Prospects of experimental tests of a fundamentally semi-classical gravitation theory

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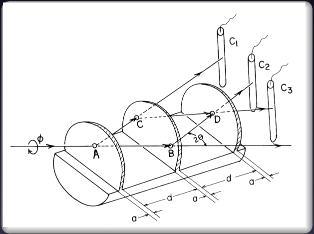
Gravitation and Quantum Mechanics?

What is the gravitational interaction of (nonrelativistic, laboratory) quantum matter?

- How does quantum matter react to an external gravitational field?
  $\rightarrow$ has an experimentally tested[1] answer:

$$\boxed{i\hbar\,\dot{\psi} = -\frac{\hbar^2}{2m}\Delta\psi + m\,g\,z\,\psi}$$



- How does quantum matter **source** the gravitational field?
  What is the gravitational field of a spatial superposition state?

[1] R. Colella, A. W. Overhauser, and S. A. Werner. Observation of Gravitationally Induced Quantum Interference. *Phys. Rev. Lett.*, 34:1472–1474, 1975

We need “Quantum Gravity”

... whatever that means?

What theory consistently combines gravity and quantum fields?
$\longrightarrow$ We don’t know (yet)

What is the **low energy limit** of this theory?
$\longrightarrow$ We don’t know either. But we can guess!

First guess

Gravity is not fundamentally different from matter fields
$\rightarrow$ it can (and must) be quantised in a similar fashion

Perturbative quantisation, in analogy to matter fields
(e. g. quantum electrodynamics) $\boxed{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}}$

Questions:

- The high energy limit must be different from known matter fields (non-renormalisability)
  $\rightarrow$ Why assume an analogy in the first place?
- **Interpretation:** Matter fields are living **on** space-time.
  $\rightarrow$ What is the gravitational field living on?

Second guess

Space-time is **fundamentally** classical
(or **more** fundamentally, i. e. at least for low energies)

$\rightarrow$ the metric tensor and curvature are classical, real valued objects

$\rightarrow$ quantum matter fields live on this classical curved space-time

$\rightarrow$ the dynamics of space-time satisfy Einstein's field equations

$$\boxed{R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = \frac{8\pi\, G}{c^4}\, T_{\mu\nu}}$$

Question:

What is $T_{\mu\nu}$, and how is it related to quantum matter fields?

Semi-classical gravity

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \langle\Psi|\hat{T}_{\mu\nu}|\Psi\rangle}$$

Weak-field non-relativistic limit: $\Delta U = 4\pi G \langle\Psi|m\hat{\psi}^\dagger\hat{\psi}|\Psi\rangle$

- **Semi-classical gravity is the mean-field limit of perturbatively quantised gravity:**
  $\Rightarrow$ The equation only makes sense for states of a large number of particles
- **Semi-classical gravity is fundamental:**
  $\Rightarrow$ One obtains the Schrödinger-Newton equation[2]

[2] M. Bahrami, et al. The Schrödinger–Newton equation and its foundations. *New J. Phys.*, 16:115007, 2014

Schrödinger–Newton equation

for *N* particles[3]**:**

$$i\hbar\,\dot{\psi}_N\left(\vec{r}^N\right) = \left[-\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\Delta_{\vec{r}_i} + V_{\mathrm{EM}}\left(\vec{r}^N\right) + U_{\mathrm{G}}\left[\Psi_N\left(\vec{r}^N\right)\right]\right]\Psi_N\left(\vec{r}^N\right)$$

$$V_{\mathrm{EM}}\left(\vec{r}^N\right) = \sum_{i=1}^{N}\sum_{j>i}\frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|}$$

$$U_{\mathrm{G}}\left[\Psi_N\left(\vec{r}^N\right)\right] = -G\sum_{i=1}^{N}\sum_{j=1}^{N} m_i m_j \int \frac{|\Psi_N(\vec{r}\,'^N)|^2}{|\vec{r}_i - \vec{r}\,'_j|}\,\mathrm{d}V'^N$$

[3] L. Diósi. Gravitation and quantum-mechanical localization of macro-objects. *Phys. Lett. A*, 105(4-5):199–202, 1984

Centre-of-mass equation

Separation ansatz: (with $\vec{r} = \sum m_i \vec{r}_i / M$ and $\vec{\rho}_i = \vec{r}_i - \vec{r}$)

$$\boxed{\Psi_N(\vec{r}^N) = \left(\frac{m_N}{M}\right)^{3/2} \psi(\vec{r})\, \chi(\vec{\rho}^{N-1})}$$

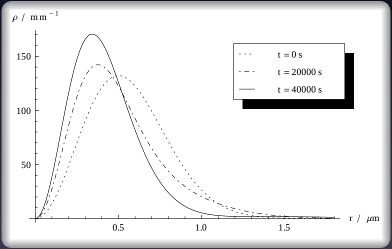
Born-Oppenheimer-type approximation yields:

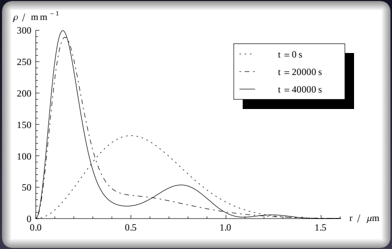
$$\boxed{\begin{aligned} i\hbar\, \dot{\psi}\,(t, \vec{r}) &= \left(-\frac{\hbar^2}{2m}\Delta - G \int d^3r' |\psi(t, \vec{r}\,')|^2\, I_\rho(\vec{r} - \vec{r}\,') \right) \psi(t, \vec{r}) \\ I_\rho(\vec{d}) &= \int d^3x\, d^3y\, \frac{\rho(\vec{x})\rho(\vec{y} - \vec{d})}{|\vec{x} - \vec{y}|} \end{aligned}}$$

Wide wave-function limit

One-particle Schrödinger–Newton equation

$$\boxed{i\hbar\,\dot{\psi}\,(t,\vec{r}) = \left(-\frac{\hbar^2}{2m}\,\Delta - G\,m^2\int\frac{|\psi(t,\vec{r}\,')|}{|\vec{r}-\vec{r}\,'|}\,\mathrm{d}^3r\,'\right)\psi(t,\vec{r})}$$





$\rho = 4\pi\,r^2\,|\psi|^2$ for masses of 7×10^9 u and 10^{10} u

Narrow wave-function limit

Expand around $\vec{r} - \vec{r}' = \vec{0}$:

$$\boxed{i\hbar\,\dot{\psi} = -\frac{\hbar^2}{2m}\Delta\psi - G\left(I_\rho(\vec{0}) + \frac{I_\rho''(\vec{0})}{2}\left(r^2 - 2\vec{r}\cdot\langle\vec{r}\rangle + \langle r^2\rangle\right)\right)\psi}$$

For a homogeneous sphere:

$$I_\rho(d) = -\frac{m^2}{R} \times \begin{cases} \frac{6}{5} - 2\left(\frac{d}{2R}\right)^2 + \frac{3}{2}\left(\frac{d}{2R}\right)^3 - \frac{1}{5}\left(\frac{d}{2R}\right)^5 & (d \leqslant 2R) \\ \frac{R}{d} & (d > 2R) \end{cases}$$

Crystalline sphere

Atomic self-energy:

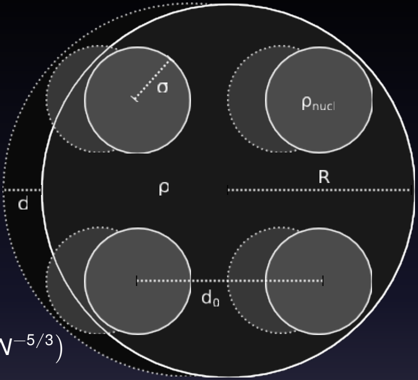
$$I_{\text{self}}(\vec{d}) = \frac{N}{\sigma}\frac{m^2}{N^2}\left(\frac{6}{5} - \frac{d^2}{2\sigma^2}\right) + \mathcal{O}\left(\frac{d^3}{\sigma^3}\right)$$

Mutual gravitational energy:

$$I_{\text{mutual}}(\vec{d}) = \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\left(\frac{(m/N)^2}{|\vec{r}_i - \vec{r}_j - \vec{d}|}\right)$$

$$= \int d^3x\, d^3y\, \frac{\rho(\vec{x})\,\rho(\vec{y}-\vec{d})}{|\vec{x}-\vec{y}|} + \mathcal{O}\left(N^{-5/3}\right)$$

$$= \frac{m^2}{R}\left(\frac{6}{5} - \frac{d^2}{2R^2}\right) + \mathcal{O}\left(\frac{d^3}{R^3}\right) + \mathcal{O}\left(N^{-5/3}\right).$$



In total:

$$\boxed{I_\rho(\vec{d}) \approx \frac{m^2}{R^3}\left(\frac{6}{5}R^2 - \gamma\frac{d^2}{2}\right)}, \qquad \gamma = 1 + \frac{\rho_{\text{nucl}}}{\rho}$$

Three plus two different regimes

1. Subatomic wave-function, $\langle r^2 \rangle \ll \sigma^2$
   $\rightarrow$ quadratic potential with $\gamma \approx \rho_{\mathrm{nucl}}/\rho$
2. Intermediate regime, $\langle r^2 \rangle \approx \sigma^2$
3. Narrow wave-function, $\sigma^2 \ll \langle r^2 \rangle \ll R^2$
   $\rightarrow$ quadratic potential with $\gamma \approx 1$
4. Intermediate regime, $\langle r^2 \rangle \approx R^2$
5. Wide wave-function, $\langle r^2 \rangle \gg R^2$
   $\rightarrow$ one-particle equation

Experimental Tests of the Schrödinger–Newton Equation

Free spreading of wave packets

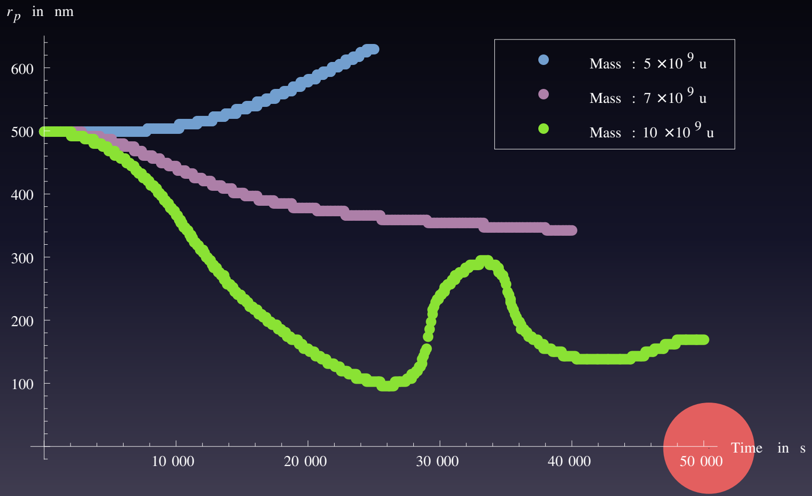


Figure: Maximum $r_p(t)$ of the radial probability density $4\pi r^2|\psi(r)|^2$

Measurable in satellite experiments?

$\Rightarrow$ Higher mass improves the time scale

With the extremal parameters from the MAQRO proposal[4]

Time for free spreading:	$100\,\mathrm{s}$
Particle mass:	$10^{10}\,\mathrm{u}$
Particle size:	$120\,\mathrm{nm}$
Initial wave-function width:	$100\,\mathrm{nm}$

$\rightarrow$ intermediate regime!

$\Rightarrow$ deviation from free wave-function: $\approx 1\%$ (i. e. $\approx 1\,\mathrm{nm}$)

(in wide wave-function limit)

Experimental accuracy of position detection: $20\,\mathrm{nm}$

[4] R. Kaltenbaek, et al. Macroscopic quantum resonators (MAQRO): 2015 Update. arXiv:1503.02640 [quant-ph]

Trapped nanospheres

Harmonic oscillator with self-gravitation

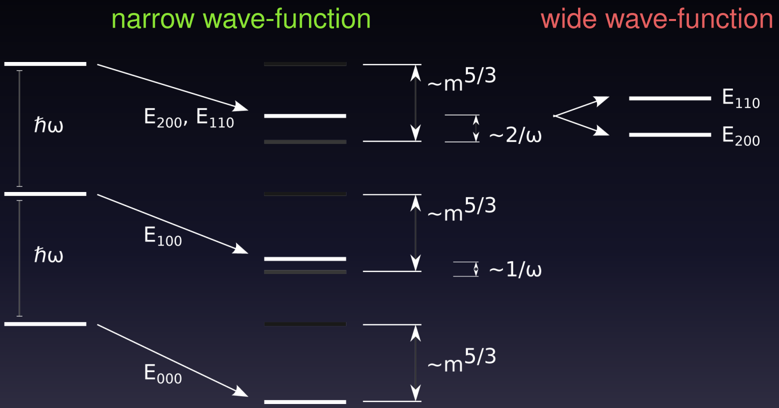
$$i\hbar\,\dot{\psi} = -\frac{\hbar^2}{2m}\Delta\psi + \frac{m\omega^2 r^2}{2}\psi + V_g[\psi]\psi$$

$V_g[\psi]$ leads to a state-dependent energy shift

$$\boxed{\Delta E = \langle\psi^{(0)}|V_g[\psi]|\psi^{(0)}\rangle}$$

$\Rightarrow$ changes the spectrum

Harmonic oscillator spectrum



$$E_{\text{trans}} = \Delta n\, \hbar\omega \left(1 + \frac{4\pi G \gamma \rho}{3\,\omega^2}\right) \qquad \text{weaker} \sim \left(R/\sqrt{\langle r^2 \rangle}\right)^3$$

$$\sim 10^{-6} \qquad (\text{for } \omega \approx 2\pi \times 10\,\text{s}^{-1},\ \gamma \approx 10^4)$$

Measuring the ionisation energy?

$$\boxed{\Delta E = -\frac{G\,m^2}{R}\left(\frac{6}{5} - \frac{\gamma}{2\,R^2}\left\langle\left(\vec{r} - \langle\vec{r}\rangle\right)^2\right\rangle\right) \approx -\frac{6}{5}\frac{G\,m^2}{R}}$$

- Much larger term, $\Delta E \approx \hbar\omega$ for $m \approx 10^{14}$ u
- Constant shift is not detectable in the spectrum
- For a wide wave-function: $\Delta E \to 0$
- Can we measure this term by kicking a state to the wide wave-function regime?
- Requires energy resolution $\Delta E/E \lesssim 10^{-15}$

Narrow wave-function dynamics

$$i\hbar\,\dot{\psi} = \left(-\frac{\hbar^2}{2m}\Delta + \frac{m\,\omega^2\,x^2}{2} - \frac{G}{2}\,I''_\rho(0)\left(x^2 - 2\vec{x}\,\langle x\rangle + \langle x^2\rangle\right)\right)\psi$$

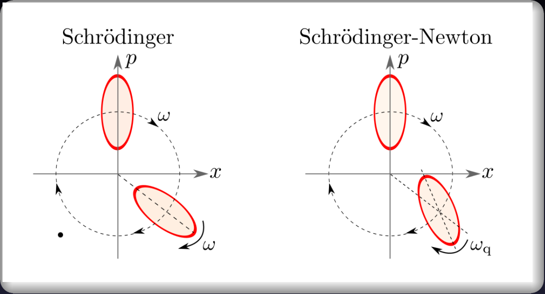
Can be solved with a general Gaussian ansatz:

$$\boxed{\begin{aligned} \langle x\rangle_t &= x_{\max}\ \sin\omega t \\ \langle x^2\rangle_t &= \langle x\rangle_t^2 + \langle x^2\rangle_0\left[1 + \sin^2\omega_{SN}t\left(\left(\frac{\langle x^2\rangle_{\text{ground}}}{\langle x^2\rangle_0}\right)^2 - 1\right)\right] \end{aligned}}$$

$\Rightarrow$ no effect on frequency of $\langle x\rangle$, only $\langle x^2\rangle$

$$\omega_{SN} = \sqrt{\omega^2 + \gamma\,\frac{4\pi}{3}\,G\,\rho}$$

Measurable in opto-mechanics?



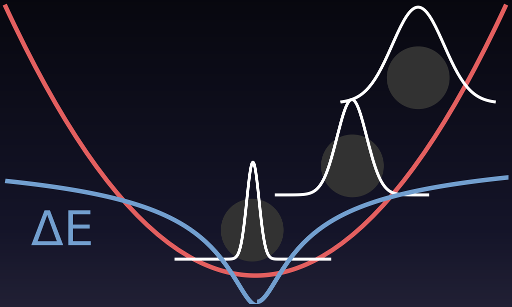
$\Rightarrow$ Rotation in phase space[5] with parameters

Silicium, density:	$\rho = 2.336\,\mathrm{g/cm^3}$
Nucleus density:	$\gamma = 8\,300 \quad (\text{at } T \sim 10\,\mathrm{K})$
Required mass (for narrow wf.):	$\sim 10^{14}\,\mathrm{u}$

$\Rightarrow$ requires $Q \gtrsim \omega^2 / \left(\gamma \frac{4\pi}{3} G \rho\right) \approx 3 \times 10^6$ for $\omega \approx 2\pi \times 10\,\mathrm{s}^{-1}$

[5] H. Yang, et al. Macroscopic Quantum Mechanics in a Classical Spacetime. *Phys. Rev. Lett.*, 110:170401, 2013

Probing the wide regime



For a squeezed state with

$$\langle x^2 \rangle - \langle x \rangle^2 \approx R$$

$\Rightarrow$ smaller masses possible

Deviation from harmonic potential $\Rightarrow$ stronger effect?

Summary

- The Schrödinger–Newton equation follows from fundamentally semi-classical gravity
- Experimental tests would provide insight into the necessity of quantising the gravitational field
- Interferometric tests as well as tests with trapped massive quantum systems seem feasible in the not too far future

Open questions

- The Schrödinger–Newton equation explains localisation of macroscopic states, but not the stochastic collapse
  $\Rightarrow$ Is there a natural way to explain the collapse as a consequence of self-gravitation?

- For the one-particle equation, probability density is radiated to infinity
  $\Rightarrow$ Is this behaviour also present for many-particle systems?

- Experimental access to the wave-function via its gravitational potential opens the possibility of faster-than-light signalling by collapsing entangled states
  $\Rightarrow$ Is there a collapse description that can prevent this?