Particle-vibration coupling effect on $\beta$-decay of magic nuclei

Yifei Niu
INFN, Milano, Italy
Institute of Fluid Physics, CAEP, China
April 19, 2015

Collaborators:
Zhongming Niu: Anhui University, China
Gianluca Colò: Università degli Studi and INFN, Milano, Italy
Enrico Vigezzi: INFN, Milano, Italy
OUTLINE

1. Introduction
2. Theory framework
3. Results and discussions
4. Summary and Perspectives
**Introduction**

Nuclear $\beta^-$ decay

\[ _NX^A_Z \rightarrow _{N-1}X^A_{Z+1} + e^- + \bar{\nu}_e \]

- mainly determined by GT transition
- energy conservation
  \[ M_i = M_f + E^* + T_e + E_\nu \]
- GT strength distribution
  - ✓ spin-orbit splitting
  - ✓ spin-isospin channel of nuclear effective interaction
- $\beta$-decay and GTR measurements complement each other

\[ Q_\beta \text{ value} = M_i \text{ (atomic)} - M_f \text{ (atomic)} \]

\[ \text{Gamow-Teller strength distribution} \]

\[ \text{Excitation energy in daughter nucleus } E^* \]

\[ \text{Q}_\beta \text{ value} = M_i \text{ (atomic)} - M_f \text{ (atomic)} \]

\[ \beta \text{ decay window} \]

\[ \text{Giant resonance} \]
The r-process

- r-process: produce the heavy elements beyond iron
- $\beta$-decay half-life:
  - govern the abundance flow from neighbouring isotopic chains
  - set the time-scale of r-process
  - ($\sim$ the sum of $\beta$-decay half-lives of nuclei in the r-process path)
development of radioactive ion-beam facilities ⇒ important advances

$^{78}$Ni and around Hosmer, et al., PRL 94, 112501, 2005; Xu, et al., PRL 113, 032505, 2014

very neutron rich Kr to Tc isotopes Nishimura, et al., PRL 106, 052502, 2011

most neutron rich nuclei relevant for r-process: out of experimental reach
Introduction

diamond ab-initio approach: for very light nuclei  
Barrett, et al. PPNP 69, 131, 2013

diamond shell model: up to $A = 40 - 50$ or around magic regions assuming a frozen core

  

diamond Random Phase Approximation (RPA)

✓ non-self-consistent
  - Quasi-particle RPA (QRPA) based on FRDM  
  Møller, et al., ADNDT 66, 131, 1997
  - DF3 + CQRPA  Borzov, PRC 67, 025802, 2003

✓ self-consistent
  - QRPA based on Skyrme density functional
  Engel, et al., PRC 60, 014302, 1999
  - QRPA based on covariant density functional
  RHB + QRPA  Nikšić, et al., PRC 71, 014308, 2005;  
  Marketin, et al., PRC 75, 024304, 2007
  RHFB + QRPA  Niu, et al., PLB 723, 172, 2013
Introduction

Problems with RPA description

◊ Skyrme HFB+QRPA

Engel, et al., PRC 60, 014302, 1999

◊ RHB+QRPA

Nikšić, et al., PRC 71, 014308, 2005

Self-consistent QRPA

✓ Half-lives are systematically overestimated
✓ Half-lives can be reduced with the inclusion of attractive isoscalar pn pairing
✓ The isoscalar pn pairing has little or no effect on closed-shell nuclei, like $^{78}\text{Ni}$. 
### Introduction

#### Possible solutions

- **Possible solution:** tensor force
  
  *Minato, et al., PRL 110, 122501, 2013*

  Skyrme RPA + tensor

  $\beta$-decay half-lives are reduced

- **Another possible solution:**

  Skyrme RPA + particle-vibration coupling (PVC)

  *Niu, et al., PRC 85, 034314, 2012*

---

<table>
<thead>
<tr>
<th>Decay</th>
<th>No Tensor</th>
<th>Tensor</th>
<th>PVC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{132}$Sn</td>
<td>$^{68}$Ni</td>
<td>$^{34}$Si</td>
<td>$^{78}$Ni</td>
</tr>
<tr>
<td>$^{122501}$</td>
<td>$^{122501}$</td>
<td>$^{122501}$</td>
<td>$^{122501}$</td>
</tr>
</tbody>
</table>

---

**RPA+PVC**

- Include correlations beyond 1p-1h configurations
- The downward shift of excitation energy and spreading of the GT strength is found.
- It is expected the PVC could help with reducing $\beta$-decay half-life.

---

**Advantage**

- Without introducing new parameters

---

*Fig. 3. Same as Fig. 2, but for $^{78}$Ni (left) and $^{34}$Si (right).*
◊ **Spreading Width**

- (Q)RPA: coherent superposition of 1p-1h states
- Spreading width $\Gamma$
  - energy and angular momentum of coherent vibrations
  - more complicated states of 2p-2h, 3p-3h \ldots character

◊ **To describe the spreading width**

- Second RPA  *Drozdz, et al., PR 197, 1, 1990*
  - configuration space 1p-1h, 2p-2h
- RPA + PVC (particle-vibration coupling)
  - configuration space 1p-1h, 1p-1h $\otimes$ phonon
    - relativistic functional
    - skyrme functional  *Colo, et al., PRC 50, 1496, 1994*
Introduction

---

In this work

---

**Goal**

- To develop self-consistent skyrme RPA + PVC model by including the whole two-body interaction in the PVC vertex
- To investigate the particle-vibration coupling effects on nuclear $\beta$-decay in magic nuclei
  - the spreading width and fragmentation of GT strength
  - the $\beta$-decay half-life
Half-life calculated in the allowed GT approximation

\[ T_{1/2} = \frac{D}{g_A^2 \int_{Q_\beta}^S(E) f(Z, \omega) dE}, \]  

(1)

- \( D = 6163.4 \text{ s} \) and \( g_A = 1 \)
- \( S(E) \): GT strength distribution with respect to daughter nucleus
- integrated phase volume

\[ f(Z, \omega_0) = \frac{1}{(m_e c^2)^5} \int_{m_e c^2}^{\omega_0} p_e E_e (\omega_0 - E_e)^2 F_0(Z + 1, E_e) dE_e. \]  

(2)

\( p_e \): momentum; \( E_e \): energy; \( F_0(Z + 1, E_e) \): Fermi function of the electron

\( \omega_0 = Q_\beta + m_e c^2 - E \)
The nuclear excitation configuration space is divided into three subspaces: $Q_1$, $Q_2$, $P$.

- $Q_1$: 1p-1h within the set $|i\rangle$ — obtained by solving the HF equation
- $P$: 1p-1h where the particle is in an unbound state ⇒ Escaping Width
- $Q_2$: “doorway states” $|N\rangle$: 1p-1h ⊗ phonon ⇒ Spreading Width

**Strength function**

$$S(\omega) = \sum_n \frac{1}{\sigma_n \sqrt{2\pi}} e^{-\frac{(\omega-\Omega_n)^2}{2\sigma_n^2}} B_n, \quad \sigma_n = \frac{\Gamma_n(\omega)}{2} + \Delta / \sqrt{2 \ln 2}, \quad (3)$$

where

$$\Omega_n(\omega) = E_{RPA} + \text{Re}[(\Sigma)_{GT,GT}(\omega)] \quad (4)$$

and

$$\Gamma_n(\omega) = -2\text{Im}[(\Sigma)_{GT,GT}(\omega)]. \quad (5)$$

$\Sigma_{GT,GT}(\omega)$ is the self-energy of the GT state.
Results and discussions

$^{208}$Pb Gamow-Teller strength distribution

Y. Niu, G. Colò, and E. Vigezzi, PRC 90, 054328 (2014)
Results and discussions

PVC effects on $\beta$-decay half-life

$T_{1/2}$ (s)


- $^{78}$Ni: bottle-neck nucleus in r-process
- Skyrme interactions are not well constrained in spin-isospin channel
- PVC reduces half-lives for all interactions.
  Reduction factor $R=42$ (SAMi), 10 (SGII), 4 (SkM*, SIII, SLy5, Skx)
- SkM*: reproduce well both GTR and $\beta$-decay half-life

How PVC reduces the half-life

Exp. : Xu, et al., PRL 113, 032505, 2014

Half life

\[ T_{1/2} = \frac{D}{g_A^2 \int_{Q,\beta}^{\infty} S(E) f(Z, \omega) dE}, \]  

(6)

Phase volume

\[ f(Z, \omega_0) = \frac{1}{(m_e c^2)^5} \int_{m_e c^2}^{\omega_0} p_e E_e (\omega_0 - E_e)^2 F_0(Z + 1, E_e) dE_e. \]  

(7)
With the inclusion of PVC, the RPA energy is shifted downwards by about 2 MeV.
The strength of each peak is basically kept conserved as the RPA case.
Due to the big increase in phase volume, the contribution to the half-life also changes a lot from RPA to PVC.

Although the PVC doesn’t change the strength of each peak, it reduces the half-life dramatically by shifting downwards the excitation energy.
Results and discussions

$\beta$-decay half-life

- **SkM***
- **SIII**
- **SGII**
- **SAMi**
The self-consistent Skyrme RPA + PVC model is developed and used for the calculation of the Gamow-Teller transitions and $\beta$-decay half-lives in magic nuclei.

- Coupling with phonons is relevant to producing a more realistic strength distribution characterized by a spreading width. As a result, very good agreement with experiment is obtained for $^{208}\text{Pb}$.
- Coupling with phonons shifts energy downwards by 1-2 MeV, and hence increases the decay phase space. As a consequence, the $\beta$-decay half-life is reduced, and reproduces the experimental data very well.

**Perspectives:**

- apply in other weak-interaction processes like electron capture.
- include the pairing correlations for open-shell nuclei
- include temperature effect
Acknowledgements

THANK YOU!
The nuclear excitation configuration space is divided into three subspaces: $Q_1$, $Q_2$, $P$.

- $Q_1$: 1p-1h within the set $|i\rangle$ — obtained by solving the HF equation
- $P$: 1p-1h where the particle is in an unbound state $\Rightarrow$ Escaping Width
- $Q_2$: “doorway states” $|N\rangle$: 1p-1h $\otimes$ phonon $\Rightarrow$ Spreading Width

In order to work inside the space $Q_1$, the effective Hamiltonian is taken

$$\mathcal{H}(\omega) = Q_1HQ_1 + W^\uparrow(\omega) + W^\downarrow(\omega)$$

$$= Q_1HQ_1 + Q_1HP\frac{1}{\omega - PHP + i\epsilon}PHQ_1 + Q_1HQ_2\frac{1}{\omega - Q_2HQ_2 + i\epsilon}Q_2HQ_1. \quad (8)$$

In this work, only the spreading width will be considered, i.e., the effective Hamiltonian

$$\mathcal{H}(\omega) = Q_1HQ_1 + W^\downarrow(\omega) \quad (9)$$

will be solved.
The creator $O_{\nu}^\dagger$ of state $|\nu\rangle$, solutions of effective Hamiltonian, is

$$O_{\nu}^\dagger = \sum_{\omega_n > 0} F_{n}^{(\nu)} O_{n}^\dagger - \bar{F}_{n}^{(\nu)} \bar{O}_{n}^\dagger, \quad (10)$$

where $O_{n}^\dagger$ and $\bar{O}_{n}^\dagger$ are creation operators of RPA states $|n\rangle$ with energy $\omega_n$, and $|\bar{n}\rangle$ with energy $-\omega_n$.

The eigenvalue equation for the effective Hamiltonian is

$$[\mathcal{H}, O_{\nu}^\dagger] = (\Omega_{\nu} - i \frac{\Gamma_{\nu}}{2}) O_{\nu}^\dagger. \quad (11)$$

The matrix form is

$$\begin{pmatrix} \mathcal{D} + A_1(\omega) & A_2(\omega) \\ -A_3(\omega) & -\mathcal{D} - A_4(\omega) \end{pmatrix} \begin{pmatrix} F^{(\nu)} \\ \bar{F}^{(\nu)} \end{pmatrix} = (\Omega_{\nu} - i \frac{\Gamma_{\nu}}{2}) \begin{pmatrix} F^{(\nu)} \\ \bar{F}^{(\nu)} \end{pmatrix}, \quad (12)$$

where $\mathcal{D}$ is a diagonal matrix with the RPA eigenvalues, and the $A_i$ matrices contain the spreading contributions.
The matrix element of the spreading term $W\downarrow$ in the basis of p-h configurations from $Q_1$ space is

$$W_{ph, p'h'}^\downarrow(\omega) = \sum_N \frac{\langle ph | V | N \rangle \langle N | V | p'h' \rangle}{\omega - \omega_N},$$

which is a sum of the following four diagrams.
Theory framework

**Introduction**

**Theory framework**

Strength function

\[ S(\omega) = -\frac{1}{\pi} \text{Im} \sum_{\nu} \langle 0 | \hat{O}_{\text{GT}} \pm \nu \rangle^2 \frac{1}{\omega - \Omega_{\nu} + i(\frac{\Gamma_{\nu}}{2} + \Delta)} \] (14)

**Diagonal Approximation**

We could take Lorentzian form

\[ S(\omega) = \sum_n \frac{1}{\pi} \frac{\frac{\Gamma_n(\omega)}{2} + \Delta}{(\omega - \Omega_n(\omega))^2 + \left(\frac{\Gamma_n(\omega)}{2} + \Delta\right)^2} B_n, \quad B_n = |\langle 0 | \hat{O}_{\text{GT}}^- | n \rangle|^2, \] (15)

or the Gaussian form of strength distribution

\[ S(\omega) = \sum_n \frac{1}{\sigma_n \sqrt{2\pi}} e^{-\frac{(\omega - \Omega_n)^2}{2\sigma_n^2}} B_n, \quad \sigma_n = \left(\frac{\Gamma_n(\omega)}{2} + \Delta\right)/\sqrt{2\ln 2}, \] (16)

where

\[ \Omega_n(\omega) = E_{\text{RPA}} + \text{Re}[\langle A_1 \rangle_{\text{GTR,GTR}}(\omega)] \] (17)

and

\[ \Gamma_n(\omega) = -2\text{Im}[\langle A_1 \rangle_{\text{GTR,GTR}}(\omega)]. \] (18)
These four diagrams are expressed by

\[
W\downarrow^J(1) = \delta_{hh'}\delta_{jp'p} \sum_{p''nL} \frac{1}{\omega - (\omega_n + \epsilon_{p''} - \epsilon_h) + i\Delta} \langle n_{p'p} | V | j_{p''}, nL \rangle \langle n_{p'p} | V | j_{p''}, nL \rangle^* \tag{19}
\]

\[
W\downarrow^J(2) = \delta_{pp'}\delta_{hh'j} \sum_{h''nL} \frac{1}{\omega - (\omega_n - \epsilon_{h''} + \epsilon_p) + i\Delta} \langle n_{h'h} | V | j_{h''}, nL \rangle \langle n_{h'h} | V | j_{h''}, nL \rangle^* \tag{20}
\]

\[
W\downarrow^J(3) = \sum_{nL} \frac{(-)^{j_p-j_{h'}+J+L}}{\omega - (\omega_n + \epsilon_p - \epsilon_{h'}) + i\Delta} \left\{ \begin{array}{ccc} j_p & j_h & J \\ j_{h'} & j_{p'} & L \end{array} \right\} \langle j_p | V | j_p, nL \rangle \langle j_{h'} | V | j_h, nL \rangle \tag{21}
\]

\[
W\downarrow^J(4) = \sum_{nL} \frac{(-)^{j_{p'}-j_h+J+L}}{\omega - (\omega_n + \epsilon_{p'} - \epsilon_h) + i\Delta} \left\{ \begin{array}{ccc} j_p & j_h & J \\ j_{h'} & j_{p'} & L \end{array} \right\} \langle j_p | V | j_{p'}, nL \rangle \langle j_h | V | j_{h'}, nL \rangle \tag{22}
\]

The evaluation of the reduced matrix element

\[
\langle i| V | j, nL \rangle = \sqrt{2L+1} \sum_{ph} X_{ph}^{nL} V_L(ihjp) + (-)^{L-j_{h'}-j_p} Y_{ph}^{nL} V_L(ipjh), \tag{23}
\]

where \( V_L \) is the p-h coupled matrix element,

\[
V_L(ihjp) = \sum_{all \ m} (-)^{j_{j''} - m_j + j_{h''} - m_h} \langle j_i m_i j_j - m_j |LM\rangle \langle j_p m_p j_h - m_h |LM\rangle \langle j_i m_i, j_h m_h |V | j_{j''} m_{j''}, j_{p'} m_{p'} \rangle \tag{24}
\]
To transform the spreading terms from p-h basis of $Q_1$ to RPA basis, we get the matrices elements $A_i$.

$$
(A_1)^{_{_{\text{spr}}}}_{mn} = \sum_{ph,p'h'} W_{ph,p'h'}(\omega) X_{ph}^{(m)} X_{p'h'}^{(n)} + W_{ph,p'h'}^{\dagger*}(-\omega) Y_{ph}^{(m)} Y_{p'h'}^{(n)},
$$

(25)

$$
(A_2)^{_{_{\text{spr}}}}_{mn} = \sum_{ph,p'h'} W_{ph,p'h'}(\omega) Y_{ph}^{(m)} Y_{p'h'}^{(n)} + W_{ph,p'h'}^{\dagger*}(-\omega) Y_{ph}^{(m)} X_{p'h'}^{(n)},
$$

(26)

$$
(A_3)^{_{_{\text{spr}}}}_{mn} = \sum_{ph,p'h'} W_{ph,p'h'}(\omega) Y_{ph}^{(m)} X_{p'h'}^{(n)} + W_{ph,p'h'}^{\dagger*}(-\omega) X_{ph}^{(m)} Y_{p'h'}^{(n)},
$$

(27)

$$
(A_4)^{_{_{\text{spr}}}}_{mn} = \sum_{ph,p'h'} W_{ph,p'h'}(\omega) Y_{ph}^{(m)} Y_{p'h'}^{(n)} + W_{ph,p'h'}^{\dagger*}(-\omega) X_{ph}^{(m)} X_{p'h'}^{(n)}.
$$

(28)
Numerical details

- HF equation: solved in coordinate space $R_{box} = 21$ fm, $dr = 0.1$ fm.
- RPA configuration space: $\epsilon_p \leq 100$ MeV
- Phonons: $0^+, 1^-, 2^+, 3^-, 4^+, 5^-, 6^+$
- Phonon energy $\omega \leq 20$ MeV
- Phonon strength (isoscalar or isovector) $\geq 5$
- Intermediate particle in the door-way state: $\epsilon_p \leq 100$ MeV
**Results and discussions — PVC effects on GTR with different interactions**

PVC Effects

- weakly dependent on the interactions
- peak energy is shifted downwards by 1.2 MeV
- A width of \( \sim 3.5 \) MeV is acquired
Results and discussions

---

$^{208}$Pb Gamow-Teller cumulative sum

<table>
<thead>
<tr>
<th>$E$ [MeV]</th>
<th>$B$(GT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>15</td>
<td>40</td>
</tr>
<tr>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>25</td>
<td>80</td>
</tr>
</tbody>
</table>

- **RPA**
- **RPA+PVC**
- **exp.**

<table>
<thead>
<tr>
<th>Ikeda sum rule</th>
<th>99.99%</th>
<th>95.2% (97.3% for $\Delta = 0.2$ MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Above 25 MeV</td>
<td>3%</td>
<td>15%</td>
</tr>
</tbody>
</table>

- $E = 25$ MeV : exp. strength/RPA+PVC strength $\sim$ 71%
\[ \Sigma = (A_1)^{spr}_{GTR,GTR} = \sum_{ph,p'h'} W_{ph,p'h'}(\omega) X_{ph}^{(GTR)} X_{p'h'}^{(GTR)} + W_{ph,p'h'}(-\omega) Y_{ph}^{(GTR)} Y_{p'h'}^{(GTR)} \]

\[ \text{Re}(\Sigma)(E_{GTR}): \text{energy shift} \quad -2\text{Im}(\Sigma)(E_{GTR}): \text{width} \]

- \(2^+, 4^+\) phonons (isoscalar): not important ⇐ cancellation between \(W_1, W_2\) and \(W_3, W_4\) diagrams
- \(1^-, 3^-\) phonons (isovector): most important phonons ⇐ no cancellation between \(W_1, W_2\) and \(W_3, W_4\) diagrams
## Appendix

---

**Pb Why are $1^-$ phonons important?**

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>Proton $^{208}$Pb</th>
<th>Neutron</th>
</tr>
</thead>
<tbody>
<tr>
<td>-25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$1i_{1/2}$</td>
<td>$1i_{1/2}$</td>
</tr>
<tr>
<td>5</td>
<td>$3p_{1/2}$</td>
<td>$3p_{1/2}$</td>
</tr>
<tr>
<td>10</td>
<td>$1g_{9/2}$</td>
<td>$1g_{9/2}$</td>
</tr>
<tr>
<td>Z=126</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2d</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2f</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1f</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Z=82$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1f</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Z=28$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1f</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Z=20$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2p</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N=126$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N=126$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N=82$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2p</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N=50$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1g</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2d</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N=50$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1g</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2f</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N=50$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Energy denominator**

$$
\begin{align*}
EGTR - (\omega_{nL} + \epsilon_{p''} - \epsilon_h) + i\Delta &= 1 \\
\epsilon_p - \epsilon_h + \Delta E - (\omega_{nL} + \epsilon_{p''} - \epsilon_h) + i\Delta &= 1 \\
\epsilon_p - \epsilon_{p''} + \Delta E - \omega_{nL} + i\Delta &= 1
\end{align*}
$$

$1^-$ phonon with $\omega_{1^-} = 12.63$ MeV (GDR) is important!
Results and discussions

— How PVC reduces the half-life