UNIVERSITY OF TRIESTE

DEPARTMENT OF PHYSICS

A new upper bound on collapse models parameters from spontaneous radiation emission

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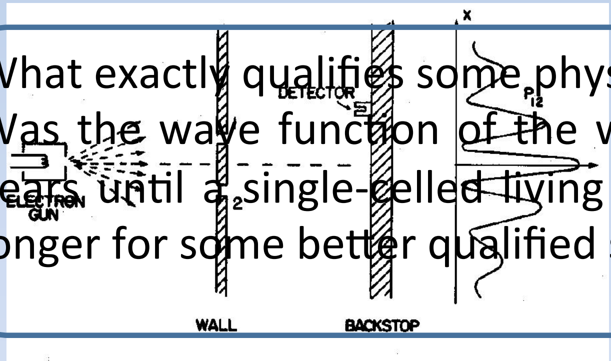
Erice, 24/03/2015

THE MEASUREMENT PROBLEM

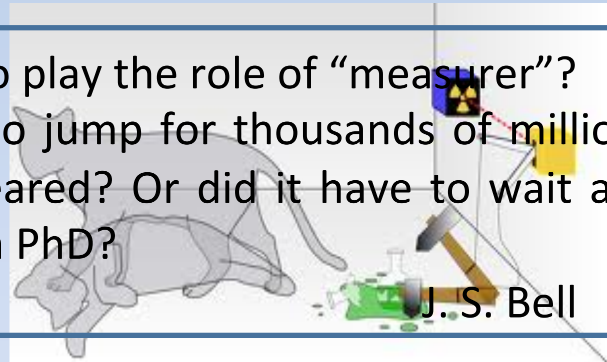
The Schrödinger equation:

- Linear
- Deterministic

$$i\hbar \frac{d}{dt} \left|\Psi(t)\right\rangle = H \left|\Psi(t)\right\rangle$$



OK



KO

What exactly qualifies some physical systems to play the role of "measurer"? Was the wave function of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer for some better qualified system...with a PhD?

J. S. Bell

The wave packet reduction postulate:

- Non Linear
- Stochastic

$$\frac{|a_1\rangle + |a_2\rangle}{\sqrt{2}} \xrightarrow{\text{measurement}} \begin{cases} \xrightarrow{\text{half of total cases}} |a_1\rangle \\ \xrightarrow{\text{half of total cases}} |a_2\rangle \end{cases}$$

There are two different laws for the evolution of the state vectors but it is not clear when to use one or the other one.

THE CSL MODEL

(CONTINUOUS SPONTANEOUS LOCALIZATIONS)

IDEA: to modify Schrödinger dynamics with one describing also the collapse:

$$d|\phi_t\rangle = \left[-\frac{i}{\hbar}H\,dt + \sqrt{\gamma}\int d\mathbf{x}\,\left(N(\mathbf{x}) - \langle N(\mathbf{x})\rangle\right)dW_t(\mathbf{x}) - \frac{\gamma}{2}\int d\mathbf{x}\,\left(N(\mathbf{x}) - \langle N(\mathbf{x})\rangle\right)^2 dt\right]|\phi_t\rangle$$

Schrödinger

stochasticity

non linearity

$$N(\mathbf{x}) = \int d\mathbf{y}\, g(\mathbf{y}-\mathbf{x})\,\psi^\dagger(\mathbf{y})\,\psi(\mathbf{y}), \qquad g(\mathbf{y}-\mathbf{x}) = \frac{1}{\left(\sqrt{2\pi}r_c\right)^3} e^{-\frac{(\mathbf{y}-\mathbf{x})^2}{2r_c^2}} \qquad \lambda = \frac{\gamma}{8\pi^{3/2}r_c^3}$$

A. Bassi and G.C. Ghirardi, Phys. Rept. **379**, 257 (2003).

- Localization in space;
- Amplification mechanism: the strength of the collapse increase with the size of the system;

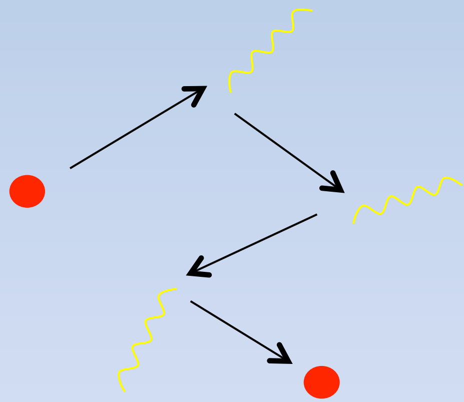
Linearized dynamics: different dynamics, same average values.

$$i\hbar\frac{d|\psi_t\rangle}{dt} = \left[H - \hbar\sqrt{\gamma}\int d\mathbf{x}\,N(\mathbf{x})w(\mathbf{x},t)\right]|\psi_t\rangle$$

$$E\left[w(\mathbf{x},t)\right] = 0 \qquad\qquad E\left[w(\mathbf{x},t)w(\mathbf{x}',t')\right] = \delta(\mathbf{x}-\mathbf{x}')f(t-t')$$

RADIATION EMISSION IN THE CSL MODEL

IDEA: the particle interaction with the noise induces radiation emission.



EMISSION RATE

$$\frac{d\Gamma}{dk} = \sum_{\mu} \int d\Omega_k \sum_{f} \frac{\partial}{\partial t} E \left| T_{fi} \right|^2$$

$$T_{fi} := \langle f; \mathbf{k}, \mu \left| U(t, t_i) \right| i; \Omega \rangle$$

$$H_{\text{tot}} = \int d\mathbf{x}\, \mathcal{H}_{\text{tot}}$$

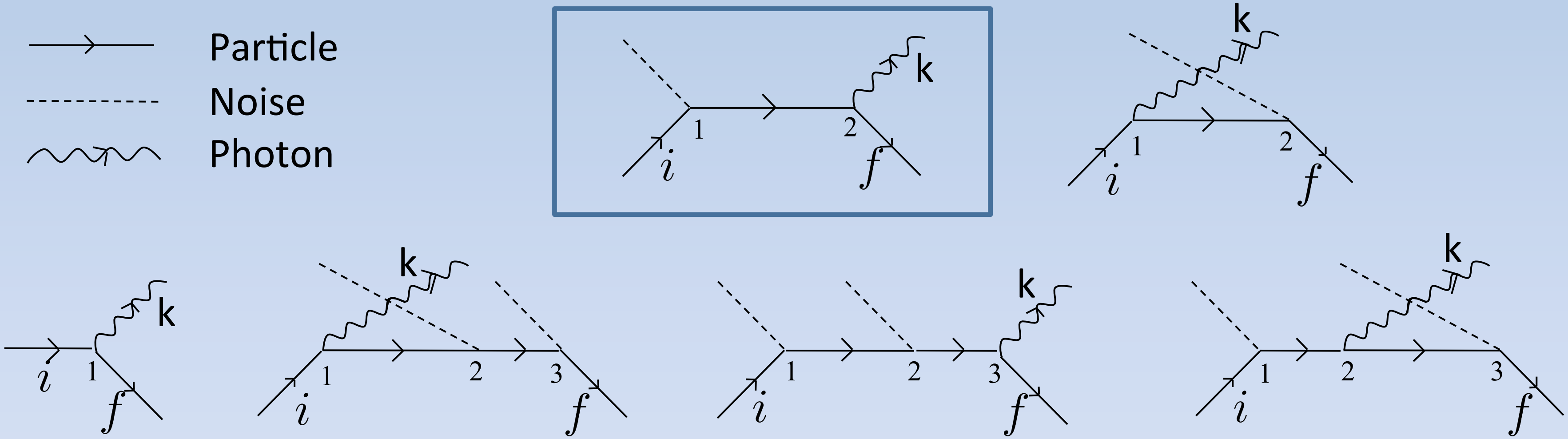
$$\mathcal{H}_{\text{tot}} = \mathcal{H}_0 + \mathcal{H}_{\text{em}} + \mathcal{H}_{\text{n}}$$

Free evolution ($\mathcal{H}_0$) — E.M. interaction ($\mathcal{H}_{\text{em}}$) — Noise interaction ($\mathcal{H}_{\text{n}}$)

$$\mathcal{H}_{\text{int}} = i\frac{\hbar e}{m}\psi^{\dagger}\mathbf{A}\cdot\nabla\psi + \cancel{\frac{e^2}{2m}\mathbf{A}^2\psi^{\dagger}\psi} - \hbar\sqrt{\gamma}w\psi^{\dagger}\psi$$

EMISSION RATE TO THE FIRST PERTURBATIVE ORDER

Relevant Feynman diagrams to the FIRST order



FREE PARTICLE RESULT

S.L. Adler, A. Bassi and S. Donadi, *J. Phys. A* **46**, 245304 (2013).

COLORED / WHITE NOISE

$$\frac{d\Gamma}{dk} = \frac{\lambda\hbar e^2}{2\pi^2\varepsilon_0 c^3 m_0^2 r_c^2 k}\left[\frac{\tilde{f}(0) + \tilde{f}(\omega_k)}{2}\right] \qquad \tilde{f}(\omega) := \int ds\, f(s)\, e^{i\omega s}$$

The zero energy component of the noise play a role in the emission of photons with arbitrary large energy!

CALCULATION WITH QMUPL MODEL: AN EXACT RESULT

A. Bassi and S. Donadi, Phys. Lett. A **378**, 761-765 (2014).

$$\frac{d\Gamma}{dk} \sim e^{-\frac{\omega_0^2 \beta}{m} t} \tilde{f}(\omega_0) \left(\frac{1 + e^{i 2 \omega_0 t}}{2} \right) + \tilde{f}(\omega_k)$$

$$\beta = \frac{e^2}{6\pi \epsilon_0 c^3}$$

ω_0 : frequency harmonic oscillator

Lowest order perturbative calculation $(\beta = 0)$ $\Longrightarrow$ presence of $\tilde{f}(0)$

REPEATING CALCULATION WITH CSL MODEL

S. Donadi, D.-A. Deckert and A. Basssi, Annals of Physics **340**, 70-86 (2014).

E.M. interaction ⟶ treated *exactly*.

NOISE interaction ⟶ treated *perturbatively*

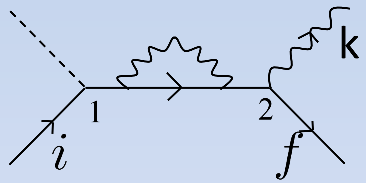
$$\frac{d\Gamma}{dk} = \frac{\lambda \hbar e^2}{4\pi^2 \varepsilon_0 c^3 m_0^2 r_c^2 k} \tilde{f}(\omega_k)$$

PERTURBATION THEORY TAKING INTO ACCOUNT PROPAGATOR DECAY

1) The previous method require to solve exactly Heisenberg equations, not always possible;

2) The damping factor resemble the decay due to the instability of the H_0 eigenstates.

For the first diagram:



$$P_{fi} \propto \int_{-\infty}^{+\infty} d\nu \tilde{f}(\nu) |\mathrm{T}|^2$$

$$\mathbf{T} = \frac{-1}{[i(\Delta_{fn}+\omega_k) - \Gamma_n]} \left\{ \frac{e^{i(\Delta_{fi}+\omega_k-\nu)t} - 1}{i(\Delta_{fi}+\omega_k-\nu)} - e^{[i(\Delta_{fn}+\omega_k)-\Gamma_n]t} \frac{e^{i(\Delta_{ni}-\nu)t} - 1}{i(\Delta_{ni}-\nu)} \right\}$$

$$\mathbf{T} = \frac{-1}{i(\Delta_{fn}+\omega_k)} \left[\frac{e^{i(\Delta_{fi}+\omega_k-\nu)t} - 1}{i(\Delta_{fi}+\omega_k-\nu)} - e^{i(\Delta_{fn}+\omega_k)t} \frac{e^{[i(\Delta_{ni}-\nu)+\Gamma_n]t} - 1}{i(\Delta_{ni}-\nu)+\Gamma_n} \right]$$

☺ $\tilde{f}(\omega_k)$ ☹ $\sim \tilde{f}(0)$

$$\left| \frac{e^{ixt} - 1}{ix} \right|^2 = \frac{\sin^2\left(\frac{xt}{2}\right)}{\left(\frac{x}{2}\right)^2} \xrightarrow[t\to\infty]{} 2\pi t \delta(x),$$

EMISSION RATE FOR A GENERIC SYSTEM USING A GENERIC MODEL

S. Donadi and A. Bassi, Journ. Phys. A: Math. Theor. **48**, 035305 (2015).

For a generic model:

$$i\hbar\frac{d|\psi_t\rangle}{dt}=\left[H-\hbar\sqrt{\gamma}\sum_{i=1}^{n}N_i w_{i,\,t}\right]|\psi_t\rangle$$

the emission rate becomes:

$$\frac{d\Gamma}{dk}=\sum_{\lambda}\int d\Omega_k\frac{\gamma}{\hbar^2}\sum_f\sum_i\left|\sum_n\frac{\langle f\,|R_k|\,n\rangle\,\langle n\,|N_i|\,i\rangle}{[i\,(\triangle_{fn}+\omega_k)-\Gamma_n]}-\frac{\langle f\,|N_i|\,n\rangle\,\langle n\,|R_k|\,i\rangle}{[i\,(\triangle_{ni}+\omega_k)+\Gamma_n]}\right|^2\tilde{f}\,(\triangle_{fi}+\omega_k)$$

$$R_k:=\ \alpha_k\sum_{j=1}^{N_p}\left(-\frac{e_j}{m_j}\right)e^{-i\mathbf{k}\cdot\mathbf{x}_j}\vec{\epsilon}_{\mathbf{k},\lambda}\cdot\mathbf{p}_j\qquad\alpha_k\equiv\sqrt{\frac{\hbar}{2\varepsilon_0\omega_k\,(2\pi)^3}}\qquad\triangle_{fn}=\frac{1}{\hbar}\,(E_f-E_n)$$

MAIN RESULTS

- We show that, in order to compute correctly the spontaneous radiation emission rate, it is fundamental to take into account the decay width of the free Hamiltonian eigenstates.

- For a free particle the emission rate predicted by the CSL model is:

$$\frac{d\Gamma}{dk} = \frac{\lambda \hbar e^2}{4\pi^2 \varepsilon_0 c^3 m_0^2 r_c^2 k} \tilde{f}\left(\omega_k\right)$$

With $\tilde{f}\left(\omega_k\right) = 1$ in the white noise case.

COMPARISON WITH
EXPERIMENTA DATA!

FINANCIAL SUPPORT