 \quad \forall k \longrightarrow A = c\mathbb{1}$

  Relaxing semigroup:
  - unique stationary solution
  - $\lim_{\tau \to \infty} \mathcal{E}_\tau(\rho) = \rho_{\mathrm{ss}} \qquad \forall \rho$

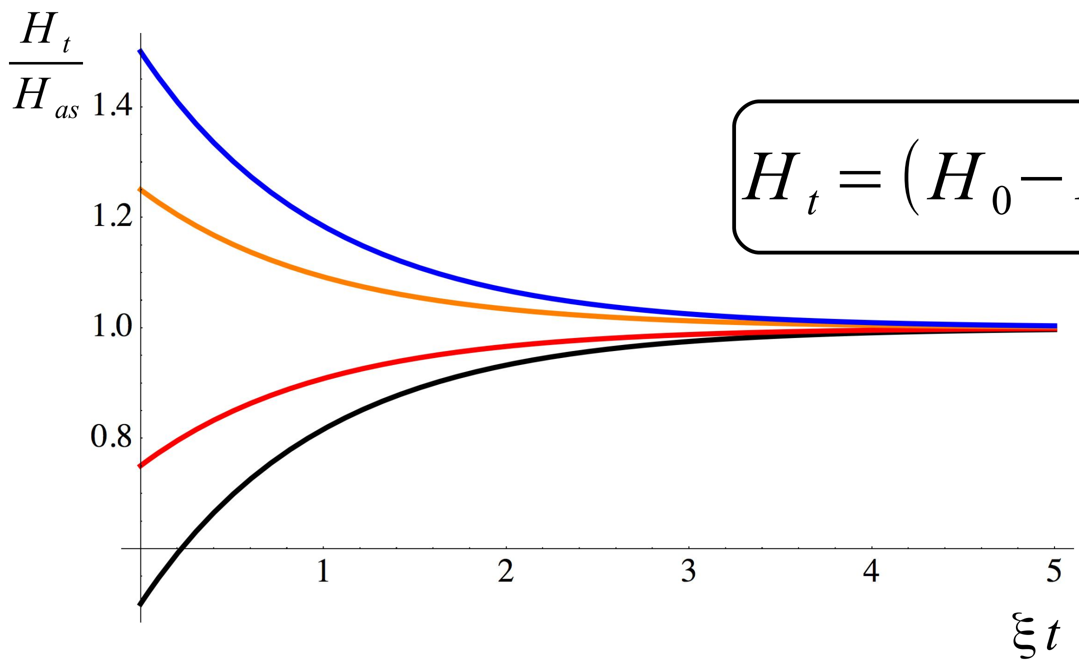
- Collisional dynamics of a tracer particle in a low density background gas (weak coupling regime)

  B. Vacchini and K. Hornberger, Phys.Rep. 478, 71 (2009)

Energy relaxation

- Mean value of the kinetic energy

$$H_t \equiv \mathbb{E}[\langle\psi(t)|\frac{\widehat{P}^2}{2m}|\psi(t)\rangle] = \mathrm{Tr}\left\{\hat{\rho}(t)\frac{\widehat{P}^2}{2m}\right\}$$



$$\boxed{H_t = (H_0 - H_{as})e^{-\xi t} + H_{as}}$$

- Rate $\qquad \frac{4\lambda k}{(1+k)^5} \approx 10^{-20} s^{-1}$

- Asymptotic energy $\qquad \frac{3\hbar^2}{16 k m r_C^2} \approx 10^{-4} eV$

Temperature of the noise

- Equipartition of the energy

$$\boxed{T=\frac{2H_{as}}{3k_B}=\frac{\hbar\nu_\eta}{4k_B r_C}}$$

It does not depend on the mass of the system

- Toward a full reestablishment of the energy conservation: energy exchanged between the system and the noise field

! The collapse model is still effective even in the presence of a low-temperature noise ($T\approx 1K \longleftrightarrow v\approx 10^5 m/s$)

Amplification mechanism (updated)

- Center of mass of an N-particle system $|\varphi_t^{(\mathrm{CM})}\rangle$
  - Crucial **assumption**: rigid body $\widehat{P}_j \approx M_j \widehat{P}_{\mathrm{T}} / M_{\mathrm{T}}$

C.o.m. and internal dynamics are ***decoupled***

- SDE (as well as master equation) as for the 1-particle system

$$\hat{\mathbb{L}}^{(\mathrm{CM})}(\mathbf{y}) = \frac{m}{(2\pi\hbar)^3} \int \mathrm{d}\mathbf{Q} \mathcal{F}_r(\mathbf{Q}) e^{\frac{i}{\hbar}\mathbf{Q}\cdot(\hat{\mathbf{x}}_{\mathrm{CM}}-\mathbf{y})} \exp\left(-\frac{r_C^2}{2\hbar^2}\left|(1+k)\mathbf{Q} + 2k\hat{\mathbf{P}}_{\mathrm{CM}}/N\right|^2\right)$$

$$\mathcal{F}_r(\mathbf{Q}) = \sum_j \exp\left(\frac{i}{\hbar}\sum_{j'} \Lambda_{jj'}\mathbf{Q}\cdot\mathbf{r}_{j'}\right)$$

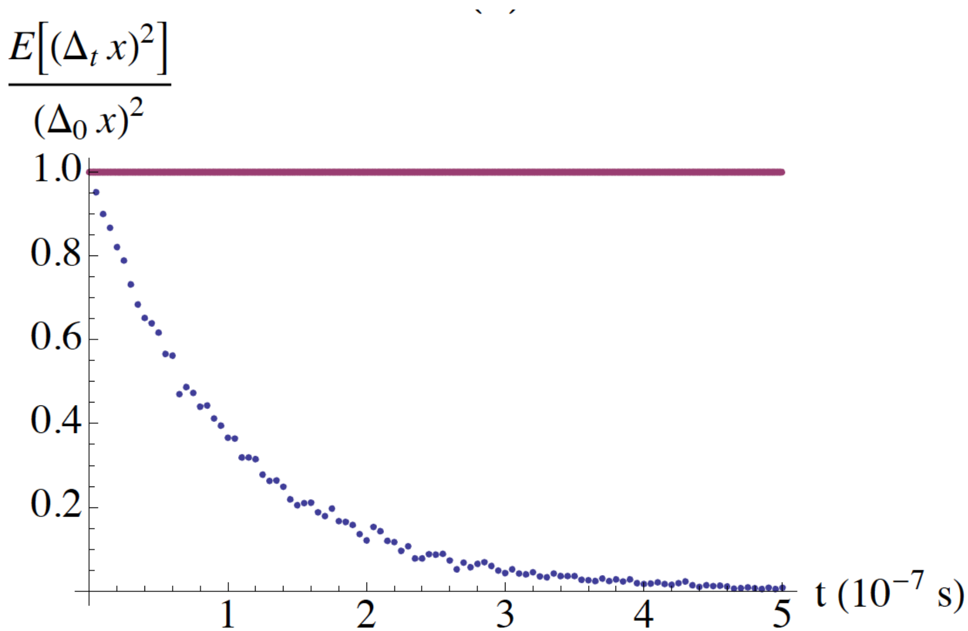
Determines the specific features of localization <u>in the CSL model</u>

- The wavefunction localization is left practically unchanged

$$\Gamma = \lambda n^2 N$$

Localization vs dissipation

- Stochastic average of the position variance



$$M = 10^{-3} Kg$$

- Finite asymptotic value of position and momentum variance

- Spherical rigid body of radius $R \gg r_C$

Localization rate

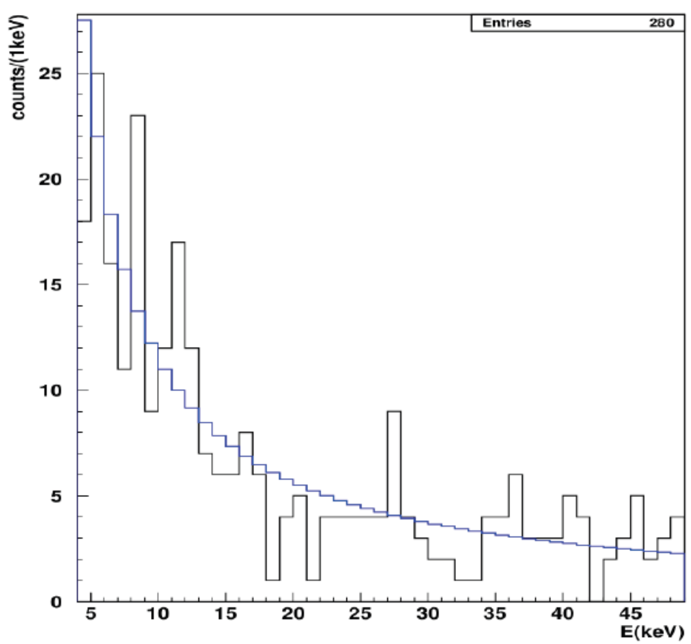
Dissipation rate

$$\frac{\Gamma}{\chi} \approx 10^4 N^2 \left(\frac{R}{r_C}\right)^2$$

Possible experimental effects

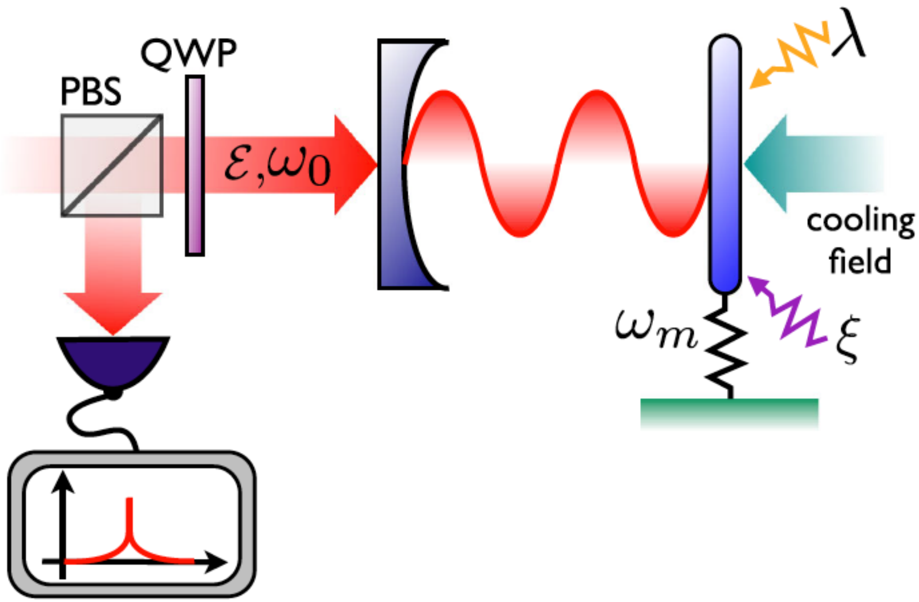
- Cosmological data: bounds to collapse parameters due to secular energy increase have to be reconsidered
- Interferometric experiments: stable against dissipative (long-time) corrections
- Dissipation could play a relevant role in:

X-ray emission



Donadi, Bassi, Deckert, Ann. Phys. 340, 70 (2014)
Curceanu et. al Int. J. Quant. Inf. 12, 1560012 (2014)

Optomechanical systems



Bahrami, Paternostro, Bassi, Ulbricht, Phys. Rev. Lett. 112, 210404 (2014)

Conclusions and outlooks

- We have extended the GRW and CSL models by means of position and **momentum dependent** localization operators, which induce **energy relaxation** to a finite asymptotic value

- Introduction of a **new parameter**, related to the temperature T of the noise. Effective models also at $T \approx 1\,K$

- **Realistic** unified framework for micro and macro systems: energy conservation principle, ***non-Markovianity***,...

- Main goal: development of a first-principle ***underlying theory,*** fixing the features of the collapse noise

Acknowledgments

COST
EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

Angelo Bassi

University of Trieste

Fundamental Problems in Quantum Physics (Action MP 1006)

nano QUESTFIT

SEVENTH FRAMEWORK PROGRAMME

EUROPEAN COMMISSION

Markus Arndt

University of Vienna

Testing quantum superposition in a mass range so far unexplored

INFN Istituto Nazionale di Fisica Nucleare

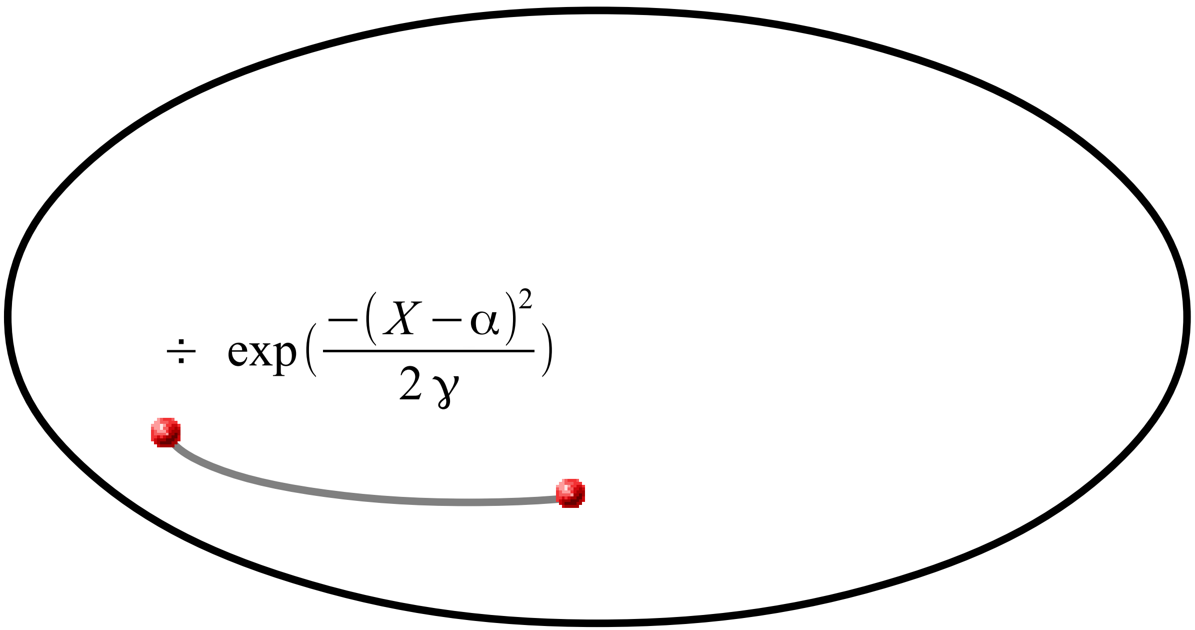
I.S. BELL

Nino Zanghì

Università di Genova

UNIVERSITÀ DEGLI STUDI DI TRIESTE

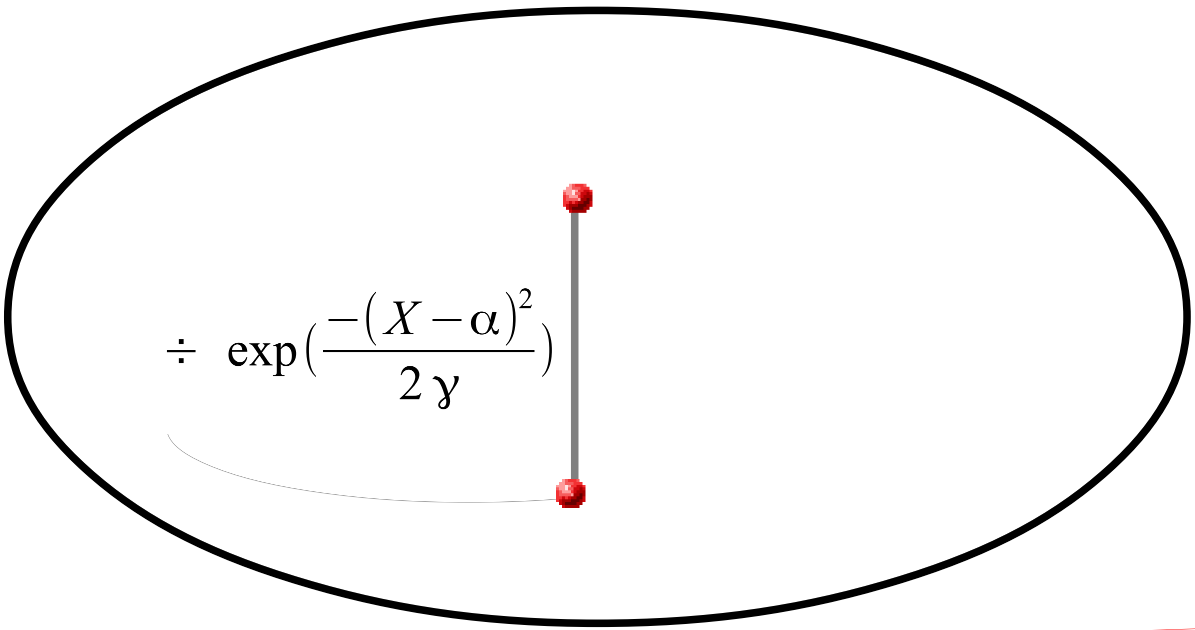
Energy divergence



- $\Re(\gamma)$ increases
- $\Im(\gamma) \neq 0$

$\langle P \rangle_t$ constant

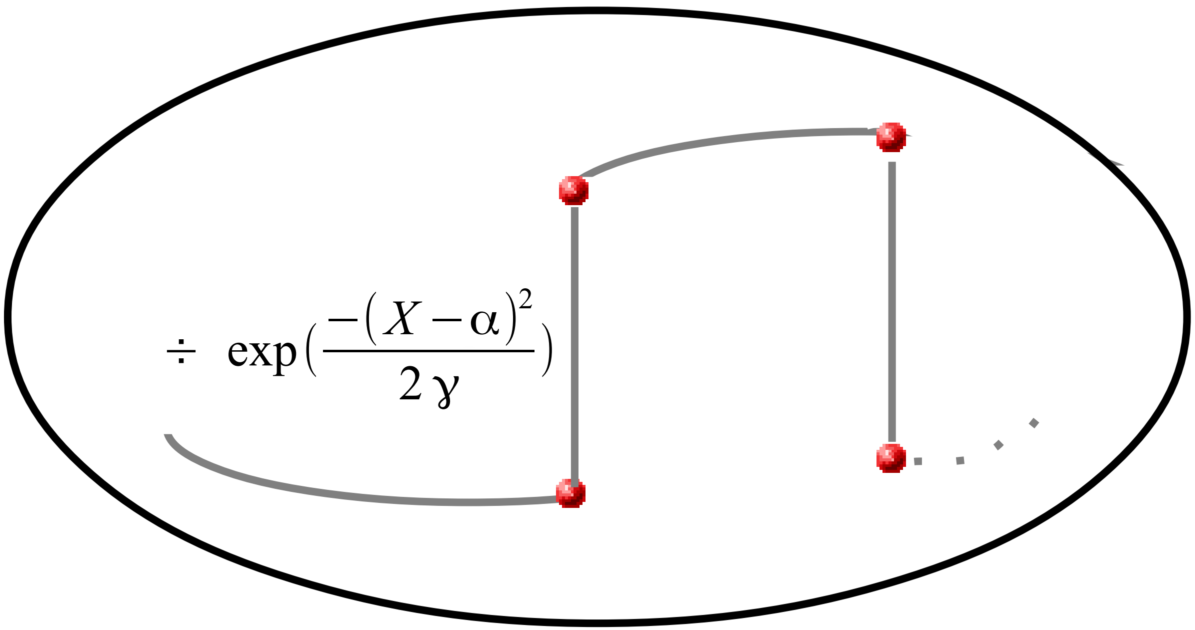
Energy divergence



$$\Im(\gamma) \neq 0$$

$$\langle P \rangle \longrightarrow \langle P \rangle + c\,\Im(\gamma)\left(\langle X \rangle - y\right)$$

Energy divergence



$$\Im(\gamma) \neq 0$$

$$\langle P \rangle \longrightarrow \langle P \rangle + c\,\Im(\gamma)\,(\langle X \rangle - y)$$

Different spatial distributions of the jumps $\Longrightarrow$ $\langle P \rangle_t \longrightarrow \pm\infty$

$$\langle H \rangle_t = ((\Delta_t P)^2 + \langle P \rangle_t^2)/(2M)$$

The average energy of the system diverges linearly in time, with a rate

$$\xi = \frac{\hbar^2 \lambda}{4 M r_c^2} \approx 10^{-25}\, eV\, s^{-1}$$