Beta decay as an absolute calibration probe for spin-isospin responses

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Today's concern

- GT strength and Spin dipole (SD) strength by charge-exchange reaction

- Extract B(GT) and B(SD)

- For that, needs calibration on reaction probe by $\beta$ decay data.
β-decay & Nuclear Reaction

- β-decay transition rate = \[ \frac{1}{t_{1/2}} = f \frac{\lambda^2}{D} B(J^\pi) \]

  \[ B(J^\pi) : \text{reduced transition strength } \propto |M|^2 \]

  ☹ Provide absolute value
  △ Q_\beta window

- Charge-exchange reaction cross-section
  = \[ K(E, A) \times B(J^\pi) \]

  △ Needs calibration
  ☺ No Q_\beta window

Reaction cross section can be calibrated by \[ B(J^\pi) \] of β decay.
$^{90}$Zr$(p, n)^{90}$Nb at 295 MeV

$^{90}$Zr$(n, p)^{90}$Y at 293 MeV

Apply the multipole decomposition analysis (MDA) analysis → $\Delta L=0$ and $\Delta L=1$ spectra

Wakasa, Sakai, PRC55(1997)2909

Yako, Sakai, PLB615(2005)193
Gamow-Teller (Op: $t_{+/−}\sigma$) strength $B(GT)$

Model independent spin sum rule (Ikeda sum rule)

\[
S_{β−} − S_{β+} = \frac{1}{2J_i + 1} \sum_f \left| \sum_{i=1}^{A} t_{−}(i)\sigma_i \right|^2 - \frac{1}{2J_i + 1} \sum_f \left| \sum_{i=1}^{A} t_{+}(i)\sigma_i \right|^2
\]

\[
= \sum B(GT−) - \sum B(GT+)
= 3(N - Z)
\]

If nucleus can be described in terms of nucleon degrees of freedom
Decomposed results

\[ \frac{d^2 \sigma_{\text{cm}}}{d\Omega d\omega} (\text{mbsr}^{-1}\text{MeV}^{-1}) \]

$^90\text{Zr}(p,n)^{90}\text{Nb}$ at 295 MeV

$0^\circ$

$4.6^\circ$

$9.8^\circ$

$^90\text{Zr}(n,p)^{90}\text{Y}$ at 293 MeV

$0^\circ-1^\circ$

$4^\circ-5^\circ$

$9^\circ-10^\circ$

\[ \left( \frac{d\sigma(0^\circ)}{d\Omega} \right)_{\Delta L=0} \Rightarrow B(GT) \]
Proportionality assumption to extract $B(GT)$

\[ \frac{d\sigma(0^\circ)}{d\Omega} \bigg|_{\Delta L=0} = \hat{\sigma}_{GT}(E_p, A) \cdot F_{GT}(q, \omega) \cdot B(GT) \]

**Unit cross section**

\[ \hat{\sigma}_{GT}(^{90}\text{Zr}) = 3.6 \pm 0.2 \text{ (mb/sr)} \]

M. Sasano et al., PRC 79, 23602 ('09)

**DWIA**

**$B(GT)$**

\[ S_{\beta^-} - S_{\beta^+} = 27.6 \]

For $0 \leq \omega \leq 50$ MeV
**Quenching factor \(^{90}\text{Zr}\)**

\[
Q = 0.92 \pm 0.07
\]

Wakasa et al., PRC 55, 2909 (1997)

**Strength spread into 2p2h coupled states**
**Small \(\Delta h^{-1}\) contribution**

**Quenching problem solved!**

However . . .

208Pb(p,n) at 300 MeV

\[
Q \sim 72 \%
\]

Wakasa et al., PRC 85, 064606 (2012)
Spin Dipole strength $B(GT)$

$$\hat{O}_{SD\pm} = \sum_{im\mu} t^i_\pm \sigma^i_m r_i Y^\mu_1 (\hat{r}_i)$$

Model independent spin sum rule

$$S_-(SD) - S_+(SD) = \frac{9}{4\pi} \left( N \langle r^2 \rangle_n - Z \langle r^2 \rangle_p \right)$$

$$\delta_{np} = \sqrt{\langle r^2 \rangle_n} - \sqrt{\langle r^2 \rangle_p}$$
Spin dipole strength

\[ \frac{d^2\sigma_{\text{em}}}{d\Omega d\omega} \text{(mbsr}^{-1}\text{MeV}^{-1}) \]

\[ ^{90}\text{Zr}(p,n)^{90}\text{Nb} \text{ at 295 MeV} \]

\[ ^{90}\text{Zr}(n,p)^{90}\text{Y} \text{ at 293 MeV} \]

\[ \Delta L = 0 \quad \Delta L = 1 \quad \Delta L = 2 \quad \Delta L = 3 \]
Proportionality relation (assumption !)

\[ \sigma_{\Delta L=1,\pm}(q, \omega) = \hat{\sigma}_{SD\pm}(q, \omega) \cdot B(\text{SD}\pm) \]

Characterized by \( \Delta S=1, \Delta L=1, \Delta J=0,1,2 \)
- \( 0+ \to 0- \) first forbidden
- \( 0+ \to 1- \) first forbidden
- \( 0+ \to 2- \) unique first forbidden

Unit cross section \( \hat{\sigma}_{SD\pm}(q, \omega) \)

⇒ Estimated with DWIA calculation at 4.5°

\[ \sigma_{\Delta L=1,\pm}(4.5^\circ, \omega) = \hat{\sigma}_{SD\pm}(4.5^\circ, \omega) \cdot B(\text{SD}\pm) \]
Spin dipole strength and sum rule value

\[ S_- - S_+ = 148 \pm 13 \text{ fm}^2 \]

\[ \sqrt{\langle r^2 \rangle_p} = 4.19 \text{ fm} \]

\[ \sqrt{\langle r^2 \rangle_n} = 4.26 \pm 0.04 \text{ fm} \]

\[ \delta_{np} = 0.07 \pm 0.04 \text{ fm} \]
Spin dipole strength in $^{208}$Pb by Wakasa (Kyushu U)

Wakasa et al., 85(2012)064606

Excellent experiment!

SD decomposed!
Spin dipole strength in $^{208}$Pb by Wakasa (Kyushu U)

Wakasa et al., 85(2012)064606

GT quenched by 30%!

$\hat{\sigma}(\text{GT})$ and $\hat{\sigma}(\text{SD})$ not calibrated

Estimated by DWIA calculation

0- and 2- quenched by 30%!
Rely on proportional relation
\[
\frac{d\sigma(\theta)}{d\Omega} = \hat{\sigma}_{GT}(E_p, A) \cdot F_{GT}(q, \omega) \cdot B(GT)
\]
\[
\frac{d\sigma(\theta)}{d\Omega} = \hat{\sigma}_{SD}(E_p, A) \cdot F_{SD}(q, \omega) \cdot B(SD)
\]

Unit cross section should be calibrated using known B(GT/SD) by β decay!

Calibration is NOT available
- \(\hat{\sigma}_{GT}\) for \(A > 130\)
- \(\hat{\sigma}_{SD}\) for \(A > 1\) (nothing)

Why no calibration?
- No good candidate with stable target
\( \beta \)-decay matrix elements for GT state

- **GT moment**

\[
\mathcal{M}(j_A, \kappa = 0, \lambda = 1, \mu) = \frac{g_A}{(4\pi)^{1/2}} \sum_k t_{-}(k)\sigma_\mu(k) \\
(3D-42)
\]

- Operator is similar to reaction probe
- need unit \( \sigma(\text{GT}) \) for \( A \sim 200 \)
Feasibility of $\sigma$(GT) calibration for $A>160$

- Possible case? $^{190}$W(p,n)
  - $\log ft = 5.12 \Rightarrow B(GT)=0.03$
  - Isolated: ?
  - Why No F-trans. to 162 keV?
  - Isomer involvement?
  - Unstable beam exp.

  *Cf.* Sasano

$^{118}$Sn, $^{120}$Sn: $B(GT)=0.34$  

Too small!

- Possible case? $^{214}$Pb(p,n)
  - $\log ft = 4.44 \Rightarrow B(GT)=0.14$
  - $\Rightarrow$ effective $B(GT)\sim0.07$
  - Isolated: many states around
  - Isomer involvement?
  - Unstable beam exp.

Still too small!

Need isolated GT decay with $B(GT)>0.5$ for $A\sim200$. 
**β-decay matrix elements of SD states**

- **SD moment**

\[
\mathcal{M}(\rho, \lambda = 0) = (4\pi)^{-1/2}\frac{g_A}{c} \sum_k t_-(k)(\sigma(k) \cdot \nu_k) \\
\mathcal{M}(j_A, \kappa = 1, \lambda = 0) = g_A \sum_k t_-(k)r_k(Y_1(\hat{f}_k)\sigma(k))_0 \\
\mathcal{M}(\rho\nu, \lambda = 1, \mu = 0) = g_\nu \sum_k t_-(k)r_k Y_{1\mu}(\hat{f}_k) \\
\mathcal{M}(j\nu, \kappa = 0, \lambda = 1, \mu) = (4\pi)^{-1/2}\frac{g_\nu}{c} \sum_k t_-(k)(\nu_k)_{1\mu} \\
\mathcal{M}(j_A, \kappa = 1, \lambda = 1, \mu) = g_A \sum_k t_-(k)r_k(Y_1(\hat{f}_k)\sigma(k))_{1\mu} \\
\mathcal{M}(j_A, \kappa = 1, \lambda = 2, \mu) = g_A \sum_k t_-(k)r_k(Y_1(\hat{f}_k)\sigma(k))_{2\mu} \\
\mathcal{M}(\lambda_\pi = 0) = 0 \\
\mathcal{M}(\lambda_\pi = 1) \\
\mathcal{M}(\lambda_\pi = 2)
\]

Operators are NOT necessarily similar to reaction probe operator \( t_{\pm} \sigma r Y_1 \)

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Bohr-Mottelson

Unique FF!
Calibration of 2- SD at A=90

\[ \mathcal{M}(j_A, \kappa = 1, \lambda = 2, \mu) = g_A \sum_k t_-(k)r_k(Y_1(f_k)\sigma(k))_{2\mu} \]

\[
\begin{align*}
0+ & \quad 0.0 \\
^90\text{Sr} & \quad \text{(p,n)}
\end{align*}
\]

\[
\begin{align*}
L=1 & \quad 0.0 \\
L=3 & \quad 0.203 \\
L=6 & \quad 0.682
\end{align*}
\]

\[
\begin{align*}
\log ft = 9.40 & \quad \text{(n,p)}
\end{align*}
\]

\[
\begin{align*}
L=1 & \quad 0.0 \\
L=3 & \quad 0.203 \\
L=6 & \quad 0.682
\end{align*}
\]

I realized we have measured!
Extraction of unit cross section for 2-\( ^{90}\text{Y} \) (n,p) 0+\( ^{90}\text{Zr} \)

\[ \log_{10} ft = 9.228 \]

\[ B(\text{SD2-}) \uparrow = 5 \frac{9}{4\pi} \frac{D}{ft} \left( \frac{g_v}{g_A} \right)^2 C^2 \]

\( D = 6143 \text{ s}, \quad C = 386 \text{ fm} \)

\( \Rightarrow B(\text{SD2-}) \uparrow = 0.74 \text{ fm}^2 \)

\[ \sigma(\text{exp}) = \hat{\sigma}(\text{SD2-}) \cdot B(\text{SD2-}) \]

\[ 0.25 \left( \frac{\text{mb}}{\text{sr}} \right) = \hat{\sigma}(\text{SD2-}) \cdot 0.74 \text{ fm}^2 \]

\( \Rightarrow \hat{\sigma}(\text{SD2-}) = 0.34 \left( \frac{\text{mb/sr}}{\text{fm}^2} \right) \)

DWIA estimation by Yako

\[ \hat{\sigma}(\text{SD2-}) = 0.29 \left( \frac{\text{mb/sr}}{\text{fm}^2} \right) \]

No peak!
Most favorable case unit-$\sigma_{SD}(2^-)$

3- 0.11 (258 s) Isomer
0- 0.0 (158 s)

$log ft=7.19$ fastest!

$^90\text{Rb} \quad log ft=7.35 \quad 0^- \rightarrow 0^+$

$2^+ \quad 0.832$

$0^+ \quad 0.0 \ (29 \text{ y})$

$log ft=9.40$

Possible case? $^90\text{Rb}(p,n)$
- $log ft = 7.19 \Rightarrow B(\text{SD}2^-)=26 \text{ fm}^2$
- Moderately isolated
- Unstable beam exp.
- Isomer involvement: Yes

$\sigma(\text{exp}) = \hat{\sigma}(\text{SD}2^-) \times B(\text{SD}2^-) = 0.34 \times 26 = 8.8 \left( \frac{\text{mb}}{\text{sr}} \right)$
Calibration of $0^-$ and $1^-$ SD

Not necessarily similar to reaction probe $t_\sigma r Y_1$

- $0^-$
  \[
  \mathcal{M}(\rho_A, \lambda = 0) = \left(4\pi\right)^{-1/2} \frac{g_A}{c} \sum_k t_-(k)(\sigma(k) \cdot v_k)
  \]

- $1^-$
  \[
  \mathcal{M}(j_A, \kappa = 1, \lambda = 0) = g_A \sum_k t_-(k)r_k(Y_1(\hat{r}_k)\sigma(k))_0
  \]

- Two terms tend to cancel.

- Involves non-spinflip.

Probably $B(0^-/1^-)$ of $\beta$ decay is unable to use as a probe calibration purpose.
Candidate of $0^-$ SD calibration

- Possible case?  \(^{96}\text{Y}(p,n)\)
  - \(\log ft = 5.59 \Rightarrow B(\text{SD}0^-) = 1.125\text{ fm}^2\!\) !
  - Isolated
  - Unstable beam exp.
  - Isomer involvement: Yes

\[
\begin{array}{c}
0^- 0.0 (5.34 \text{ s}) \quad 96\text{Y} \\
\text{Isomer} \\
2^+ 1.581 \\
0^- \rightarrow 0^+ \\
96\text{Zr}
\end{array}
\]

\[
8^+ 1.14 (9.65 \text{ s}) \quad 96\text{Y}
\]

\[
\log ft = 5.59
\]

\[
0^- \rightarrow 0^+
\]

\[
\sim 2\text{ enhancement due to MEC}
\]

- Assume: \(B(\text{SD}0^-; \text{SF}) = 0.1 \times B(\text{SD}0^-)\)

\[
\sigma(\text{exp}) = \sigma(\text{SD}2^-) \times B(\text{SD}0^-; \text{SF}) \approx 10 \text{ (mb/sr)}
\]

- Feasible with RI beam exp.
- Proportionality ???
Difficulties of $1^-$ SD calibration

- Shell model estimate of $1^-$ SD

$\beta$ decays of isotones with neutron magic number of $N = 126$ and $r$-process nucleosynthesis

Toshio Suzuki,$^{1,2,3}$ Takashi Yoshida,$^4$ Toshitaka Kajino,$^{3,4}$ and Takaharu Otsuka$^{5,6}$

\[
B(SD\lambda) = \frac{1}{2J_i + 1} |\langle f || r[Y^{(1)} \times \tilde{\sigma}]^\lambda t_- || i \rangle|^2
\]

\[
O(1^-) = \left[ g_V \frac{\tilde{p}}{M_N} \Xi (g_A \tilde{\sigma} \times \tilde{r} - ig_V \tilde{r}) \right] t_- ,
\]

- Strong non-spinflip strength
- $\log ft$ is large
- Small branching ratio

⇒ Probably non-realistic to use $\beta B(1^-)$ for calibration
Summary

1. GT and SD: important spin-isospin responses
   - (p,n) reaction could provide B(GT) and B(SD)

2. (p,n) reaction must be calibrated by $\beta$ B(GT/SD)
   - No B(GT) for $A>160$
   - Nothing for B(SD)

3. RI beam is now available $\Rightarrow$ open new possibilities

4. GT:
   - $B(GT) > 0.5$ is needed for $A>160$

5. SD: with 0- or 1-
   - $B(2-): ^{90}$Rb(p,n) may be feasible with log $ft=7.19$
   - $B(1-):$ Maybe essential difficulty with non-spinflip
   - $B(0-):$ $^{96}$Y(p,n) may be with log $ft=5.59$ but $\gamma_5$ term?

6. SPES project
   - $B(GT)/B(SD)$ are always precious for structure study
   - $\beta$ decay measurement?