Collapse models, the quantum-classical transition, and experimental searches

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Quantum theory, amazingly successful; but what's the catch?

- The quantum measurement has been the source of unease from the very beginning.
- To Explain what is observed, the Schrödinger dynamics MUST be supplemented by:
  - The reduction (i.e. **collapse**) postulate
  - **Born rule**

Two add-ons of the Copenhagen Interpretation

The problem of quantum-to-classical transition (or macro-objectification)

- At what point do quantum **superpositions** break downs and **definite** outcomes appears?
- Where is the dividing line between micro (quantum coherence) and macro (**localized** states)?
- What is **responsible** for this division?
- Is there a quantitative criterion (e.g., size) governing the transition?

Leggett, J. Phys.: Condens. Matter 14, R415 (2002).
Adler and Bassi, Science 325, 275 (2009).
Arndt and Hornberger Nat. Phys. 10, 271 (2014).

Collapse models

Collapse models explain:

- Why **random definite** outcomes?
- Why **Born rule?**
- Why micro is quantum and macro is classical? (**amplification mechanism**)

Also it allows:

- Preservation of the norm of wave function.
- No faster-than-light-signalling.

Collapse models assumptions:

- Space is filled with a **very low energy, purely classical random field** that couples to matter with an **anti-Hermtian** Hamiltonian.

$$\frac{\mathrm{d}}{\mathrm{d}t}|\psi_t\rangle = \left(-\frac{i}{\hbar}\hat{H} + \sqrt{\gamma}\int \mathrm{d}^3\mathbf{x}\,\hat{L}(\mathbf{x})\,\xi(t,\mathbf{x}) + \hat{O}_\psi\right)|\psi(t)\rangle$$

- The wave function normalization is preserved.
- There is no faster than light signaling.

Dynamical equation of collapse models

$$d|\psi\rangle \;=\; (\alpha\, dt + \beta\, dW_t)|\psi\rangle,$$

Bassi and Ghirardi, Phys. Rep. 379, 257 (2003).
Bassi, *et al* Rev. Mod. Phys. 85, 471 (2013).

dW_t is the stochastic Ito differential, which obeys the rules:

$$(dW_t)^2 = dt, dW_t\, dt = 0$$

dW_t is a fluctuating variable with magnitude$(dt)^{\frac{1}{2}}$

$$\alpha \;=\; -\frac{i}{\hbar}\hat{H}_0 - \frac{1}{8}\gamma^2\left(\hat{L} - \langle\psi|\hat{L}|\psi\rangle\right)^2$$

$$\beta \;=\; \frac{1}{2}\gamma\left(\hat{L} - \langle\psi|\hat{L}|\psi\rangle\right)$$

Lindblad self-adjoint operator that can describe decoherence effect or, as it is the case here, intrinsic nonlinearities (i.e., collapse) in the dynamics for the wave function.
In collapse models, it is usually the local mass density operator.

The collapse random field

- *W (t, x)* is standard Wiener process (Brownian motion) giving a random field:

$$\xi(t, \mathbf{x}) = \mathrm{d}W(t, \mathbf{x})/\mathrm{d}t$$

- Assuming that the random field is Gaussian, then the only relevant terms are:

$$\mathbb{E}(\xi(t, \mathbf{x})) \text{ and } \mathbb{E}(\xi(t_1, \mathbf{x})\xi(t_2, \mathbf{y}))$$

with $\mathbb{E}(\cdots)$ the stochastic average.

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Statistical properties of the collapse random field

- Without losing the generality, we can set the mean value of the noise as zero: $\mathbb{E}(\xi(t,\mathbf{x}))=0$

- Naively speaking, one might separate the spatial and temporal parts:

$$\mathbb{E}(\xi(t_1,\mathbf{x})\xi(t_2,\mathbf{y})) \quad = \quad f(t_1,t_2)\, g(\mathbf{x},\mathbf{y})$$

- Accordingly, the two-point correlation is determined by two parameters:
  - The time correlation
  - The length correlation

$$\mathbb{E}(\xi(t_1,\mathbf{x})\xi(t_2,\mathbf{y})) \propto \exp\left(-\frac{(t_1-t_2)^2}{2\tau_c^2}\right)\exp\left(-\frac{(\mathbf{x}-\mathbf{y})^2}{2r_c^2}\right)$$

Statistical properties of the collapse random field

- In collapse models, the noise usually is considered as a white noise, meaning that the length correlation is the only free parameter.

$$\mathbb{E}(\xi(t_1,\mathbf{x})\xi(t_2,\mathbf{y})) \propto \delta(t_1 - t_2)\ \exp\left(-\frac{(\mathbf{x}-\mathbf{y})^2}{2r_C^2}\right)$$

Free parameters of collapse models

- The length correlation
- The coupling constant of the random field with matter

These parameters are usually set phenomenologically.

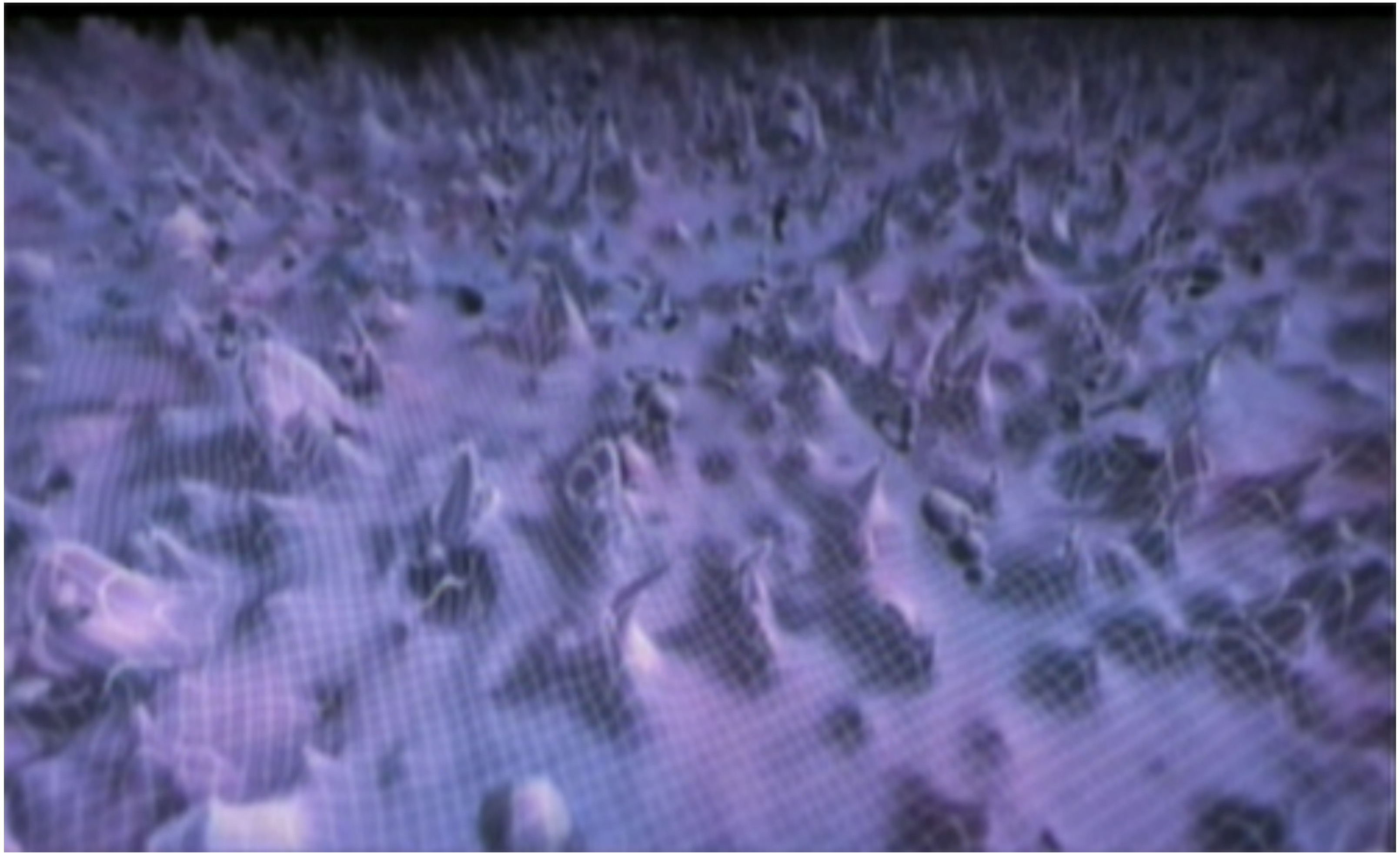
Physical picture in brief

- A universal random field is postulated in collapse models, inducing appropriate Brownian-motion corrections to standard quantum dynamics.
- The strength of collapse-driven Brownian fluctuations depend on:

  (i) parameters characterizing the system (e.g., mass, size, density).

  (ii) two phenomenological parameters.

A purely-classical random field $\xi(t, \mathbf{x})$



Conventional proposals for experimental test of collapse models

- Preparation of large systems in a spatial quantum superposition.
- Observing the position of system after some time. The distribution shows an interference pattern.
- The collapse manifests as the loss of visibility in the observed inference pattern.

Marquardt F and Girvin S M 1993 Physics 2, 40.
Aspelmeyer M, Gr¨blacher S, Hammerer K, Kiesel N 2010 J. Opt. Soc. Am. B 27, A189.
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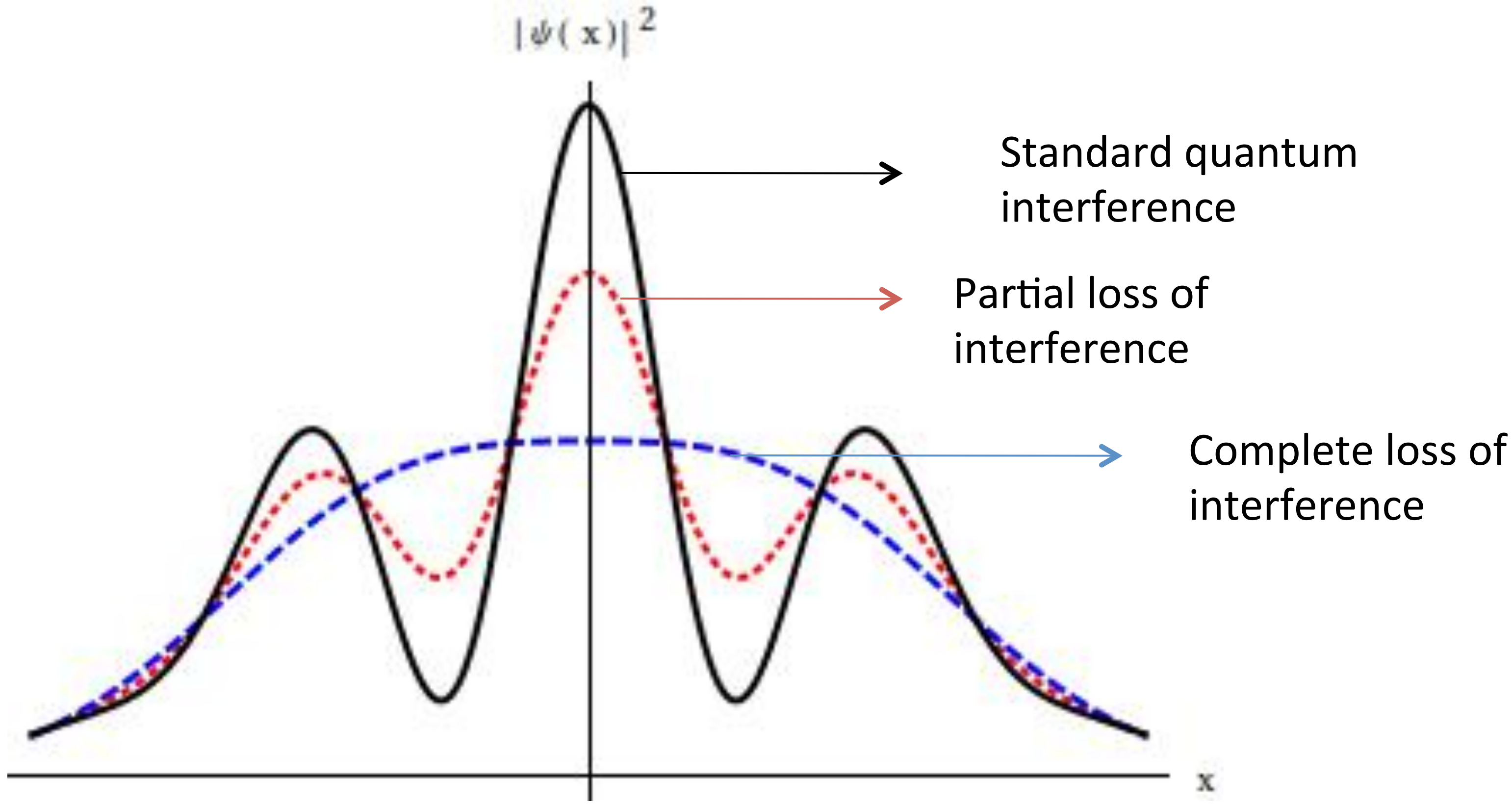
Romero-Isart O 2011 Phys. Rev. A 84, 052121.
Hornberger K, Gerlich S, Haslinger P, Nimmrichter S and Arndt M 2012 Rev. Mod. Phys. 84, 157.
Juffmann T, Ulbricht H and Arndt M 2013 Rep. Prog. Phys. 76, 086402.

Conventional way for experimental test of collapse models

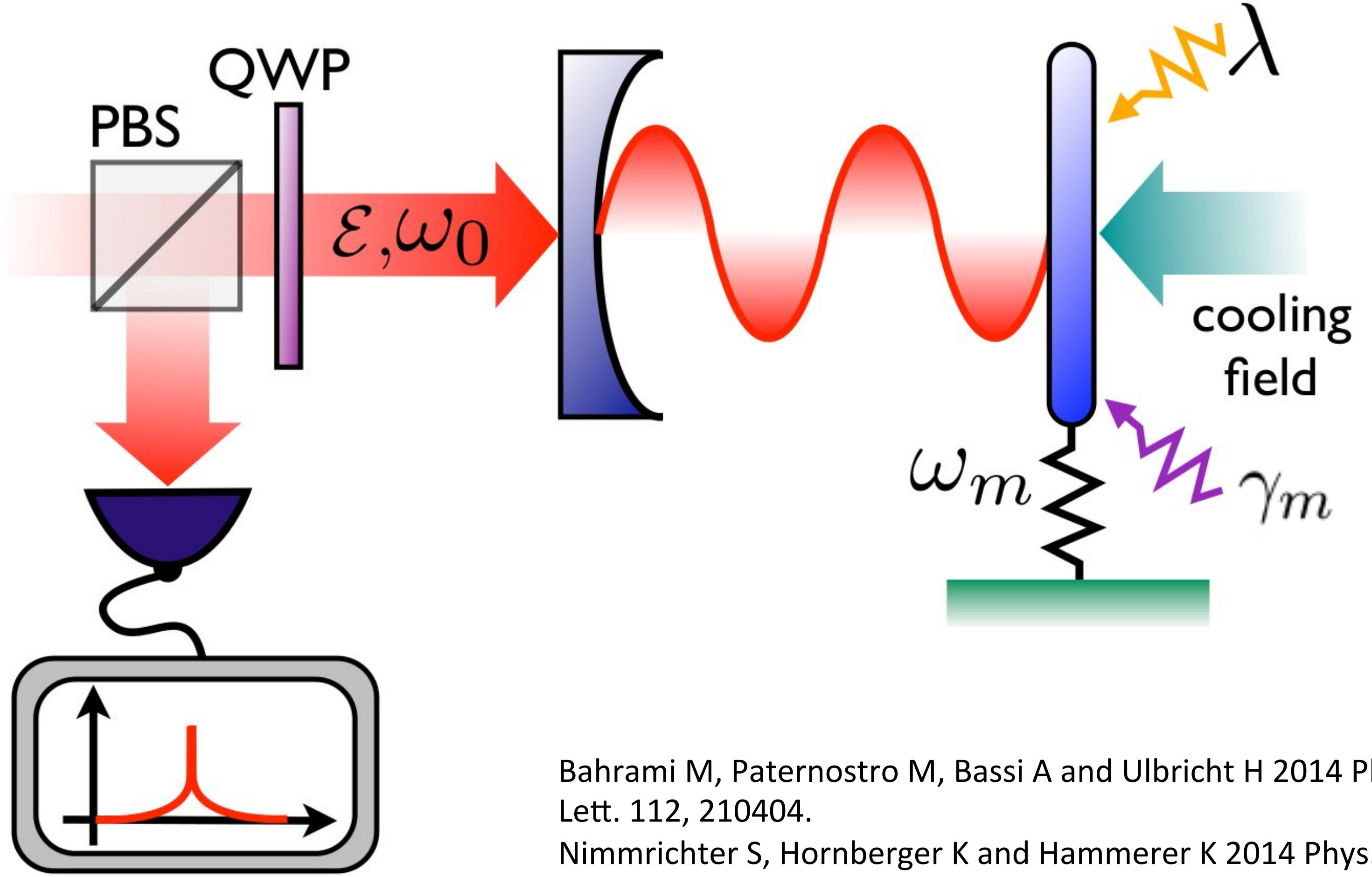


Alternative prescription to observe the collapse noise

- We take the fluctuating quantum system as the source of radiation.
- So any fluctuations at the source are written at the light interacting with it.
- Then, by studying the fluctuating properties of the light, we can infer about the fluctuations at the source.

Bahrami M, Bassi A and Ulbricht H 2014 Phys. Rev. A 89, 032127.
Bahrami M, Paternostro M, Bassi A and Ulbricht H 2014 Phys. Rev. Lett. 112, 210404.
Nimmrichter S, Hornberger K and Hammerer K 2014 Phys. Rev. Lett. 113, 020405.

Radiative corrections of the collapse field



Bahrami M, Paternostro M, Bassi A and Ulbricht H 2014 Phys. Rev. Lett. 112, 210404.
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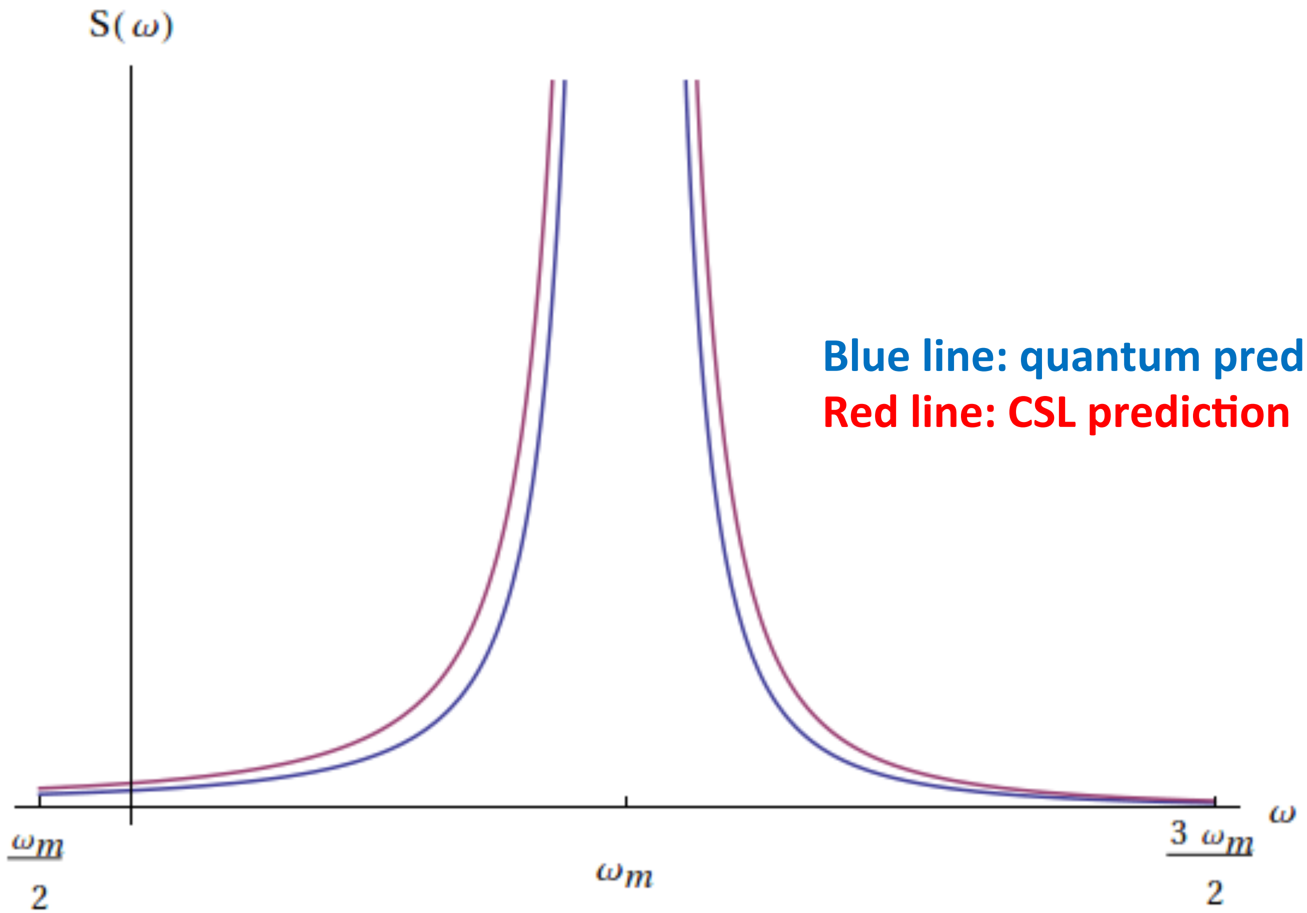


Blue line: quantum prediction
Red line: CSL prediction

Collapse equation for center-of-mass

$$\frac{\mathrm{d}}{\mathrm{d}t}|\psi_t\rangle = \left[-\frac{i}{\hbar}\hat{H}_{\mathrm{com}} + \sqrt{\gamma}\sum_{k=1}^{3}(\hat{q}_k - \langle\hat{q}_k\rangle)\,\mathrm{d}W_k(t) - \frac{\gamma}{2}\sum_{k,l=1}^{3} \eta_{k,l}\,(\hat{q}_k - \langle\hat{q}_k\rangle)\,(\hat{q}_l - \langle\hat{q}_l\rangle) \right] |\psi_t\rangle$$

where $\hat{H}_{\mathrm{com}}$ is the standard quantum Hamiltonian of the center-of-mass,

$$\eta_{k,l} = \iint \mathrm{d}^3\mathbf{r}\,\mathrm{d}^3\mathbf{r}'\,\frac{\exp\left(-\frac{|\mathbf{r}-\mathbf{r}'|^2}{4r_C^2}\right)}{(2\sqrt{\pi}\,r_C)^3}\,\frac{\partial\varrho(\mathbf{r})}{\partial r_k}\,\frac{\partial\varrho(\mathbf{r}')}{\partial r_l'},$$

and $w_k(t) = \mathrm{d}W_k(t)/\mathrm{d}t$ are white noises:

$$w_k(t) = \int \mathrm{d}^3\mathbf{x}\,\xi(t,\mathbf{x}) \int \mathrm{d}^3\mathbf{r}\,g(\mathbf{x}-\mathbf{r}-\langle\hat{\mathbf{q}}\rangle_{\Psi_t})\,\frac{\partial\varrho(\mathbf{r})}{\partial r_k},$$

with a zero mean (i.e. $\mathbb{E}(w_k(t)) = 0$) and correlation: $\mathbb{E}(w_k(t)\,w_l(s)) = \delta(t-s)\,\eta_{k,l}$.

Linear random potential

- Instead of working with the stochastic nonlinear dynamics, we work with the Schrodinger equation with a stochastic potential as follows:

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = (\hat{H}_{\mathrm{com}} + \hat{V}(t))|\psi(t)\rangle, \qquad \hat{V}(t) = -\hbar\sqrt{\gamma}\sum_{k=1}^{3}\hat{q}_k\, w_k(t),$$

  which is a linear stochastic differential equation in Stratonovich form.
- The effects of nonlinear terms in collapse equation, at the statistical level, can be mimicked also by linear random potentials.
- For individual realizations of the noise, the predictions of a linear dynamics vs. those of a nonlinear one are very different, while at the statistical level they coincide.

Theoretical modeling of optomechanical oscillator

- The oscillator is the moving mirror of a Fabry-Perot cavity, that couples to an external laser field and is immersed in a finite-temperature bath.
- The noise sources are:
  - **Thermal-driven Brownian motion of the oscillator interacting with the bath.**
  - **The input laser noise.**
  - **The CSL collapse noise.**
- We use the **quantum Langevin formalism** to account for the dynamics of the oscillator.

Quantum Langevin equation of optomechanical oscillator

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{Q} = \omega_m \hat{P}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{P} = \omega_m \bar{\chi}\, \hat{a}^\dagger \hat{a} - \omega_m \hat{Q} + \sqrt{\lambda}\, w(t) - \gamma_m \hat{P} + \hat{W}(t)$$

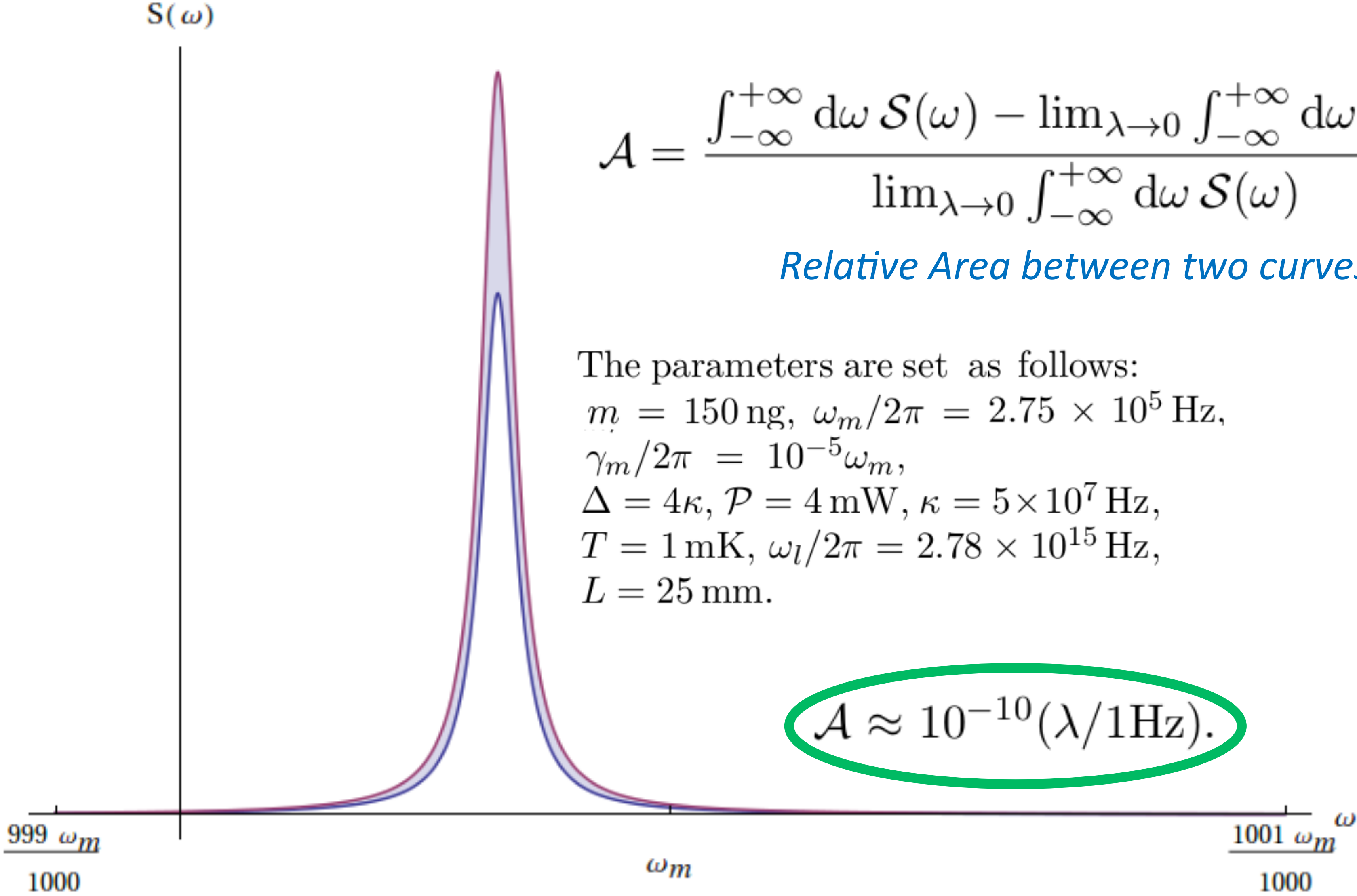
$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{a} = -i(\omega_c - \omega_l - \omega_m \bar{\chi}\, \hat{Q})\hat{a} - \kappa\, \hat{a} + \varepsilon + \hat{a}_{\mathrm{in}}\sqrt{2\kappa}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{a}^\dagger = i(\omega_c - \omega_l - \omega_m \bar{\chi}\, \hat{Q})\hat{a}^\dagger - \kappa\, \hat{a}^\dagger + \varepsilon + \hat{a}_{\mathrm{in}}^\dagger\sqrt{2\kappa}$$

$$\langle\, \delta\hat{a}_{\mathrm{in}}(t)\, \delta\hat{a}_{\mathrm{in}}^\dagger(s)\rangle = \delta(t-s),$$

We write each operator as follows: $\hat{A} = A_s + \delta\hat{A}$, where A_s is the classical steady-state value of the operator $\hat{A}$, and $\delta\hat{A}$ is small quantum fluctuation around this steady-state value.

Spectral density



$$\mathcal{A} = \frac{\int_{-\infty}^{+\infty} \mathrm{d}\omega\, \mathcal{S}(\omega) - \lim_{\lambda\to 0} \int_{-\infty}^{+\infty} \mathrm{d}\omega\, \mathcal{S}(\omega)}{\lim_{\lambda\to 0} \int_{-\infty}^{+\infty} \mathrm{d}\omega\, \mathcal{S}(\omega)}.$$

Relative Area between two curves

The parameters are set as follows:
$m = 150\,\mathrm{ng}$, $\omega_m/2\pi = 2.75 \times 10^5\,\mathrm{Hz}$,
$\gamma_m/2\pi = 10^{-5}\omega_m$,
$\Delta = 4\kappa$, $\mathcal{P} = 4\,\mathrm{mW}$, $\kappa = 5\times 10^7\,\mathrm{Hz}$,
$T = 1\,\mathrm{mK}$, $\omega_l/2\pi = 2.78 \times 10^{15}\,\mathrm{Hz}$,
$L = 25\,\mathrm{mm}$.

$$\mathcal{A} \approx 10^{-10}(\lambda/1\mathrm{Hz}).$$

The collapse frequency λ

- The collapse frequency is

$$\lambda = \frac{\hbar\,\eta}{m\omega_m} = \frac{\hbar}{m\omega_m}\,\gamma \iint \mathrm{d}^3\mathbf{r}\,\mathrm{d}^3\mathbf{r}'\,\frac{\exp\left[-\frac{|\mathbf{r}-\mathbf{r}'|^2}{4r_C^2}\right]}{(2\sqrt{\pi}\,r_C)^3}\,\frac{\partial\varrho(\mathbf{r})}{\partial\mathbf{r}_1}\,\frac{\partial\varrho(\mathbf{r}')}{\partial\mathbf{r}_1'}.$$

- It is similar to the famous Joos/Zeh decoherence rate:

$$\lambda = \eta\, x_0^2$$

η is the localization strength (or the localization rate), and it depends on the mass and the geometry of the oscillator. $x_0 = \sqrt{\hbar/m\omega_m}$ is the delocalization distance, which is also the zero-point fluctuation of the oscillator.

Results

- Very similar to vacuum fluctuations, the collapse random field produces two types of radiative corrections in the light spectrum:

  (i) frequency shift and (ii) **broadening**.
- Both phenomena appear naturally in analysis of the **spectrum** of light interacting with the system.
- Accurate-enough spectroscopic experiments are within reach, with current technology.

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