Fundamental issues in gravity theory and its relation to quantum mechanics

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Fundamental Problems in Quantum Physics
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Naïve questions concerning gravity

1. What is/are Einstein's equation/s?
2. How can it be used to answer physical questions?
3. What singularities are acceptable?
4. What does it really predict?
5. How does classical matter move?
6. How does quantum matter move?
7. How does quantum matter gravitate?
8. Can we tell a quantum from a classical gravitational field?
9. Can we produce direct evidence for gravitons?
10. Does (quantum-)gravity continue to relate to geometry in a natural way?

Einstein's Field Equation (EFE)

"Matter tells spacetime how to curve and spacetime tells matter how to move."

- Matter tells ...

$$G_{\mu\nu}[g] = \kappa T_{\mu\nu}[g,\phi] \qquad (1a)$$

- Spacetime tells ... (integrability condition)

$$\nabla_\mu[g]T^{\mu\nu}[g,\phi] = 0 \qquad (\Rightarrow \text{eq. of motion}) \qquad (1b)$$

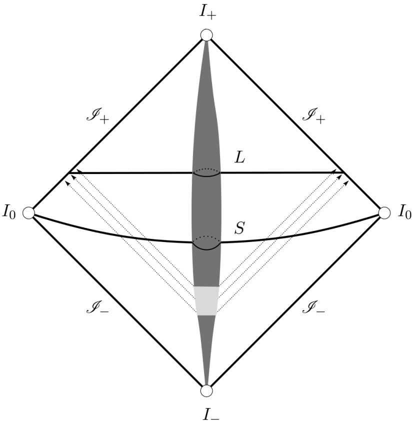
- Evolutionary form of EFE:

$$\mathfrak{g}^{\alpha\beta}\partial_\alpha\partial_\beta\mathfrak{g}^{\mu\nu} + (\text{terms} \propto \partial_\alpha\mathfrak{g}^{\alpha\beta}) = -2\kappa(g)T^{\mu\nu} \qquad (2)$$

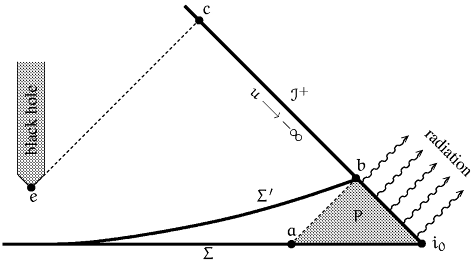
- Geometric form of EFE. For all timelike n have:

$$\sum_{\text{planes} \perp n} \textbf{Sec} = \kappa T(n,n) \qquad (3)$$

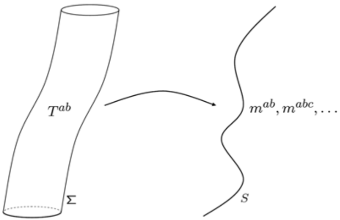
Localisation and radiation



Initial-value problem for fields: controlling “junk radiation”



Initial-value problem for matter: what and where is a "body"?



- In order to talk about "motion" we need to define "position".
- This is ambiguous in SR-theories, though in a well defined way (→ group theory).
- In GR-theories many additional ambiguities enter.

- Example: Mathisson-Papapetrou equation (pole-dipole-approximation)

$$\frac{Dp^\alpha}{ds} = \tfrac{1}{2} R^\alpha{}_{\beta\mu\nu} u^\beta S^{\mu\nu} \tag{4a}$$

$$\frac{DS^{\alpha\beta}}{ds} = (p \wedge u)^{\alpha\beta} \tag{4b}$$

- Need extra (suplementary-) condition, like $S^{\alpha\beta} u_\beta = 0$ (Frenkel, Pirani) or $S^{\alpha\beta} p_\beta = 0$ (Tulczyjew, Dixon) to select centre-of-mass wordline with respect to which u^μ is tangent. Given any of them, have

$$u^\alpha = \hat{p}^\alpha + \frac{2 S^{\alpha\beta} S^{\mu\nu} R_{\mu\nu\beta\gamma} \hat{p}^\gamma}{4M^2 + S^{\mu\nu} R_{\mu\nu\alpha\beta} S^{\alpha\beta}} \tag{5}$$

Einstein's Equivalence Principle (EEP)

- **Universality of Free Fall (UFF):** "Test bodies" determine path structure on spacetime (not necessarily of Riemannian type). UFF-violations are parametrised by the Eötvös factor

$$\eta(A,B) := 2\,\frac{|a(A)-a(B)|}{|a(A)+a(B)|} \tag{6}$$

- **Local Lorentz Invariance (LLI):** Local non-gravitational experiments exhibit no preferred directions in spacetime, neither timelike nor spacelike. Possible violations of LLI concern, e.g., variations in $\Delta c/c$.
- **Universality of Gravitational Redshift (UGR):** "Standard clocks" are universally affected by the gravitational field. UGR-violations are parametrised by the α-factor

$$\frac{\Delta\nu}{\nu} = (1+\alpha)\frac{\Delta U}{c^2} \tag{7}$$

$\Rightarrow$ **Geometrisation of gravity and unification with inertial structure. Gravity ceases to be a force!**

Levels of verification of EEP

- **UFF:** Torsion-balance experiments ("Eöt-Wash" 1994-2008)

$$\eta(Al, Pt) = (-0.3 \pm 0.9) \times 10^{-12}\,, \quad \eta(Be, Ti) = (0.3 \pm 1.8) \times 10^{-13} \quad (8)$$

Next expected improved level is $5 \cdot 10^{-16}$ (MICROSCOPE 2016-18)

- **LLI:** Currently best MM-type experiments (Nagel et al. 2015)

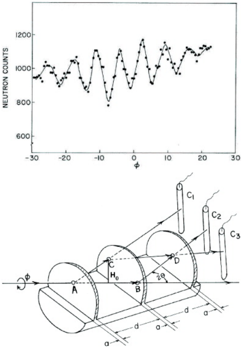
$$\frac{\Delta c}{c} < 10^{-17} \quad (9)$$

- **UGR:** Absolute redshift with H-maser clocks in space (1976, $h = 10\,000\,\mathrm{Km}$) and relative redshifts using precision atomic spectroscopy (2007) give

$$\alpha_{\mathrm{abs}} < 2 \times 10^{-4} \qquad \alpha_{\mathrm{rel}} < 4 \times 10^{-6} \quad (10)$$

  - In Feb. 2010 Müller *et. al.* claimed improvements by 10^4 (disputed). Long-term expectation for future space missions is to get to 10^{-10} level.
  - In Sept. 2010 Chou *et al.* report measurability of gravitational redshift on Earth for $h = 33\,\mathrm{cm}$ using Al^+-based optical clocks ($\Delta t/t < 10^{-17}$).

QM & Gravity: Tested so far



Colella Overhauser Werner, PRL 1975

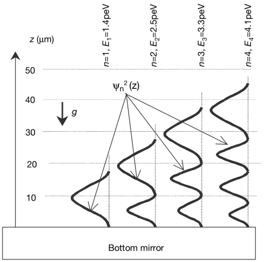


Figure 1 Wavefunctions of the quantum states of neutrons in the potential well formed by the Earth's gravitational field and the horizontal mirror. The probability of finding neutrons at height *z*, corresponding to the *n*th quantum state, is proportional to the square of the neutron wavefunction $\psi_n^2(z)$. The vertical axis *z* provides the length scale for this phenomenon. E_n is the energy of the *n*th quantum state.

Nesvizhevsky et al., Nature 2002

$$i\hbar\dot{\Psi} = -\frac{\hbar^2}{2m_i}\Delta\Psi + V_{\mathrm{grav}}\Psi$$

$$V_{\mathrm{grav}} = m_g g z$$

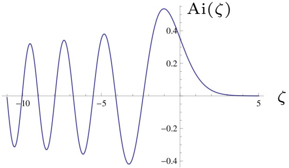
Homogeneous static gravitational field

- Time independent Schrödinger equation in linear potential $V(z) = m_g g z$ is equivalent to:

$$\left(\frac{d^2}{d\zeta^2} - \zeta\right)\psi = 0\,, \quad \zeta := \kappa z - \varepsilon \tag{11}$$

where

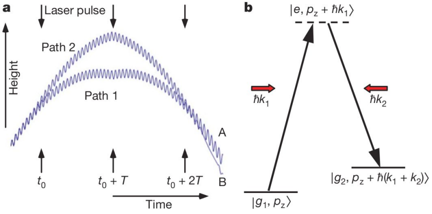
$$\kappa := \left[\frac{2m_i\, m_g\, g}{\hbar^2}\right]^{\frac{1}{3}}\,, \quad \varepsilon := E\cdot\left[\frac{2m_i}{m_g^2\, g^2\, \hbar^2}\right]^{\frac{1}{3}} \tag{12}$$



- Complement by hard (horizontal) wall $V(z) = \infty$ for $z \leq 0$ get energy eigenstates from boundary condition $\psi(z = 0) = 0$, hence $\varepsilon = -z_n$:

$$E(n) = -z_n\left[\frac{m_g^2}{m_i}\cdot\frac{g^2\hbar^2}{2}\right]^{\frac{1}{3}} \tag{13}$$

Quantum gravimeters and an alleged 10^4-improvement of UGR-tests



(Müller *et al.*, Nature 2010)

Have (using $k := \Delta p/\hbar$)

$$\Delta\phi = k\,T^2 \cdot g^{(\mathrm{Cs})} = k\,T^2 \cdot \frac{m_g^{(\mathrm{Cs})}}{m_i^{(\mathrm{Cs})}} \cdot g^{\mathrm{Earth}}$$
$$= k\,T^2 \cdot \frac{m_g^{(\mathrm{Cs})}}{m_i^{(\mathrm{Cs})}} \cdot \frac{m_i^{(\mathrm{Ref})}}{m_g^{(\mathrm{Ref})}} \cdot g^{(\mathrm{Ref})} = \eta(\mathrm{Cs}, \mathrm{Ref}) \cdot kT^2 \cdot g^{(\mathrm{Ref})} \qquad (14)$$

- Proportional to (1+Eötvös-factor) in UFF-violating theories.
- $\mathbb{Q}$ Dependence on α in UGR-violating theories? Müller *et al.* argue for $\propto (1+\alpha)$ by interpretation of $\Delta\phi$ as a mere redshift.

The "clocks-from-rocks" dispute

- A clock ticking at frequency ω suffers gravitational phase-shift in Kasevich-Chu situation of

$$\begin{aligned}\Delta\phi &= \Delta\omega T \\ &= \omega\frac{\Delta U}{c^2}T \\ &= \omega\frac{g\,\Delta h}{c^2}T \\ &= \omega\frac{g\,\Delta p}{mc^2}\,T^2 \\ &= \left(\frac{\omega}{mc^2/\hbar}\right)\,g\,T^2\,\frac{\Delta p}{\hbar}\end{aligned} \tag{15}$$

This equals (14) if

$$\omega = mc^2/\hbar \tag{16}$$

- Objection!

UFF in QM

- Consider a particle of mass m in spatially homogeneous force field $\vec{F}(t)$. The classical trajectories solve

$$\ddot{\vec{\xi}}(t) = \vec{F}(t)/m \tag{17}$$

Let $\xi(t)$ denote a solution with $\vec{\xi}(0) = \vec{0}$ and some initial velocity. Its flow-map $\Phi : \mathbb{R}^4 \to \mathbb{R}^4$ defines a *freely-falling frame*:

$$\Phi(t, \vec{x}) = \big(t, \vec{x} + \xi(t)\big) \tag{18}$$

- **Proposition:** ψ solves the forced Schrödinger equation

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m_i}\Delta - \vec{F}(t)\cdot\vec{x}\right)\psi \tag{19}$$

iff

$$\psi = \big(\exp(i\alpha)\,\psi'\big)\circ\Phi^{-1} \tag{20}$$

where ψ' solves the free Schrödinger equation and

$$\alpha(t,\vec{x}) = \frac{m_i}{\hbar}\left\{\dot{\vec{\xi}}(t)\cdot\big(\vec{x}+\vec{\xi}(t)\big) - \frac{1}{2}\int^t dt'\,\|\dot{\vec{\xi}}(t')\|^2\right\} \tag{21}$$

S & KG: Inertial motion

- Galilei symmetry is a suitable $1/c \to 0$ limit (contraction) of Poincaré symmetry. Likewise, the Schrödinger equation for ψ is a suitable $1/c \to 0$ limit of the Klein-Gordon equation for ϕ if we set

$$\phi(t, \vec{x}) = \exp\{-imc^2\, t/\hbar\}\, \psi(t, \vec{x}) \tag{22}$$

- The Klein-Gordon field transforms as scalar

$$\phi'(t', \vec{x}') = \phi(t, \vec{x}) \tag{23}$$

Hence (22) implies

$$\psi'(t', \vec{x}') = \exp\{-imc^2\, (t - t')/\hbar\}\, \psi(t, \vec{x}) \tag{24}$$

- Using

$$t = \frac{t' + \vec{x}' \cdot \vec{v}/c^2}{\sqrt{1 - v^2/c^2}} = t' + c^{-2}\big(\vec{x}' \cdot \vec{v} + t'v^2/2\big) + \mathcal{O}(1/c^4) \tag{25}$$

the $1/c \to 0$ limit of Poincare symmetry by proper representations turns into Galilei symmetry by non-trivial ray representations:

$$\psi'(t', \vec{x}') = \exp\{-im(\vec{x}' \cdot \vec{v} + t'v^2/2)/\hbar\}\; \psi(t, \vec{x}) \tag{26}$$

S & KG: Rigid accelerations

- In Minkowski space, rigid motions in x-direction and of arbitrary acceleration of a body parametrised by ξ are given by family of timelike lines $\tau \mapsto \big(ct(\tau,\xi)\,,\,x(\tau,\xi)\big)$, where

$$ct(\tau,\xi) = c\int^{\tau} d\tau' \cosh\chi(\tau') + \xi\,\sinh\chi(\tau) \tag{27a}$$

$$x(\tau,\xi) = c\int^{\tau} d\tau' \sinh\chi(\tau') + \xi\,\cosh\chi(\tau) \tag{27b}$$

Here τ is eigentime of body element $\xi = 0$ and $\chi(\tau) = \tanh^{-1}(v/c)$ is rapidity of all body elements at τ.

- The Minkowski metric in co-moving coordinates (τ,ξ) reads $(g := c\dot{\chi})$

$$ds^2 = c^2\,dt^2 - d\vec{x}^2 = \left(1 + \frac{g(\tau)\,\xi}{c^2}\right)\,c^2\,d\tau^2 - d\vec{\xi}^2 \tag{28}$$

- Write down Klein-Gordon equation in co-moving coordinates

$$\left\{\Box_g + m^2\right\}\phi = \left\{(-\det g)^{-1/2}\ \partial_a\left[(-\det g)^{1/2}\ g^{ab}\partial_b\right] + m^2\right\}\phi = 0 \tag{29}$$

S & KG: Rigid accelerations

- In analogy to (22) write

$$\phi(t,\vec{x}) = \exp\{-imc^2\,\tau/\hbar\}\,\psi(t,\vec{x}) \tag{30}$$

and take $1/c^2 \to 0$ limit; get

$$i\hbar\partial_\tau\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\vec{\xi}^2} + mg(\tau)\xi\right)\,\psi \tag{31}$$

This corresponds to particle in homogeneous but time-dependent gravitational field pointing in negative ξ-direction.

- Note that again ϕ transformed as scalar (compare (23))

$$\phi^{\rm inert}(t,\vec{x}) = \phi^{\rm acc}(\tau,\vec{\xi}) \tag{32}$$

but that again this is not true for ψ, where (compare (22))

$$\begin{aligned}\phi^{\rm inert}(t,\vec{x}) &= \exp\{-imc^2\,t/\hbar\}\,\psi^{\rm inert}(t,\vec{x})\\ \phi^{\rm acc}(\tau,\vec{\xi}) &= \exp\{-imc^2\,\tau/\hbar\}\,\psi^{\rm acc}(\tau,\vec{\xi})\end{aligned} \tag{33}$$

- Hence (compare (24))

$$\psi^{\rm acc}(\tau,\vec{\xi}) = \exp\{-imc^2\,(t-\tau)/\hbar\}\;\psi^{\rm inert}(t,\vec{x}) \tag{34}$$

Schrödinger-Newton equation

- Consider Einstein – Klein-Gordon system

$$R_{ab} - \tfrac{1}{2} g_{ab} R = \tfrac{8\pi G}{c^4}\; T^{KG}_{ab}(\phi)\,, \quad \bigl(\Box_g + m^2\bigr)\phi = 0 \tag{35}$$

- Make WKB-like ansatz

$$\phi(\vec{x},t) = \exp\left(\frac{ic^2}{\hbar} S(\vec{x},t)\right) \sum_{n=0}^{\infty} \left(\frac{\sqrt{\hbar}}{c}\right)^n a_n(\vec{x},t) \tag{36}$$

and perform $1/c$ expansion (D.G. & A. Großardt 2012).

- Obtain

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m}\Delta + mV\right)\,\psi \tag{37}$$

where

$$\Delta V = 4\pi G\bigl(\rho + m|\psi|^2\bigr) \tag{38}$$

- Ignoring self-coupling, this just generalises previous results and conforms with expectations.

Schrödinger-Newton equation

- Without external sources get **"Schrödinger-Newton equation"** (Diosi 1984, Penrose 1998):

$$i\hbar\,\partial_t\psi(t,\vec{x}) = \left(-\frac{\hbar^2}{2m}\Delta - Gm^2\int\frac{|\psi(t,\vec{y})|^2}{\|\vec{x}-\vec{y}\|}\,d^3y\right)\psi(t,\vec{x}) \tag{39}$$

- It can be derived from the action

$$\begin{aligned}\mathcal{S}[\psi,\psi^*] = \int dt\Big\{&\frac{i\hbar}{2}\int d^3x\Big(\psi^*(t,\vec{x})\dot{\psi}(t,\vec{x}) - \psi(t,\vec{x})\dot{\psi}^*(t,\vec{x})\Big)\\ &-\frac{\hbar^2}{2m}\int d^3x\,(\vec{\nabla}\psi(t,\vec{x}))\cdot(\vec{\nabla}\psi^*(t,\vec{x}))\\ &+\frac{Gm^2}{2}\iint d^3x\,d^3y\,\frac{|\psi(t,\vec{x})|^2\,|\psi(t,\vec{y})|^2}{\|\vec{x}-\vec{y}\|}\Big\}\end{aligned} \tag{40}$$

- More on SNE ⇒ talk by André Großardt.

Conclusion

1. What is meant by "quantum tests of the equivalence principle"?
2. How do we systematically couple QM to GR?
3. Can we test vector- and tensor-couplings in the laboratory?
4. Can we test gravitational self-couplings in laboratory/space experiments?
5. Is quantum gravity necessary?
6. Is quantum gravity quantum geometry?

THANKS!

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