AdS/CFT duality

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Fundamental Problems in Quantum Physics
Erice

What is it about?

AdS=Anti de Sitter

Maximally symmetric solution of the Einstein equations with negative cosmological constant

CFT=Conformal Field Theory

Relativistic quantum field theory invariant under scaling transformations

Conformal symmetry

- A conformal transformation is a function that preserves angles locally.
- Generators:
  - $M_{\mu\nu}$: Lorentz transformations
  - P_μ: translations
  - D: dilatations
  - K_μ: special conformal transformations
- Generators satisfy an algebra $SO(d,2) \supset \boxed{\text{SO(d)}} \times \boxed{\text{SO(2)}}$
  - spin
  - dimension

Superconformal symmetry

- Generalisation of Poincare group $\rightarrow$ supersymmetry
- Add fermionic generators Q which anticommute with P_μ
- Supersymmetric and conformal group can be joined in some d and for some number of supersymmetries
- Conformal group $+$
  - supersymmetry generators (superpartners of translations) Q and $\bar{Q}$
  - special superconformal generators(superpartners of special conformal transformations) S and $\bar{S}$
  - R-symmetry (global symmetry which does not commute with SUSY) generators T

Anti de Sitter spacetime

- It is the solution of Einstein equations (in vacuum)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

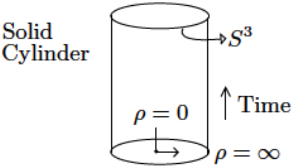
  - For $\Lambda = 0$ the solution is Minkowski spacetime
- Maximally symmetric $R = \frac{2d}{d-2}\Lambda$
- Global parametrisation

$$ds^2 = -R^2 \cosh^2 \rho dt^2 + R^2 d\rho^2 + R^2 \sinh^2 \rho d\Omega_{d-2}^2$$

- there is a boundary at $\rho = \infty$ and it has a topology $\mathbb{R} \times S^{d-2}$

Anti de Sitter spacetime

- We can take out a factor of $\cosh^2 \rho$ and define $dx = \frac{d\rho}{\cosh \rho}$
- The range of x is finite
- Penrose diagram is a cylinder and for $d = 5$ the boundary is $\mathbb{R} \times S^3$
- On the boundary the isometries of AdS act like the conformal group in 4 dimensions!

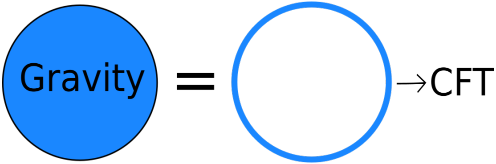


Duality

It is an equality between two (apparently) completely different theories

Gravitational theory in d dimensional AdS spacetime

Conformal field theory in $d-1$ dimensions



A succesful and fruitful idea

The original idea by J. Maldacena goes back to 1997 and through the years it has sparked a lot of interest in many aspects of theoretical physics (not only high energy physics).

13. **The Large N limit of superconformal field theories and supergravity**
Juan Martin Maldacena (Harvard U.). Nov 1997. 19 pp.
Published in **Int.J.Theor.Phys. 38 (1999) 1113-1133, Adv.Theor.Math.Phys. 2 (1998) 231-252**
HUTP-97-A097, HUTP-98-A097
DOI: 10.1023/A:1026654312961
e-Print: **hep-th/9711200** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 10595 records 1000+

Holographic principle

- Bekenstein bound: the maximum entropy in a region of space is $S_{max} = \frac{Area}{4G_N}$ where $Area$ is the area of the boundary region.
- If we have a state with $S \geq S_{max}$ then it violates the second law of thermodynamics.
- The number of degrees of freedom inside some region grows as the area not as the volume!
- Holographic principle: in a quantum theory of gravity all physics within some volume can be described in terms of some theory on the boundary which has less than one degree of freedom per Planck area.

Holographic principle

The AdS/CFT correspondence provides the most concrete example of the holographic principle!

Heuristic idea

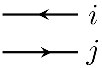
- Spin-two graviton could arise as a composite of two spin-one gauge bosons.
  - this is forbidden if the graviton and the gauge bosons live in the same Lorentian spacetime
- The gauge theory has to contain the graviton and an extra dimension $\rightarrow$ a parameter with respect to which the physics behaves locally.
- This parameter in a gauge theory is the energy scale $\rightarrow$ renormalization group equations describe the flow of the coupling constants with energy scale.

Large N limit and strings

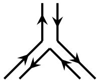
- Consider a theory where the basic field is represented by a hermitian matrix M, *e.g.* $U(N)$ gauge theory with matter fields in the adjoint rep.
- The lagrangian is $U(N)$ invariant $M \to UMU^\dagger$

$$L = \frac{1}{g^2} Tr\left[(\partial M)^2 + V(M)\right]$$

- We can introduce a double line notation for M^i_j



Propagator



Vertex

Large N limit and strings

- Each propagator contributes with a factor of g^2 while each vertex contributes with $\frac{1}{g^2}$
- Each closed line gives a factor of N (summation of gauge indices)
- If we draw Feynman diagrams on a two dimensional surface of genus h, each diagram contributes with

$$N^{2-2h}(g^2 N)^{\#}$$

- The 't Hooft limit consists in taking

$$N \to \infty \quad \text{with} \quad \lambda = g^2 N \quad \text{fixed}$$

In this limit only planar diagrams contribute and at large λ they become dense $\to$ discretized worldsheet of some string theory!

Which string theory?

- If we want to study 4-dimensional Yang Mills theory we need to include a dimension more: what is the most general string theory in 5 dimensions?
  - Poincare symmetry $+$reparametrization invariance:

$$ds^2 = w(z)^2 \left(dx_{1+3}^2 + dz^2\right)$$

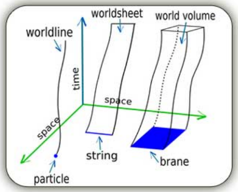
  - Scale invariance implies that $x \to \lambda x$ has to be a symmetry
  - String theory has a scale, the scaling has to be is an isometry, $w \to \frac{R}{z}$

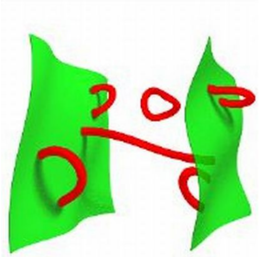
$$ds^2 = R^2 \frac{dx^2 + dz^2}{z^2}$$

5-dimensional Anti de Sitter spacetime!

D-branes

- String theory describes also extended objects: Dp-branes
- Dp-branes preserve half supersymmetries, they are BPS states $\rightarrow$ there is no force between parallel branes
- The tension of the brane $T_p \propto \frac{\alpha'^{(-1-p)/2}}{g_s}$
- Dp-branes couple to $(p+1)$-form potentials
- The worldvolume effective action of a single Dp-brane is (p+1)-dimensional SYM with gauge group $U(1)$





Different limits

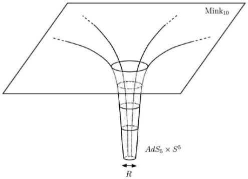
Consider N coincident $D3$-branes in 10 d type IIB string theory

Weak coupling $g_s N \ll 1$

- Description in term of zero-thickness objects in flat space
- $D3$-branes as boundary conditions for open strings

Strong coupling $g_s N \gg 1$

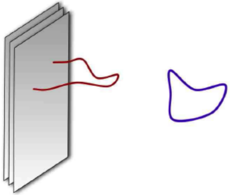
- The backreaction of the branes on a finite region of the spacetime cannot be neglected
- They source a geometry



Weak coupling

- Excitations are:
  - open strings: 3+1 dimensional $\mathcal{N} = 4$ SYM in 4 dimensions (end points of the strings are confined on the brane)+massive states
  - closed strings: massless graviton supermultiplet + massive excitations
- At low energies closed strings become non interacting

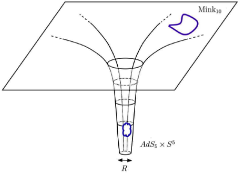
At low energies: interacting $\mathcal{N} = 4$ SYM in 3+1 dimensions and free gravity in 10 dimensions



Strong coupling

- Excitations are:
  - propagating in the flat Minkowski region
  - propagating in the throat
- At low energies:
  - massless 10 dimensional graviton multiplet
  - the whole tower of massive string excitations, deeper and deeper in the throat

At low energies: interacting closed strings in $AdS_5 \times S^5$ and free gravity in 10 dimensional spacetime



AdS/CFT duality

String theory on $AdS_5 \times S^5$ is equivalent to $\mathcal{N}$=4 U(N) gauge group gauge theory in 4 dimensions.

The two sides of the duality are simply different languages which describe the same physics.

$\mathcal{N}$=4 Super Yang-Mills

- Theory with 4 supersymmetries in 4 spacetime dimensions $\rightarrow$ 16 real supercharges
- Field content:
  - 1 vector field
  - 6 scalars
  - 4 fermions
- Global $SO(6)$ R-symmetry
- The theory is conformal
  - $\beta(g) = 0$ at all orders
- Parameters:
  - N: rank of the gauge group
  - g_{YM}: coupling constant

Parameters

String theory

- string coupling: g_s
- curvature scale: $\frac{R}{\ell_s}$

Gauge theory

- rank of the gauge group: N
- coupling constant: g_{YM}

$$4\pi g_s = g_{YM}^2 \qquad \frac{R}{\ell_s} = (4\pi g_s N)^{\frac{1}{4}}$$

- The gravity description is a good approximation of ST when $R \gg \ell_s$ since ℓ_s is the intrinsic size of the graviton

$$R \gg \ell_s \qquad g_{YM}^2 N = \lambda \gg 1$$

$\boxed{\text{Weakly coupled}}$ $\boxed{\text{Strongly coupled}}$

Weak/strong duality

- When the gauge theory is strongly coupled, and hence all perturbative techniques fail, one can study it using the weakly coupled string theory description and viceversa.
- This makes the correspondence very powerful (and not obviously contradictory!) and very hard, if not impossible, to prove.
- Early checks: due to supersymmetry, there are quantities which are protected (do not depend on the coupling constant) and they can be computed in both side of the correspondence and they match.

Practical use

- Each field in the 5d bulk $\rightarrow$ operator in the dual field theory
  - The graviton is associated to the stress tensor
  - The dilaton is associated to the lagrangian
- The partition function of the ST is equal to the generating function of the correlation functions of the corresponding operators

$$\mathcal{Z}_{bulk}\left[\phi(\vec{x},z)|_{z=0}=\phi_0(\vec{x})\right]=\langle e^{\int d^4x\phi_0(\vec{x})\mathcal{O}(\vec{x})}\rangle_{CFT}$$

- $\phi_0(\vec{x})$ is an arbitrary function which specifies the boundary values of the field ϕ
- By taking derivatives with respect to ϕ_0 and equating them to zero one gets the correlation functions.

Tests and generalisations

- Tests:
  - Spectrum of operators
  - Correlation functions
  - Wilson loops
  - ...
- Applications:
  - Finite temperature $\mathcal{N} = 4$ SYM
  - Quark Gluon Plasma physics
  - Condensed matter physics
  - ...
- Generalisations:
  - Other dimensions
  - Asymptotically locally AdS
  - Other asymptotics
  - ...

Conclusions

- We have two equivalent descriptions of the same physics
  - string theory on $AdS_5 \times S^5$
  - $\mathcal{N}$=4 with $U(N)$ gauge group gauge in 4 dimensions
- AdS/CFT correspondence is a powerful tool to study gauge theories in a regime in which perturbation theory is not applicable
- We can understand some general features of strongly coupled gauge theories

Some references

Some references that I used to prepare the talk (time ordered!):

- `The AdS/CFT correspondence`, by Veronika E. Hubeny, gr-qc/1501.0007
- `Gauge/gravity duality`, by Gary T.Horowitz and Joseph Polchinski, gr-qc/0602037
- `TASI 2003 lectures on AdS/CFT`, by Juan Maldacena, hep-th/0309246
- `Large` N `Field Theories, String Theory and Gravity`, by O.Aharony, S.S. Gubser, J.Maldacena, H.Ooguri and Y.Oz, hep-th/9905111

`Disclaimer`: Almost all the pictures are "borrowed" from the web or from papers.